| Name: | Register No.: | Class: |
|-------|---------------|--------|
|       |               |        |



# CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION

 MATHEMATICS
 4048/01

 Paper 1
 14 Aug 2018

Candidates answer on the Question Paper.

#### **READ THESE INSTRUCTIONS FIRST**

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

#### Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 80.

|       | For Examiner's Use |    |                     |    |    |    |    |    |    |    |
|-------|--------------------|----|---------------------|----|----|----|----|----|----|----|
| Qn    |                    |    |                     |    |    |    |    |    |    |    |
| No.   | 1                  | 2  | 3                   | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| Marks | 2                  | 2  | 2                   | 2  | 2  | 2  | 2  | 3  | 4  | 3  |
| Qn    |                    |    |                     |    |    |    |    |    |    |    |
| No.   | 11                 | 12 | 13                  | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Marks |                    | 3  | 3                   | 3  | 4  | 3  | 6  | 7  | 4  | 4  |
| Qn    |                    |    | Total No. of Marks  |    |    |    |    |    |    |    |
| No.   | 21                 | 22 | i otal No. Of Marks |    |    |    |    |    |    |    |
| Marks | 9                  | 8  |                     | 80 |    |    |    |    | 30 |    |

2 hours

## **MATHEMATICAL FORMULAE**

Compound Interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =  $\pi rl$ 

Surface area of a sphere =  $4\pi r^2$ 

Volume of a cone = 
$$\frac{1}{3}\pi r^2 h$$

Volume of a sphere = 
$$\frac{4}{3}\pi r^3$$

Area of triangle 
$$ABC = \frac{1}{2}ab\sin C$$

Arc length =  $r\theta$ , where  $\theta$  is in radians

Sector area = 
$$\frac{1}{2}r^2\theta$$
, where  $\theta$  is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Statistics** 

$$Mean = \frac{\sum fx}{\sum f}$$

Standard Deviation = 
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

# Answer all the questions.

| 1 | By approximating each number to 2 significant figures, estimate the value of $\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}}$ . Show your working and give your answer to a reasonable degree of accuracy. |  |  |  |
|---|--|--|--|--|
|   | <i>Answer</i> [2]  |  |  |  |
| 2 | (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.<br>(b) If $5a = 4b$ and $8b = 3c$ , find $a:b:c$ .  |  |  |  |
|   | Answer (a)   |  |  |  |

*Answer* ......days [2]

Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same

rate. After 3 days, two workers left. How many days would the remaining workers take to

3

finish building the wall?

| 4 | The value of a new house depreciated 10% each year for 3 years in a row. Then, for the next 3 years, the value of the house increased 10% each year. Did the value of the house increase or decrease after 6 years? Explain your answer. |
|---|--|
|   | Answer:  |
|   |  |
|   | [2]  |
| 5 | The exterior angles of a hexagon are in the ratio $2:3:4:4:8$ . Find the smallest interior angle of the hexagon.   |
|   |  |
|   |  |
|   |  |
|   | Answerº [2]  |
| 6 | Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form. (1 nano unit = $1 \times 10^{-9}$ unit; 1 mega unit = $1 \times 10^{6}$ unit)   |
|   |  |
|   |  |
|   |  |
|   |  |
|   | Answer: megaseconds [2]  |

| 7 | On a particular day at noon, the temperature 15 m above the sea level is 4° C. The temperature 30 m below the sea level is -23° C. Calculate  (a) the difference between these temperatures, |  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|--|
|   | (b)  | Answer $^{\circ}$ C [1] the temperature at sea level at noon, assuming that the temperature changes  |  |  |  |  |  |  |
|   |  | uniformly with height.   |  |  |  |  |  |  |
|   |  | Answer° C [1]  |  |  |  |  |  |  |
| 8 | A ta (a)   | blet is sold at \$1450 after a discount of 20%.  Find the marked price of the tablet.  |  |  |  |  |  |  |
|   |  | Answer \$ [1]  |  |  |  |  |  |  |
|   | (b)  | A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of 60% and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of $x$ % per annum. Given that the total interest he paid is \$34.80, find $x$ . |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |  |
|   |  | Answer $x =$   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |  |

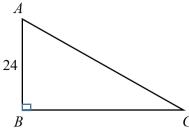
9 (a) Factorise completely  $4+6ab-9a^2-b^2$ .

| Answer | <br>[2] | ] |
|--------|---------|---|
|        |         |   |

**(b)** Without using a calculator, show that  $2^{17} - 2^{14}$  is divisible by 7.

Answer .....[2]

- In triangle ABC,  $\angle ABC = 90^{\circ}$ , AB = 24 cm and  $\sin \angle ACB = \frac{3}{5}$ . Without the use of calculator, find the value of
  - (i) *AC*
  - (ii)  $5\cos(180^{\circ} \angle ACB)$



Answer (i) ......[1]

(ii) ...... [2]

| 11 | Solve the following inequality $6 < 2x + \frac{3x - 5}{4} \le 4x + 2$ |
|----|---|
|    | 4   |

| Answer |  | [2] |
|--------|--|-----|
|--------|--|-----|

| 12 | (a) | A designer has to design a box in the shape of a cube of length <i>l</i> cm so as to store |
|----|-----|--|
|    |     | rectangular bricks of dimensions 45 cm by 21 cm by 15 cm. To save cost, he must            |
|    |     | ensure that the bricks fit exactly into the box, leaving no gaps in between. What is       |
|    |     | the smallest possible value of <i>l</i> ?  |

**(b)** How many bricks can fit into the box?

| <b>13</b> | A map | is | drawn | to | a scal | le o | f 1 | : | 20 | 000 |
|-----------|-------|----|-------|----|--------|------|-----|---|----|-----|
|-----------|-------|----|-------|----|--------|------|-----|---|----|-----|

(a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.

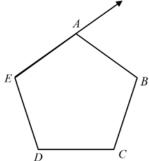
| Answer |  | km | [1 | ] |
|--------|--|----|----|---|
|--------|--|----|----|---|

(b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [1 hectare =  $10\ 000\ m^2$ ]

Answer ...... cm<sup>2</sup> [2]

In the diagram, ABCDE is a regular pentagon where A is due north of E. Find the bearing of

- (i) B from A,
- (ii) D from A,
- (iii) C from E.



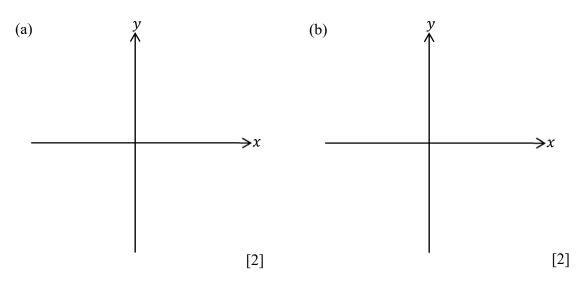
| Answer | (i) °             | [1] |
|--------|-------------------|-----|
|        | (ii) <sup>o</sup> | [1] |
| (      | (iii)°            | Г1  |

15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.

(a) 
$$y = (x-1)^3$$
,

**(b)** 
$$y = \frac{x+1}{x}$$
.

Answer



(a) Consider the sequence  $\frac{2}{3}$ ,  $\frac{5}{8}$ ,  $\frac{13}{21}$ ,  $\frac{34}{55}$ , .....

Write down the next two terms of the sequence.

*Answer* .....[1]

(b) Write down an expression, in terms of n, for the nth term of the sequence 3, 6, 10, 15, 21, .....

| 17 | The volume of cylinder A of radius $r$ cm and height $h$ cm is 240 cm <sup>3</sup> .   |
|----|--|
|    | (a) Find the volume of cylinder B of radius $2r$ cm and height $\frac{1}{3}h$ cm.  |
|    | 3  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    | Answer cm <sup>3</sup> [2]   |
|    | <b>(b)</b> Cylinder C is similar to cylinder A. If the radius of cylinder C is $\frac{1}{2}r$ cm, find   |
|    | its volume.  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    | <i>Answer</i>  |
|    | AnswerCIIF [2]   |
|    | (c) A cone $D$ has the same volume as cylinder $A$ . If the height of cone $D$ is $h$ cm, find the ratio of the radius of cone $D$ to that of cylinder $A$ . |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    |  |
|    | <i>Answer</i> [2]  |
|    |  |

18 Given that  $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$ ,  $\overrightarrow{OB} = \begin{pmatrix} -6 \\ 12 \end{pmatrix}$  and C is the point on OB such that OC: CB = 1:2.

(a) Find

(i) 
$$\begin{vmatrix} \overrightarrow{AB} \end{vmatrix}$$
, [1]

(ii) the position vector of C, [1]

(iii) 
$$\overrightarrow{AC}$$
 [2]

- (b) Given that  $\binom{2}{m}$  and  $\stackrel{\longrightarrow}{AB}$  are parallel vectors. Find the value of m. [1]
- (c) Find the coordinates of D, such that ABCD is a parallelogram. [2]

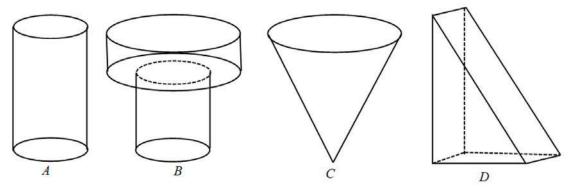
Answer (a)(i) ..... units [1]

(ii) ...... [1]

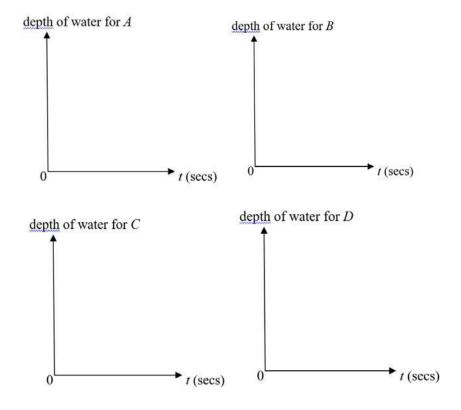
(iii) 
$$\overrightarrow{AC} = \dots [2]$$

- (b)  $m = \dots [1]$
- (c) D is ......[2]

19 The diagrams below show four containers (not drawn to scale), A, B, C and D each with a height of h cm. The containers are initially empty. It takes t seconds to fill each container with water at a constant rate.



(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.

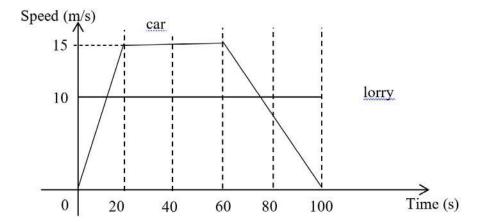


[2]

**(b)** It takes 12 seconds to fill container *D* to the brim. Find the time it takes to fill container *D* to half its height.

*Answer* ..... secs [2]

The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of 15 m/s for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of 10 m/s throughout.



(a) Find the time when the speed of the car is 10 m/s.

| Answer | sec | [1] | 1 |
|--------|-----|-----|---|
|        |     |     |   |

**(b)** Find the speed of the car 10 seconds before it comes to rest.

| Answer | <br>m/s | $\lceil 1 \rceil$ |  |
|--------|---------|-------------------|--|
|        |         |                   |  |

(c) Find the time when the car overtakes the lorry.

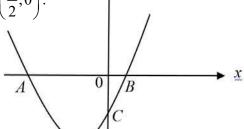
*Answer*..... sec [2]

| 21 | drav | (a) A box contains five slips of paper. Each slip has one of the numbers 4, 6, 7, 8 or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins. |  |  |  |  |  |
|----|------|---|--|--|--|--|--|
|    | (b)  | Answer:   |  |  |  |  |  |
|    |      | Answer[2]   |  |  |  |  |  |
|    |      | Suppose now that the game is such that the same die is rolled repeatedly until two '6's are obtained. Find the probability that  (i) the game ends on the third roll,  (ii) the game ends on the third roll and the sum of the scores is odd.   |  |  |  |  |  |
|    |      | Answer (i)[1]   |  |  |  |  |  |
|    | (c)  | <ul> <li>(ii)</li></ul>   |  |  |  |  |  |
|    |      | Answer (i)[2]   |  |  |  |  |  |
|    |      | (ii) [1]  |  |  |  |  |  |

The diagram shows part of the graph of  $5(y+3) = ax^2 + bx$ , where a and b are 22 constants.

The graph cuts the x-axis at  $A\left(-2\frac{1}{2},0\right)$  and  $B\left(\frac{1}{2},0\right)$ .

The graph meets the y-axis at the point C. Find



- (i) the value of a and of b,
- (ii) the coordinates of C,
- (iii) the coordinates of the minimum point,
- (iv) the equation of the line of symmetry,
- (v) the area of triangle ABC.

#### **END OF PAPER**

Answer Key

| Answer | Key                                     |       |  |
|--------|---|-------|--|
| Qn     |   | Qn    |  |
| No.    |   | No.   |  |
| 1      | 28 or 30                                | 17    | (a) $320 \text{ cm}^3$   |
| 2      | (a) 1:40                                |       | (b) $30 \text{ cm}^3$  |
|        | (b) 12:15:40                            |       | (c) $\sqrt{3}:1$   |
| 3      | 7.2 days                                | 18    | (a) (i) 10 units   |
| 4      | 0.9703P < P, decreased                  |       |  |
| 5      | 60°                                     |       | (ii) $\begin{pmatrix} -2\\4 \end{pmatrix}$   |
| 6      | 9.52×10 <sup>-18</sup> megaseconds      |       |  |
| 7      | (a) $27^{\circ} C$ (b) $-5^{\circ} C$   |       | $\begin{array}{cc} - & \text{(iii)} & \begin{pmatrix} 12 \\ -14 \end{pmatrix} \end{array}$ |
| 8      | (a) \$1812.50 (b) 4.5                   |       | (b) $-1\frac{1}{2}$  |
| 9      | (a) $(2-3a+b)(2+3a-b)$                  |       | (c) $D(-10,10)$  |
|        | (b) $2^{14} \times 7$ is divisible by 7 |       |  |
| 10     | (i) 40 (ii) -4                          | 19(a) | depth of water for A   |
| 11     |   | 17(0) | 1  |
|        | $x > 2\frac{7}{11}$                     |       |  |
| 12     | (a) 315                                 |       | /  |
|        | (b) 2205                                |       |  |
|        |   |       |  |
| 1.0    | ( ) 1.61                                |       | 0 t (secs)   |
| 13     | (a) 1.6 km<br>(b) 0.625 cm <sup>2</sup> |       | $\frac{\text{depth}}{\uparrow} \text{ of water for } B$                                    |
| 14     | (i) 072°                                |       |  |
|        | (ii) 144°                               |       |  |
|        | (iii) 072°                              |       |  |
|        | (111) 072                               |       |  |
|        |   |       | 0 $t$ (secs)   |
| 15     | (a)                                     |       | depth of water for C   |
|        |   |       | <b>†</b>   |
|        |   |       |  |
|        |   |       |  |
|        | x                                       |       |  |
|        | -1 1                                    |       | $0 \longrightarrow t \text{ (secs)}$   |
|        |   |       | depth of water for D   |
|        | (b) Y                                   |       | 1  |
|        |   |       | ,  |
|        |   |       |  |
|        | v = 1                                   |       |  |
|        |   |       |  |
|        | -1\ -1\                                 |       | 0 t (secs)   |
|        |   |       | - 89   |
|        |   |       |  |
|        |   |       |  |

| 16 | (a) $\frac{89}{144}, \frac{233}{377}$<br>(b) $T_n = \frac{1}{2}(n+1)(n+2)$   | 19(b) | 9 secs  |
|----|--|-------|---|
| 20 | (a) 13.3 sec , 73.3 sec<br>(b) 3.75 m/s<br>(c) 30 sec  | 21    | (a) $\frac{2}{5}$<br>(b) $\frac{671}{1296}$<br>(i) $\frac{5}{108}$ (ii) $\frac{1}{36}$<br>(c) (i) $x-1$<br>(ii) $A \cap B'$ |
| 22 | (i) $a = 12$ , $b = 24$<br>(ii) $C(0,-3)$<br>(iii) $\left(-1,-5\frac{2}{5}\right)$<br>(iv) $x = -1$<br>(v) $4.5 \text{ units}^2$ |       |   |

Name: MARX SCHEME Register No.: Class:



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|-------|--------------------|----|---------------------|----|----|----|----|----|----|----|
| Qn    |                    |    |                     |    |    |    |    |    |    |    |
| No.   | 1                  | 2  | 3                   | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| Marks | 2                  | 2  | 2                   | 2  | 2  | 2  | 2  | 3  | 4  | 3  |
| Qn    |                    |    |                     |    |    |    |    |    |    |    |
| No.   | 11                 | 12 | 13                  | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Marks | 2                  | 3  | 3                   | 3  | 4  | 3  | 6  | 7  | 4  | 4  |
| Qn    |                    |    | Total No. of Marks  |    |    |    |    |    |    |    |
| No.   | 21                 | 22 | i Otal NO. Of Walks |    |    |    |    |    |    |    |
| Marks | 9                  | 8  |                     |    |    |    |    |    | 8  | 30 |

#### MATHEMATICAL FORMULAE

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Arc length =  $r\theta$ , where  $\theta$  is in radians

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$$\frac{1}{2}r^2\theta$$
, where  $\theta$  is in radians

**Trigonometry** 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Statistics** 

$$Mean = \frac{\sum fx}{\sum f}$$

Standard Deviation = 
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

## Answer all the questions.

By approximating each number to 2 significant figures, estimate the value of  $\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}}$ . Show your working and give your answer to a reasonable degree of accuracy.

$$\frac{12.1 \times \sqrt{48.8}}{\sqrt[3]{27.3}} \approx \frac{12 \times \sqrt{49}}{\sqrt[3]{27}}$$

$$= \frac{12 \times 7}{3}$$

$$= 28$$

$$\approx 30$$
B1 for estimating all values to 2 s.f.

Answer ......28 or 30......[2]

- 2 (a) Express the ratio of 1 minute and 30 seconds to 1 hour in its simplest form.
  - (b) If 5a = 4b and 8b = 3c, find a:b:c.
  - (a)  $1 \min 30 \text{ s} : 1 \text{ h} = 90 \text{ s} : 3600 \text{ s}$ = 1 : 40
  - (b) a:b:c 4 x 3:5 x 3 3 x 5:8 x 5

3 Twelve workers are hired to build a wall in 9 days, assuming that they all work at the same rate. After 3 days, two workers left. How many days would the remaining workers take to finish building the wall?

After 3 days, number of man-day needed to complete the job =  $6 \times 12$ 

$$= 72$$
Hence number of days needed for remaining workers =  $\frac{72}{10}$ 

$$= 7.2$$
M1A1

Answer .......7.2......days [2]

The value of a new house depreciated 10% each year for 3 years in a row. Then, for the 4 next 3 years, the value of the house increased 10% each year. Did the value of the house increase or decrease after 6 years? Explain your answer. [2]

Answer: Let original value of house be P. Price after 3 years is  $P(0.9)^3$ . After another 3 years the value of the house will be  $P(0.9)^3(1.1)^3$ , which equals approximately 0.9703P. Since 0.9703P < P, the value of the house decreased after 6 years, by approximately 3%

B1 for  $P(0.9)^3$ ; B1 for 0.9703P

5 The exterior angles of a hexagon are in the ratio 2:3:3:4:4:8. Find the smallest interior angle of the hexagon.

Largest exterior angle = 
$$\frac{8}{24} \times 360^{\circ} = 120^{\circ}$$
 B1

Hence smallest interior angle =  $180^{\circ} - 120^{\circ} = 60^{\circ}$ B1

Express 0.00952 nanoseconds in megaseconds, giving your answer in standard form. 6 (1 nano unit =  $1 \times 10^{-9}$  unit; 1 mega unit =  $1 \times 10^{6}$  unit)

$$0.00952 \times 10^{-9} \text{ seconds} = k \times 10^{6} \text{ seconds}$$
 Or  $0.00952 \times 10^{-9} \text{ seconds}$ 

$$k = 0.00952 \times 10^{-15} = 0.00952 \times 10^{-9-6} \text{ megaseconds}$$

$$= \frac{9.52}{1000} \times 10^{-15} = 9.52 \times 10^{-3} \times 10^{-15}$$

$$= 9.52 \times 10^{-18} = 9.52 \times 10^{-18}$$
M1 A 1

M1A1

Answer: .....  $9.52 \times 10^{-18}$  ... megaseconds [2]

- On a particular day at noon, the temperature 15 m above the sea level is  $4^{\circ}$  C. The temperature 30 m below the sea level is  $-23^{\circ}$  C. Calculate
  - (a) the difference between these temperatures,

difference = 
$$4-(-23)$$
  
=  $27^{\circ}$  C

(b) the temperature at sea level at noon, assuming that the temperature changes uniformly with height.

Rate of temperature increase = 
$$\frac{27}{45} = 0.6^{\circ} C$$
  
Hence temperature at sea level =  $-23 + 30 \times 0.6$   
=  $-5$  °  $C$ 

- **8** A tablet is sold at \$1450 after a discount of 20%.
  - (a) Find the marked price of the tablet.

$$\frac{1450}{80} \times 100 = 1812.50$$

(b) A customer bought the tablet at the discounted price and he paid for it using a hire purchase scheme according to the following terms: a down-payment of 60% and the remaining to be paid in monthly instalments over 16 months at a simple interest rate of x% per annum. Given that the total interest he paid is \$34.80, find x.

$$(40\% \times 1450) \times \frac{x}{100} \times \frac{16}{12} = 34.80$$
 M1A1  
$$x = \frac{34.80}{580} \times 100 \times \frac{12}{16}$$
  
= 4.5

Factorise completely  $4+6ab-9a^2-b^2$ . 9 (a)

$$4+6ab-9a^{2}-b^{2}$$

$$=4-(9a^{2}-6ab+b^{2})$$

$$=4-(3a-b)^{2}$$

$$=(2-3a+b)(2+3a-b)$$

M1 for 
$$(3a-b)^2$$

Answer 
$$(2-3a+b)(2+3a-b)$$
 ...... [2]

Without using a calculator, show that  $2^{17} - 2^{14}$  is divisible by 7. **(b)** 

$$2^{17} - 2^{14}$$

$$= 2^{14} \cdot 2^3 - 2^{14}$$

$$= 2^{14} (8 - 1)$$

$$= 2^{14} \times 7$$
M1A1

**10** In triangle ABC,  $\angle ABC = 90^{\circ}$ , AB = 24 cm and  $\sin \angle ACB = \frac{3}{5}$ . Without the use of calculator, find the value of



(ii) 
$$5\cos(180^{\circ} - \angle ACB)$$



(ii) 
$$BC^2 = 40^2 - 24^2$$
  
 $BC = 32$   
 $5\cos(180^\circ - \angle ACR) = -5$ 

$$5\cos(180^{\circ} - \angle ACB) = -5\cos \angle ACB \qquad M1$$
$$= -5 \times \frac{32}{40}$$
$$= -5 \times \frac{32}{40} \qquad A1$$

Solve the following inequality  $6 < 2x + \frac{3x - 5}{4} \le 4x + 2$ .

$$6 < 2x + \frac{3x - 5}{4} \text{ and } 2x + \frac{3x - 5}{4} \le 4x + 2$$

$$24 < 8x + 3x - 5$$

$$29 < 11x$$

$$x > \frac{29}{11}$$

$$x > 2\frac{7}{11}$$

$$x \ge -2\frac{3}{5}$$

deduct 1 mark from full mark of 2, if 'and' is missing

B1 for any one inequality correct

Answer ..... 
$$x > 2\frac{7}{11}$$
..... [2]

12 (a) A designer has to design a box in the shape of a cube of length *l* cm so as to store rectangular bricks of dimensions 45 cm by 21 cm by 15 cm. To save cost, he must ensure that the bricks fit exactly into the box, leaving no gaps in between. What is the smallest possible value of *l*?

$$45 = 3^{2} \times 5$$

$$21 = 3 \times 7$$

$$15 = 3 \times 5$$

$$LCM = 3^{2} \times 5 \times 7$$

$$= 315$$
A1

**(b)** How many bricks can fit into the box?

Number of bricks = 
$$\frac{315^3}{45 \times 21 \times 15}$$
$$= 2205$$

- A map is drawn to a scale of 1: 20 000
  - (a) The distance from one end of a grassland to the other end is represented by a line of length 8 cm on the map. Calculate the actual distance, giving your answer in kilometres.

1 cm : 0.2 km 8 cm : 1.6 km

- *Answer* ......1.6...... km [1]
- (b) The actual area of the grassland is 2.5 hectares. Calculate the area on the map, giving your answer in square centimetres. [1 hectare =  $10\ 000\ m^2$ ]

**B**1

2.5 hectares = 25 000 m<sup>2</sup>  
= 
$$\frac{25000}{10^6}$$
  
= 0.025 km<sup>2</sup> B1

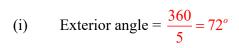
Area on the man =  $\frac{0.025}{0.025}$ 

Area on the map = 
$$\frac{0.025}{0.04}$$
  
= 0.625 cm<sup>2</sup>

- 14 In the diagram, ABCDE is a regular pentagon where A is due north of E.

Find the bearing of

- (i) B from A,
- (ii) D from A,
- (iii) C from E.

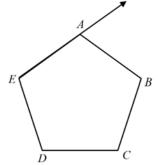


Bearing =  $072^{\circ}$ 



Bearing =  $144^{\circ}$ 

(iii) Bearing = 072°

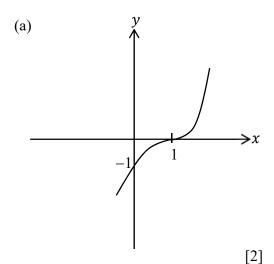


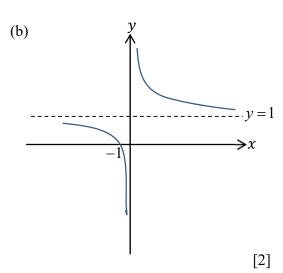
15 In the axes provided below, sketch the graphs of the following. State intercept(s) if any.

(a) 
$$y = (x-1)^3$$
,

**(b)** 
$$y = \frac{x+1}{x}$$
.  $y = 1 + \frac{1}{x}$ 

Answer





(a) Shape with point of inflexion on x-axis i.e. check that graph curve towards origin B1; y-intercept and x-intercept B1

(b) Shape and dash asymptote B1; x-intercept and y = 1 B1

16

(a) Consider the sequence  $\frac{2}{3}$ ,  $\frac{5}{8}$ ,  $\frac{13}{21}$ ,  $\frac{34}{55}$ , .....

Write down the next two terms of the sequence.

$$\frac{34+55}{55+89} = \frac{89}{144}$$
,  $\frac{233}{377}$ 

Answer .......
$$\frac{89}{144}$$
,  $\frac{233}{377}$ ......[1]

(b) Write down an expression, in terms of n, for the nth term of the sequence  $3, 6, 10, 15, 21, \dots$ 

1+2, 1+2+3, 1+2+3+4, 1+2+3+4+5, ...... 
$$T_n = (n+2)\frac{n+1}{2}$$
 M1A1

Answer .... 
$$T_n = (n+2)\frac{n+1}{2}$$
.....[2].

- The volume of cylinder A of radius r cm and height h cm is 240 cm<sup>3</sup>.
  - (a) Find the volume of cylinder B of radius 2r cm and height  $\frac{1}{3}h$  cm.

Volume of cylinder 
$$B = \pi (2r)^2 \frac{1}{3}h$$
 M1A1  

$$= \frac{4}{3}\pi r^2 h$$

$$= \frac{4}{3} \times 240$$

$$= 320$$

**(b)** Cylinder C is similar to cylinder A. If the radius of cylinder C is  $\frac{1}{2}r$  cm, find its volume.

$$\frac{\text{volume of } C}{240} = \left(\frac{1}{2}\right)^3$$

Volume of 
$$C = \frac{1}{8} \times 240$$
 M1A1
$$= 30$$

(c) A cone D has the same volume as cylinder A. If the height of cone D is h cm, find the ratio of the radius of cone D to that of cylinder A.

$$\frac{1}{3}\pi \left(r_1\right)^2 h = \pi r^2 h$$

M1A1

$$\left(\frac{r_1}{r}\right)^2 = 3$$

$$\frac{r_1}{r} = \sqrt{3}$$

Accept 1.73:1

Given that  $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$ ,  $\overrightarrow{OB} = \begin{pmatrix} -6 \\ 12 \end{pmatrix}$  and C is the point on OB such that OC: CB = 1:2.

(a) Find

(i) 
$$\begin{vmatrix} \overrightarrow{AB} \end{vmatrix}$$
, [1]

(ii) the position vector of 
$$C$$
, [1]

(iii) 
$$\overrightarrow{AC}$$
 [2]

- (b) Given that  $\binom{2}{m}$  and  $\stackrel{\longrightarrow}{AB}$  are parallel vectors. Find the value of m. [1]
- (c) Find the coordinates of D, such that ABCD is a parallelogram. [2]

(a) (i) 
$$\begin{vmatrix} \overrightarrow{AB} \end{vmatrix} = 10$$
 units

(ii) 
$$\overrightarrow{OC} = \frac{1}{3} \overrightarrow{OB}$$

$$= \begin{pmatrix} \frac{-6}{3} \\ \frac{12}{3} \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

(b) 
$$\frac{m}{2} = \frac{-6}{8}$$
$$= -1\frac{1}{2}$$

(iii) 
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BO} + \overrightarrow{OC}$$
  

$$= \begin{pmatrix} 8 \\ -6 \end{pmatrix} - \begin{pmatrix} -6 \\ 12 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

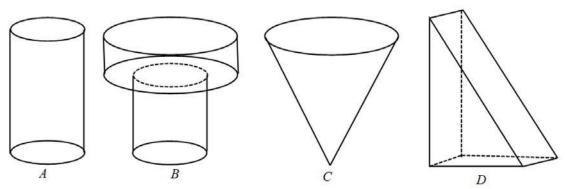
$$= \begin{pmatrix} 12 \\ -14 \end{pmatrix}$$

(c) 
$$\overrightarrow{CD} = \overrightarrow{BA} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$
  
Hence  $D$  is  $(-2-8, 4+6)$   
 $= (-10, 10)$ 

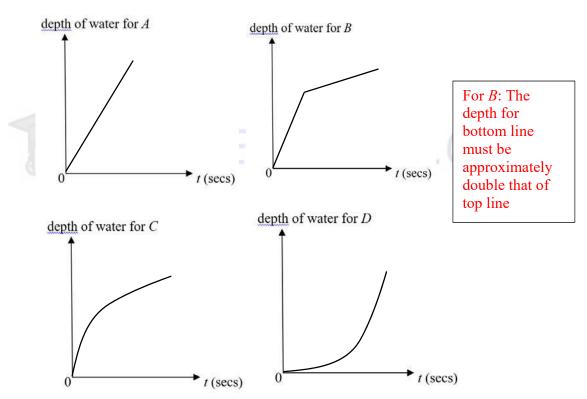
M1A1

M1A1

19 The diagrams below show four containers (not drawn to scale), A, B, C and D each with a height of h cm. The containers are initially empty. It takes t seconds to fill each container with water at a constant rate.



(a) On the axes in the answer space below, sketch the graph of the depth of the water against time for each of the four containers.



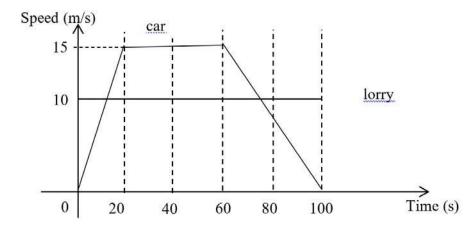
B2 or B1(any two correct) [2]

**(b)** It takes 12 seconds to fill container *D* to the brim. Find the time it takes to fill container *D* to half its height.

Ratio of volume of prisms =  $\frac{1}{4}$ . Required time =  $\frac{3}{4} \times 12 = 9$  seconds M1A1

Answer ......9...... secs [2]

The diagram shows the speed-time graphs of a car and a lorry travelling on the road for a period of 100 seconds. After accelerating, the car travels at a constant speed of 15 m/s for the next 40 seconds before decelerating to a stop, while the lorry travels at a constant speed of 10 m/s throughout.



(a) Find the time when the speed of the car is 10 m/s.

Acceleration = 
$$\frac{15}{20}$$
 = 0.75 m/s<sup>2</sup> Deceleration = 0.375 m/s in 1 sec

Hence time = 
$$\frac{10}{0.75}$$
 = 13.3 Hence time =  $100 - \frac{10}{0.375}$  = 73.3

(b) Find the speed of the car 10 seconds before it comes to rest.

Deceleration = 
$$\frac{15}{40}$$
 m/s<sup>2</sup>

Required speed = 
$$\frac{15}{40} \times 10 = 3.75$$

(c) Find the time when the car overtakes the lorry.

$$\frac{1}{2}(t+t-20) \times 15 = 10t$$

$$\left(t - 10\right) = \frac{2}{3}t$$

$$t = 30$$
 M1A1

*Answer*.......30...... sec [2]

21 (a) A box contains five slips of paper. Each slip has one of the numbers 4, 6, 7, 8 or 9 written on it. There are two players for the game. The first player reaches into the box and draws two slips and adds the two numbers. If the sum is even, the player wins. If the sum is odd, the player loses. What is the probability that the first player wins.

P(even and even) + P(odd and odd) = 
$$\left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{1}{4}\right)$$
  
=  $\frac{2}{5}$  M1A1  
Answer: .....  $\frac{2}{5}$  ...... [2]

(b) A game is such that a fair die is rolled respectively until a '6' is obtained. Find the probability that the game ends by the fourth roll.

Suppose now that the game is such that the same die is rolled repeatedly until two '6's are obtained. Find the probability that

- (i) the game ends on the third roll,
- (ii) the game ends on the third roll and the sum of the scores is odd.

(c) Of the 33 students in a class, 25 own tablet PC and 9 own desktop computers. It is given that

 $\xi = \{ \text{ students in the class } \},$ 

 $A = \{ \text{ students who own a tablet PC } \},$ 

 $B = \{ \text{ students who own a desktop computer } \} \text{ and } n(A \cap B) = x \}$ 

- (i) Express  $n(A' \cap B')$  in terms of x.
- (ii) Express in set notation { students who own tablet PC but not desktop computer}.

**22** (a) The diagram shows part of the graph of  $5(y+3) = ax^2 + bx$ , where a and b are constants. The graph cuts the x-axis at  $A\left(-2\frac{1}{2},0\right)$  and  $B\left(\frac{1}{2},0\right)$ . The graph meets the y-axis at the point C.

Find

- (i) the value of a and of b,
- (ii) the coordinates of C,
- (iii) the coordinates of the minimum point,
- (iv) the equation of the line of symmetry,
- (v) the area of triangle ABC.

(i) 
$$a = 60 - 2b$$
 ---- (1)  $12 = 5a - 2b$  ---- (2)

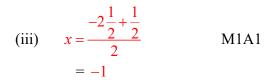
Sub (1) into (2)

$$b = 24$$

$$a = 12$$

M1A1





Hence minimum point is  $\left(-1, -5\frac{2}{5}\right)$ 

(iv) 
$$x = -1$$

(v) Area of triangle 
$$ABC = \frac{1}{2} \times 3 \times 3 = 4.5 \text{ units}^2$$
 M1A1

Answer (i) ... a = 12...., ... b = 24... [2]

0

x

(iii) .......
$$\left(-1, -5\frac{2}{5}\right)$$
.......[2]

$$(iv) \dots x = -1 \dots [1]$$

### **END OF PAPER**

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| Name: | Register No.: | Class: |  |
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# CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION 2018

**MATHEMATICS** 

4048/02 17 August 2018 2 hours 30 minutes

Paper 2

Additional Materials: Answer Paper

Graph Paper (1 sheet)

Mark Sheet

#### READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

## Answer all the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

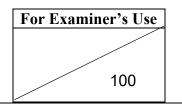
The use of a scientific calculator is expected, where appropriate.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 100.



### Mathematical Formulae

Compound Interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =  $\pi r l$ 

Surface area of a sphere =  $4\pi r^2$ 

Volume of a cone =  $\frac{1}{3}\pi r^2 h$ 

Volume of a sphere =  $\frac{4}{3}\pi r^3$ 

Area of triangle  $ABC = \frac{1}{2}ab \sin C$ 

Arc length =  $r\theta$ , where  $\theta$  is in radians

Sector area =  $\frac{1}{2}r^2\theta$ , where  $\theta$  is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation = 
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) Solve the equation 
$$\frac{x}{x^2 - 3x - 4} - \frac{x + 5}{4 - x} = 1$$
. [3]

**(b)** Make t the subject in the formula 
$$x = 2t\sqrt{\frac{k^2}{2k^2 + 3t^2}}$$
. [3]

(c) Simplify the expression 
$$\sqrt[3]{\frac{y}{x^2}} \times \frac{y}{x} \div \sqrt{\frac{x^3}{9y^{-2}}}$$
. [2]

2 A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies. Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

|  | Saturday |        |        | Sunday |        |        |
|--|----------|--------|--------|--------|--------|--------|
| Size                                     | Small    | Medium | Large  | Small  | Medium | Large  |
| Cost of ice-cream per cup                | \$2.50   | \$3.20 | \$4.50 | \$2.50 | \$3.20 | \$4.50 |
| Number of cups of<br>Rum Raisin sold     | 12       | 17     | 8      | 14     | 12     | 10     |
| Number of cups of<br>Super Chunkies sold | 18       | 15     | 11     | 13     | 21     | 16     |

The information for Saturday's sales can be represented by the matrix,

$$\mathbf{M} = \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix}$$
 and the cost of each flavour for each size can be represented by the matrix

$$C = \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix}$$
. The information for the Sunday's sales can be represented by a 2 × 3 matrix **N**.

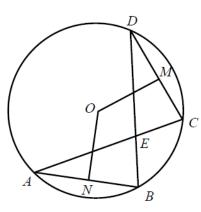
(a) Write down the matrix 
$$N$$
. [1]

(b) Calculate 
$$P = (M + N)$$
. [1]

(d) Calculate 
$$\mathbf{Q} = \frac{1}{2} \mathbf{PC}$$
. [2]

(f) Calculate and describe what is represented by the elements of 
$$\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{PC}$$
. [2]

3

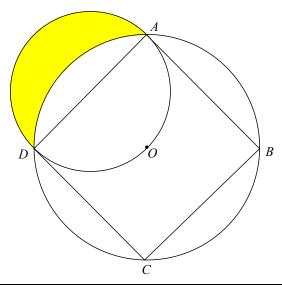


The points A, B, C, D lie on a circle, centre O. N and M are midpoints of AB and CD respectively. It is given that ON = OM.

- (a) Show that the triangles ABE and DCE are congruent. [3]
- (b) It is given that AB = 6 cm and ON = (r-1) cm, where r is the radius of the circle. Find the value of r. [2]
- 4 In the diagram, ABCD is a square whose diagonals are each 2 cm long. Given that O is the centre of the bigger circle and BD and AD are the diameters of the bigger and smaller circle respectively.

Find the area of the shaded region.

[4]



5 The first four terms in a sequence of numbers,  $p_1, p_2, p_3, p_4, ...$ , are given below.

$$p_1 = 1^2 + 2^2 + 2^2 = 3^2$$

$$p_2 = 2^2 + 3^2 + 6^2 = 7^2$$

$$p_3 = 3^2 + 4^2 + 12^2 = 13^2$$

$$p_4 = 4^2 + 5^2 + 20^2 = 21^2$$

- (a) Write down an expression for  $p_5$  and show that  $p_5 = 961$ .
- (b) Given that  $p_{10} = 10^2 + 11^2 + s^2 = k$ , express k as a perfect square in terms of s. [1]
- (c) Given that  $p_w = w^2 + (w + 1)^2 + r^2 = 5257^2$ , find the value of r and of w. [2]
- (d) Show that  $p_n = n^4 + 2n^3 + 3n^2 + 2n + 1$ . [3]

### 6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation  $y = x + \frac{7}{x} - 6$ .

The table below gives some values of x and the corresponding values of y.

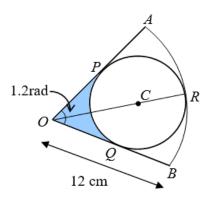
| х | 0.5 | 1 | 2    | 3.5 | 5   | 6    | 7 | 8    |
|---|-----|---|------|-----|-----|------|---|------|
| у | p   | 2 | -0.5 | q   | 0.4 | 1.17 | 2 | 2.88 |

- (a) Find the values of p and q. [1]
- (b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of  $y = x + \frac{7}{x} 6$  for the values of x in the range  $0 < x \le 8$ . [3]
- (c) Draw the line  $y = -\frac{1}{2}x + 3$ . [1]
- (d) Use your graph to find the x-coordinate of a point on the curve  $y = x + \frac{7}{x} 6$  at which the gradient of the tangent is equal to -0.5.
- (e) Find the range of values of x for which  $x^2 6x + 7 \le 2.5x$ . [2]

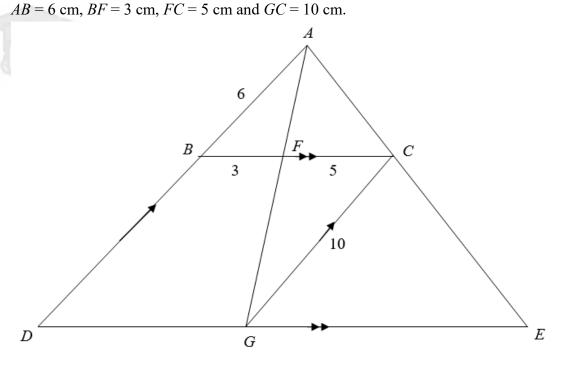
[2]

(f) By drawing a suitable line on your graph, find the solutions of the equation  $2x^2 - 13x + 14 = 0$ . [3]

7 (a) In the diagram, OARB is a sector of a circle with centre O, radius 12 cm and angle AOB = 1.2 radians. C is the centre of the circle enclosed inside the sector, OCR is a straight line and the circle touches the sector at P, Q and R.



- (i) Show that the radius of the enclosed circle is 4.3305 cm, correct to 4 decimal places.
- decimal places. [3]
  (ii) Calculate the perimeter of the shaded region *POQ*. [2]
- (b) In the diagram below, ABD, AFG, ACE, BFC and DGE are straight lines. BFC is parallel to DGE and DBA is parallel to GC.



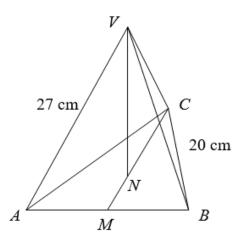
- (i) Prove that triangle BFA is similar to triangle CFG. [2]
- (ii) Calculate GE. [2]
- (iii) Given the area of triangle ABF is 6.4 cm<sup>2</sup>, find the area of trapezium ACGD. [3]

- 8 Amanda ran the 21km of a half-marathon race at an average speed of x km/h.
  - Write down, in terms of x, an expression for the number of hours it took her to complete the race. [1]
  - Deborah ran the same race at an average speed which is 3km/h faster than Amanda's **(b)** speed. Write down, in terms of x, an expression for the number of hours which Deborah took.

[1]

- Given that the difference between the two times was 20 minutes, write down an (c) equation in x and show that it reduces to  $x^2 + 3x - 189 = 0$ . [3]
- (d) Solve the equation  $x^2 + 3x - 189 = 0$ , giving your answers correct to 2 decimal places. [2]
- Find, in hours and minutes, the time it took Amanda to complete the race. (e) [1]

9



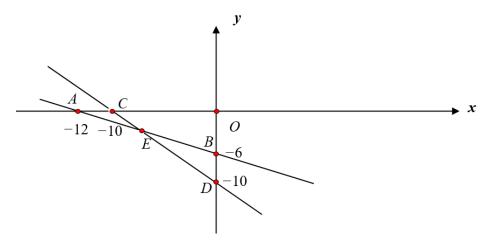
[The volume of tetrahedron =  $\frac{1}{3} \times \text{base area} \times \text{height}$ ]

The diagram shows a tetrahedron, VABC, which has a horizontal equilateral triangular base ABC of side 20 cm.

The slant edge of the tetrahedron (VA, VB and VC) are each of length 27 cm. M is the mid-point of AB and the vertical line VN meets the plane ABC at N where MN : NC = 1 : 2. Calculate

- CM, (a) [2]
- **(b)** the angle of elevation of V from A, [3]
- the volume of the tetrahedron. (c) [3]

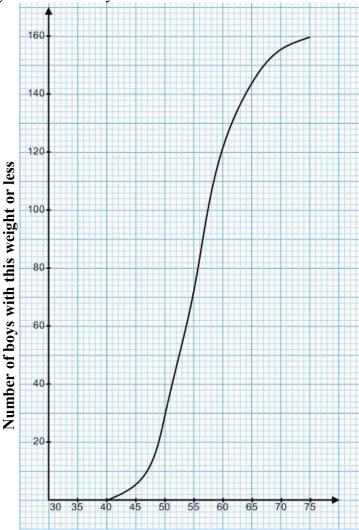
10 In the figure below, the x-intercept and y-intercept of the line AB are -12 and -6 respectively. Both the x-intercept and y-intercept of the line CD are -10.



Find

- (a) the equation of the line AB and CD, [2]
- (b) the coordinates of E, [2]
- (c) the area of *OCEB*, [2]
- (d) the coordinates of F given that point F lies on AB produced such that AF: FB = 5:3, [2]
- (e) find the coordinates of point G where G is the point on the x axis such that OE is parallel to GD.

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.



- (a) Use the graph to find an estimate for
  - (i) the number of boys whose weight is more than 60kg,

the median weight, [1]

[1]

[1]

[2]

(iii) the percentage of boys whose weight is less than or equal to 52 kg.

(iii) the percentage of boys whose weight is less than of equal to 32 kg.

**(b)** The lightest 25% of the boys are classified as "slim".

The heaviest 25% of the boys are classified as "big-sized".

The remaining boys are classified as "medium".

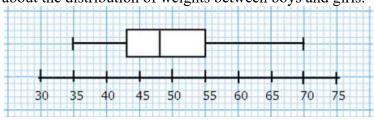
Find an estimate for

(ii)

(i) the greatest possible difference in weight between 2 boys classified as "big-sized",

(ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized". [2]

(c) The following box-and-whisker plot shows the distribution of weights of 160
Secondary One girls from the same school. Compare the data provided and record 2
observations about the distribution of weights between boys and girls. [2]



12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

# Price of formula milk in Singapore has soared

The average price of a 900g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

| Brand<br>Date | Similae | Friso | Nan  | S26  | Mamil |
|---------------|---------|-------|------|------|-------|
| Dec 2012      | 5.71    | 5.22  | 5.20 | 5.13 | 4.96  |
| Mar 2017      | 7.05    | 6.56  | 7.45 | 6.36 | 6.41  |
| % increase    |         |       | 43.3 | 24.0 | 29.2  |

Coffee stain was found on the newspaper article covering some of the information.

- (a) (i) Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain.
  - ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason.

A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

| Brand<br>Country | Similac | Friso | Nan  | S26  | Mamil |
|------------------|---------|-------|------|------|-------|
| Singapore        | 7.05    | 6.56  | 7.45 | 6.36 | 6.41  |
| Malaysia         | 3.92    | 3.54  | 4.29 | 4.13 | 3.51  |
| China            | 4.79    | 5.58  | 9.06 | 4.25 | 4.00  |

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China.

[2]

[2]

Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

| Weight of parcel, x (kg) | Shipping Rate            |
|--------------------------|--------------------------|
| weight of parcer, x (kg) | 1st kg 150 RMB follow by |
| <i>x</i> ≤ 10            | 75 <i>RMB</i> / kg       |
| $10 < x \le 20$          | 35 <i>RMB</i> / kg       |
| $20 < x \le 50$          | 31 <i>RMB</i> / kg       |
| $50 < x \le 75$          | 27 <i>RMB</i> / kg       |
| $75 < x \le 100$         | 25 <i>RMB</i> / kg       |
| $100 < x \le 200$        | 24 <i>RMB</i> / kg       |
| x > 200                  | 22 <i>RMB</i> / kg       |

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.





Mrs Tan intends to spend at most S\$650 for both the formula milk and shipping fee.

**(b)** Calculate the maximum number of cans of formula milk that Mrs Tan can buy.

[5]

## **END OF PAPER**

# **ANSWER KEY**

| 1 | a | $r = -\frac{9}{2}$   |
|---|---|--|
|   |   | 10   |
|   | b | $x = -\frac{9}{10}$ $t = \pm \sqrt{\frac{-2k^2x^2}{3x^2 - 4k^2}}  \text{or}  t = \pm \sqrt{\frac{2k^2x^2}{4k^2 - 3x^2}}$ |
|   | c | $\frac{3y^{\frac{1}{3}}}{x^{\frac{19}{6}}}$  |
| 2 | a | $\mathbf{N} = \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix}$  |
|   | b | $\mathbf{P} = \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix}$  |
|   | d | $\begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix}$   |
|   | f | (553)  |
| 3 | b | r = 5cm  |
| 4 |   | $\frac{1}{2}$ or 0.500 cm <sup>2</sup>   |
| 5 | a | 961  |
|   | b | $k = (s + 1)^2$  |
|   | c | r = 5256   |
| 6 | a | w = 72 $p = 8.5$   |
|   |   | q = -0.5   |
|   | b | y  |
|   | e | $0.9(\pm 0.1) \le x \le 7.6(\pm 0.1)$  |
|   | f | $x = 0.6 \pm 0.1$ and $= 5.9 \pm 0.1$  |
|   |   | 1 0.0 = 0.0 = 0.0 = 0.0  |

|    | 1    |   |
|----|------|---|
| 7  | aii  | 21.1 cm                                 |
|    | bii  | $13\frac{1}{3} \text{ cm}$              |
|    | L    | <b>5</b>                                |
|    | biii | 74.0 cm <sup>2</sup>                    |
| 8  | a    | $\frac{21}{\text{hr}}$                  |
|    |      | x                                       |
|    | b    | 21 .                                    |
|    |      | $\frac{21}{x+3}$ hr                     |
|    | d    | x = 12.33  (2dp) or $x = -15.33  (2dp)$ |
|    |      |   |
|    | e    | 1hr 42 mins                             |
| 9  | a    | 17.3 cm                                 |
|    | b    | 64.7°                                   |
|    | c    | 1410 cm <sup>3</sup>                    |
| 10 | a    | y = -x - 10                             |
|    | b    | (-8, -2)                                |
|    | c    | 34 units <sup>2</sup>                   |
|    | d    | (18,-15)                                |
|    | e    | (40,0)                                  |
| 11 | ai   | 40                                      |
|    | aii  | 56 kg                                   |
|    | aiii | 27.5%                                   |
|    | bi   | 15 kg                                   |
|    | bii  | 8.5 kg                                  |
| 12 | ai   | % increase of Similac = 23.5%           |
|    |      | % increase of the Friso = 25.7%         |
|    | b    | 13 tins                                 |

| Name:       | Register No.: | Class: |
|-------------|---------------|--------|
| MARK SCHEME |               |        |



## CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION 2018

**MATHEMATICS** 

4048/02 17 August 2018 2 hours 30 minutes

Paper 2

Additional Materials: Answer Paper

Graph Paper (1 sheet)

Mark Sheet

#### READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

#### Answer all the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

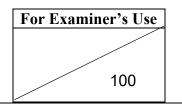
The use of a scientific calculator is expected, where appropriate.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 100.



#### Mathematical Formulae

Compound Interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =  $\pi r l$ 

Surface area of a sphere =  $4\pi r^2$ 

Volume of a cone =  $\frac{1}{3}\pi r^2 h$ 

Volume of a sphere =  $\frac{4}{3}\pi r^3$ 

Area of triangle  $ABC = \frac{1}{2}ab \sin C$ 

Arc length =  $r\theta$ , where  $\theta$  is in radians

Sector area =  $\frac{1}{2}r^2\theta$ , where  $\theta$  is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Statistics** 

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation = 
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) Solve the equation 
$$\frac{x}{x^2 - 3x - 4} - \frac{x + 5}{4 - x} = 1$$
. [3]

**(b)** Make t the subject in the formula 
$$x = 2t\sqrt{\frac{k^2}{2k^2 + 3t^2}}$$
. [3]

(c) Simplify the expression 
$$\sqrt[3]{\frac{y}{x^2}} \times \frac{y}{x} \div \sqrt{\frac{x^3}{9y^{-2}}}$$
. [2]

1 (a) 
$$\frac{x}{x^2 - 3x - 4} - \frac{x + 5}{4 - x} = 1$$

$$\frac{x}{(x - 4)(x + 1)} - \frac{x + 5}{4 - x} = 1$$

$$\frac{x}{(x - 4)(x + 1)} + \frac{x + 5}{x - 4} = 1$$
Multiply throughout by  $(x - 4)(x + 1)$ 

$$x + (x + 5)(x + 1) = (x - 4)(x + 1)$$

$$x + x^2 + 6x + 5 = x^2 - 3x - 4$$
M1

$$10x + 9 = 0$$

$$x = -\frac{9}{10}$$
A1

(b) 
$$x = 2t\sqrt{\frac{k^2}{2k^2 + 3t^2}}$$

$$\frac{x}{2t} = \sqrt{\frac{k^2}{2k^2 + 3t^2}}$$

$$\frac{x^2}{4t^2} = \frac{k^2}{2k^2 + 3t^2}$$

$$x^2(2k^2 + 3t^2) = 4k^2t^2$$

$$2k^2x^2 + 3t^2x^2 = 4k^2t^2$$

$$t^2(3x^2 - 4k^2t^2 = -2k^2x^2$$

$$t = \pm \sqrt{\frac{-2k^2x^2}{3x^2 - 4k^2}} \quad \text{or} \quad t = \pm \sqrt{\frac{2k^2x^2}{4k^2 - 3x^2}}$$
Al (No marks if  $\pm$  is not shown)

(c) 
$$\sqrt[3]{\frac{y}{x^2}} \times \frac{y}{x} \div \sqrt{\frac{x^3}{9y^{-2}}}$$

$$= \frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \times \frac{y}{x} \div \frac{x^{\frac{3}{2}}}{3y^{-1}}$$

$$= \frac{y^{\frac{1}{3}}}{x^{\frac{3}{3}}} \times \frac{y}{x} \times \frac{3y^{-1}}{x^{\frac{3}{2}}}$$

$$= \frac{3y^{\frac{1}{3}}}{x^{\frac{19}{6}}}$$
M1 – indices of variables correct

A1

A shop sells two flavours of ice-cream, Rum Raisin and Super Chunkies. 2 Each flavour is sold in cups of three different sizes, small, medium and large, and of different prices. The sales in two successive days are given in the table below.

|  |        | Saturday |        |        | Sunday |        |  |
|--|--------|----------|--------|--------|--------|--------|--|
| Size                                     | Small  | Medium   | Large  | Small  | Medium | Large  |  |
| Cost of ice-cream per cup                | \$2.50 | \$3.20   | \$4.50 | \$2.50 | \$3.20 | \$4.50 |  |
| Number of cups of Rum Raisin sold        | 12     | 17       | 8      | 14     | 12     | 10     |  |
| Number of cups of<br>Super Chunkies sold | 18     | 15       | 11     | 13     | 21     | 16     |  |

The information for Saturday's sales can be represented by the matrix,

 $\mathbf{M} = \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix}$  and the cost of each flavour for each size can be represented by the matrix

 $\mathbf{C} = \begin{pmatrix} 2.5 \\ 3.2 \end{pmatrix}$ . The information for the Sunday's sales can be represented by a 2 × 3 matrix  $\mathbf{N}$ .

(b) Calculate 
$$P = (M + N)$$
. [1]

(d) Calculate 
$$\mathbf{Q} = \frac{1}{2} \mathbf{PC}$$
. [2]

(f) Calculate and describe what is represented by the elements of 
$$\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{PC}$$
. [2]

(a) 
$$N = \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix}$$
 Sizes B1

(a) 
$$\mathbf{N} = \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix}$$
 Sizes

(b)  $\mathbf{P} = \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix}$  ← Flavours B1

(c) Total number of cups of Rum Raisin and Super

(d) 
$$\mathbf{Q} = \frac{1}{2}\mathbf{PC}$$

$$= \frac{1}{2} \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix}$$

$$= \begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix} \leftarrow \text{Flavours}$$
A1

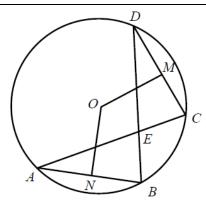
A1

(f) 
$$\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} = \begin{pmatrix} 553 \end{pmatrix}$$

It represents the total amt. collected from the sales of all flavours and cup sizes of ice-cream on Saturday and Sunday. **B**1

В1

3



The points A, B, C, D lie on a circle, centre O. N and M are midpoints of AB and CD respectively. It is given that ON = OM.

(a) Show that the triangles ABE and DCE are congruent.

[3]

(b) It is given that AB = 6 cm and ON = (r-1) cm, where r is the radius of the circle. Find the value of r.

[2]

(a)  $\angle ABE = \angle DCE$  (angles in the same segment)

 $\angle BAE = \angle CDE$  (angles in the same segment)

ON = OM(given)

 $\therefore AB = CD$  (equal chords, equidistant from centre)

 $\therefore \Delta ABE \equiv \Delta DCE \text{ (ASA)}$ 

M2 – All 3 reasons are correctly given

M1 – Any 2 correct reasons

are given

A1 – congruency statement

$$\mathbf{(b)} \qquad OB^2 = ON^2 + NB^2$$

$$r^2 = 3^2 + (r-1)^2$$

 $r^2 = 9 + r^2 - 2r + 1$ 

2r = 10

r = 5 cm

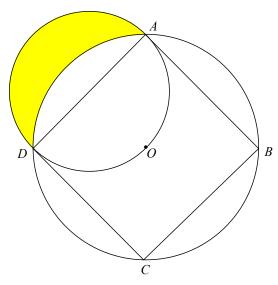
M1

**A**1

4 In the diagram, ABCD is a square whose diagonals are each 2 cm long. Given that O is the centre of the bigger circle and BD and AD are the diameters of the bigger and smaller circle respectively.

Find the area of the shaded region.

[4]



By Pythagoras' theorem

$$AD = \sqrt{2}$$
  $\Rightarrow$  Radius of small circle  $= \frac{\sqrt{2}}{2}$  M1

Area of semi-circle = 
$$\frac{1}{2}\pi \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{4}$$
 or 0.78539 cm<sup>2</sup>

Area of segment 
$$AD = \frac{1}{2}(1)^2 \left[\frac{\pi}{2} - \sin\frac{\pi}{2}\right] = \frac{\pi}{4} - \frac{1}{2} \text{ or } 0.28539 \text{ cm}^2$$
 M1

Area of shaded region = 
$$\frac{\pi}{4} - 0.28539 = \frac{1}{2} \text{ or } 0.500 \text{ cm}^2 \text{ (3sf)}$$

5 The first four terms in a sequence of numbers,  $p_1, p_2, p_3, p_4, ...$ , are given below.

$$p_1 = 1^2 + 2^2 + 2^2 = 3^2$$

$$p_2 = 2^2 + 3^2 + 6^2 = 7^2$$

$$p_3 = 3^2 + 4^2 + 12^2 = 13^2$$

$$p_4 = 4^2 + 5^2 + 20^2 = 21^2$$

- (a) Write down an expression for  $p_5$  and show that  $p_5 = 961$ . [1]
- (b) Given that  $p_{10} = 10^2 + 11^2 + s^2 = k$ , express k as a perfect square in terms of s. [1]
- (c) Given that  $p_w = w^2 + (w + 1)^2 + r^2 = 5257^2$ , find the value of r and of w. [2]

(d) Show that 
$$p_n = n^4 + 2n^3 + 3n^2 + 2n + 1$$
. [3]

(a) 
$$p_5 = 5^2 + 6^2 + 30^2$$
  
 $= 25 + 36 + 900$   
 $= 961$ 
B1
(b)  $k = (s + 1)^2$ 
B1
(c)  $r = 5256$ 

$$w(w+1) = 5256$$

$$= 72 \times 73$$

$$w = 72$$
A1

(d) 
$$p_n = n^2 + (n+1)^2 + [n(n+1)]^2$$

$$= n^2 + n^2 + 2n + 1 + (n^2)(n+1)^2$$

$$= 2n^2 + 2n + 1 + n^2(n^2 + 2n + 1)$$

$$= 2n^2 + 2n + 1 + n^4 + 2n^3 + n^2$$

$$= n^4 + 2n^3 + 3n^2 + 2n + 1$$
A1

# 6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation  $y = x + \frac{7}{x} - 6$ .

The table below gives some values of x and the corresponding values of y.

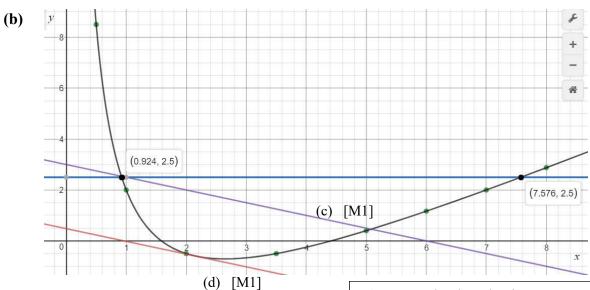
| x | 0.5 | 1 | 2    | 3.5 | 5   | 6    | 7 | 8    |
|---|-----|---|------|-----|-----|------|---|------|
| У | p   | 2 | -0.5 | q   | 0.4 | 1.17 | 2 | 2.88 |

- (a) Find the values of p and q. [1]
- (b) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of  $y = x + \frac{7}{x} 6$  for the values of x in the range  $0 < x \le 8$ . [3]

(c) Draw the line 
$$y = -\frac{1}{2}x + 3$$
. [1]

- (d) Use your graph to find the x-coordinate of a point on the curve  $y = x + \frac{7}{x} 6$  at which the gradient of the tangent is equal to -0.5. [2]
- (e) Find the range of values of x for which  $x^2 6x + 7 \le 2.5x$ . [2]
- (f) By drawing a suitable line on your graph, find the solutions of the equation  $2x^2 13x + 14 = 0$ . [3]

(a) 
$$p = 8.5$$
 A1 – both correct answer  $q = -0.5$ 



P1 – correctly plotted points

M1 - smooth, neat + labelled curve

M1 – labelled axis and correct scale

(d) 
$$x = 2.15 \pm 0.2$$

M1

(e) 
$$x^2 - 6x + 7 \le 2.5x$$
  
 $x - 6 + \frac{7}{x} \le 2.5$   
 $y \le 2.5$ 

M1

$$0.9(\pm 0.1) \le x \le 7.6(\pm 0.1)$$

A1 – both correct answer

(f) 
$$2x^2 - 13x + 14 = 0$$
  
 $x + \frac{7}{x} - 6.5 = 0$   
 $x + \frac{7}{x} - 6 = 0.5$ 

M1

Draw y = 0.5 on the graph

y = 0.5

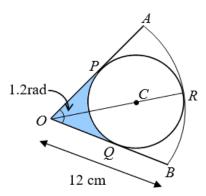
M1

$$x = 0.6 \pm 0.1$$

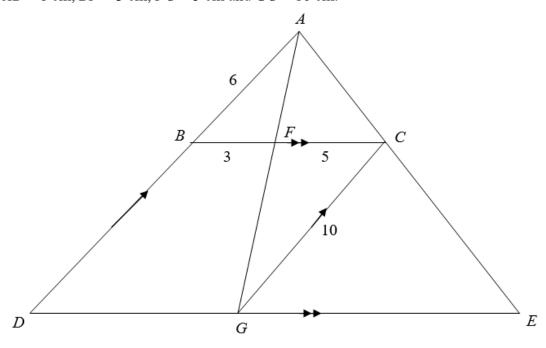
$$=5.9\pm0.1$$

 $A1-both\ correct\ answer$ 

In the diagram, *OARB* is a sector of a circle with centre *O*, radius 12 cm and angle 7 (a) AOB = 1.2 radians. C is the centre of the circle enclosed inside the sector, OCR is a straight line and the circle touches the sector at P, Q and R.



- **(i)** Show that the radius of the enclosed circle is 4.3305 cm, correct to 4 decimal places.
- [3] [2] Calculate the perimeter of the shaded region *POQ*. (ii)
- **(b)** In the diagram below, ABD, AFG, ACE, BFC and DGE are straight lines. BFC is parallel to DGE and DBA is parallel to GC. AB = 6 cm, BF = 3 cm, FC = 5 cm and GC = 10 cm.



- Prove that triangle *BFA* is similar to triangle *CFG*. (i)
- [2] Calculate *GE*. [2] (ii)
- Given the area of triangle ABF is 6.4 cm<sup>2</sup>, find the area of trapezium ACGD. [3] (iii)
- (i) Let r be the radius of the enclosed circle. In  $\triangle COP$ ,

$$\sin 0.6 = \frac{CP}{OC}$$

$$= \frac{r}{12 - r}$$

$$\sin 0.6(12 - r) = r$$

$$r \sin 0.6 + r = 12 \sin 0.6$$

$$r = \frac{12 \sin 0.6}{\sin 0.6 + 1}$$
M1

M1

$$\therefore r = 4.3305 \text{ cm } (4 \text{ dp}) (\text{shown})$$
 A1

(ii) 
$$\angle PCQ = 2\pi - \frac{\pi}{2} - \frac{\pi}{2} - 1.2 = \pi - 1.2 = 1.9415 \text{ rad}$$
 M1

In  $\triangle COP$ , tan  $0.6 = \frac{4.3305}{OP}$ 
 $OP = 6.3298 \text{ cm} = OQ$ 

Perimeter of shaded region = 
$$2 (6.3298) + (4.3305)(1.9415)$$
  
=  $21.0672$   
 $\approx 21.1 \text{ cm (3sf)}$  A1

(b)(i) 
$$\angle BFA = \angle CFG \text{ (vertically opposite angles)}$$
 
$$\angle ABF = \angle GCF \text{ (alternate angles)}$$
 
$$\therefore \Delta BFA \text{ is similar to } \Delta CFG$$
 A1

(b)(ii) 
$$\frac{AB}{AD} = \frac{BC}{DE} \quad (\Delta ABC \text{ is similar to } \Delta ADE)$$

$$\frac{6}{6+10} = \frac{8}{8+GE}$$

$$\frac{6}{16} = \frac{8}{8+GE}$$

$$8+GE = \frac{8\times16}{6}$$

$$GE = 13\frac{1}{3} \text{ cm}$$
A1

OR

$$\frac{GE}{BC} = \frac{BC}{AB} \quad (\Delta ABC \text{ is similar to } \Delta CGE)$$

$$\frac{GE}{10} = \frac{8}{6} \qquad M1$$

$$GE = 13\frac{1}{3} \text{ cm} \qquad A1$$

(b)(iii)

Area of 
$$\triangle ABC = \frac{8}{3} \times \text{Area of } \triangle ABF$$
  

$$= \frac{8}{3} \times 6.4$$

$$= 17 \frac{1}{15} \text{ cm}^2$$
M1

Area of trapezium  $ACGD = \frac{1}{2}(CG + AD) \times$  perpendicular distance from CG to AD

Area of triangle  $ABC = \frac{1}{2}(AB) \times \text{perpendicular distance from } CG \text{ to } AD$ 

$$\frac{\text{Area of trapezium } ACGD}{\text{Area of triangle } ABC} = \frac{CG + AD}{AB}$$

Area of trapezium 
$$ACGD = \frac{16+10}{6} \times 17\frac{1}{15}$$

$$= 74.0 \text{ cm}^2 \text{ (3sf)}$$
M1

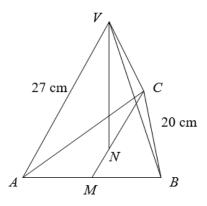
- 8 Amanda ran the 21km of a half-marathon race at an average speed of x km/h.
  - (a) Write down, in terms of x, an expression for the number of hours it took her to complete the race. [1]
  - (b) Deborah ran the same race at an average speed which is 3km/h faster than Amanda's speed. Write down, in terms of x, an expression for the number of hours which Deborah took. [1]
  - (c) Given that the difference between the two times was 20 minutes, write down an equation in x and show that it reduces to  $x^2 + 3x 189 = 0$ . [3]
  - (d) Solve the equation  $x^2 + 3x 189 = 0$ , giving your answers correct to 2 decimal places. [2]
  - (e) Find, in hours and minutes, the time it took Amanda to complete the race. [1]

M1

- $T_1 = \frac{21}{r} \operatorname{hr}$  A1
- (b)  $T_2 = \frac{21}{x+3} \text{hr}$  A1
- (c)  $T_1 T_2 = \frac{20}{60}$   $\frac{21}{x} \frac{21}{x+3} = \frac{1}{3}$ 
  - $\frac{21(x+3)-21x}{x(x+3)} = \frac{1}{3}$  M1
  - $x^{2} + 3x = 3 \times 63$  $x^{2} + 3x - 189 = 0$  (shown) A1
- (d)  $x^2 + 3x 189 = 0$  $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-189)}}{2}$   $x = 12.33 \text{ (2dp)} \quad \text{or} \quad x = -15.33 \text{ (2dp)}$ A1
- (e) x = -15.33 (rejected)

Time taken =  $\frac{21}{12.33}$ = 1 hr 42 mins

9



[The volume of tetrahedron =  $\frac{1}{3} \times \text{base area} \times \text{height}$ ]

The diagram shows a tetrahedron, *VABC*, which has a horizontal equilateral triangular base *ABC* of side 20 cm.

The slant edge of the tetrahedron (VA, VB and VC) are each of length 27 cm. M is the mid-point of AB and the vertical line VN meets the plane ABC at N where MN:NC=1:2. Calculate

(a) 
$$CM$$
, [2]

(b) the angle of elevation of V from A, [3]

[3]

(c) the volume of the tetrahedron.

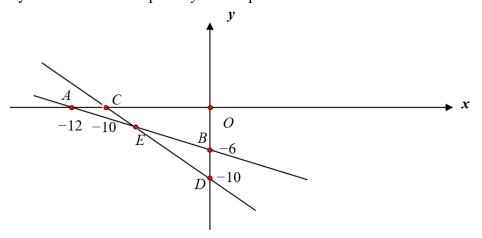
(a) 
$$CM^2 = CB^2 - MB^2$$
 (Pythagoras Theorem)  
 $= 20^2 - 10^2$  M1  
 $= 300$   
 $CM = 17.3 \text{ cm (3sf)}$ 

(b) 
$$AN^2 = AM^2 + MN^2$$
 (Pythagoras Theorem)  
 $= 10^2 + \left[\frac{1}{3}(17.320)\right]^2$   
 $= 11.546 \text{ cm}$  M1  
 $\cos \angle VAN = \frac{11.546}{27}$  M1  
 $\angle VAN = 64.7^{\circ} \text{ (1dp)}$ 

(c) 
$$VN = VA \sin \angle VAN$$
$$= 27 \sin 64.68$$
$$= 24.4 \text{ cm}$$
M1

Volume of tetrahedron = 
$$\frac{1}{3} \times \text{Area of base} \times VN$$
  
=  $\frac{1}{3} \times \left(\frac{1}{2} \times 20 \times 20 \sin 60^{\circ}\right) \times 24.406$  M1  
=  $1410 \text{ cm}^{3}$  A1

10 In the figure below, the x-intercept and y-intercept of the line AB are -12 and -6 respectively. Both the x-intercept and y-intercept of the line CD are -10.



Find

(a) the equation of the line AB and CD, [2]

(b) the coordinates of 
$$E$$
, [2]

(c) the area of *OCEB*, [2]

(d) the coordinates of F given that point F lies on AB produced such that AF: FB = 5:3, [2]

(e) find the coordinates of point G where G is the point on the x – axis such that OE is parallel to GD.

(a) Gradient of the line 
$$AB = \frac{-6-0}{0-(-12)} = -\frac{1}{2}$$

Equation of the line AB:  $y = -\frac{1}{2}x - 6$  A1

Gradient of the line  $CD = \frac{-10 - 0}{0 - (-10)} = -1$ 

Equation of the line *CD*: y = -x - 10

(b) 
$$y = -\frac{1}{2}x - 6$$
 (1)

$$y = -x - 10 \tag{2}$$

$$(1)-(2)$$

$$x = -8$$

$$y = -2$$

Coordinates of E = (-8, -2)

M1

**A**1

(c) Area of 
$$OCEB$$
 = Area of  $\triangle CDO$  - Area of  $\triangle EBD$ 

$$= \frac{1}{2} \times 10 \times 10 - \frac{1}{2} \times 4 \times 8$$

$$= 34 \text{ units}^2$$
 A1

$$\frac{AF}{FB} = \frac{5}{3}$$

Using similar triangles,

$$\frac{-12-x}{0-x} = \frac{5}{3}$$
$$x = 18$$

$$\frac{0-y}{-6-y} = \frac{5}{3}$$

$$y = -15$$

Coordinates of F = (18, -15)

A1

A1

M1

(e) Let the coordinates of G be (k, 0)

$$\frac{0 - (-2)}{0 - (-8)} = \frac{0 - (-10)}{k - 0}$$
1 10

$$\frac{1}{4} = \frac{10}{k}$$

$$k = 40$$
  
Coordinates of  $G = (40,0)$ 

**A**1

OR

$$\overrightarrow{OE} = m\overrightarrow{GD}$$

$$\begin{pmatrix}
-8 \\
-2
\end{pmatrix} = m \begin{pmatrix}
-k \\
-10
\end{pmatrix}$$

$$-2 = -10m$$
M1

$$m = \frac{1}{5}$$

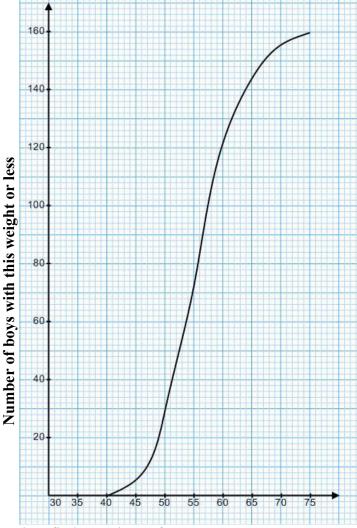
$$-8 = -mk$$

$$k = 40$$

Coordinates of 
$$G = (40,0)$$

**A**1

11 The cumulative frequency curve below shows the weights of a sample of 160 boys from a school when they enter Secondary One.



- (a) Use the graph to find an estimate for
  - (i) the number of boys whose weight is more than 60kg,

[1]

[1]

[1]

[2]

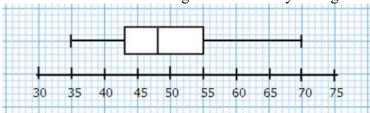
- (ii) the median weight,
- (iii) the percentage of boys whose weight is less than or equal to 52 kg.
- **(b)** The lightest 25% of the boys are classified as "slim".

The heaviest 25% of the boys are classified as "big-sized".

The remaining boys are classified as "medium".

Find an estimate for

- (i) the greatest possible difference in weight between 2 boys classified as "big-sized".
- (ii) the least possible difference between the weight of a boy classified as "slim" and the weight of a boy classified as "big-sized". [2]
- (c) The following box-and-whisker plot shows the distribution of weights of 160
  Secondary One girls from the same school. Compare the data provided and record 2
  observations about the distribution of weights between boys and girls. [2]



- 11 (a) (i) Number of boys whose weight is more than 60 kg = 40.
- B1

(ii) median weight = 56 kg.

- В1
- (iii) Percentage of boys whose weight  $\leq 52 \text{ kg} = \frac{44}{160} \times 100\%$ 
  - = 27.5% B1
- (b) (i) Based on distribution of number of boys Greatest possible difference = 75-60

M1

=15 kg

A1

(ii) Based on distribution of weight

Least possible difference = 60 - 51.5

M1

 $=8.5 \,\mathrm{kg}$ 

A1

- (c) Median weight of girls lower than boys; girls are lighter; For boys,  $Q_3 - Q_1 = 8.5$ kg. For girls,  $Q_3 - Q_1 = 12$ kg; Girls' weight has more variation.
- 12 Mrs Tan, a mother of two children, saw a newspaper article as shown.

# Price of formula milk in Singapore has soared

The average price of a 900g tin of formula milk has increased sharply over the last 5 years, outstripping the price increases of other dairy products and household staples.

On Monday, the Government announced it is tightening rules to encourage greater price competition.

The table below shows the price (in SGD) per 100 grams of different brands of formula milk in year 2012 and 2017.

| Brand<br>Date | Similae | Friso | Nan  | S26  | Mamil |
|---------------|---------|-------|------|------|-------|
| Dec 2012      | 5.71    | 5.22  | 5.20 | 5.13 | 4.96  |
| Mar 2017      | 7.05    | 6.56  | 7.45 | 6.36 | 6.41  |
| % increase    |         |       | 43.3 | 24.0 | 29.2  |

Coffee stain was found on the newspaper article covering some of the information.

(a) (i) Calculate the percentage increase in the price of the Similac and Friso formula milk covered by the coffee stain..

[2]

(ii) Hence, do you agree with the headline of the newspaper article? Support your answer with a reason.

[2]

A few days later, Mrs Tan saw another article regarding the price of similar brands of formula milk sold in Singapore, Malaysia and China.

| Brand<br>Country | Similac | Friso | Nan  | S26  | Mamil |
|------------------|---------|-------|------|------|-------|
| Singapore        | 7.05    | 6.56  | 7.45 | 6.36 | 6.41  |
| Malaysia         | 3.92    | 3.54  | 4.29 | 4.13 | 3.51  |
| China            | 4.79    | 5.58  | 9.06 | 4.25 | 4.00  |

Price (in SGD) per 100 grams of formula milk in Singapore, Malaysia and China. Upon seeing the article, Mrs Tan intends to purchase some cans of formula milk in China and ship them back during her holidays.

Mrs Tan did an online research and found the following shipping rate from China to Singapore by SHIPPER Company.

| Weight of parcel, x (kg) | Shipping Rate            |
|--------------------------|--------------------------|
| weight of parcel, x (kg) | 1st kg 150 RMB follow by |
| <i>x</i> ≤ 10            | 75 <i>RMB</i> / kg       |
| $10 < x \le 20$          | 35 <i>RMB</i> / kg       |
| $20 < x \le 50$          | 31 <i>RMB</i> / kg       |
| $50 < x \le 75$          | 27 <i>RMB</i> / kg       |
| $75 < x \le 100$         | 25 <i>RMB</i> / kg       |
| $100 < x \le 200$        | 24 <i>RMB</i> / kg       |
| x > 200                  | 22 <i>RMB</i> / kg       |

The information below shows a can of the 900 g formula milk that Mrs Tan intends to purchase in China and the online currency conversion.





[5]

Mrs Tan intends to spend at most S\$650 for both the formula milk and shipping fee.

**(b)** Calculate the maximum number of cans of formula milk that Mrs Tan can buy.

12 (a) Percentage increase of Similac = 
$$\frac{7.05-5.71}{5.71} \times 100\%$$
  
= 23.5% B1  
Percentage increase of the Friso =  $\frac{6.56-5.22}{5.22} \times 100\%$   
= 25.7% B1

(ii) Mean of percentage increase = 
$$\frac{23.5 + 25.7 + 43.3 + 24.0 + 29.2}{5}$$
$$= 29.14\%$$

Agree.

B1
as the mean of percentage increase is greater than 29% which is much higher than price increase of general food consumption items.

(accept any logical answer)

(b) 
$$S$650 = 4.89 \times 650$$
  
= 3178.50 RMB

Let *x* be the number of can of milk powder.

$$\underbrace{4.25 \times 4.89 \times 9 \times x + 150 + 35 \left(\frac{900x}{1000}\right)}_{\text{M1}} \le 3178.50$$
 M1-accept if students use equation

$$187.0425x + 31.5x \le 3028.50$$
  
 $218.5425x \le 3078.50$  M1  
 $x \le 13.857$   
 $x = 13$  tins of formula milk A1

# Or working in term of Singapore Dollar

Shipping rate of 150 RMB = S\$30.675 35 RMB = S\$7.157

Let *x* be the number of can of milk powder.

$$4.25 \times 9 \times x + 30.675 + 7.157 \left(\frac{900x}{1000}\right) \le 650$$
M1-accept if students use equation
$$M1$$

$$44.6913x < 619.325$$

$$44.6913x \le 619.325$$
 M1  $x \le 13.857$   $x = 13$  tins of formula milk A1

# **END OF PAPER**