Name : _____ Class | Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

MID-YEAR EXAMINATION 2017
Secondary 4

Monday 15 May 2017

MATHEMATICS
Paper 1

4048/01

2 h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

80

This question paper consists of 19 printed pages.

Mathematical Formulae

For Examiner's Use

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions

For Examiner's Use

1 **(a)** Calculate $\dfrac{23-\left(-4\frac{3}{5}\right)^3}{\sqrt[3]{(7.2)^2+\sqrt{6.81}}}$.

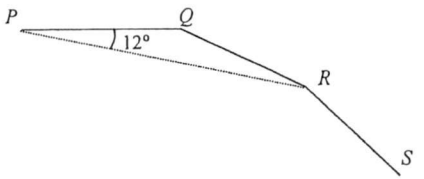
Write down the first five digits on your calculator display.

Answer (a) ..... [1]

(b) Write your answer to part **(a)** correct to 1 significant figure.

Answer (b) ..... [1]

2



PQ, QR and RS are adjacent sides of a regular polygon.

Given that $\angle QPR = 12°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.

Answer (a) ..... ° [1]

(b) ..... [1]

(c) $\angle PRS$ = ... ° [1]

For Examiner's Use

3 The masses of 20 apples were measured, to the nearest grams.

The results are shown in the stem and leaf diagram below.

6	0 5
7	4 4 6 8 x 9
8	1 1 2 5 5
9	4 4 4 5 6
10	1 7

Key 7 | 4 means 74 grams

(a) Given that the percentage of apples whose weights are less than 80 grams is 40%, state a possible value of x.

Answer (a) ..... [1]

(b) Find the median weight.

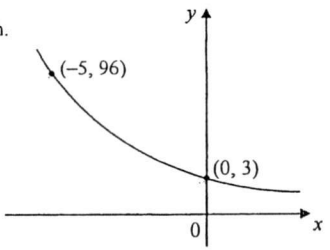
Answer (b) .....grams [1]

(c) The same information is to be shown on a pie chart.
Calculate the angle of the sector which would represent the number of apples whose weights are between 80 grams and 100 grams.

Answer (c) ..... ° [1]

4 The sketch shows the graph of $y = ka^x$.
The points (0, 3) and (–5, 96) lie on the graph.

Find the values of k and a.

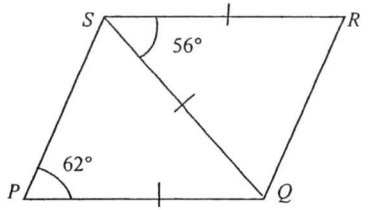


Answer k = ... [1]

a = ... [1]

5 In the diagram, PQS and RSQ are isosceles triangles in which $PQ = QS = SR$, $\angle SPQ = 62°$ and $\angle QSR = 56°$

For Examiner's Use



(a) Prove that ΔPQS and ΔRSQ are congruent.

(b) Write down the special name of quadrilateral $PQRS$.

Answer (a) .....

.....

.....

..... [2]

(b) ..... [1]

6 **(a)** Find the least values of x and y such that $2^3 \times 3^x \times 5^y$ is a multiple of 10.

Answer (a) x = ... [1]

y = ... [1]

(b) Written as a product of its prime factors, $201684 = 2^2 \times 3 \times 7^5$.
Find the smallest positive integer k such that $201684k$ is a cube number.

Answer (b) k = ... [1]

7 A map is drawn to the scale of 1 : 25 000.

For Examiner's Use

(a) The representation of a lake on the map has a perimeter of 8 cm.
How many kilometres will a girl cover if she walks around the lake?

Answer (a) ..... km [1]

(b) The area of a plantation is 75 000 m^2. Calculate the area of the plantation on the map in square centimetres.

Answer (b) ..... cm^2 [2]

8

	Singapore	Indonesia	China
Population	5.7×10^6	260×10^6	1.38×10^9
Area (km^2)	719.1	1.9×10^6	9.6×10^6

For Examiner's Use

Use as much information from the table as necessary to answer the following.

(a) Find the ratio of the population of Indonesia to the population of China.
Give your answer in the form $a : b$.

(b) How many times is the population of Indonesia that of Singapore?
Give your answer to the nearest whole number.

(c) Calculate the average number of people per square kilometre in China.
Give your answer to the nearest whole number.

Answer (a) ..... [1]

(b) ..... [1]

(c) ..... [1]

9 $\varepsilon = \{ x : x \text{ is an integer and } 4 \le x \le 25 \}$
$A = \{ x : x \text{ is divisible by } 5 \}$
$B = \{ x : x \text{ is a perfect square} \}$
$C = \{ x : x \text{ is a prime number} \}$

For Examiner's Use

(a) List the element/s contained in the set $A \cap B$,

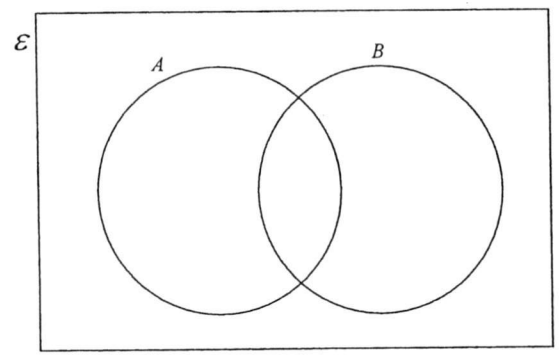
Answer (a) ..... [1]

(b) Write down in set notation an equation involving B and C.

Answer (b) ..... [1]

(c) In the Venn diagram below, shade the region representing $(A \cap B')'$.

Answer



[1]

10 Solve the equation $\frac{x}{x+1}-\frac{x+1}{3x-1}=\frac{1}{4}$.

For Examiner's Use

Answer $x =$ ....., ... [3]

11 **(a)** Express $x^2-6x+11$ in the form $(x-p)^2+q$.

Answer (a) ..... [2]

(b) **Hence**, write down the minimum value of $x^2-6x+11$.

Answer (b) ..... [1]

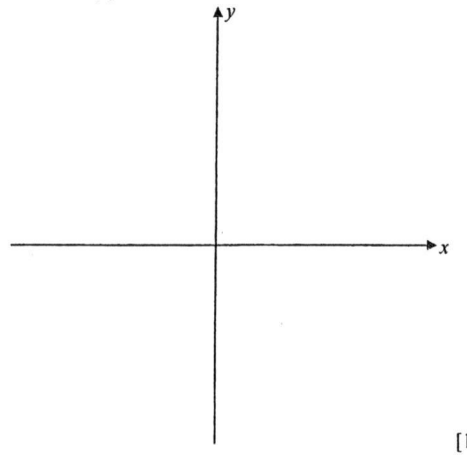
(c) Write down the equation of the line of symmetry of the graph of $y=x^2-6x+11$.

Answer (c) ..... [1]

12 On the axes given, sketch the graphs of

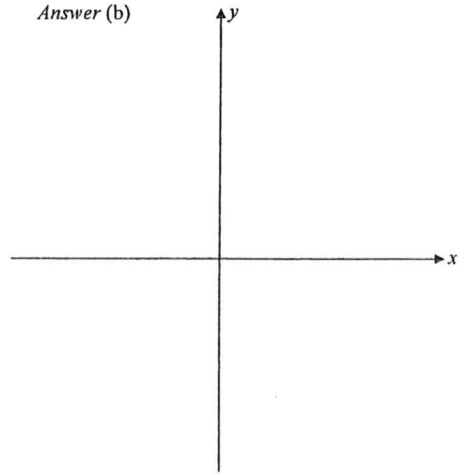
For Examiner's Use

(a) $y=2-x^3$, *Answer* (a)



[1]

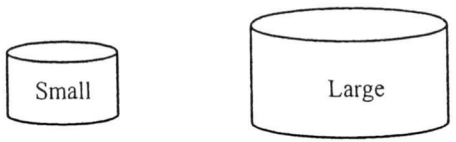
(b) $y=\frac{1}{x^2}$. *Answer* (b)



[1]

13 The base areas of two geometrically similar tins of sardines labelled Small and Large are 240 cm^2 and 540 cm^2 respectively.

For Examiner's Use



(a) The height of the Large tin is 18 cm. Find the height of the Small tin.

Answer (a) ..... cm [2]

(b) The price of the Small tin of sardines is \$3.20. A shopkeeper used the ratio of the base areas given above to price the Large tin of sardines.
Explain, with clear working, why the Large tin is better value for money.

Answer (b) .....
.....
.....
.....
.....
.....
..... [2]

14 Two outlets sold the following number of cups of drinks on a particular day.

For Examiner's Use

	Number of cups of drinks		
	Tea	Coffee	Milo
Outlet 1	32	67	56
Outlet 2	24	56	43

It costs \$0.50, \$0.80 and \$0.60 for the outlets to prepare a cup of tea, coffee and milo respectively. Each cup of tea, coffee and milo was sold for \$3.50, \$4.50 and \$4.00 respectively. It is given that

$$\mathbf{P} = \begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0.50 \\ 0.80 \\ 0.60 \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} 3.50 \\ 4.50 \\ 4.00 \end{pmatrix}$$

(a) Find $\mathbf{P(S - C)}$.

Answer (a) = ... [1]

(b) Explain what your answer to **(a)** represents.

Answer (b) .....
.....
.....
..... [1]

(c) Using matrix multiplication of a 1×2 matrix with the matrix obtained in part **(a)**, calculate the total profit made from the two outlets for the day.

Answer (c) \$..... [2]

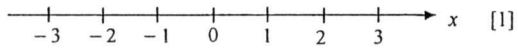
For Examiner's Use

15 **(a)** Solve the inequalities $\frac{x-1}{4} < \frac{2x+1}{3} \leq 3x-2$.

Answer (a) ..... [3]

(b) Represent solution set on the number line below.

Answer (b)



[1]

(c) Write down the smallest prime number which satisfies the above inequalities.

Answer (c) ..... [1]

16 **(a)** Factorise completely $4x + 4xy - 12y^2 - 12y$.

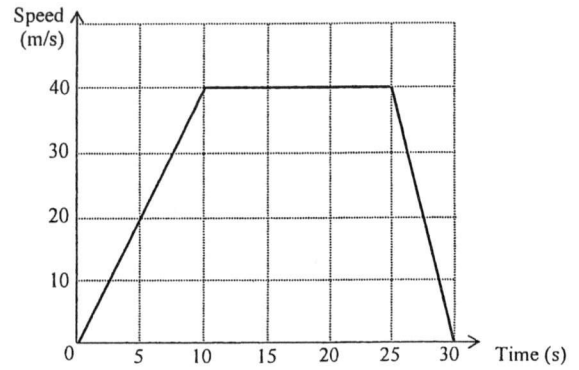
Answer (a) ..... [2]

(b) Simplify $\frac{a^4}{b} \times \left(\frac{b^6}{a^9}\right)^{-\frac{1}{3}}$, giving your answer in positive indices only.

Answer (b) ..... [2]

For Examiner's Use

17 The diagram is the speed-time graph for the first 30 seconds of a car's journey.



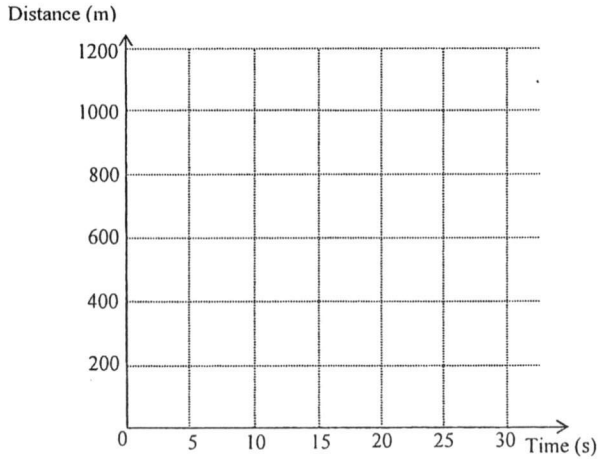
(a) Calculate the speed when the time is 7 seconds.

Answer (a) ..... m/s [1]

(b) Calculate the deceleration in the last 5 seconds.

Answer (b) ..... m/s^2 [1]

(c) On the axes below, sketch the distance-time graph for the journey.



[3]

18 A source of light is observed from a distance of d metres.
The amount of light received, L units, is inversely proportional to square of the distance from the source.

For Examiner's Use

(a) When the distance is 2 m apart, the amount of light received is 9 units.
Find the relationship between L and d.

Answer (a) ..... [2]

(b) Find the amount of light received when the distance is 5 m.

Answer (b) ..... units [1]

(c) When the source is at a certain distance, the amount of light received is p units.
Find the amount of light received, in terms of p, when the distance is doubled.

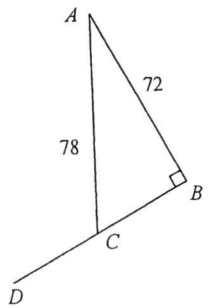
Answer (c) ..... units [1]

19 In the diagram, $\angle ABC = 90°$ and BCD is a straight line.
All measurements are in centimetres.

Without the use of calculators, find
(a) sin $\angle BAC$,

(b) cos $\angle ACD$.

Give both answers in the simplest form of $\frac{a}{b}$,
where a and b are integers.

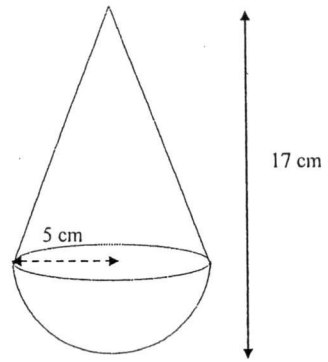


Answer (a) sin $\angle BAC$ = ... [2]

(b) cos $\angle ACD$ = ... [1]

20 The diagram shows a solid wooden toy made from a cone and a hemisphere of radius 5cm.
The total height of the toy is 17 cm.
The cost of painting this wooden toy is 0.5 cents per cm^2.

For Examiner's Use

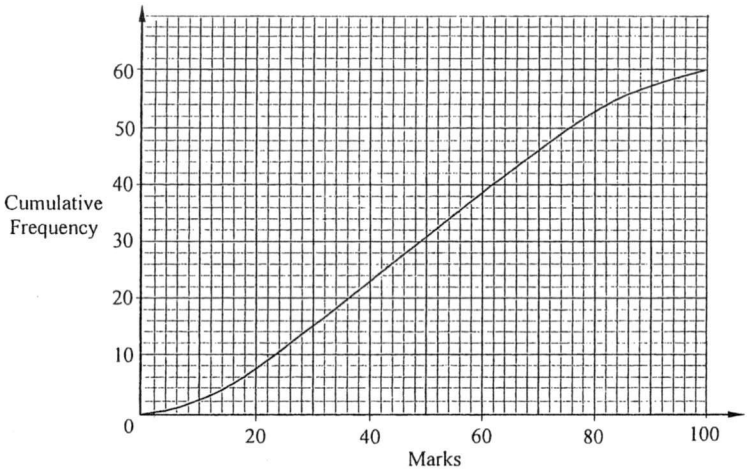


Find the total cost of painting the wooden toy in dollars.

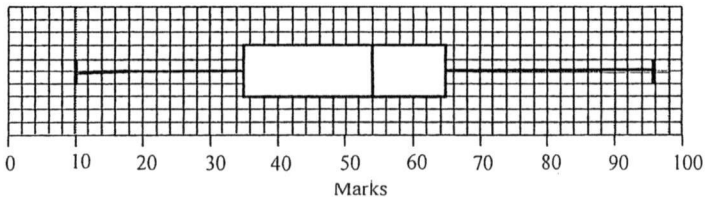
Answer $ ..... [3]

21 The cumulative frequency curve below shows the marks obtained, out of 100, by 60 students in an Additional Mathematics test.

For Examiner's Use



The same 60 students also set for the Physics paper.
The box-and-whisker diagram below illustrates the marks obtained.
The maximum mark was again 100.



Use the two diagrams to complete this table for the two tests.

Subject	Lower Quartile	Median	Upper Quartile	Interquartile Range
Add. Mathematics		49	68	
Physics	35		65	

[2]

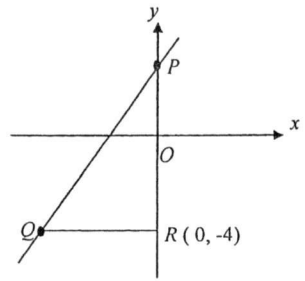
A student commented that the results for Additional Mathematics were better than for Physics.
Do you agree? Give a reason for your answer.

Answer .....

..... [1]

22 In the diagram below, R is the point (0, - 4) and P is a point on the y-axis. The line PQ meets the horizontal line through R at Q.

For Examiner's Use



(a) Write down the equation of line QR.

(b) Given that the equation of the line PQ is $3y - 7x = 9$, find the coordinates of Q.

(c) Find the area of triangle PQR.

(d) Calculate the perpendicular distance from R to PQ, giving your answer correct to two decimal places.

Answer (a) ..... [1]

(b) (,) [1]

(c) ..... units^2 [2]

(d) ..... units [2]

23 A playground is in the shape of a quadrilateral $ABCD$.
It is given that $AB = 90$ m, B is due East of A, $AD = 70$ m, $\angle BAD = 115°$, $\angle ADC = 85°$ and the bearing of C from $B = 010°$.

For Examiner's Use

(a) Complete the scale drawing of the playground, using a scale of 1cm to 10 m. [2]

(b) On your drawing, construct

(i) the perpendicular bisector of CD, [1]

(ii) the angle bisector of $\angle ABC$. [1]

(c) A statue , P, is to be built inside the playground such that it is nearer to D than C and equidistant from AB and BC.

Mark and label a possible position of the statue on your drawing. [1]

End of Paper

Answer all the questions

For Examiner's Use

1 **(a)** Calculate $\dfrac{23-\left(-4\frac{3}{5}\right)^3}{\sqrt[3]{(7.2)^2+\sqrt{6.81}}}$.

Write down the first five digits on your calculator display.

31.749

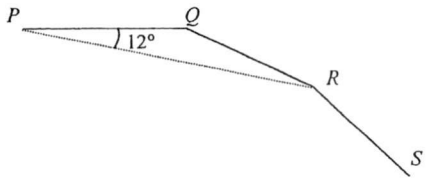
Answer (a) ..... [1]

(b) Write your answer to part **(a)** correct to 1 significant figure.

30

Answer (b) ..... [1]

2



PQ, QR and RS are adjacent sides of a regular polygon.

Given that $\angle QPR = 12°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.

(a) an exterior angle of the polygon = 12° + 12° = 24°

(b) $\dfrac{360°}{24°} = 15$ (c) $\angle PRS$ = 156° - 12° =144°

Answer (a) 24 [1]

(b) 15 ° [1]

(c) $\angle PRS$ = 144 ° [1]

For Examiner's Use

3

The masses of 20 apples were measured, to the nearest grams.

The results are shown in the stem and leaf diagram below.

6	0 5
7	4 4 6 8 *x* 9
8	1 1 2 5 5
9	4 4 4 5 6
10	1 7

Key 7 | 4 means 74 grams

(a) Given that the percentage of apples whose weights are less than 80 grams is 40%, state a possible value of x.

8, 9

Answer (a) ..... [1]

(b) Find the median weight.

81.5

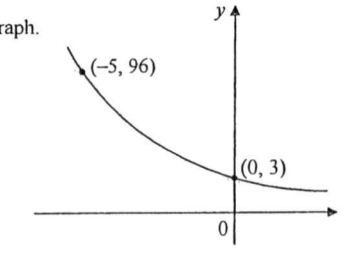
Answer (b) .....grams [1]

(c) The same information is to be shown on a pie chart.
Calculate the angle of the sector which would represent the number of apples whose weights are between 80 grams and 100 grams.

216 °

Answer (c) ..... [1]

4 The sketch shows the graph of $y = ka^x$.
The points (0, 3) and (–5, 96) lie on the graph.



Find the values of k and a.

When $x = 0$, $y = 3$, $3 = k\,a^0$,
$k = 3$

$y = 3a^x$
$96 = 3a^{-5}$
$a^5 = \dfrac{3}{96}$
$a = \dfrac{1}{2}$

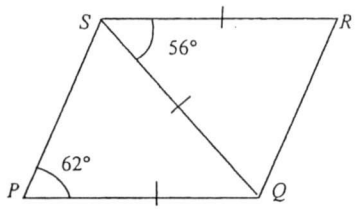
Answer k = 3 [1]

$\frac{1}{2}$

a = ... [1]

5 In the diagram, PQS and RSQ are isosceles triangles in which $PQ = QS = SR$ $\angle SPQ = 62°$ and $\angle QSR = 56°$

For Examiner's Use



(a) Prove that ΔPQS and ΔRSQ are congruent.

(b) What special type of quadrilateral is $PQRS$?

Answer (a) $\angle PQS = 180^0 - 62^0 - 62^0 = 56^0$ [$\angle$ sum of Δ]
$\therefore \angle PQS = \angle RSQ$
SQ is common
$PQ = RS$ [Given]
$\therefore \Delta PQS \equiv \Delta RSQ$ [SAS] [2]

(b) parallelogram [1]

6 **(a)** Find the least values of x and y such that $2^3 \times 3^x \times 5^y$ is a multiple of 10.

Answer (a) x = ...0..... [1]
y = ...1..... [1]

(b) Written as a product of its prime factors, $201684 = 2^2 \times 3 \times 7^5$.
Find the smallest positive integer k such that $201684k$ is a cube number

$k = 2 \times 3^2 \times 7$

Answer (b) k = ...126..... [1]

7 A map is drawn to the scale of 1 : 25 000.

For Examiner's Use

(a) The representation of a lake on the map has a perimeter of 8 cm.
How many kilometres will a girl cover if she walks around the lake?

1 cm : 25 000 cm
: 250 m
: 0.25 km
8 cm : 8 X 0.25 km
: 2 km

Answer (a) .....2..... km [1]

(b) The area of a plantation is 75 000 m^2. Calculate the area of the plantation on the map in square centimetres.

1 cm : 250 m
1cm^2 : 62500 m^2
1.2 cm^2 : 75000 m^2

Answer (b) .....1.2..... cm^2 [2]

8

	Singapore	Indonesia	China
Population	5.7×10^6	260×10^6	1.38×10^9
Area (km^2)	719.1	1.9×10^6	9.6×10^6

For Examiner's Use

Use as much information from the table as necessary to answer the following.

(a) Find the ratio of the population of Indonesia to the population of China.
Give your answer in the form $a : b$.

(b) How many times is the population of Indonesia that of Singapore?
Give your answer to the nearest whole number.

(c) Calculate the average number of people per square kilometre in China.
Give your answer to the nearest whole number.

(a) $260 \text{ X } 10^6 : 1.36 \text{ X } 10^9$

(b) $\frac{260 \times 10^6}{5.7 \times 10^6} = 45.6$

(c) $\frac{1.38 \times 10^9}{9.6 \times 10^6} = 1.4375 \times 10^2$

Answer (a) ..... 13 : 69 ..... [1]

(b) ..... 46 ..... [1]

(c) 144 [1]

9

$\varepsilon = \{ x : x \text{ is an integer and } 4 \le x \le 25 \}$
$A = \{ x : x \text{ is divisible by } 5 \}$
$B = \{ x : x \text{ is a perfect square} \}$
$C = \{ x : x \text{ is a prime number} \}$

For Examiner's Use

(a) List the element/s contained in the set $A \cap B$,

Answer (a) {25} [1]

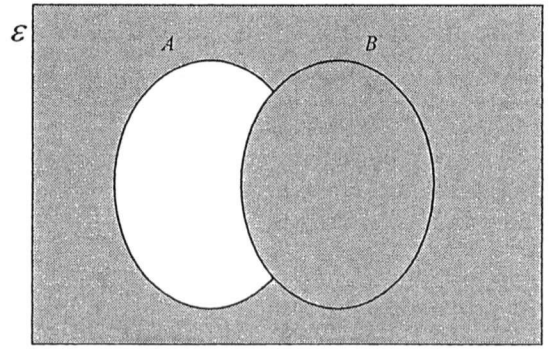
(b) Write down in set notation an equation involving B and C.

$B \cap C = \{\}$ or $B \cap C = \phi$

Answer (b) ..... [1]

(c) In the Venn diagram below, shade the region representing $(A \cap B')'$.

Answer



[1]

10 Solve the equation $\frac{x}{x+1}-\frac{x+1}{3x-1}=\frac{1}{4}$.

For Examiner's Use

$4x(3x-1)-4(x+1)^2=(x+1)(3x-1)$

$12x^2-4x-4(x^2+2x+1)=3x^2+2x-1$

$12x^2-4x-4x^2-8x-4-3x^2-2x+1=0$

$5x^2-14x-3=0$

$(5x+1)(x-3)=0$

$x=-\frac{1}{5}$ or $x=3$

Answer $x=-\frac{1}{5}$, 3 [3]

11 **(a)** Express $x^2-6x+11$ in the form $(x-p)^2-q$.

$x^2-6x+11$

$=(x-3)^2-9+11$

$=(x-3)^2+2$

Answer (a) $(x-3)^2+2$ [2]

(b) Hence write down the minimum value of $x^2-6x+11$.

Answer (b) 2 [1]

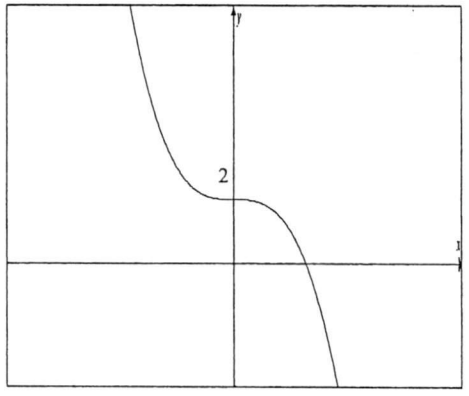
(c) Write down the equation of the line of symmetry of the graph of $y=x^2-6x+11$.

Answer (c) $x=3$ [1]

12 On the axes given, sketch the graphs of

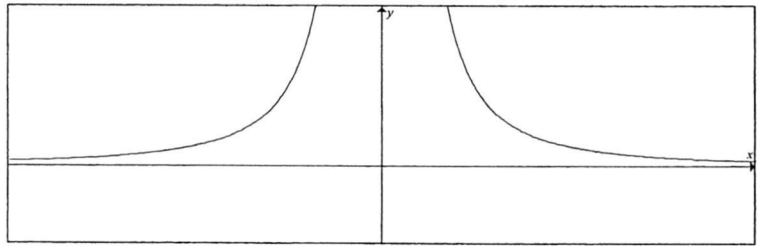
For Examiner's Use

(a) $y=2-x^3$ *Answer* (a)



[1]

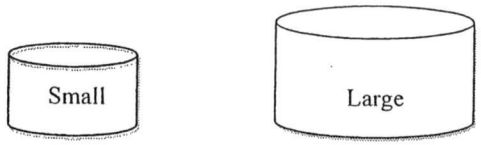
(b) $y=\frac{1}{x^2}$ *Answer* (b)



[1]

13 The base areas of two geometrically similar tin of sardines labelled Small and Large are 240 cm^2 and 540 cm^2 respectively.

For Examiner's Use



(a) The height of the Large tin is 18 cm. Find the height of the Small tin.

Let the height of the Small tin be x cm.

$$\left(\frac{x}{18}\right)^2 = \frac{240}{540}$$

$$\frac{x}{18} = \sqrt{\frac{4}{9}}$$

$$\frac{x}{18} = \frac{2}{3}$$

Answer (a) .....12..... cm [2]

(b) The price of the Small tin of sardines is \$3.20. A shopkeeper used the ratio of the base areas given above to price the Large tin of sardines. Explain , with clear working, why the Large tin is value for money.

Price of Large tin based on ratio of area $= \left(\frac{3}{2}\right)^2 \times \$3.20 = \$7.20$

Ratio of volume $= \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Price of Large tin based on volume $= \frac{27}{8} \times \$3.20 = \$10.80$

Answer (b) Based on the ratio of areas, the price paid for the Large tin is less.

$\therefore$ it is more value for money. [2]

14 Two outlets sold the following drinks on a particular day.

For Examiner's Use

	Tea	Coffee	Milo
Outlet 1	32	67	56
Outlet 2	24	56	43

It costs \$0.50, \$0.80 and \$0.60 for the outlets to prepare a cup of tea, coffee and milo respectively. Each cup of tea, coffee and milo was sold for \$3.50, \$4.50 and \$4.00 respectively. It is given that

$$\mathbf{P} = \begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0.50 \\ 0.80 \\ 0.60 \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} 3.50 \\ 4.50 \\ 4.00 \end{pmatrix}$$

(a) Find **P(S – C)**.

$$\begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix} \begin{pmatrix} 3.00 \\ 3.70 \\ 3.40 \end{pmatrix}$$

Answer (a) $= \begin{pmatrix} 534.30 \\ 425.40 \end{pmatrix}$ [1]

(b) Explain what your answer to **(a)** represents.

Answer (b) Profit from selling all 3types of drinks at each outlet respectively. [1]

(c) Using matrix multiplication of a 1 X 2 matrix with the matrix obtained in part **(a)**, calculate the total profit made from the two outlets for the day.

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 534.30 \\ 425.40 \end{pmatrix} = (534.30 + 425.40) = (959.70)$$

Answer (c) \$959.70 [2]

For Examiner's Use

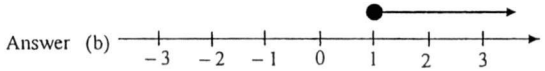
15 **(a)** Solve the inequalities $\frac{x-1}{4} < \frac{2x+1}{3} \leq 3x-2$.

$3(x-1) < 4(2x+1)$
$3x-3 < 8x+4$
$3x-8x < 7$
$-5x < 7$
$x > -\frac{7}{5}$

$2x+1 \leq 9x-6$
$2x-9x \leq -6-1$
$-7x \leq -7$
$x \geq 1$

Answer (a) $x \geq 1$ [3]

(b) Represent solution set on the number line below.

Answer (b) 

[1]

(c) Write down the smallest prime number which satisfies the above inequalities.

Answer (c) 2 [1]

16 **(a)** Factorise completely $4x+4xy-12y^2-12y$,

$4x(1+y)-12y(y+1)$
$=4(y+1)(x-3y)$

Answer (a) $4(y+1)(x-3y)$ [2]

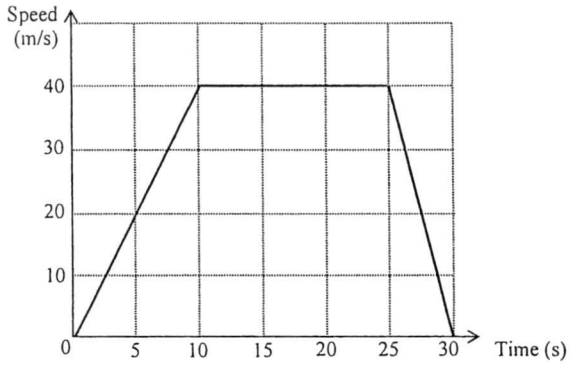
(b) Simplify $\frac{a^4}{b} \times \left(\frac{b^6}{a^9}\right)^{-\frac{1}{3}}$, giving your answer in positive indices only.

$\frac{a^4}{b} \times \left(\frac{a^9}{b^6}\right)^{\frac{1}{3}}$
$= \frac{a^4}{b} \times \frac{a^3}{b^2}$

Answer (b) $\frac{a^7}{b^3}$ [2]

For Examiner's Use

17 The diagram is the speed-time graph for the first 30 seconds of a car's journey.



(a) Calculate the speed when the time is 7 seconds.

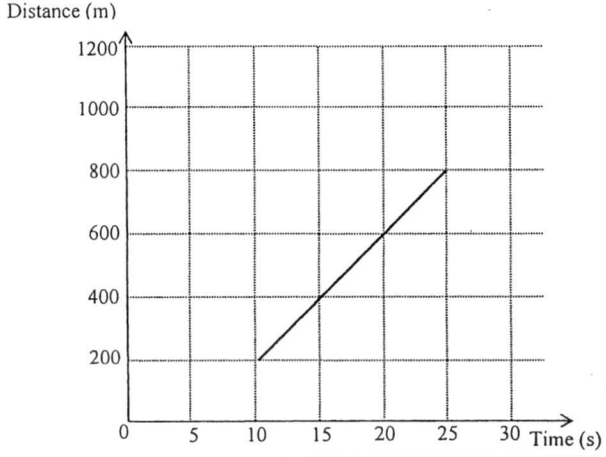
$\frac{v}{40} = \frac{7}{10}$ Answer (a) 28 m/s [1]

(b) Calculate the deceleration in the last 5 seconds.

$\frac{40}{5} = 8$

Answer (b) 8 m/s² [1]

(c) On the axes below, sketch the distance-time graph for the journey.



[3]

18 A source of light is observed from a distance of d metres.
The amount of light received, L units, is inversely proportional to square of the distance from the source.

For Examiner's Use

(a) When the distance is 2 m apart, the amount of light received is 9 units.
Find the relationship between L and d.

$$L = \frac{k}{d^2}$$

$$9 = \frac{k}{2^2}$$

$$k = 36$$

Answer (a) $L = \dfrac{36}{d^2}$ [2]

(b) Find the amount of light received when the distance is 5 m.

$$L = \frac{36}{5^2} = \frac{36}{25}$$

Answer (b) 1.44 units [1]

(c) When the source is at a certain distance, the amount of light received is p units.
Find the amount of light received, in terms of p, when the distance is doubled.

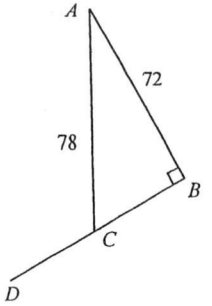
$$p = \frac{36}{d^2}$$

$$\frac{36}{(2d)^2} = \frac{36}{4d^2}$$

Answer (c) $\dfrac{1}{4}p$ units [1]

19 In the diagram, $\angle ABC = 90°$ and BCD is a straight line.
All measurements are in centimetres.

For Examiner's Use



Without the use of calculators, find
(a) $\sin \angle BAC$,

(b) $\cos \angle ACD$.

Give both answers in the simplest form of $\dfrac{a}{b}$,

where a and b are integers.

$BC = \sqrt{78^2 - 72^2} = 30$

(a) $\sin \angle BAC = \dfrac{30}{78} = \dfrac{5}{13}$

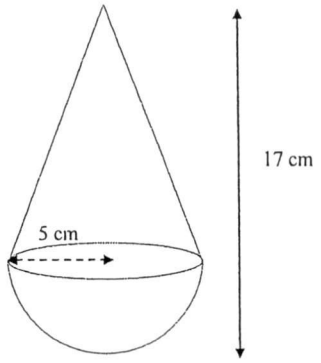
(b) $\cos \angle ACD = -\cos \angle ACB = -\dfrac{30}{78} = -\dfrac{5}{13}$

Answer (a) $\sin \angle BAC = \dfrac{5}{13}$ [2]

(b) $\cos \angle ACD = -\dfrac{5}{13}$ [1]

20 The diagram shows a solid wooden toy made from a cone and a hemisphere of radius 5cm.
The total height of the toy is 17 cm.
The cost of painting this wooden toy is 0.5 cents per cm^2.

For Examiner's Use



Find the total cost of painting the wooden toy in dollars.

Height of cone = 12

Curved surface of cone = $\pi(5)(13) = 65\pi$

Curved surface area of hemisphere = $2\pi(5)^2 = 50\pi$

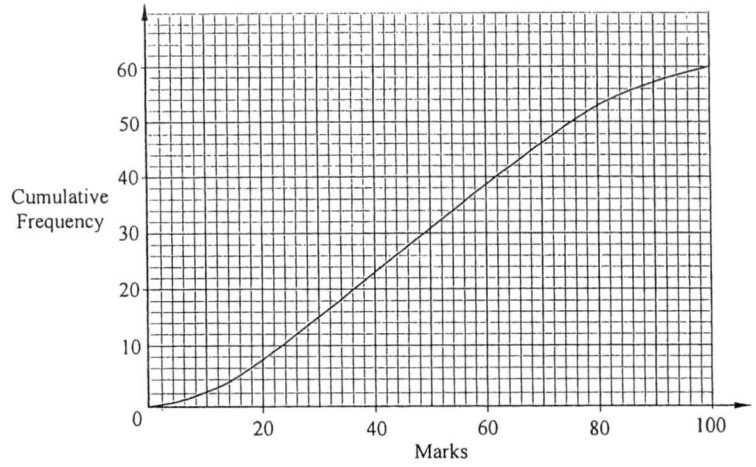
Total Surface Area = 115π

Cost of painting = $115\pi \times 0.5$ cents
= 180.64 cents

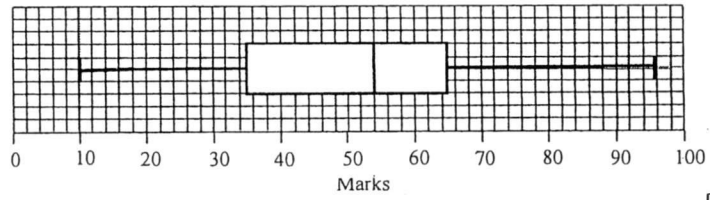
Answer $ 1.81 [3]

21 The cumulative frequency curve below shows the marks obtained, out of 100, by 60 students in an Additional Mathematics test.

For Examiner's Use



The same 60 students also set for the Physics paper.
The box-and-whisker diagram below illustrates the marks obtained.
The maximum mark was again 100.



[2]

Use the two diagrams to complete this table for the two tests.

Subject	Lower Quartile	Median	Upper Quartile	Interquartile Range
Add. Mathematics	30	49	68	38
Physics	35	54	65	30

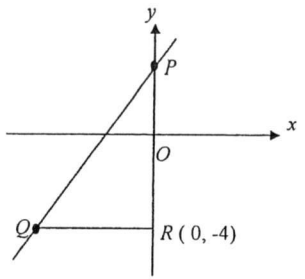
A student commented that the results for Additional Mathematics was better than for Physics.
Do you agree? Give a reason for your answer.

Answer (c) No, Physics has a higher median and a smaller interquartile range.

[1]

22 In the diagram below, R is the point (0, - 4) and P is a point on the y-axis. The line PQ meets the horizontal line through R at Q.

For Examiner's Use



(a) Write down the equation of line QR.

(b) Given that the equation of the line PQ is $3y - 7x = 9$, find the coordinates of Q.

(c) Find the area of triangle PQR.

(d) Calculate the perpendicular distance from R to PQ, giving your answer correct to two decimal places.

(b)

when $y = -4$, $\quad -4(3) - 7x = 9$

$$-7x = 21$$

$$x = -3$$

(c) when $x = 0$, $3y = 9$

$y = 3$

Coordinates of P = (0, 3)

Area of $\Delta PQR = \frac{1}{2} \times 3 \times 7 = 10.5$ units^2

Let d be the perpendicular distance from R to PQ

$PQ = \sqrt{3^2 + 7^2} = \sqrt{58}$

(d) $\frac{1}{2} \times \sqrt{58} \times d = 10.5$

$d = \frac{21}{\sqrt{58}}$

Answer (a) $y = -4$ [1]

(b) (−3 , −4) [1]

(c) 10.5 units^2 [2]

(d) 2.76 units [2]

23 A playground is in the shape of a quadrilateral $ABCD$.
It is given that $AB = 90$ m, B is due East of A, $AD = 70$ m, $\angle BAD = 115°$, $\angle ADC = 85°$ and the bearing of C from $B = 010°$.

For Examiner's Use

(a) Complete the scale drawing of the playground, using a scale of 1cm to 10 m. [2]

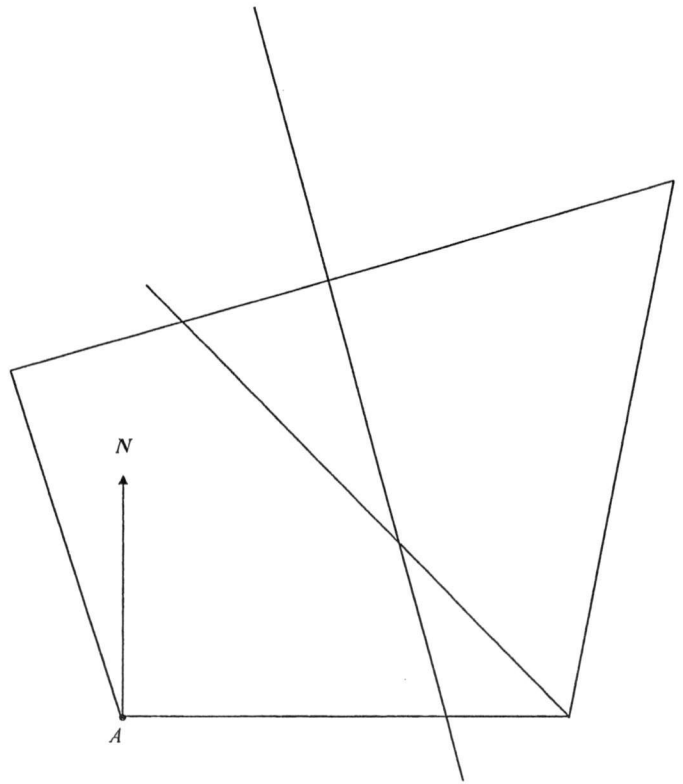
(b) On your drawing, construct

(i) the perpendicular bisector of CD, [1]

(ii) the angle bisector of $\angle ABC$. [1]

(c) A statue , P, is to be built inside the playground such that it is nearer to D than C and equidistant from AB and BC.

Mark and label a possible position of the statue on your drawing. [1]



End of Paper

Answer **all** the questions.

1 **(a)** **(i)** Factorise completely $2a^2 - 11a - 21$. [1]

(ii) **Hence**, factorise completely $2(2b+1)^2 - 22b - 32$. [2]

(b) Express $\frac{6t-8}{3t^2-5t+2} \div \frac{4-3t}{3t-3}$ as a single fraction in its simplest form. [3]

(c) It is given that $w = \frac{x+2t^2}{a}$.

(i) Find w when $x = -2$, $t = -3$ and $a = 19$. [1]

(ii) Express t in terms of w, x and a. [2]

2 The table below shows the distribution of the heights of class of 30 Primary One students.

Height x (cm)	Frequency
$100 \le x < 105$	2
$105 \le x < 110$	6
$110 \le x < 115$	15
$115 \le x < 120$	7

(a) State the modal class of the distribution. [1]

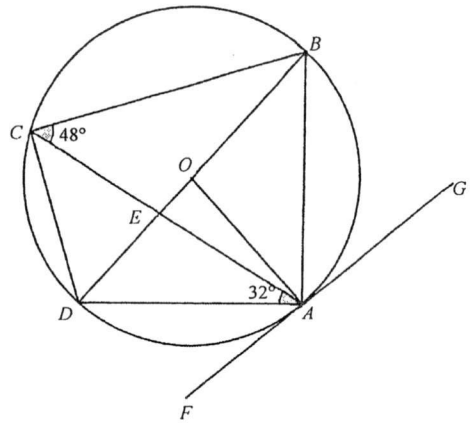
(b) Calculate an estimate of the mean height of the Primary One students. [2]

(c) Calculate an estimate of the standard deviation of their heights. [1]

(d) The standard deviation of the heights of a class of Primary Two students is 3.52. Use this information to comment on one difference between the two distributions. [1]

(e) It was found that the heights of all the Primary One students were measured incorrectly. The correct heights were all 3 cm more than those recorded. Explain how the estimated mean and standard deviation of the heights have been affected by this error. [2]

3 In the diagram, O is the centre of the circle through A, B, C and D. FG is the tangent to the circle at A. AC intersects BD at E. $\angle ACB = 48°$ and $\angle CAD = 32°$.



(a) Calculate the following angles, showing reasons,

(i) $\angle ABO$, [2]

(ii) $\angle BDA$, [1]

(iii) $\angle CDA$, [2]

(iv) $\angle GAB$. [2]

(b) Is BD parallel to GF? Justify your answer. [2]

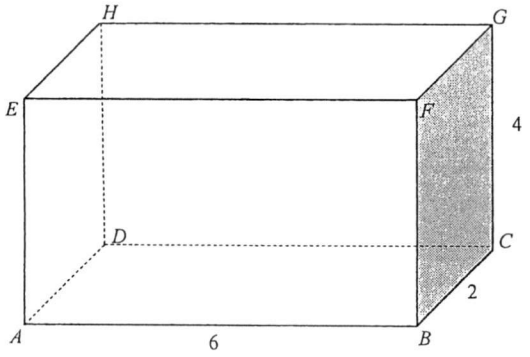
4 **(a)** A sofa can be bought locally at \$1 200 before a discount of 5%. The same sofa can be purchased from China at RMB5000. However, this overseas purchase will incur a shipping charge of S\$50 and a 7% Goods and Service Tax (GST) based on total charges of the item. The exchange rate is RMB4.84 = S\$1.
Determine, with clear working, the cheaper option to buy the sofa. [4]

(b) Mr Tan wants to deposit \$50 000 into Bank ABC. ABC offers two deposit plans.

Plan A	Plan B
Simple interest: 3% per annum	Half-yearly compounded interest: x% per annum

Mr Tan calculated that the interest yield from either plan is the same when he deposits the sum for 3 years. Calculate the value of x. [4]

5 The diagram shows a rectangular cuboid $ABCDEFGH$.
$AB = 6$ cm, $BC = 2$ cm and $CG = 4$ cm.



(a) Show that the angle $HBD = 32.3°$, correct to 1 decimal place. [2]

(b) Calculate angle AFC. [3]

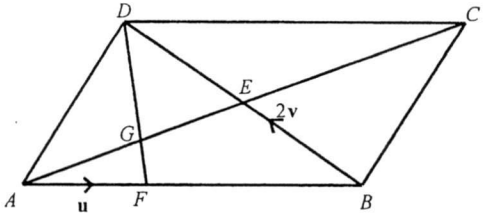
(c) Calculate the greatest angle of elevation of the point H when viewed from any point along AB. [1]

6 **(a)** J is the point $(-2, 4)$. The point K is the result of the translation of J by $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

(i) Find the position vector of K. [1]

(ii) Find the equation of line JK. [2]

(b) In the diagram $ABCD$ is a parallelogram. The diagonals AC and BD intersect at E. F is a point on AB such that $AB = 3AF$. G is the midpoint of AE. $\overrightarrow{AF} = \mathbf{u}$ and $\overrightarrow{BD} = 2\mathbf{v}$.



(i) Express the following, as simply as possible, in terms of **u** and/or **v**,

(a) $\overrightarrow{FB}$, [1]

(b) $\overrightarrow{AG}$, [1]

(c) $\overrightarrow{AD}$, [1]

(d) $\overrightarrow{DF}$, [1]

(e) $\overrightarrow{DG}$. [2]

(ii) State two facts about the points D, G and F. [2]

(iii) Calculate the values of

(a) $\dfrac{Area\ of\ \Delta ADF}{Area\ of\ \Delta ADG}$, [1]

(b) $\dfrac{Area\ of\ \Delta ADG}{Area\ of\ ABCD}$. [2]

7 **(a)** The diagram shows part of a number grid.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

A rectangle outlining 6 numbers, as shown, can be placed anywhere on the grid.

(i) If n represents the number in the top left corner of the rectangle, write down an expression, in terms of n, for the number in the bottom right of the rectangle. [1]

(ii) Show that the difference in the squares of any two numbers in the same column is always a multiple of 9. [2]

(iii) Find the number in the top right corner of the rectangle given that the sum of the six numbers in the rectangle is 777. [3]

(b) A bag contains 5 red and 7 green balls. Joan removes a ball from the bag. Paul then removes another ball from the bag.

(i) Draw a tree diagram to show the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) both Joan and Paul pick red balls, [1]

(b) Paul picks a green ball, [2]

(c) both Joan and Paul pick different coloured balls. [2]

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{4}{x}$.

Some corresponding values of x and y, correct to one decimal place, are given in the following table.

x	0.5	1	2	3	4	5	6	7	8
y	8.5	5	4	4.3	5	a	6.7	7.6	8.5

(a) Find the value of a. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.

Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $0 \le y \le 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve $5 - x = \frac{4}{x}$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(1, 5)$. [2]

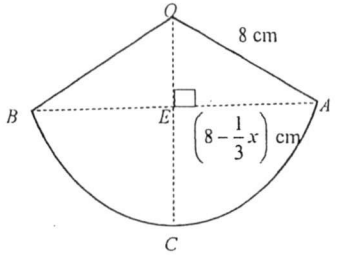
(e) **(i)** On the same axes, draw the graph of $y = -x + 7$. [1]

(ii) Write down the x-coordinate of the points at which the two graphs intersect. [1]

(iii) These values are solutions of the equation $2x^2 + ax + b = 0$.

Find the values of a and b. [2]

9 OAB is a sector of a circle, with centre O and radius 8 cm.
OC cuts AB at E. It is given that $AE = (8 - \frac{1}{3}x)$ cm and $\angle OEA = 90°$.



(a) Given that $EC = x$ cm, express OE in terms of x. [1]

(b) Hence, by making use of your answer in **(a)**, write an equation in terms of x and show that it reduces to $5x^2 - 96x + 288 = 0$. [3]

(c) Solve the equation $5x^2 - 96x + 288 = 0$, giving your answers correct to two decimal places. [3]

(d) Calculate the length of AB. [1]

(e) Calculate angle AOB in terms of radians. [1]

(f) A cone is made from using the sector by joining OA to OB. Calculate the radius of the circular base of this cone. [3]

10 Mr Loh bought a new car.

(a) Mr Loh made a downpayment of \$60 000 and had to pay 48 months of instalments of \$1 200 each for the car.
Find the original price of the car given that the simple interest of the loan amount was 2.5% per annum, giving your answer to the nearest dollar. [3]

The following shows some information he found from the internet about his new car's fuel capacity and fuel efficiency.

Fuel Tank Capacity: 15 gallons

Fuel Efficiency:

City Driving: 22 miles per gallon
Expressway Driving: 26 miles per gallon

(b) In Singapore, fuel is sold by the litre and distance is measured by kilometre. Convert the above information for the fuel tank capacity to litres and fuel efficiency for both the city and expressway driving to kilometres per litre given that 1 gallon = 3.7854 litres and 1 mile = 1.60934 kilometres. [3]

(c) Mr Loh is deciding between a self-drive trip to Melaka in Malaysia or going by takin the bus coach for his family of 5 people.
He calculated that the driving distance from Singapore to Melaka is about 240km, of which 210km is via expressway driving. Petrol is priced at \$1.50 per litre. Toll fee is about \$50 per way. Coach fare per person per way is \$35.
Determine, with clear working, which mode of travelling should Mr Loh decide to save money. [4]

End of Paper 2

1(a)(i) $2a^2 - 11a - 21$

$= (2a + 3)(a - 7)$ [B1]

(ii) $2(2b + 1)^2 - 22b - 32$

$= [2(2b + 1) + 3][2b + 1 - 7]$ [M1]

$= (4b + 5)(2b - 6)$

$= 2(4b + 5)(b - 3)$ [A1]

(b) $\dfrac{6t - 8}{3t^2 - 5t + 2} \div \dfrac{4 - 3t}{3t - 3}$

$= \dfrac{2(3t - 4)}{(3t - 2)(t - 1)} \times \dfrac{3(t - 1)}{4 - 3t}$ [factorise first denominator M1]

$= \dfrac{2(3t - 4)}{(3t - 2)(t - 1)} \times \dfrac{3(t - 1)}{-(3t - 4)}$ [change sign M1]

$= \dfrac{2(3)}{-(3t - 2)}$

$= \dfrac{6}{2 - 3t}$ [A1]

(c)(i) $w = \dfrac{x + 2t^2}{a}$

$w = \dfrac{-2 + 2(-3)^2}{19}$

$w = 0.842 (3sf)$ *or* $\dfrac{16}{19}$ [A1]

(ii) $w = \dfrac{x + 2t^2}{a}$

$wa = x + 2t^2$

$2t^2 = wa - x$ [M1]

$t = \pm\sqrt{\dfrac{wa - x}{2}}$ [A1]

2(a) $110 \leq x \leq 115$ [B1]

(b) Estimated mean $= \dfrac{102.5(2) + 107.5(6) + 112.5(15) + 117.5(7)}{30}$ [M1]

$= 112$ cm [A1]

(c) Standard deviation = 4.15 [B1]

(d) The heights of the Primary 2 students are more consistent [smaller spread] as the standard deviation is smaller. [B1]

(e) Estimated Mean will remain the same. [A1]

Standard deviation will reduce to 4.08. [A1]

3(a)(i) $\angle BOA = 48^{\circ} \times 2$ (angle at centre = 2 angle at circumference)

$= 96^{\circ}$ [M1]

$\angle ABO = (180° - 96°) \div 2$ (base angles of isos triangle)

$= 42°$ [A1]

Or $\angle ADB = 48^{\circ}$ (angles in same segment)

$\angle BAD = 90^{\circ}$ (right angle in semi-circle) [M1]

$\angle ABO = 180° - 48° - 90°$ (angles sum of triangle)

$= 42°$ [A1]

(ii) $\angle BDA = 48^{\circ}$ (angles in same segment) [A1]

(iii) $\angle DCA = 42^{\circ}$ (angles in same segment) [M1]

$\angle CDA = 180° - 42° - 32°$ (angles sum of triangle)

$= 106°$ [A1]

Or $\angle CBD = 32^{\circ}$ (angles in same segment) [M1]

$\angle CDA = 180° - 42° - 32°$ (angles in opposite segment)

$= 106°$ [A1]

(iv) $\angle GAB = 48°$ (alt segment theorem) [B2]

Or $\angle OAG = 90°$ (tan ⊥ rad) [M1]

$\angle OAB = 42°$ (base angle of isoc triangle)

$\angle GAB = 90° - 42° = 48°$ [A1]

(b) No. $\angle GAB = 48°$ $\angle DBA = 42°$ [M1]

Since they are not equal, they cannot be alternate angles of a set of parallel lines, hence, BD is not parallel to GF. [A1]

4(a) Cost of sofa when bought locally = $\$1200 \times \frac{95}{100} = \$1140$ [M1]

Cost of sofa when bought in China = $\left(\frac{5000}{484} + 50\right) \times 1.07 = \$1158.87 (2dp)$ [M2]

Cheaper option is to buy locally as the cost is lower. [A1]

3(b) Interest from Plan A = $\frac{3}{100} \times \$50000 \times 3$

$= \$4500$ [M1]

$4500 + 50000 = 50000(1 + \frac{x}{2 \times 100})^{6}$ [M1]

$\frac{109}{100} = (1 + \frac{x}{200})^{6}$

$1 + \frac{x}{200} = \sqrt[6]{\frac{109}{100}}$ [M1]

$\frac{x}{200} = \sqrt[6]{\frac{109}{100}} - 1$

$x = 2.89$ $(3sf)$ [A1]

5(a) $BD = \sqrt{(6)^2 + (2)^2} = \sqrt{40}$ [M1]

$\tan \angle HBD = \frac{4}{\sqrt{40}}$ [M1]

$\angle HBD = 32.3°$ (1dp)

(b) $AC = BD = \sqrt{40}$

$AF = \sqrt{6^2 + 4^2} = \sqrt{52}$

$FC = \sqrt{2^2 + 4^2} = \sqrt{20}$ [Any 2, M1]

$\cos \angle AFC = \frac{40 - 52 - 20}{-2\sqrt{52}\sqrt{20}}$ [M1]

$\angle AFC = 60.3°$ (1dp) [A1]

c) Greatest angle of elevation = $\tan^{-1}\left(\frac{4}{2}\right) = 63.4°$ (1dp) [A1]

6(a) (i) $\overrightarrow{JO} + \overrightarrow{OK} = \overrightarrow{JK}$

$\begin{pmatrix} 2 \\ -4 \end{pmatrix} + \overrightarrow{OK} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\overrightarrow{OK} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ [A1]

(ii) Gradient of $JK = -\frac{1}{4}$ [M1]

$y - 4 = -\frac{1}{4}(x + 2)$

$y = -\frac{1}{4}x + \frac{7}{2}$ [A1]

(b)(i)(a) $\overrightarrow{FB} = 2\mathbf{u}$ [B1]

(b) $\overrightarrow{AG} = \frac{1}{2}\overrightarrow{AE} = \frac{1}{2}(3\mathbf{u} + \mathbf{v})$ [A1]

(c) $\overrightarrow{AD} = 3\mathbf{u} + 2\mathbf{v}$ [B1]

(d) $\overrightarrow{DF} = -2(\mathbf{u} + \mathbf{v})$ [B1]

(e) $\overrightarrow{DG} = -3\mathbf{u} - 2\mathbf{v} + \frac{1}{2}(3\mathbf{u} + \mathbf{v})$ [M1]

$= -\frac{3}{2}\mathbf{u} - \frac{3}{2}\mathbf{v}$

$= -\frac{3}{2}(\mathbf{u} + \mathbf{v})$ [A1]

(ii) $\overrightarrow{DG} = \frac{3}{4}\overrightarrow{DF}$

D, F and G are collinear. [A1]

$DG = \frac{3}{4}DF$ [A1]

(iii)(a) $\frac{\text{Area of } \Delta ADF}{\text{Area of } \Delta ADG} = \frac{4}{3}$ [B1]

(b) $\frac{\text{Area of } \Delta ADG}{\text{Area of } \Delta DE} = \frac{1}{2}$

$\frac{\text{Area of } \Delta ADG}{\text{Area of ABCD}} = \frac{1}{2 \times 4}$ [M1]

$= \frac{1}{8}$ [A1]

7 (a)(i) $n+11$ [B1]

(ii) $(n+9)^2 - n^2$ [M1]

$= n^2 + 18n + 91 - n^2$

$= 18n + 81$

$= 9(2n+9)$ [M1]

It is a multiple of 9.

(iii) Sum of numbers = $n+n+1+n+2+n+9+n+10+n+11$

$= 6n+33$ [M1]

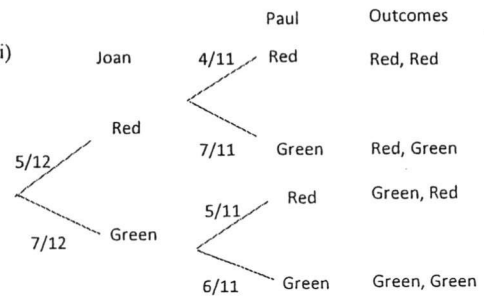
$6n+33 = 777$

$6n = 744$

$n = 124$ [M1]

Number in top right corner of rectangle = 126 [A1]

(b) (i)



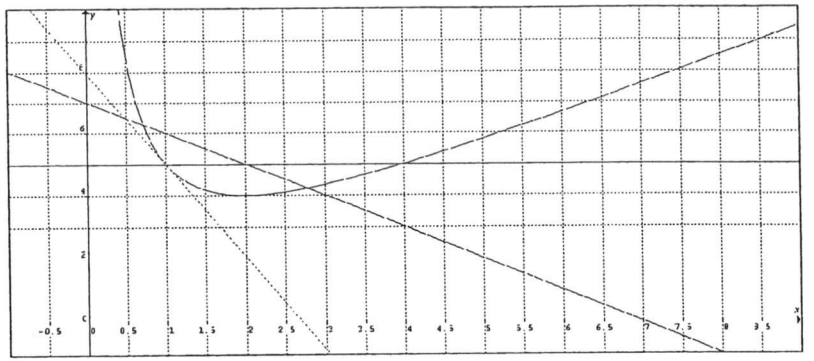
A1 for fractions, A1 for outcomes

(ii)(a) P(both red balls) $= \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ [A1]

(b) P(Paul picks green ball) $= \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{6}{11}$ [M1]

$= \frac{7}{12}$ [A1]

(c) P(both different coloured balls) $= \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11}$ [M1] $= \frac{35}{66}$ [A1]



Q9 (a). $a = 5.8$ [B1]

(c) insert y= 5, therefore x = 1 and 4 [A1 each]

(d) Gradient = -3 Accept range from -3.938 to -2.3057 [A1] , tangent line [M1]

(e)(ii) x= 0.7 and x =2.8

(iii) $x + \frac{4}{x} = -x + 7$

$2x + \frac{4}{x} - 7 = 0$

$2x^2 - 7x + 4 = 0$ [M1]

a = -7, b =4 [A1]

9(a) $OE = (8- x)$cm [B1]

(b) $8^2 = (8-x)^2 + (8-\frac{1}{3}x)^2$ [M1]

$64 = 64 - 16x + x^2 + 64 - \frac{16}{3}x + \frac{1}{9}x^2$ [M1[

$10x^2 - 192x + 576 = 0$ [M1]

$5x^2 - 96x + 288 = 0$ (shown)

(c) $x = \frac{-(-96) \pm \sqrt{(-96)^2 - 4(5)(288)}}{2(50}$ [M1]

= 15.48 or 3.72 (2dp) [A1, A1]

(d) AB = $2\left[8 - \frac{1}{3}(3.72122\right] = 13.5cm(3sf)$ [A1]

(e) $\sin \angle EOA = \frac{8 - \frac{1}{3}(3.72122)}{8}$ [M1]

$\angle EOA = \sin^{-1}\left[\frac{8 - \frac{1}{3}(3.72122)}{8}\right.$

$\angle AOB = 2\angle EOA = 2.01rad$ [A1]

(f) arc AB =circumference of base of cone = 8 (2.0143) cm

Let radius be r.

$8(2.0143) = 2\pi r$

r =2.56 cm (3sf)

10(a) Total instalment paid = $48 \times 1200 = \$57600$ [M1]

Total Interest paid $= \frac{57600}{110} \times 10$ [M1]

=\$5236.3636..

Original car price = \$60000 +\$57600-\$5236.36363

=\$112364 (nearest dollar) [A1]

(b) Fuel capacity = 15×3.7854 =56.781 litres [B1]

City driving fuel efficiency= $\frac{22 \times 1.60934}{3.7854} = 9.35316.. = 9.35$ km/l [A1]

Expressway driving fuel efficiency = $\frac{26 \times 1.60934}{3.7854} = 11.0537.. = 11.1$ km/l [A1]

(c) Cost of coach fare $= 5 \times 35 \times 2 = \$350$ [M1]

Cost of self-drive $= \left[\frac{210}{11.0537} + \frac{30}{9.35316}\right] \times 1.5 \times 2 + 100]$

= \$166 (nearest dollar) [M1]

Should self-drive as it is cheaper. [A1]