CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1

1 **(a)** Given $x^9 = 9^0$, find the value of *x*.

Answer ………………………….. [1]

(b) Simplify $\frac{x^2}{3y} \div \frac{x}{9y^2}$.

Answer ………………………….. [1]

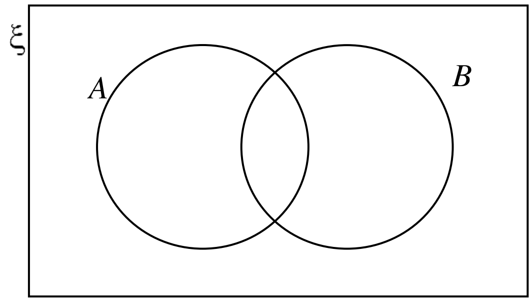
2 Factorise $2p - 2q - p^2 + pq$.

Answer ………………………….. [2]

3 Write as a single fraction in its simplest form $\frac{5x}{(3-x)^2} - \frac{1}{x-3}$.

Answer ………………………….. [2]

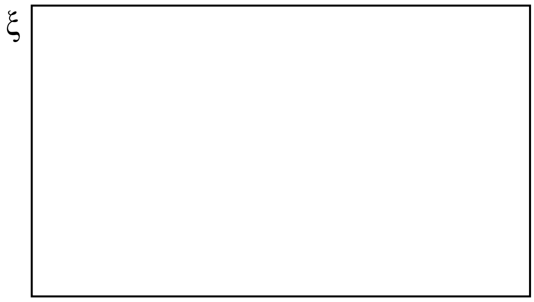
4 **(a)** On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

(b) Given that P is a subset of Q, simplify $(P \cup Q)'$.

You may use the space below to help in your investigation.



Answer $(P \cup Q)' =$ …………….. [1]

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5 A shuttle bus is due to arrive at the ABC station at a certain time every morning.

The numbers of minutes by which the bus was late on ten successive days are shown below.

3 0 −2 −3 8 4 10 5 −4 9

(a) Explain the meaning of the number '−2' in the list of numbers of minutes.

Answer ....

....

.... [1]

(b) Find the mean number of minutes by which the bus was late.

Answer ………………………….. [1]

6 Given that p is a positive integer,

(a) write down expressions for the next two even numbers after $2(p - 1)$.

Answer ………… and ………….. [1]

(b) **(i)** find, in its simplest form, an expression for the sum of the squares of these three even numbers,

Answer ………………………….. [2]

(ii) explain why this sum is a multiple of 4.

Answer ....

....

.... [1]

---

7 **(a)** Express 40 and 138 as the product of their prime factors.

Answer 40 = …………..............……, 138 = ...………….. [2]

(b) Hence, find the smallest positive integer k such that $138k$ is divisible by 40.

Answer smallest positive integer k = ………....…………. [1]

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8 A wooden cube with side 8 cm is cut into two-centimetre cubes.
All of the two-centimetre cubes are then arranged to form a cuboid with height greater than 8 cm.
The perimeter of the top of the cuboid is 36 cm.

Find the height of the cuboid.

Answer ……………………… cm [2]

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9 A map is drawn to a scale of 1 : 40 000.

(a) This scale can be expressed as 1 cm represents n km.

Find n.

Answer n = ……….....………….. [1]

(b) The distance between a seaport and an airport on the map is 60 cm.

Find the actual distance, in kilometres, between the seaport and the airport.

Answer ……….....………….. km [1]

(c) A bus depot has an actual area of 8 km^2.

Find the area, in square centimetres, of the bus depot on the map.

Answer ……….....………….. cm^2 [2]

---

10 **(a)** Fynn deposited $\$m$ into an account that paid a compound interest of 1.85% per annum. He made no other deposits or withdrawals for three years. At the end of three years, he had $2509.26 in his account.

Find the value of m, giving your answer correct to the nearest dollar.

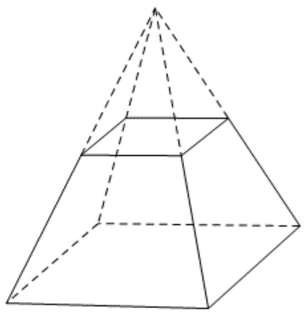
Answer m = …………....………….. [2]

(b) Fynn withdrew all his money from the bank and used 30% of it to buy a watch. Subsequently he sold the watch for a profit of 60%.

Find the selling price of the watch.

Answer $…………....………….. [2]

11 The diagram shows a frustum obtained by removing a small pyramid with height half of that of the original pyramid.
[A frustum is a portion of a pyramid that is left after a smaller pyramid is removed from the top.]



Find the ratio of the volume of the frustum to the volume of the original pyramid.

Answer ……......… : …......…... [2]

12 **(a)** Express $3x^2 - 12x$ in the form $3[(x+a)^2 + b]$.

Answer ………………………….. [2]

(b) Write down the smallest value of $3x^2 - 12x$.

Answer ………………………….. [1]

---

13 Cooking oil is sold in two sizes:

$4.80 for each 2 kg-bottle

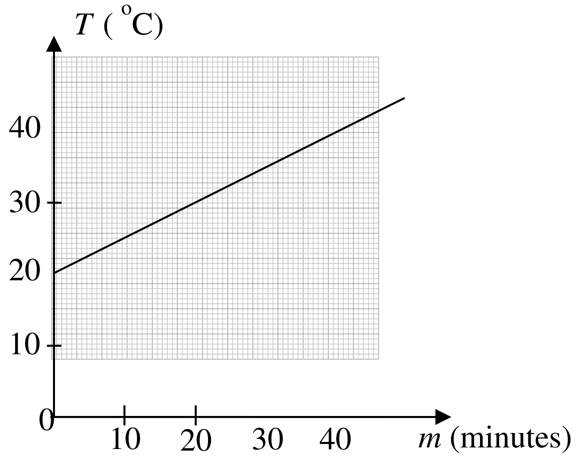
$6.95 for each 3 kg-bottle

Which bottle gives the better value?

You must show your calculations.

Answer The ……………… kg-bottle gives the better value. [2]

14 The graph shows the temperature, $T\,^{\circ}$C, of the water in a hot water tank after the heater is switched on for m minutes.



Use the graph to find

(a) the increase in temperature of the heater when it is switched on for 20 minutes,

Answer ……………………..……. °C [1]

(b) an equation for T in terms of m.

Answer ……………….…………….. [1]

---

15 The distance between the points M (k, 7) and N (9, k) is $\sqrt{20}$.

Given that k > 10, find the value of k.

Answer k = ………......………….. [3]

16 The table shows some corresponding values of x and y of the equation of a line.

x	-1	0	b	3
y	2	a	0	-18

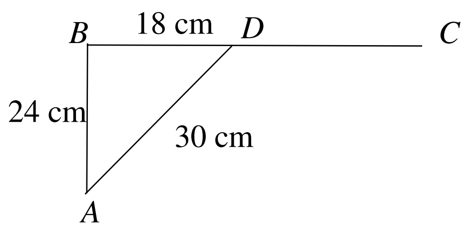
(a) Find the equation of the line.

Answer ………………………….. [2]

(b) Hence find the value of a and of b.

Answer a = ……………, b = …………….. [2]

17



In triangle ABD, AB = 24 cm, BD = 18 cm, AD = 30 cm. BD is produced to C.

(a) Explain why angle ABD is a right angle.

Answer ……………………………………………………………………………..

……………………………………………………………………………..

……………………………………………………………………………..

……………………………………………………………………………..

……………………………………………………………………………..

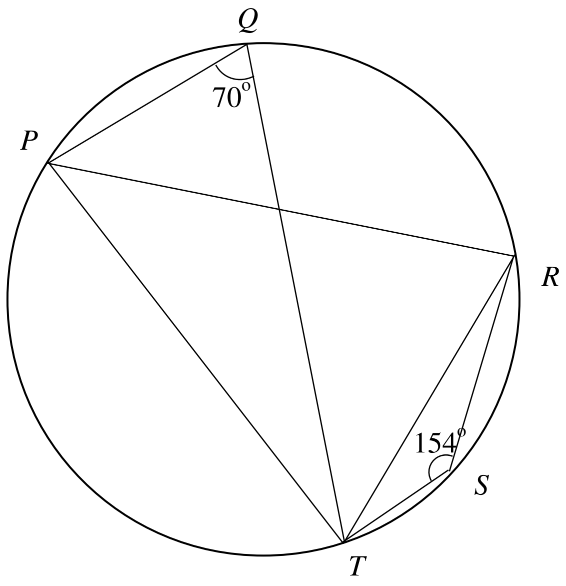
……………………………………………………………………………..

…………………………………………………………………………….. [1]

(b) Write down the value of $\cos\angle ADC$.

Answer ………………………….. [1]

18



In the diagram, P, Q, R, S and T are points on the circumference of a circle.
Angle $TQP = 70^{\circ}$ and angle $TSR = 154^{\circ}$.

(a) Find angle PTR.
Give a reason for each step of your working.

Answer angle PTR =………………………….. [3]

(b) There is a point A on the same side of PT as point R. Angle $TAP = 90^{\circ}$.

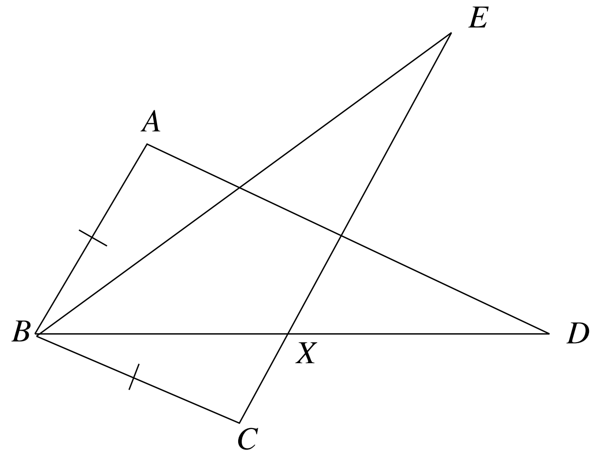
Determine if point A lies on the circumference of the circle, inside or outside the circle.
Justify your answer.

Answer Point A lies ……………………………………. the circle because ....

....

.... [1]

19



(a) In the diagram, $BA = BC$, angle ABE = angle CBD and angle BEC = angle BDA.

Explain why triangles ABD and CBE are congruent.

Answer ……………..................……………………………………………..…….………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

………………………………………………………………….....…………………. [2]

(b) Given further that angle ABE = angle BEC, what type of quadrilateral is $ABCE$? Justify your answer.

Answer Quadrilateral $ABCE$ is a …………………………… .

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

………………………………………………………………….....…………………. [2]

20 The table shows the number of people in groups of 1, 2, 3 and 4 people who attended a travel fair exhibition during a two-hour period.

No. of people in each group	1	2	3	4
No. of groups	20	94	85	26

Find

(a) the total number of people who attended the exhibition during the two-hour period,

Answer ……………....………….. [1]

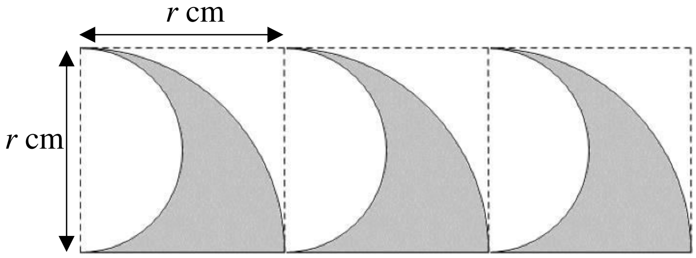
(b) the median number of people per group,

Answer ……………....………….. [1]

(c) the percentage of groups with at least 2 people.

Answer …………....…………..% [1]

21



A surfing brand's logo consists of 3 waves.
Each wave is made up of a quadrant with a semicircle removed.

(a) Find the perimeter of the logo in terms of r.

Answer ……………....………… cm [3]

The logo is drawn and then cut from a piece of fabric measuring $3r$ cm by r cm.

(b) Given that the area of the remaining fabric is 16.4 cm^2, find the value of r.

Answer r = ……………....………… [3]

22 A is the point (–4, 11). The position vector of B is $\begin{pmatrix}10\\4\end{pmatrix}$.

(a) Express $\overrightarrow{BA}$ as a column vector.

Answer ………………......…………..[1]

(b) Calculate $|\overrightarrow{AB}|$.

Answer ………………......…… units [1]

$\overrightarrow{BC} = \begin{pmatrix}0\\6\end{pmatrix}$ and D is the point (0, d).

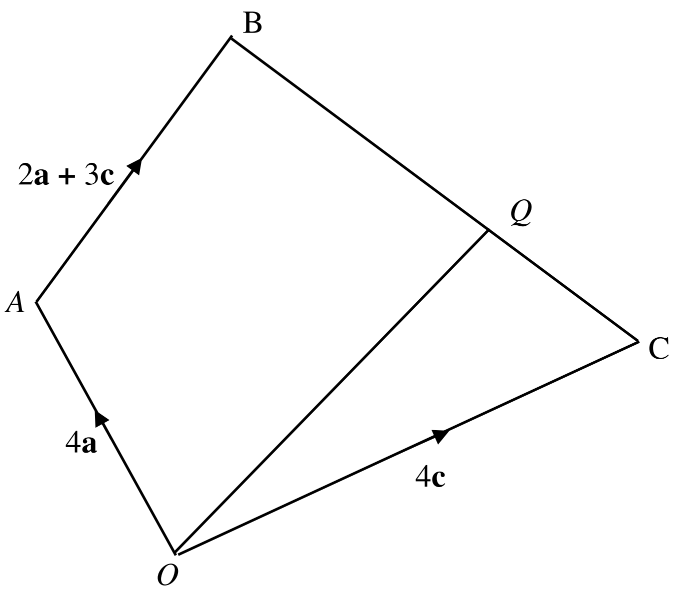
(c) **(i)** Find the column vector $\overrightarrow{OC}$.

Answer ………………......…………..[1]

(ii) If $\overrightarrow{BA}$ is parallel to $\overrightarrow{CD}$, find the value of d.

Answer d = ……......…....………..[3]

23



OABC is a quadrilateral.

$\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OC} = 4\mathbf{c}$, *and* $\overrightarrow{AB} = 2\mathbf{a} + 3\mathbf{c}$.

$CQ : QB = 2 : 3$.

(a) Write each of the following in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{BC}$,

Answer …………….....…………..[1]

(ii) $\overrightarrow{OQ}$.

Answer …………….....…………..[1]

(b) Use your answer to part **(a) (ii)** to explain why AB is parallel to OQ.

Answer ....

....

.... [1]

(c) Find

(i) $OQ : AB$,

Answer ……………......…………..[1]

(ii) $\dfrac{\text{Area of triangle } OAB}{\text{Area of triangle } OQB}$,

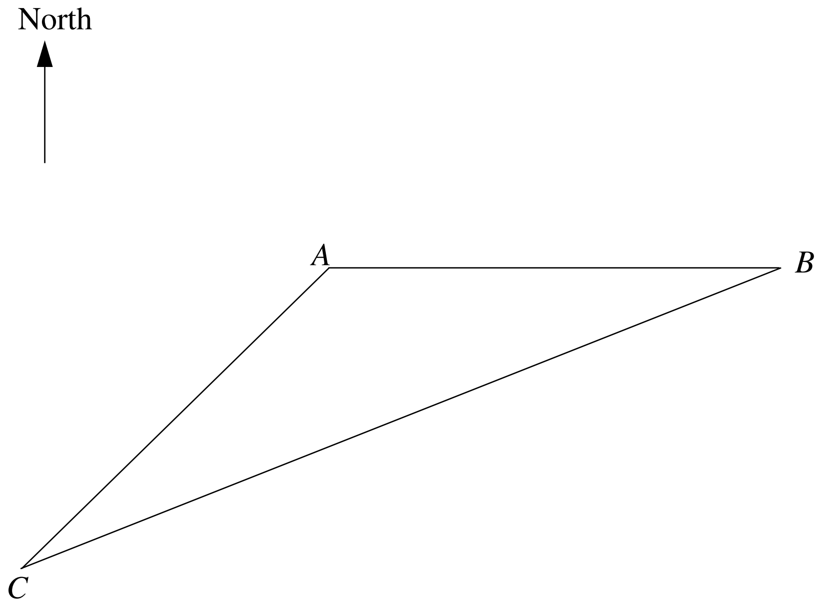
Answer ……………......…………..[1]

(iii) $\dfrac{\text{Area of triangle } OQC}{\text{Area of triangle } OBC}$.

Answer ……………......…………..[1]

24 The diagram below shows a scale drawing of triangle *ABC*.

Answer (a), (b), (c) **Scale: 1 cm to 10 m**



(a) Measure the bearing of *B* from *C*.

Answer ……………....…………..[1]

Points *A*, *B* and *C* are on the ground and a WiFi router is placed at *B*.

The WiFi router's signal can reach a distance of up to 42 m.

(b) Construct the range of the WiFi signal from the WiFi router at *B*. [1]

(c) Stacia is currently at *C* and starts walking along a path that is equidistant to *AC* and *BC*.
She stops at a point that is equidistant from *A* and *B*.

(i) Locate this point by construction and label it *S*. [3]

(ii) Hence state if Stacia is able to receive the Wifi signal at *S*, giving a reason for your answer.

Answer Stacia is .... to receive the Wifi signal at *S*, ...

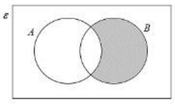
....

....

.... [1]

CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1

Answers

1.(a) $x = 1$, (b) $3xy$	15. $k = 11$
2. $(p - q)(2 - p)$	16. (a) $y = -5x - 3$
3. $\frac{4x+3}{(x-3)^2}$ or $\frac{4x+3}{(3-x)^2}$	(b) a = − 3, b = $-\frac{3}{5}$
4. (a)  (b) Q'	17.(a) $(AB)^2 + (BD)^2 = 24^2 + 18^2$ $= 900$ $= (30)^2$ $= (AD)^2$ angle $ABD = 90°$ by Pythagoras Theorem
5. (a) The number '−2' means the bus was early by 2 minutes	(b) $-\frac{3}{5}$
(c) 3 minutes	18. (a) angle $PTR = 84°$
6. (a) $2p$ and $2p + 2$	(b) Point A lies inside the circle because if A lies on the circumference, $\angle TAP = 70°$, and since $\angle TAP = 90° > 70°$, A lies inside the circle.
(b) $12p^2 + 8$	
(b) $12p^2 + 8 = 4(3p^2 + 2)$, hence the sum has a factor of 4, this means the sum is a multiple of 4.	
7. (a) $40 = 2^3 \times 5$, $138 = 2 \times 3 \times 23$	19(a) $\angle ABE = \angle CBD$ (given)
(b) smallest positive integer $k = 20$	$\angle ABE + \angle EBD = \angle CBD + \angle EBD$
8. height = 16 cm	$\therefore \angle ABD = \angle CBE$
9. (a) n = 0.4	$\angle BDA = \angle BEC$ (given)
(b) 24 km	$BA = BC$ (given)
(c) 50 cm^2	$\Delta ABD \equiv \Delta CBE$ (AAS)
10. (a) $m = 2375$	(b) $ABCE$ is a trapezium. AB is parallel to CE because alternate angles, $\angle ABE$ and $\angle BEC$, are equal.
(b) \$1204.44	
11. ratio = 7 : 8	
12. (a) $3\left((x-2)^2 - 4\right)$	20.(a) 567
(b) − 12	(b) 2 people per group
13. The 3 kg-bottle	(c) $91\frac{1}{9}$ % or 91.1 % (to 3 s.f.)
14. (a) 10°C	21. (a) $(3\pi r + 3r)$ cm or $12.4r$ cm
(b) $T = 20 + \frac{1}{2}m$	(b) r = 3.00 (to 3 s.f.)

CHIJ St Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 1 Answers

22.(a) $\overrightarrow{BA} = \begin{pmatrix} -14 \\ 7 \end{pmatrix}$

(b) 15.7 units (to 3 s.f.)

(c) (i) $\overrightarrow{CA} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

(ii) $d = 15$

23. (a) (i) $\overrightarrow{BC} = -6\mathbf{a} + \mathbf{c}$

(ii) $\overrightarrow{OQ} = \frac{12}{5}\mathbf{a} + \frac{18}{5}\mathbf{c}$

or $\frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$

23(b) $\overrightarrow{OQ} = \frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$

$\therefore \overrightarrow{OQ} = \frac{6}{5}\overrightarrow{AB}$

Hence AB is parallel to OQ.

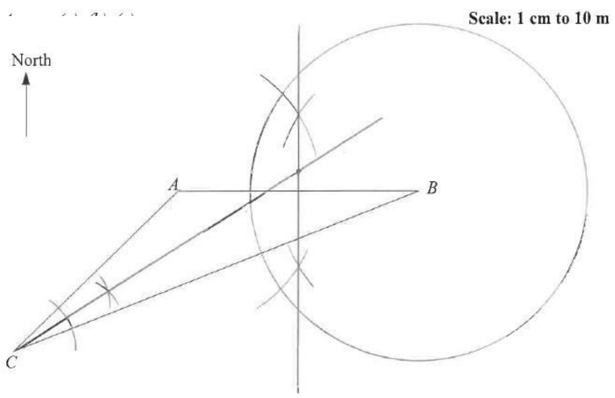
(d) (i) 6 : 5

(ii) $\frac{5}{6}$

(iii) $\frac{2}{5}$

24. (a) 068°

(b),(c) (i)



(c) (ii) Stacia is <u>able</u> to receive the Wifi signal at S, <u>because she is within the 42 m radius circular region, centre at the router position at B.</u>

1 **(a)** Given $x^9 = 9^0$, find the value of x.

$x^9 = 9^0$

$x^9 = 1$

$x = 1$ A1

Answer ………………………….. [1]

(b) Simplify $\frac{x^2}{3y} \div \frac{x}{9y^2}$.

(b) $\frac{x^2}{3y} \div \frac{x}{9y^2}$

$= \frac{x^2}{3y} \times \frac{9y^2}{x}$

$= 3xy$ A1

Answer ………………………….. [1]

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2 Factorise $2p - 2q - p^2 + pq$.

$2p - 2q - p^2 + pq$

$= 2(p-q) - p(p-q)$ M1

$= (p-q)(2-p)$ A1

Answer ………………………….. [2]

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3 Write as a single fraction in its simplest form $\frac{5x}{(3-x)^2} - \frac{1}{x-3}$.

$\frac{5x}{(3-x)^2} - \frac{1}{x-3}$

$= \frac{5x}{(x-3)^2} - \frac{1}{x-3}$ B1 for $3 - x = -(x-3)$

$= \frac{5x - x + 3}{(x-3)^2}$

$= \frac{4x+3}{(x-3)^2}$ or $\frac{4x+3}{(3-x)^2}$ A1

Answer ………………………….. [2]

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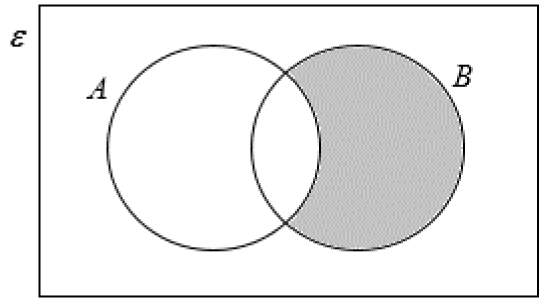
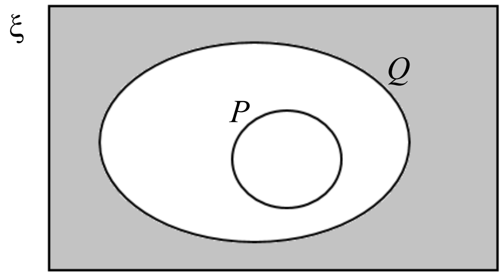
4 **(a)** On the Venn diagram, shade the region which represents $A' \cap B$.



[1]

(b) Given that P is a subset of Q, simplify $(P \cup Q)'$.

You may use the space below to help in your investigation.



Answer $(P \cup Q)' =$ ....Q'........ [1]

---

5 A shuttle bus is due to arrive at the ABC station at 09 30 daily.

The numbers of minutes by which the bus was late on ten successive days are shown below.

3 0 −2 −3 8 4 10 5 −4 9

(a) Explain the meaning of the number '−2' in the list of numbers of minutes.

Answer ....

The number '−2' means the bus was early by 2 minutes

....

.... [1]

(b) Find the mean number of minutes by which the bus was late.

$$\text{(a)} \quad \bar{x} = \frac{30}{10} = 3 \text{ min} \quad \text{A1}$$

Answer .... [1]

6 Given that p is a positive integer,

(a) write down expressions for the next two even numbers after $2(p - 1)$.

$2p$ and $2p + 2$ or $2(p + 1)$ A1

Answer ………… and ………….. [1]

(b) **(i)** find, in its simplest form, an expression for the sum of the squares of these three even numbers,

$$(2p-2)^2+(2p)^2+(2p+2)^2$$
$$=4p^2-8p+4+4p^2+4p^2+8p+4 \quad \text{M1 follow thru}$$
$$=12p^2+8 \quad \text{A1}$$

Answer ………………………….. [2]

(ii) explain why this sum is a multiple of 4.

Answer $12p^2 + 8 = 4(3p^2 + 2)$, hence the sum has a factor of 4, this means the sum is a multiple of 4.

OR $4(3p^2 + 2)$ is a multiple of 4. [1]

7 **(a)** Express 40 and 138 as a product of their prime factors.

$40 = 2^3 \times 5$ A1

$138 = 2 \times 3 \times 23$ A1

Answer 40 = ………….........……, 138 = ...………….. [2]

(b) Hence, find the smallest positive integer k such that $138k$ is divisible by 40.

$$\frac{138k}{40} = \frac{2\times3\times23\times k}{2^3\times5}$$

$$k = 2^2 \times 5$$

$$= 20 \quad \text{A1}$$

Answer smallest positive integer k = ………....………….. [1]

8 A wooden cube with side 8 cm is cut into two-centimetre cubes.
All of the two-centimetre cubes are then arranged to form a cuboid with height greater than 8 cm.
The perimeter of the top of the cuboid is 36 cm.

Find the height of the cuboid.

Total volume = $8^3 = 512$ cm^3

Total number of cubes = $4^3 = 64$ cubes

Breadth of cuboid = 2 cm

Length of cuboid = 16 cm

Height of cuboid = $512 \div 2 \div 16$

= 16 cm

B1 for breadth or length

B1 for height

Answer ……………………… cm [2]

9 A map is drawn to a scale of 1 : 40 000.

(a) This scale can be expressed as 1cm represents n km.

Find n.

1 : 40 000
1 cm : 0.4 km
$n = 0.4$ A1

Answer n = ……….....………….. [1]

(b) The distance between a seaport and an airport on the map is 60 cm.

Find the actual distance, in kilometres, between the seaport and airport.

$0.4 \times 60 = 24$ km A1

Answer ……….....………….. km [1]

(c) A bus depot has an actual area of 8 km^2.

Find the area, in square centimetres, of the bus depot on the map.

1 cm^2 : 0.16 km^2 B1

$\frac{8}{0.16} = 50\,\text{cm}^2$ A1

Answer ……….....………….. cm^2 [2]

10 **(a)** Fynn deposited $\$m$ into an account that paid a compound interest of 1.85% per annum. He made no other deposits or withdrawals for three years.
At the end of three years, he had \$2509.26 in his account.

Find the value of m, giving your answer correct to the nearest dollar.

$$2509.26 = m\left(1+\frac{1.85}{100}\right)^3 \quad \text{B1}$$

$m = \$2374.994..$

$m = \$2375$ (nearest dollar) A1

Answer m = ……….....………….. [2]

(b) Fynn withdrew all his money from the bank and used 30% of it to buy a watch. Subsequently he sold the watch for a profit of 60%.

Find the selling price of the watch.

2509.26×0.3 M1

$= \$752.778$

$$\frac{160}{100} \times \$752.788$$

$= \$1204.44$ (2 d.p.) A1

Answer \$……………....………….. [2]

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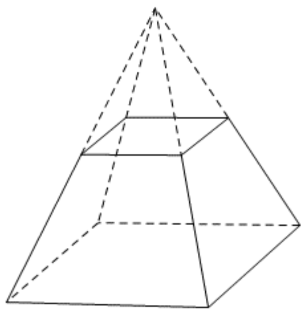
11 The diagram shows a frustum obtained by removing a small pyramid with height half of that of the original pyramid.
[A frustum is a portion of a pyramid that is left after a smaller pyramid is removed from the top.]



Find the ratio of the volume of the frustum to the volume of the original pyramid.

$$V_{top} : V_{original} = \left(\frac{1}{2}\right)^3 : 1^3 \quad \text{B1 for cube}$$

$$= \frac{1}{8} : 1 = 1 : 8$$

$$V_{frustum} : V_{original} = 1 - \frac{1}{8} : 1$$

$$= \frac{7}{8} : 1$$

$= 7 : 8$ B1

Answer ……….....… : …........…... [2]

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12 **(a)** Express $3x^2 - 12x$ in the form $3\left[(x+a)^2 + b\right]$.

(a) $3x^2 - 12x$ $= 3(x^2 - 4x)$ B1 for factor 3 $= 3((x-2)^2 - 4)$ A1

Answer ………………………….. [2]

(b) Write down the smallest value of $3x^2 - 12x$.

(b) – 12 A1

Answer ………………………….. [1]

---

13 Cooking oil is sold in two sizes:

$4.80 for each 2 kg bottle

$6.95 for each 3 kg bottle

Which bottle gives the better value?

You must show your calculations.

$\frac{4.80}{2} = \$2.40$ per kg $\frac{6.95}{3} = \$2.32$ per kg The 3kg bottle gives the better value	} B1 for either B1 for answer

OR

$\frac{2}{4.80} =$ 0.416666... kg per \$ $\frac{3}{6.95} =$ 0.431654... kg per \$ The 3kg bottle gives the better value	} B1 for either B1 for answer

Answer The ……………… kg-bottle gives the better value. [2]

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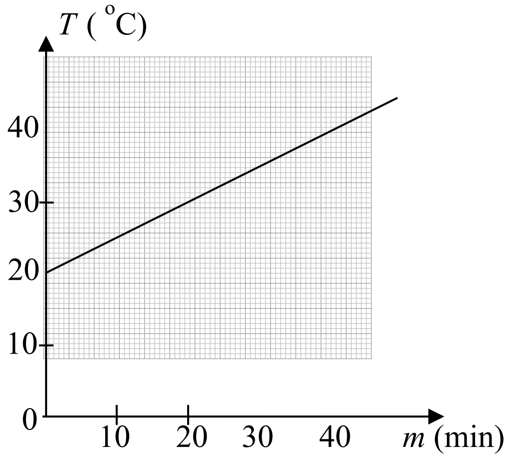
14 The graph shows the temperature, $T\,^{\circ}$C, of the water in a hot water tank after the heater is switched on for m minutes.



Use the graph to find

(a) the increase in temperature of the heater when it is switched on for 20 minutes,

(a) 10 °C A1

Answer ……………….……… °C [1]

(b) an equation for T in terms of m.

(b) $T = 20 + \frac{1}{2}m$ A1

Answer …………….…………….. [1]

---

15 The distance between the points $M\,(k,7)$ and $N\,(9,k)$ is $\sqrt{20}$.

Given that $k > 10$, find the value of k.

$\sqrt{(k-9)^2+(7-k)^2}=\sqrt{20}$	B1	
$k^2-18k+81+49-14k+k^2=20$ $2k^2-32k+110=0$ $k^2-16k+55=0$		
$(k-5)(k-11)=0$ $k=5$ *or* $k=11$ (NA)	M1 DA1	For factorising

Answer k = ……….....………….. [3]

16 The table shows some corresponding values of x and y of the equation of a line.

x	-1	0	b	3
y	2	a	0	-18

(a) Find the equation of the line.

$m = \dfrac{-18-2}{3-(-1)} = -5$ M1

$y = -5x + c$

$2 = -5(-1) + c$

$\therefore c = -3$

$y = -5x - 3$

A1

Answer ………………………….. [2]

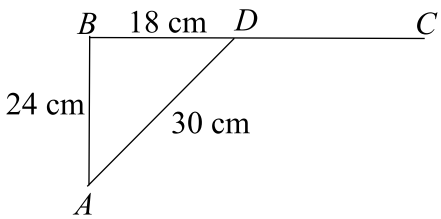
(b) Hence find the values of a and b.

$a = -3$ A1

$b = -\dfrac{3}{5}$ A1

Answer $a =$ ……………, $b =$ …………….. [2]

17



In triangle ABD, $AB = 24$ cm, $BD = 18$ cm, $AD = 30$ cm. BD is produced to C.

(a) Explain why angle ABD is a right angle.

$$\begin{aligned}(AB)^2 + (BD)^2 &= 24^2 + 18^2 \\ &= 900 \\ &= (30)^2 \\ &= (AD)^2\end{aligned}$$

angle $ABD = 90°$ by Pythagoras Theorem A1

Answer ………………………….. [1]

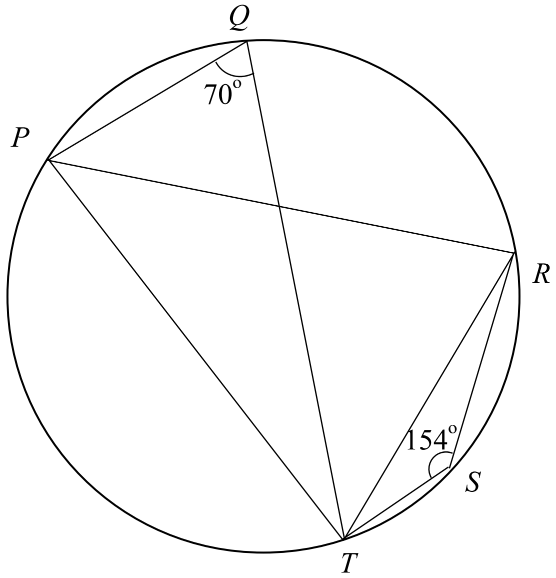
(b) Write down $\cos\angle ADC$.

$\cos\angle ADC = -\cos\angle ADB$

$= -\dfrac{18}{30} = -\dfrac{3}{5}$ A1

Answer ………………………….. [1]

18



In the diagram, P, Q, R, S and T are points on the circumference of a circle.
Angle $TQP = 70^{\circ}$ and angle $TSR = 154^{\circ}$.

(a) Find angle PTR.
Give a reason for each step of your working.

$\angle TPR = 180^{o} - \angle TSR$ ($\angle$s in opp segment are supp)
$=180^{o} - 154^{o}$ M1
$= 26^{o}$

$\angle PRT = \angle TQP = 70°$ ($\angle$s in same segment) M1

$\angle PTR = 180° - 70° - 26°$ ($\angle$ sum of Δ) (can don't see this reason)
$= 84°$ A1

Answer angle PTR =………………………….. [3]

(b) There is a point A on the same side of PT as point R. Angle $TAP = 90^{\circ}$.

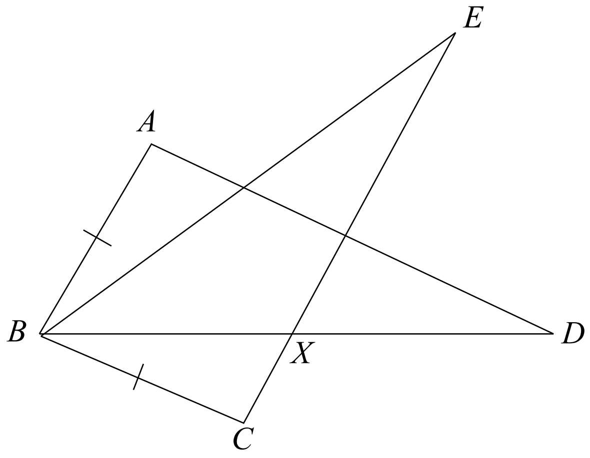
Determine if point A lies on the circumference of the circle, inside or outside the circle. Justify your answer.

Answer Point A lies ……inside…… the circle because ....

if A lies on the circumference, $\angle TAP = 70°$ ($\angle$s in the same segment), and since

$\angle TAP = 90° > 70°$, A lies inside the circle. A1 .... [1]

19



(a) In the diagram, $BA = BC$, angle ABE = angle CBD and angle BEC = angle BDA.

Explain why triangles ABD and CBE are congruent.

Answer ……………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………………………… [2]

(b) Given further that angle ABE = angle BEC, what type of quadrilateral is $ABCE$? Justify your answer.

19(b) $ABCE$ is a trapezium. A1

AB is parallel to CE because alternate angles, as angle ABE and angle BEC, are equal. A1

Answer $ABCE$ is a …………………………….. [2]

19(a) $\angle ABE = \angle CBD$ (given)

$\angle ABE + \angle EBD = \angle CBD + \angle EBD$

$\therefore \angle ABD = \angle CBE$

$\angle BDA = \angle BEC$ (given)

$BA = BC$ (given) M1 for any correct two conditions

$\Delta ABD \equiv \Delta CBE$ (AAS) A1

20 The table shows the number of people in groups of 1, 2, 3 and 4 people who attended a travel fair exhibition during a two -hour period.

No. of people in each group	1	2	3	4
No. of groups	20	94	85	26

Find

(a) the total number of people who attended the exhibition during the two-hour period,

(a)	$20 + 2\times94 + 3\times85 + 4\times26$ $= 567$	A1	

Answer ………………......………….. [1]

(b) the median number of people per group,

(b)	225 groups in total, 113th group is the middle Median is 2 people per group	A1	

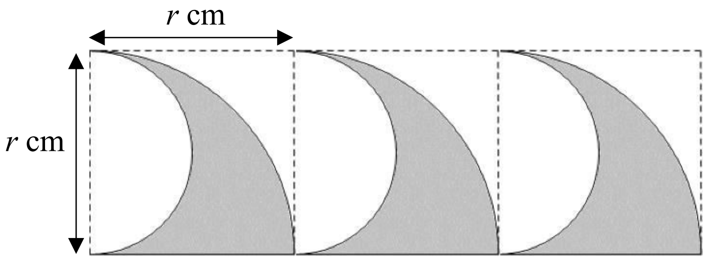
Answer ………………......………….. [1]

(c) the percentage of groups with at least 2 people,

(c)	Total groups = 20 + 94 + 85 + 26 = 225 205 groups with at least 2 people. $\frac{205}{225}\times100\%$ $= 91\frac{1}{9}\%$ or 91.1 % (to 3 s.f.)	A1	

Answer ……………......…………..% [1]

21



A surfing brand's logo consists of 3 waves.
Each wave is made up of a quadrant with a semicircle removed.

(a) Find the perimeter of the logo in terms of r.

Arc length of quadrant $= \frac{1}{4}(2\times\pi\times r)$ B1

$= \frac{\pi r}{2}$ cm

Arc length of semi-circle

$= \frac{1}{2}\left(2\pi r\times\frac{1}{2}r\right)$ B1 *no / wrong unit, –1 per paper*

$= \frac{\pi r}{2}$ cm

Perimeter $= 3\left(\frac{\pi r}{2}+\frac{\pi r}{2}+r\right)$

$= 3(\pi r + r)$ cm B1

or $(3\pi r + 3r)$ cm

Answer ……………......………… cm [3]

The logo is drawn and then cut from a piece of fabric measuring $3r$ cm by r cm.

(b) Given that the area of the remaining fabric is 16.4 cm^2, find the value of r.

$r^2\left(1-\frac{\pi}{8}\right)=\frac{16.4}{3}$ M1√ for atempt to factorise

$r=\sqrt{\frac{16.4}{3\left(1-\frac{\pi}{8}\right)}}$, r is positive

r = 3.00 (to 3 s.f.) A1 cannot '3'

$r^2\left(1-\frac{\pi}{8}\right)=\frac{16.4}{3}$ M1√ for atempt to factorise

$r=\sqrt{\frac{16.4}{3\left(1-\frac{\pi}{8}\right)}}$, r is positive

r = 3.00 (to 3 s.f.) A1 cannot '3'

Area of logo $= 3\left[\frac{1}{4}\pi r^2-\frac{1}{2}\pi\left(\frac{r}{2}\right)^2\right]$ $= = \frac{3}{8}\pi r^2$ cm^2 B1

$3r^2 - \frac{3}{8}\pi r^2 = 16.4$ M1√ for attempting to factorise

$r=\sqrt{\frac{16.4}{\left(3-\frac{3\pi}{8}\right)}}$, where r >0 , r = 3.00 (to 3 s.f.) A1

Answer r = ……………......………… [3]

22 A is the point (–4, 11). The position vector of B is $\begin{pmatrix}10\\4\end{pmatrix}$.

(a) Express $\overrightarrow{BA}$ as a column vector.

(a)	$\overrightarrow{BA} = \begin{pmatrix}-4\\11\end{pmatrix} - \begin{pmatrix}10\\4\end{pmatrix} = \begin{pmatrix}-14\\7\end{pmatrix}$	A1	

Answer ……………......…………..[1]

(b) Calculate $|\overrightarrow{AB}|$.

(b)	$\lvert\overrightarrow{AB}\rvert = \lvert\overrightarrow{BA}\rvert = \sqrt{(-14)^2 + 7^2}$ $= \sqrt{245} = 15.7$ units (3 s.f.)	A1	

Answer ……………......…… units [1]

$\overrightarrow{BC} = \begin{pmatrix}0\\6\end{pmatrix}$ and D is the point (0, d).

(c) (i) Find the column vector $\overrightarrow{OC}$.

(c)	$\therefore\ \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix}0\\6\end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix}0\\6\end{pmatrix} + \begin{pmatrix}10\\4\end{pmatrix}$ $= \begin{pmatrix}10\\10\end{pmatrix}$	A1	

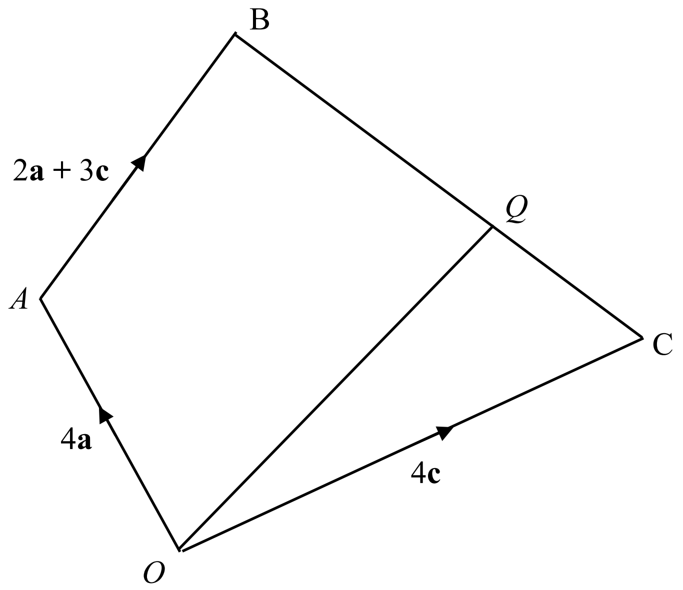
Answer ……………......…………..[1]

(ii) If $\overrightarrow{BA}$ is parallel to $\overrightarrow{CD}$, find the value of d.

(iii)	$\overrightarrow{BA} = h\ \overrightarrow{CD}$ $\begin{pmatrix}-14\\7\end{pmatrix} = h\left[\begin{pmatrix}0\\d\end{pmatrix} - \begin{pmatrix}10\\10\end{pmatrix}\right]$	M1	
	$= \begin{pmatrix}-10h\\h(d-10)\end{pmatrix}$ $-10h = -14$ $h = \frac{7}{5}$	M1	can be $h = \frac{5}{7}$ if $h\overrightarrow{BA} = \overrightarrow{CD}$
	$\frac{7}{5}(d-10) = 7$ $d - 10 = 5$, $\quad d = 15$	A1	

Answer $d =$ ……......…......………..[3]

23



$OABC$ is a quadrilateral.
$\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OC} = 4\mathbf{c}$, *and* $\overrightarrow{AB} = 2\mathbf{a} + 3\mathbf{c}$.
$CQ : QB = 2 : 3$.

(a) Write each of the following in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{BC}$,

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC} \\ &= -\overrightarrow{AB} + (-\overrightarrow{OA}) + \overrightarrow{OC} \\ &= -2\mathbf{a} - 3\mathbf{c} - 4\mathbf{a} + 4\mathbf{c} \\ &= -6\mathbf{a} + \mathbf{c}\end{aligned}$$

Answer ……………......…………..[1]

(ii) $\overrightarrow{OQ}$.

(ii)	$\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$ $= 4\mathbf{c} + \frac{2}{5}\overrightarrow{CB}$ $= 4\mathbf{c} + \frac{2}{5}(6\mathbf{a} - \mathbf{c})$ $= \frac{12}{5}\mathbf{a} + \frac{18}{5}\mathbf{c}$ or $\frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$ A1

Answer …………………..……………..[1]

(b) Use your answer to part **(a) (ii)** to explain why AB is parallel to OQ.

Answer ……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………………………… [1]

(b)	$\overrightarrow{OQ} = \frac{6}{5}(2\mathbf{a} + 3\mathbf{c})$ $\therefore \overrightarrow{OQ} = \frac{6}{5}\overrightarrow{AB}$ Hence AB is parallel to OQ.	A1	

(c) Find

(i) $OQ : AB$,

$\boxed{6 : 5}$

Answer …………………..……………..[1]

(ii) $\dfrac{\text{Area of triangle } OAB}{\text{Area of triangle } OQB}$,

$\boxed{\frac{5}{6}}$

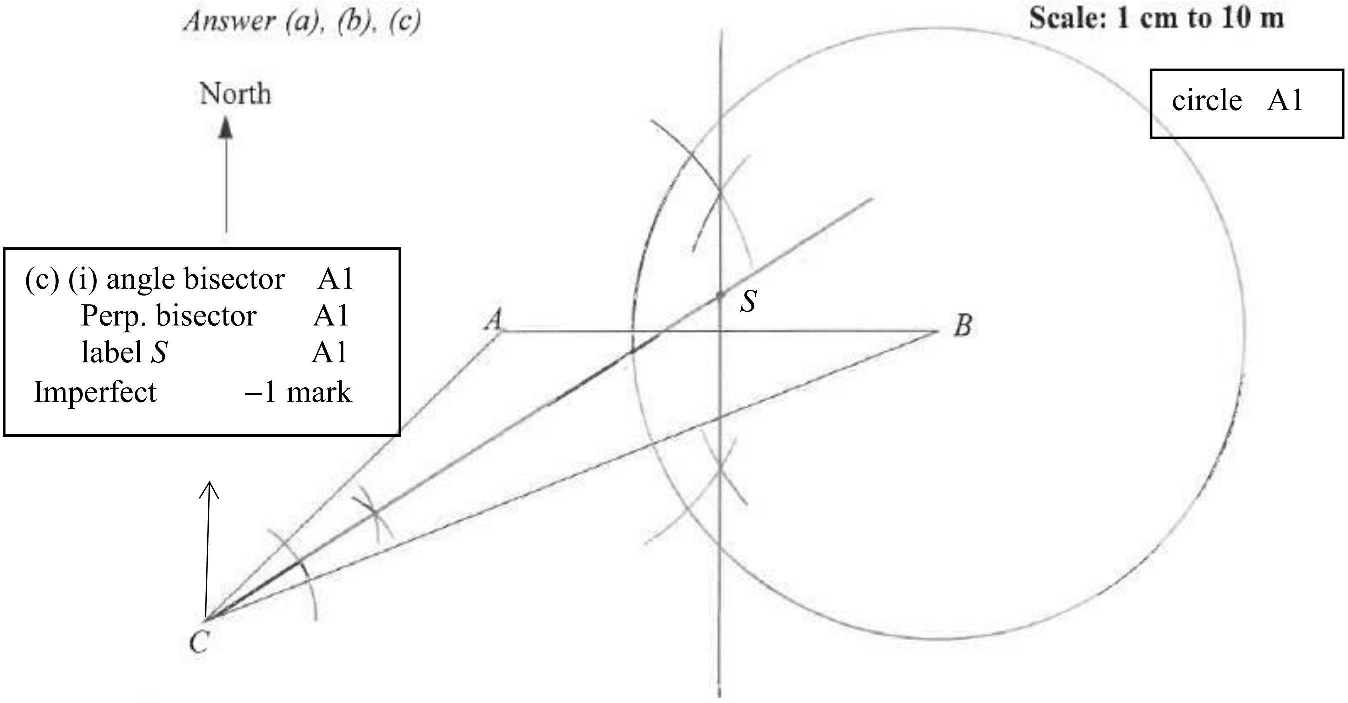
Answer …………………..……………..[1]

(iii) $\dfrac{\text{Area of triangle } OQC}{\text{Area of triangle } OBC}$.

$\boxed{\frac{2}{5}}$

Answer …………………..……………..[1]

24 The diagram below shows a scale drawing of triangle *ABC*.



Measure the bearing of *B* from *C*.

Answer ……068° A1……….[1]

Points *A*, *B* and *C* are on the ground and a WiFi router is placed at *B*.

The WiFi router's signal can reach a distance of up to 42 m.

(a) Construct the range of the WiFi signal from the WiFi router at *B*. [1]

(b) Stacia is currently at *C* and starts walking along a path that is equidistant to *AC* and *BC*.
She stops at a point that is equidistant from *A* and *B*.

(i) Locate this point by construction and label it *S*. [3]

(ii) Hence state if Stacia is able to receive the Wifi signal at *S*, giving a reason for your answer.

Answer Stacia is ……able…… to recieve the Wifi signal at S, ……because……

……she is within the 42 m radius circular region, centre at the router positon at *B*.……

…………………………………………………… [1]

CHIJ St. Nicholas Girls' School
2018 Preliminary Examination 2018 Mathematics Paper 2

1 **(a)** It is given that $h = \dfrac{k+h}{3h-k}$.

(i) Find the positive value of h when $k = 2h$. [1]

(ii) Express k in terms of h. [3]

(b) Solve the equation $\dfrac{3x}{4} + \dfrac{1}{x} = 2$. [3]

(c) Solve these simultaneous equations

$$x + 4y + 3 = 0,$$
$$5x - 2y - 29 = 0.$$ [2]

(d) Simplify $\dfrac{2 - 5x - 7x^2}{1 - x^2}$. [3]

---

2 **(a)** The interior angles of a hexagon are

$(2x + 17)°$, $(3x - 4)°$, $(2x + 49)°$, $(x + 40)°$, $(x - 17)°$, $(3x - 25)°$.

Find the smallest exterior angle. [3]

(b) The areas of the two similar octagons are 25 cm^2 and 576 cm^2.
The length of the sides of the octagons are x cm and 7 cm.

Find the two possible values of x. [3]

---

3 A group of volunteers pack goodie bags for the residents of a nursing home.
The table shows the contents of one of each type of goodie bag.

	Bag Type		
	P	*Q*	*R*
Number of buns	5	3	4
Number of toothbrushes	2	1	2
Number of packets of Milo	2	3	2
Number of packets of coffee	1	2	3

This information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

There are 20 bags of type *P*, 30 bags of type *Q* and 10 bags of type *R*.

(a) **(i)** Represent the numbers of the three types of goodie bags in a 3×1 column matrix **B**. [1]

(ii) Evaluate the matrix **C** = **AB**. [2]

(iii) State what the elements of **C** represent. [1]

(b) A bun costs $1.
A toothbrush costs $1.50.
A packet of Milo costs $6.40.
A packet of coffee costs $5.60.

The elements of the matrix **E**, where **E** = **DA**, represent the costs, in dollars, of each bag of *P*, of *Q* and of *R* respectively.

(i) Write down the matrix **D**. [1]

(ii) Evaluate the matrix **E**. [1]

(c) Evaluate the matrix **F** = **EB.** [1]

(d) State what the element(s) of **F** represent. [1]

4 The first four terms in a sequence of numbers are given below.

$T_1 = 1^2 + 8 \ \ = 9$

$T_2 = 2^2 + 12 = 16$

$T_3 = 3^2 + 16 = 25$

$T_4 = 4^2 + 20 = 36$

(a) Find T_5. [1]

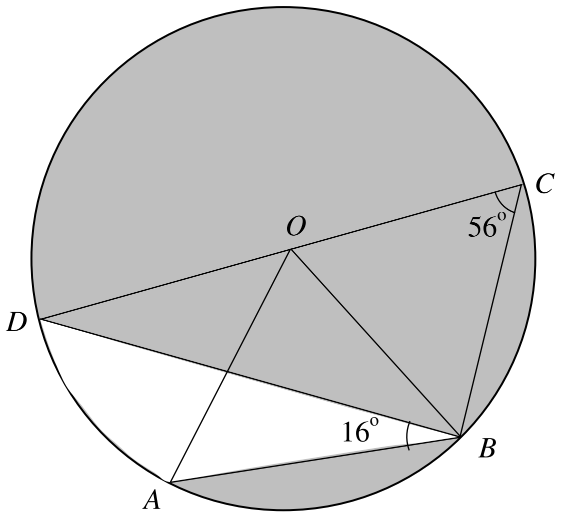
(b) Find an expression, in terms of n, for the nth term, T_n , of the sequence. [2]

(c) T_p and T_{p+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of p, for $T_{p+1} - T_p$. [2]

(d) Explain why two consecutive terms of the sequence cannot have a difference of less than 7. [1]

5



The diagram shows a circle $ABCD$, centre O and radius 4 cm.
COD is a diameter of the circle.

Angle ABD = 16° and angle BCD = 56°.

(a) Find the reflex angle DOB. [2]

(b) Find angle AOB. [2]

(c) Find the shaded area. [4]

---

6 Answer the whole of this question on a sheet of graph paper.

A ball was thrown from the top of a building.
The height, h metres, of the ball above ground level t seconds after it was thrown was measured every second.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6	7
h	210	250	250	237	206	155	84	0

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \leq t \leq 7$.
Using a scale of 4 cm to represent 100 metres, draw a vertical h-axis for $0 \leq h \leq 300$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Explain what the h-intercept of the curve represents. [1]

(c) Use your graph to estimate

(i) the maximum height of the ball, [1]

(ii) the time taken for the ball to reach its maximum height. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (4, 206). [2]

(ii) Use your answer in **(d)(i)** to explain what was happening to the ball at $t = 4$. [1]

7 **(a)** A shopkeeper mixed 30 kg of Brand *A* tea, which he bought at \$32 per kg, with 20 kg of Brand *B* tea, which he bought at \$35 per kg.
He sold all the mixture at \$40 per kg.

Determine whether the shopkeeper made a gain or loss from this transaction.
Show your working clearly. [2]

(b) Mrs Tan bought some packets of coffee for \$800. Each packet of coffee costs \$$x$.

(i) Write down an expression, in terms of x, for the number of packets of coffee bought. [1]

It was found that 2 packets were damaged and had to be thrown away.
Mrs Tan then sold each of the remaining packets of coffee for \$2 more than what she had paid for.

(ii) Write down an expression, in terms of x, for the total sum received from the sale of the packets of coffee. [1]

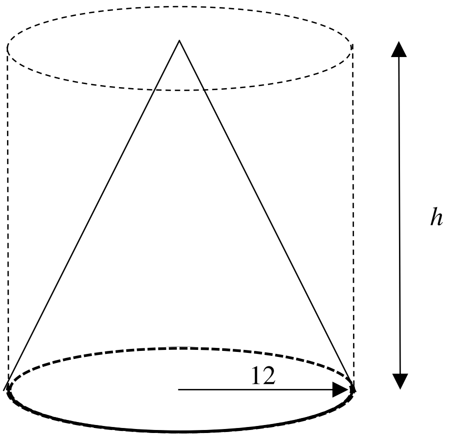
(iii) Given that Mrs Tan made a profit of \$99 from the sale of the packets of coffee, form an equation in x and show that it reduces to

$$2x^2 + 103x - 1600 = 0.$$ [3]

(iv) Solve the equation $2x^2 + 103x - 1600 = 0$. [3]

(v) Find the number of packets of coffee sold. [1]

8



The diagram shows a solid cone of radius 12 cm and height h cm cut from a solid cylindrical steel block of the same radius and height.

(a) The cylinder has a volume of 4320π cm^3. Find the value of h. [2]

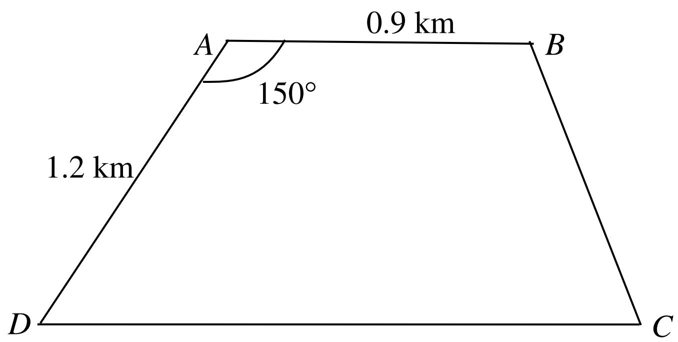
(b) Find the total surface area of the cone. [3]

(c) After the cone is cut from the steel block, the remaining steel is melted down and made into a solid sphere.

(i) Find the radius of the sphere. [3]

(ii) Find the surface area of the sphere. [1]

9



The diagram shows four towns A, B, C and D on a piece of horizontal land.
$ABCD$ is a trapezium.

$AB = 0.9$ km, $AD = 1.2$ km and angle $BAD = 150°$.

(a) Calculate the distance between Town B and Town D. [2]

(b) Calculate the value of angle BDC. [2]

(c) A tower is standing at Town B.
The greatest angle of elevation of the top of the tower, T, from the path CD is 18°.

Find the height of the tower in metres. [3]

10 (a) A chicken farmer fed 15 new-born chicks with a new variety of grain.

The stem-and-leaf diagram shows the weight gains of the chicks after three weeks.

37	8			
38	1	9		
39	0	5	6	
40	2	3	7	9
41	8	9		
42	5	7		
43	9			

Key 37 | 8 means 378 grams

(i) Find the median weight gain. [1]

(ii) Find the interquartile range. [2]

(iii) Calculate

(a) the mean of the weight gain, [1]

(b) the standard deviation. [2]

Chicks fed on the standard variety of grain had weight gains after three weeks.

The mean of these weight gains was 392 grams while the standard deviation was 12 grams.

(c) State briefly how the new variety of grain compares to the standard variety. [2]

(b) Box A contains 6 red cards, 4 blue cards and 2 green cards.
Box B contains 3 red cards and 5 blue cards.

A card is drawn at random from Box A and put into Box B.
Next, a card is drawn at random from Box B.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

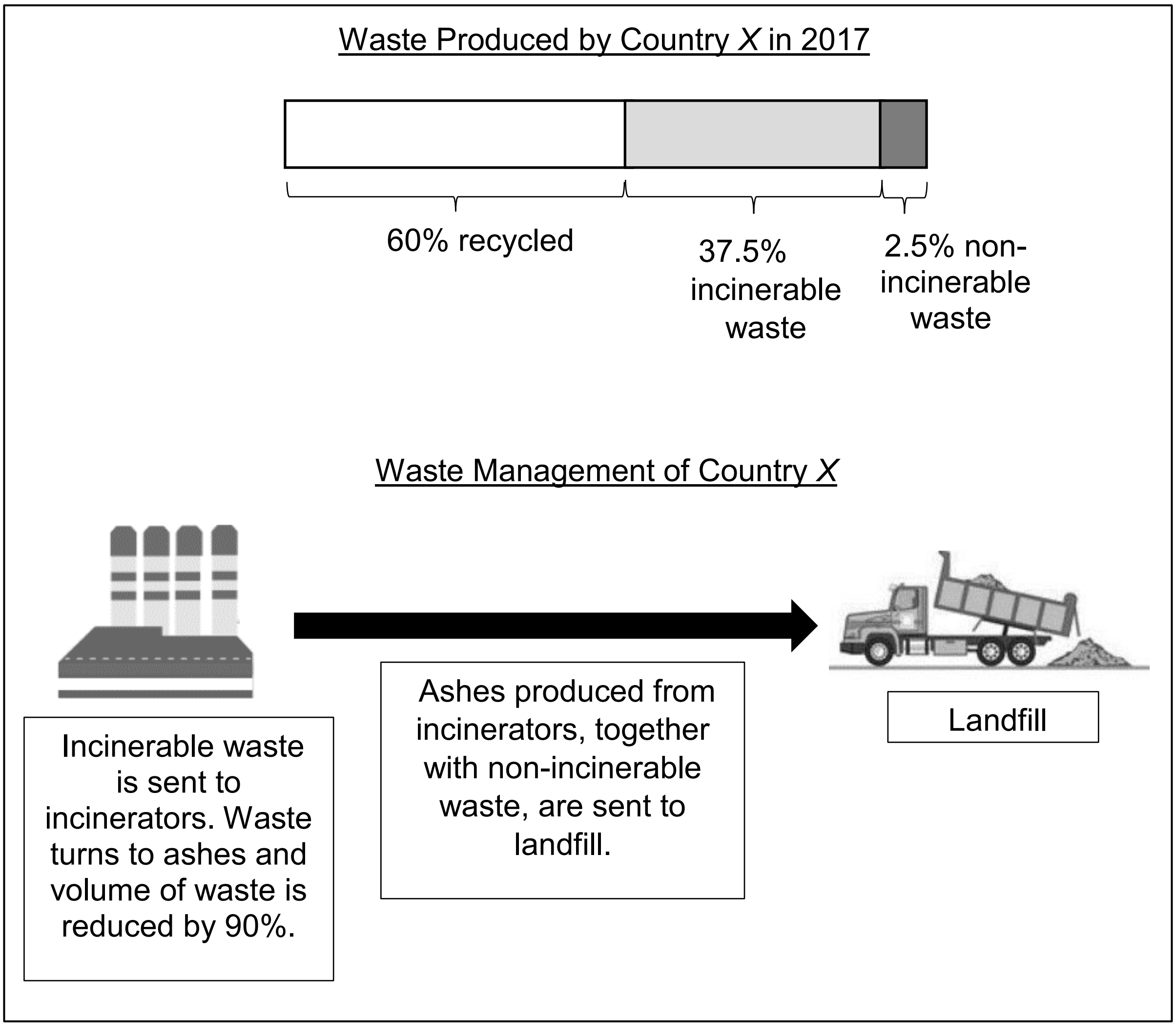
(ii) Find, as a fraction in its simplest form, the probability that

(a) two green cards are drawn, [1]

(b) neither of the cards is green, [1]

(c) the two cards are of different colours. [2]

11 Country *X* produced 3 million tonnes of waste in 2017. The infographic below shows more information on the waste produced and the waste management of Country *X*.



Images from: https://www.dreamstime.com/illustration/dumptruck.html,
https://www.mewr.gov.sg/topic/landfill

(a) Given that the density of waste is 125 kg/m^3 and 1 tonne = 1000 kg, calculate the volume of waste, in m^3, that was incinerated in 2017.
Give your answer in standard form. [3]

The landfill used by Country *X* has a total capacity of 42 000 000 m^3.
By the end of 2017, 30% of the landfill has already been used.
A news article claims it will take another 32 years before the landfill is completely used.

(b) Is the news article correct?
Justify your decision with calculations. [6]

(c) State one assumption you made in your calculations in **(b)**. [1]

CHIJ St. Nicholas Girls' School 2018 Preliminary Examination Mathematics Paper 2 Answers

1 (a) (i) 3 (ii) $k = \dfrac{h(3h-1)}{(1+h)}$ (b) $x = 2,\ x = \frac{2}{3}$ (c) $x = 5, y = -2$ (d) $\dfrac{2-7x}{1-x}$

2 (a) 19° (b) 1.46 , 33.6

3 (a) (i) $\begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix}$ (ii) $\begin{pmatrix} 230 \\ 90 \\ 150 \\ 110 \end{pmatrix}$

(iii) The elements of **C** represent the **<u>total numbers</u>** of buns, of toothbrushes, of packets of Milo and of packets of coffee respectively, needed to pack **<u>all</u>** the bags.

(b) (i) $\begin{pmatrix} 1 & 1.5 & 6.4 & 5.6 \end{pmatrix}$ (ii) $\begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix}$

(c) $\begin{pmatrix} 1941 \end{pmatrix}$

(d) The element in F represents the **<u>total cost</u>** in dollars for packing the goodie bags.

4 (a) 49 (b) $n^2 + 4n + 4$ (c) $2p + 5$

(d) As the difference between two consecutive terms is $(2p + 5)$, and p is a positive integer, the smallest difference is 2(1) + 5, which is 7. Hence the difference cannot be less than 7.

5 (a) 248^o (b) 80^o (c) 45.3 cm$^{2.}$

6 (b) h-intercept represents the height of the building is 210 m.

(c) (i) 255 m (ii) 1.5 s

(d) (i) −36.8 (ii)The ball is falling at a speed of 36.8 m /s.

7 (a) Cost per kg \$33.20 < \$40 , Gain

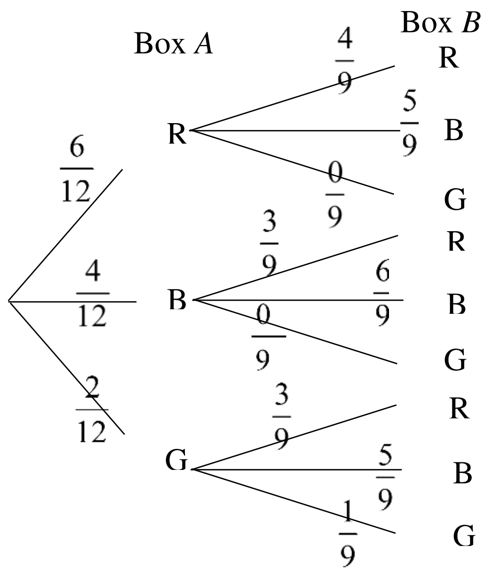
(b)(i) $\dfrac{800}{x}$ (ii) $\$\left(\dfrac{800}{x} - 2\right)(x+2)$ (iv) 12.5, −64 (v) 62

8 (a) 30 (b) 1670 cm^2 (c) (i) 12.9 cm (ii) 2100 cm^2

9 (a) 2.03 km (b) 17.2° (c) 195 m

10 (a) (i) 403 grams (ii) 29 grams (iii)(a) 405.2 grams (b) 17.1 grams
(a) (iii) (c) As 392 < 405.3, chicks had more weight gain when fed with the new variety of grain.
As 12 < 17.1, the weight gain from the new variety of grain shows more spread in the results.

10 (b) (i)



(ii) (a) $\frac{1}{54}$ (b) $\frac{5}{6}$ (c) $\frac{29}{54}$

11 (a) 9×10^6 m^3

(b) No, the news article is incorrect.
(with working to show it takes less than 32 yrs)
(1 possible solution is the waste produced per year will take only 19.6 years before the landfill is completely used)

(c) Possible answers:
- Amount of incinerated and non-incinerable waste remains the same every year
- The percentage breakdown of waste remains the same every year.

1 **(a)** It is given that $h=\dfrac{k+h}{3h-k}$.

(i) Find the positive value of h when $k = 2h$. [1]

(ii) Express k in terms of h. [3]

(b) Solve the equation $\dfrac{3x}{4}+\dfrac{1}{x}=2$. [3]

(c) Solve these simultaneous equations

$$x + 4y + 3 = 0,$$
$$5x - 2y - 29 = 0.$$ [2]

(d) Simplify $\dfrac{2-5x-7x^2}{1-x^2}$. [3]

1(a)	**(i)** $h=\dfrac{2h+h}{3h-(2h)}$ $h=\dfrac{3h}{h}$ $h=3$	A1	
	(ii) $h=\dfrac{k+h}{3h-k}$ $k+h=h(3h-k)$ $k+h=3h^2-hk$ $k+hk=3h^2-h$ $k(1+h)=3h^2-h$ $k=\dfrac{h(3h-1)}{(1+h)}$ or $\dfrac{3h^2-h}{(1+h)}$	M1 M1 A1	No fraction group like terms
(b)	$\dfrac{3x}{4}+\dfrac{1}{x}=2$ $\dfrac{3x^2+4}{4x}=2$ $3x^2-8x+4=0$ $3x^2-8x+4=0$ $(x-2)(3x-2)=0$ $x=2,\ x=\frac{2}{3}$	M1 B1 A1	single fraction
(c)	$x+4y+3=0$ ..... (1) $5x-2y-29=0$ .... (2) (1)×5, $5x + 20y + 15 = 0$ ..... (3) (3) – (2), $22y + 44 = 0$ $\therefore y=-2,\quad x=5$	M1 A1	
(d)	$\dfrac{2-5x-7x^2}{1-x^2}=\dfrac{(2-7x)(1+x)}{(1-x)(1+x)}$ $=\dfrac{2-7x}{1-x}$	B1, B1 A1	for each factorisation

2 **(a)** The interior angles of a hexagon are

$(2x + 17)°$, $(3x - 4)°$, $(2x + 49)°$, $(x + 40)°$, $(x - 17)°$, $(3x - 25)°$.

Find the smallest exterior angle. [3]

(b) The areas of the two similar octagons are 25 cm^2 and 576 cm^2.
The length of the sides of the octagons are x cm and 7 cm.

Find the two possible values of x. [3]

(a)	sum of interior angles = $12x + 60$ $12x + 60 = (6 - 2) \times 180$ $12x = 720 - 60$ $x = \frac{660}{12}$ $= 55$ smallest exterior angle $= 180° -$ largest interior angle $= 180° - (3\times55-4)°$ $= 180° - 161°$ $= 19°$	M1✓ M1✓ A1	
(b)	$\frac{x}{7} = \sqrt{\frac{25}{576}}$ $\frac{x}{7} = \frac{5}{24}$ $\therefore x = 1.46$ or $1\frac{11}{24}$ $\frac{x}{7} = \sqrt{\frac{576}{25}}$ $\frac{x}{7} = \frac{24}{5}$ $\therefore x = 33.6$ or $33\frac{3}{5}$	M1 A1 A1	Either sq rt

3 A group of volunteers pack goodie bags for the residents of a nursing home.
The table shows the contents of one of each type of goodie bag.

	Bag Type		
	P	*Q*	*R*
Number of buns	5	3	4
Number of toothbrushes	2	1	2
Number of packets of Milo	2	3	2
Number of packets of coffee	1	2	3

This information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

There are 20 bags of type P, 30 bags of type Q and 10 bags of type R.

(a) **(i)** Represent the numbers of the three types of goodie bags in a 3×1 column matrix **B**. [1]

(ii) Evaluate the matrix $\mathbf{C} = \mathbf{AB}$. [2]

(iii) State what the elements of **C** represent. [1]

(b) A bun costs \$1.
A toothbrush costs \$1.50.
A packet of Milo costs \$6.40.
A packet of coffee costs \$5.60.

The elements of the matrix **E**, where $\mathbf{E} = \mathbf{DA}$, represent the costs, in dollars, of each bag of P, of Q and of R respectively.

(i) Write down the matrix **D**. [1]

(ii) Evaluate the matrix **E**. [1]

(c) Evaluate the matrix $\mathbf{F} = \mathbf{EB}$. [1]

(d) State what the element(s) of **F** represent. [1]

3(a)	(i) $\mathbf{B} = \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix}$	B1	
(a)	(ii) $\mathbf{C} = \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \end{array} \begin{array}{c} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \end{array} \begin{array}{c} P \\ Q \\ R \end{array}$ $= \begin{pmatrix} 230 \\ 90 \\ 150 \\ 110 \end{pmatrix} \begin{array}{c} \text{buns} \\ \text{toothbrush} \\ \text{Milo} \\ \text{coffee} \end{array}$	M1√ A1	
(a)	(iii)The elements of **C** represent the **<u>total numbers</u>** of buns, of toothbrushes, of packets of Milo and of packets of coffee respectively, needed to pack **<u>all</u>** the bags.	A1	accept 'no packet'
(b)	(i) $\mathbf{D} = \begin{pmatrix} 1 & 1.5 & 6.4 & 5.6 \end{pmatrix}$	A1	accept 6.40, 5.60
(b)	(ii) $\mathbf{E} = \begin{array}{c} \begin{array}{cccc} \text{B} & \text{TB} & \text{Milo} & \text{coffee} \end{array} \\ \begin{pmatrix} 1 & 1.50 & 6.40 & 5.60 \end{pmatrix} \end{array} \begin{array}{c} \begin{array}{ccc} \text{P} & \text{Q} & \text{R} \end{array} \\ \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \end{array} \begin{array}{c} \text{B} \\ \text{TB} \\ \text{Milo} \\ \text{coffee} \end{array}$ $= \begin{array}{c} \begin{array}{ccc} \text{P} & \text{Q} & \text{R} \end{array} \\ \begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix} \end{array}$	A1	accept 26.40, 34.90, 36.60
(c)	$\mathbf{F} = \begin{array}{c} \begin{array}{ccc} \text{P} & \text{Q} & \text{R} \end{array} \\ \begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix} \end{array} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \begin{array}{c} \text{P} \\ \text{Q} \\ \text{R} \end{array}$ $= \begin{pmatrix} 1941 \end{pmatrix}$	A1	cannot 1940, cannot (\$1941)
(d)	The element of **F** represents the **<u>total cost</u>** in dollars of ***all the <u>items</u>*** needed to pack **<u>all the goodie bags</u>** altogether. OR The element in F represents the **<u>total cost</u>** in dollars for packing the goodie bags.	A1	

4 The first four terms in a sequence of numbers are given below.

$T_1 = 1^2 + 8 \;\; = 9$

$T_2 = 2^2 + 12 = 16$

$T_3 = 3^2 + 16 = 25$

$T_4 = 4^2 + 20 = 36$

(a) Find T_5. [1]

(b) Find an expression, in terms of n, for the nth term, T_n , of the sequence. [2]

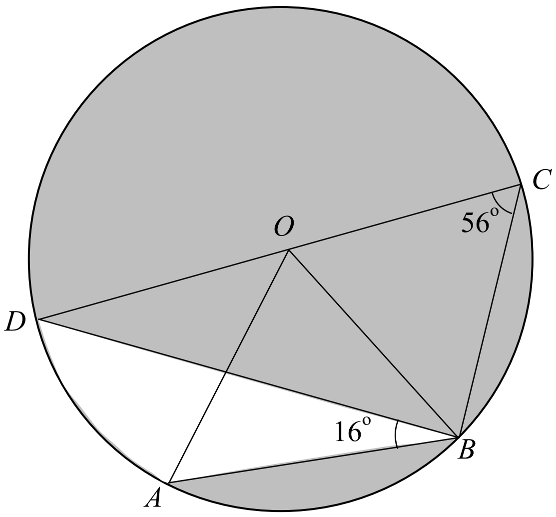
(c) T_p and T_{p+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of p, for $T_{p+1} - T_p$. [2]

(d) Explain why two consecutive terms of the sequence cannot have a difference of less than 7. [1]

4(a)	$T_5 = 5^2 + 24 = 7^2 = 49$	A1	accept just 49
(b)	$T_\text{n} = (n + 2)^2$ or $T_\text{n} = n^2 + 4(n + 1)$ $= n^2 + 4n + 4$	A1 + A1	1 mark for $(n+2)$, 1 mark for perfect or 1 mark for n^2 or $4(n + 1)$ 1 mark for perfect
(c)	$T_{p+1} - T_p = (p + 3)^2 - (p + 2)^2$ $= (p^2 + 6p + 9) - (p^2 + 4p + 4)$ $= 2p + 5$ or $T_{p+1} - T_p = (p + 1)^2 + 4(p+2) - p^2 - 4p - 4$ $= 2p + 5$	M1 A1 or M1 A1	1 mark for $(p + 3)^2$ or $(p + 2)^2$. 1 mark for answer 1 mark for $(p+1)^2+4(p+2) -p^2-4p-4$
(d)	As the difference between two consecutive terms is $(2p + 5)$, and p is a positive integer, the smallest difference is $2(1) + 5$, which is 7. Hence the difference cannot be less than 7.	A1	

5



The diagram shows a circle *ABCD*, centre *O* and radius 4 cm.
COD is a diameter of the circle.

Angle *ABD* = 16° and angle *BCD* = 56°.

(a) Find the reflex angle *DOB*. [2]

(b) Find angle *AOB*. [2]

(c) Find the shaded area. [4]

5(a)	$\angle DOB = 2\times 56^o \quad = 112^o$ ($\angle$at the centre=2$\angle$ at circumf) reflex angle $\angle DOB = 360^o - 112^o$ $= 248^o$	M1 A1	
(b)	$\angle DOA = 2\times 16^o$ $= 32^o$ ($\angle$ at centre $= 2\times$ $\angle$ at circumf) $\angle AOB = 112^o - 32^o$ $= 80^o$	M1 A1	
(c)	Area of major sector *DOB* $=\dfrac{248^o}{360^o}\times\pi(4)^2$ $= 34.627\ \text{cm}^2$ Area of minor sector *AOB* $=\dfrac{80^o}{360^o}\times\pi(4)^2$ $= 11.170\ \text{cm}^2$ Area of ΔDOB $=\dfrac{1}{2}(4)^2\sin 112^\circ$ $= 7.417\ \text{cm}^2$ Area of ΔAOB $=\dfrac{1}{2}(4)^2\sin 80^\circ$ $= 7.878\ \text{cm}^2$ Total area $= 34.627 + 7.417 + 11.170 - 7.878$ $= 45.3\ \text{cm}^2$	M1 M1 M1 A1	1 mark for either sector, 1 mark for either area of triangle 1 mark for either segment 1 mark for total area

6 Answer the whole of this question on a sheet of graph paper.

A ball was thrown from the top of a building.
The height, h metres, of the ball above ground level t seconds after it was thrown was measured every second.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6	7
h	210	250	250	237	206	155	84	0

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \leq t \leq 7$.
Using a scale of 4 cm to represent 100 metres, draw a vertical h-axis for $0 \leq h \leq 300$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Explain what the h-intercept of the curve represents. [1]

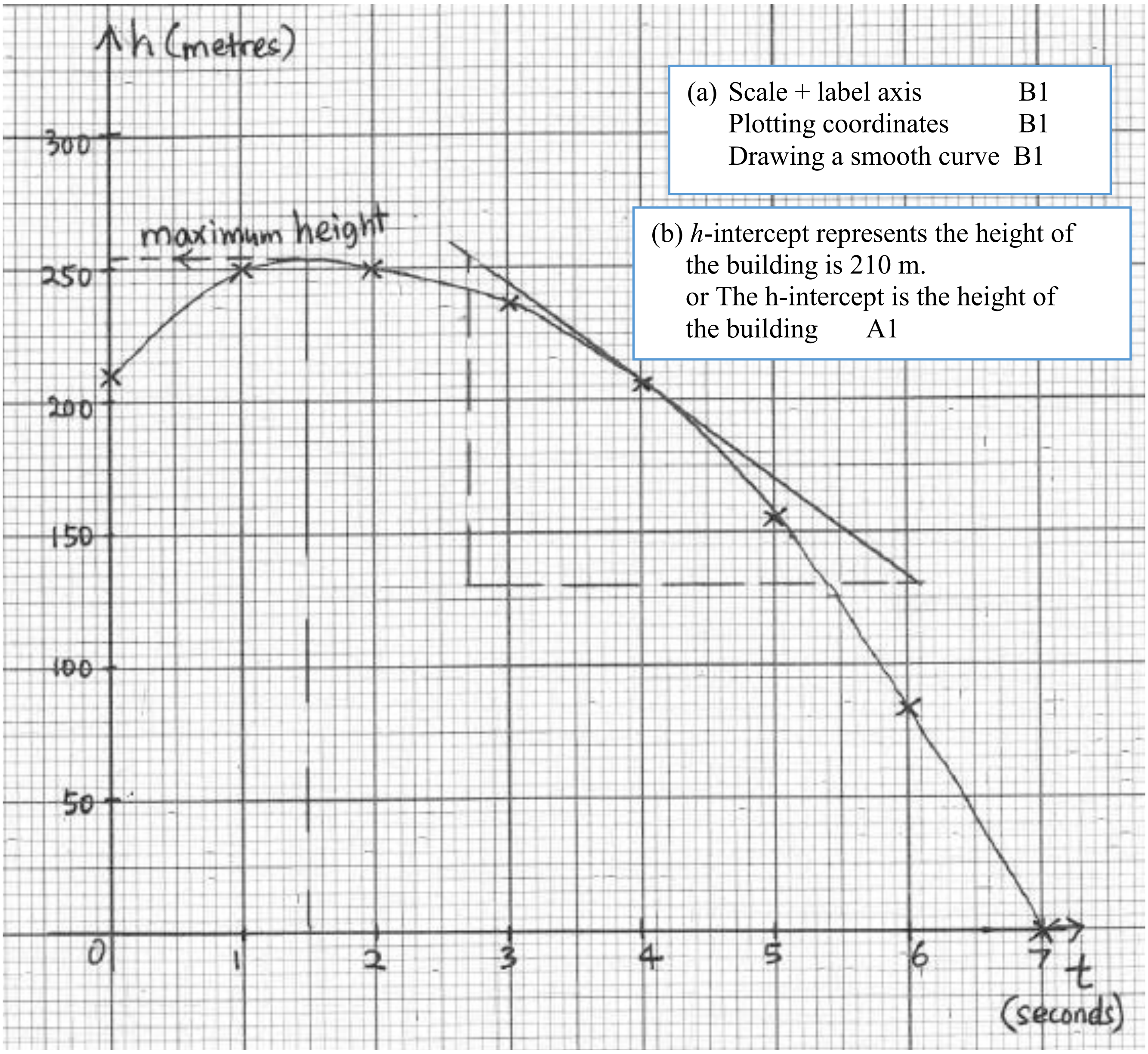
(c) Use your graph to estimate

(i) the maximum height of the ball, [1]

(ii) the time taken for the ball to reach its maximum height. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at (4, 206). [2]

(ii) Use your answer in **(d)(i)** to explain what was happening to the ball at $t = 4$. [1]



(a) Scale + label axis B1
Plotting coordinates B1
Drawing a smooth curve B1

(b) *h*-intercept represents the height of the building is 210 m.
or The h-intercept is the height of the building A1

(c) (i) maximum height
= 2.55 m A1
(ii) time taken = 1.5 s A1

(d) (i) Drawing tangent at x = (4, 206) B1

$$\text{gradient} = \frac{125}{3.4}$$

$$= -36.8$$ A1

(ii) The ball is falling at a speed of 36.8 m /s. A1

7 **(a)** A shopkeeper mixed 30 kg of Brand *A* tea, which he bought at \$32 per kg, with 20 kg of Brand *B* tea, which he bought at \$35 per kg.
He sold all the mixture at \$40 per kg.
Determine whether the shopkeeper made a gain or loss from this transaction.
Show your working clearly. [2]

(b) Mrs Tan bought some packets of coffee for \$800. Each packet of coffee costs \$$x$.

(i) Write down an expression, in terms of x, for the number of packets of coffee bought. [1]

It was found that 2 packets were damaged and had to be thrown away.
Mrs Tan then sold each of the remaining packets of coffee for \$2 more than what she had paid for.

(ii) Write down an expression, in terms of x, for the total sum received from the sale of the packets of coffee. [1]

(iii) Given that Mrs Tan made a profit of \$99 from the sale of the packets of coffee, form an equation in x and show that it reduces to

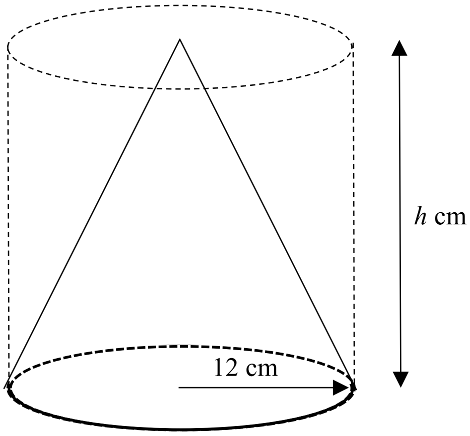
$$2x^2+103x-1600=0\ .$$ [3]

(iv) Solve the equation $2x^2+103x-1600=0$. [3]

(v) Find the number of packets of coffee sold. [1]

(a)	$\dfrac{30\times32+20\times35}{30+20}$ \$33.20 per kg \$33.20 < \$40 $\therefore$ Gain Alternative method For 50 kg, cost is \$1660 Selling price is \$2000 \$1660 < \$2000	M1 A1	method for finding cost per kg of mixure Answer of 'gain'
(b)	(i) $\dfrac{800}{x}$	A1	
(ii)	$\left(\dfrac{800}{x}-2\right)(x+2)$	A1	
(iii)	$\left(\dfrac{800}{x}-2\right)(x+2)-800=99$ $800+\dfrac{1600}{x}-2x-4-800=99$ $\dfrac{1600}{x}-2x-103=0$ $-2x^2-103x+1600=0$ $2x^2+103x-1600=0$	M1 M1 B1	
(iv)	$2x^2+103x-1600=0$ $(2x-25)(x+64)=0$ $\therefore x=12.5,\ x=-64$	M1 DA1, DA1	
(v)	$\dfrac{800}{12.5}-2\ =62$	A1	

8



The diagram shows a solid cone of radius 12 cm and height h cm cut from a solid cylindrical steel block of the same radius and height.

(a) The cylinder has a volume of 4320π cm^3. Find the value of h. [2]

(b) Find the total surface area of the cone. [3]

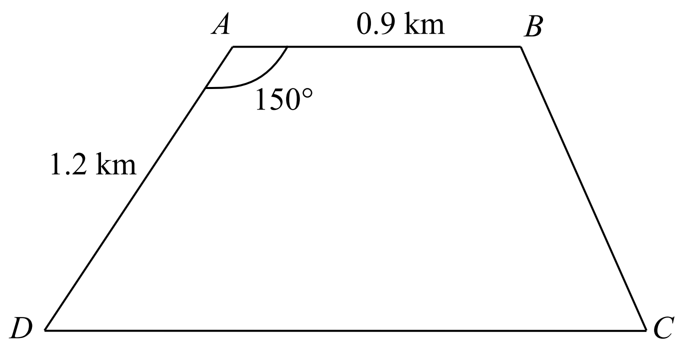
(c) After the cone is cut from the steel block, the remaining steel is melted down and made into a solid sphere.

(i) Find the radius of the sphere. [3]

(ii) Find the surface area of the sphere. [1]

8(a)	$\pi(12)^2 h = 4320\pi$ $h = \dfrac{4320}{144} = 30$	B1 A1	units overall − 1
(b)	Slant height $= \sqrt{12^2 + 30^2}$ $= \sqrt{1044}$ $= 32.31$ cm Total surface area $= \pi(12)^2 + \pi(12)(32.31)$ $= 1670$ cm^2 (3 s.f)	M1√ M1 A1	
(c)(i)	Volume of remaining steel $= 4320\pi - \dfrac{1}{3}\pi\left(12^2\right)30$ or $\dfrac{2}{3}\pi(12)^2(30)$ $= 2880\pi$ cm^3 $\dfrac{4}{3}\pi r^3 = 2880\pi$ $r^3 = 2160$ $r = 12.9$ cm (3 s.f)	M1 B1√ A1	
(ii)	Surface area of sphere $= 4\pi(12.92)^2$ $\approx 2\,097.6$ $= 2100$ cm^2 (3 s.f.)	A1	

9



The diagram shows four towns A, B, C and D on a piece of horizontal land.
$ABCD$ is a trapezium.
$AB = 0.9$ km, $AD = 1.2$ km and angle $BAD = 150°$.

(a) Calculate the distance between Town B and Town D. [2]

(b) Calculate the value of angle BDC. [2]

(c) A tower is standing at Town B.
The greatest angle of elevation of the top of the tower, T, from the path CD is 18°.
Find the height of the tower in metres. [3]

9(a)	$BD^2 = (0.9)^2 + (1.2)^2 - 2(0.9)(1.2)\cos 150°$ $BD = 2.0299$ $= 2.03$ km (3 s.f.)	M1 A1	
(b)	$\dfrac{\sin \angle ABD}{1.2} = \dfrac{\sin 150°}{2.0299}$ $\sin \angle ABD = \dfrac{1.2 \sin 150°}{2.0299}$ $\angle ABD = 17.19°$ $= 17.2°$ (to 1 d.p.) $\angle BDC = \angle ABD$ (alt $\angle$s, AB // DC) $= 17.2°$	M1√ A1	accept no mention of angle property
(c)	Let the shortest distance from B to CD be d km. $\sin 17.19° = \dfrac{d}{2.0299}$ $d = 0.5999$ km Let x m be the height of the tower. $\dfrac{x}{0.5999} = \tan 18°$ $x = 0.5999 \tan 18°$ $= 0.1949$ km $= 195$ m (to 3 s.f.)	M1√ B1√ A1	

10 **(a)** A chicken farmer fed 15 new-born chicks with a new variety of grain.

The stem-and-leaf diagram shows the weight gains of the chicks after three weeks.

37	8			
38	1	9		
39	0	5	6	
40	2	3	7	9
41	8	9		
42	5	7		
43	9			

Key: 37 | 8 means 378 grams

(i) Find the median weight gain. [1]

(ii) Find the interquartile range. [2]

(iii) Calculate

(a) the mean of the weight gain, [1]

(b) the standard deviation. [2]

Chicks fed on the standard variety of grain had weight gains after three weeks. The mean of these weight gains was 392 grams while the standard deviation 12 grams.

(c) State briefly how the new variety of grain compares to the standard variety. [2]

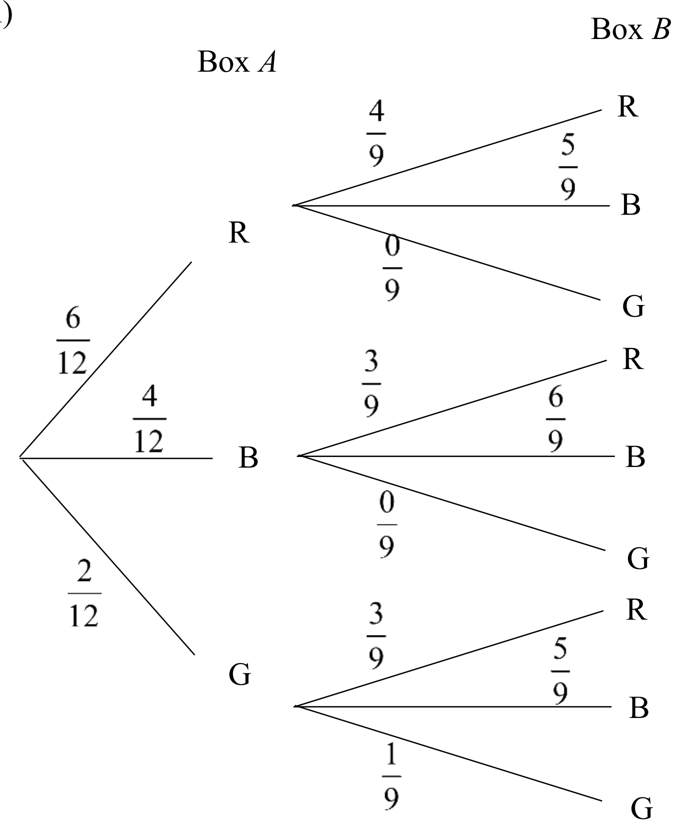
10 (a)	(i) median weight gain = 403 grams	A1	
	(ii) interquartile range = 419 − 390 = 29 grams	M1 A1	
	(iii)(a) mean weight gain= $\frac{6078}{15}$ = 405.2 grams	A1	
	(iii)(b) standard deviation = $\sqrt{\frac{2\,467\,210}{15}-\left(\frac{6078}{15}\right)^2}$ = 17.135 538..... = 17.1 grams (to 3 s.f.)	M1 A1	
	(iii)(c) 392 < 405.3, chicks had more weight gain when fed with the new variety of grain. 12 < 17.1, the weight gain from the new variety of grain shows more spread / more variation / less consistent results.	A1 A1	

(b) Box *A* contains 6 red cards, 4 blue cards and 2 green cards.
Box *B* contains 3 red cards and 5 blue cards.
A card is drawn at random from Box *A* and put into Box *B*.
Next, a card is drawn at random from Box *B*.

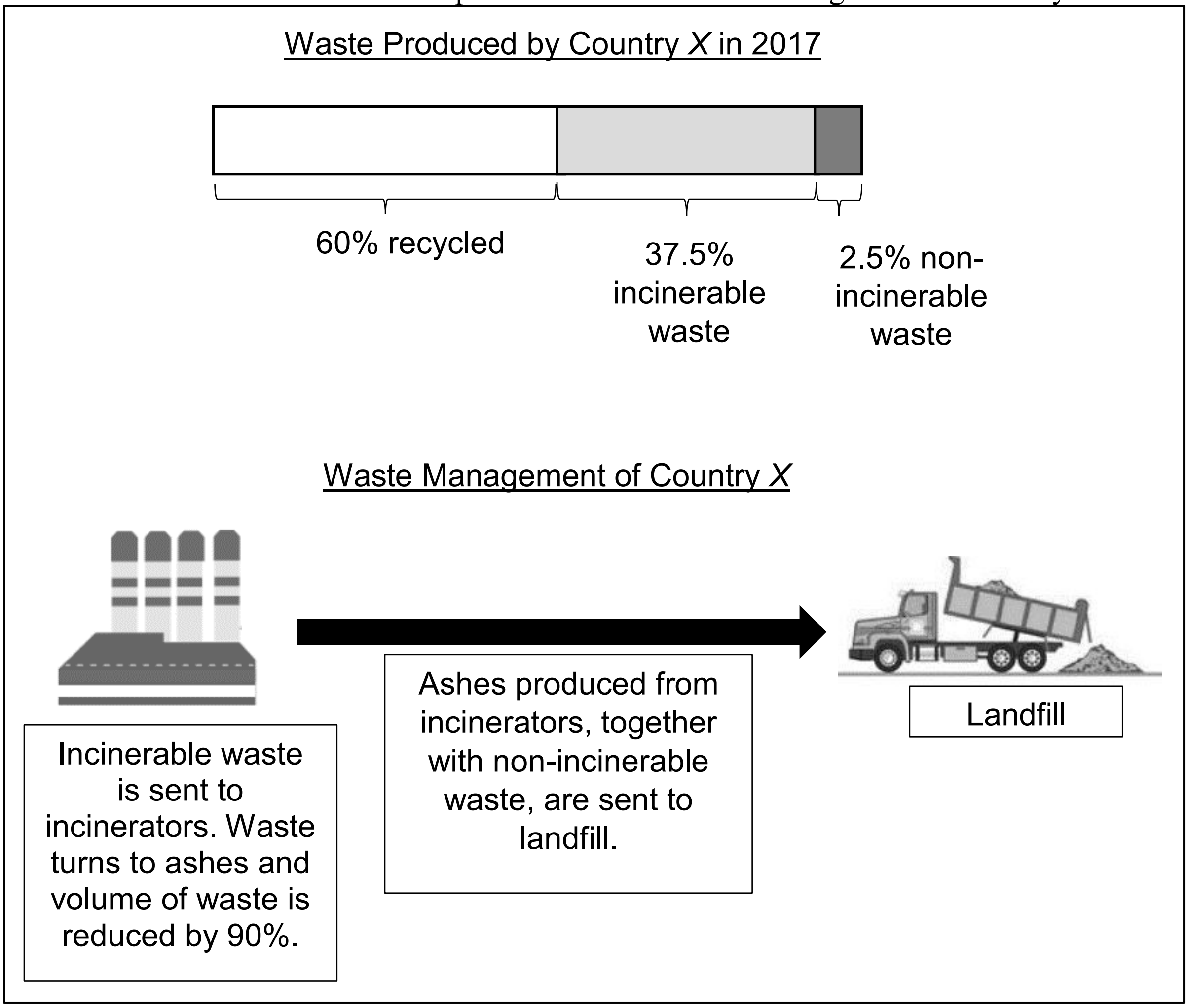
(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

- **(a)** two green cards are drawn, [1]
- **(b)** neither of the cards is green, [1]
- **(c)** the two cards are of different colours. [2]

10(b)	(i) 	A1 A1	1 mark for $\frac{6}{12}, \frac{4}{12}, \frac{2}{12}$ (fractions can be in lowest terms too) 1 mark for the prob of the draws from Box *B*.
10(b)	(ii)(a) P(GG) $= \frac{2}{12} \times \frac{1}{9}$ or $\frac{1}{6} \times \frac{1}{9}$ $= \frac{1}{54}$	A1	
	(ii)(b) P(neither is G) $= \frac{6}{12} + \frac{4}{12} = \frac{5}{6}$	A1	
	(ii)(c) P(different colour) = P(RR′) + P(BB′) + P(GG′) $= \frac{6}{12} \times \frac{5}{9} + \frac{4}{12} \times \frac{3}{9} + \frac{2}{12} \times \frac{8}{9}$ $= \frac{58}{108}$ $= \frac{29}{54}$	M1 A1	1 mark for any of the P(RR′), P(BB′), P(GG′)

11 Country X produced 3 million tonnes of waste in 2017. The infographic below shows more information on the waste produced and the waste management of Country X.



Images from: https://www.dreamstime.com/illustration/dumptruck.html,
https://www.mewr.gov.sg/topic/landfill

(a) Given that the density of waste is 125 kg/m^3 and 1 tonne = 1000 kg, calculate the volume of waste, in m^3, that was incinerated in 2017.
Give your answer in standard form. [3]

The landfill used by Country X has a total capacity of 42 000 000 m^3.
By the end of 2017, 30% of the landfill has already been used.
A news article claims it will take another 32 years before the landfill is completely used.

(b) Is the news article correct?
Justify your decision with calculations. [6]

(c) State one assumption you made in your calculations in **(b)**. [1]

10	(a) Mass of incinerable waste = 3 000 000 × 37.5 % tonnes = 1 125 000 tonnes $= 1.125 \times 10^9$ kg Volume of incinerable waste = $\dfrac{1.125\times10^9}{125}$ m^3 = 9 000 000 m^3	B1 M1√ A1	
	(b)Volume of non-incinerable waste = $\dfrac{3\,000\,000\times1000\times2.5\%}{125}$ = 600 000 m^3 Volume of ashes from incinerated waste = 9 000 000 × 10% = 900 000 m^3 Total volume of waste to be landfilled = 600 000 + 900 000 = 1500 000 m^3 Volume of landfill left = 42 000 000 × 70% = 29 400 000 m^3 Years left = $\dfrac{29\,400\,000}{1\,500\,000}$ = 19.6 No, the news article is incorrect.	M1 M1√ M1√ B1 A1 DA1	√ vol of incin waste √ vol of non-incin and incin waste
	(d) Amount of incinerated and non-incinerable waste remains the same every year. Or The percentage breakdown of waste remains the same every year.	A1	