	Class	Index No.
Candidate Name: ______________________________		

FUHUA SECONDARY SCHOOL

Secondary Four Express / Five Normal (Academic)

4E & 5NA

Mid-Year Examination 2017

MATHEMATICS **4048/01**

PAPER 1

DATE: **8 May 2017**

TIME: **10 30 – 12 30**

DURATION: **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		/ 80
	Statements/Accuracy		
	Poor Presentation		

Setter: Ms Er Ying Ying Vetter: Ms Winnifred Lim

This question paper consists of 21 printed pages including this page.

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $2y-3(x-y+1)$.

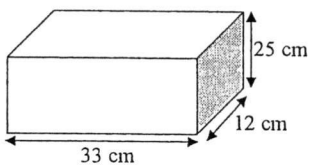
Answer (a) .....[1]

(b) Factorise $3p^2 - pq - 4q^2$.

Answer (b) .....[1]

---

2 The diagram below shows a wooden block measuring 33 cm by 12 cm by 25 cm.
An engineer wants to build cubes using these wooden blocks.

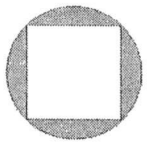


Find the smallest possible length of side of a cube that can be built.

Answer .....cm [2]

[Turn Over

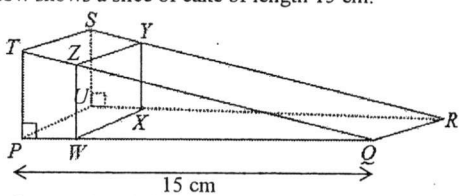
3 The diagram shows a square inscribed in a circle. The area of the circle is 1386 cm^2.
Find the area of the square. (Take $\pi = \frac{22}{7}$)



Answer .....cm^2 [2]

---

4 The diagram below shows a slice of cake of length 15 cm.

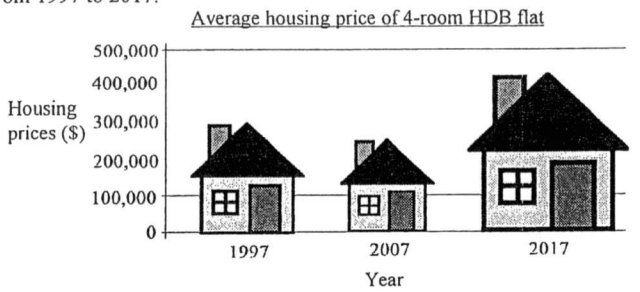


The slice of cake is cut at *WXYZ* into two pieces such that the ratio of area of the two slices of cake *WXYZ* to *PSTU* is 2 : 3. Calculate the length of the smaller piece of cake, *WQ*.

Answer .....cm [2]

---

5 The graph below shows the changes in average housing price of a 4-room HDB flat from 1997 to 2017.



State one aspect of the graph that may be misleading and explain how it may lead to a misinterpretation of the graph.

Answer

.....

.....

..... [2]

---

6 On a particular day, the temperature ranged between −11°C and 7 °C.

(a) Find the difference between the highest and lowest temperature.

Answer (a) .....°C [1]

(b) The temperature at 6 am was −11°C and the temperature at 2 pm was 7 °C. Given that the temperature rises at a constant rate, find the time when the temperature was 1.375 °C.

Answer (b) .....[2]

[Turn Over

7 Write $\frac{2m-1}{21-3m}+\frac{3m+5}{5m-35}$ as a fraction in its simplest form.

Answer .....[3]

---

8 Kelvin wishes to loan $125000 to pay off the remaining cost of his HDB flat. A bank charges an interest rate of 2.35% per annum which is compounded half-yearly.

(a) Calculate the total amount he has to pay back in 7 years if he loans from the bank.

Answer (a) $.....[2]

(b) Calculate the monthly installment he has to pay.

Answer (b) $.....[1]

---

9 Given that $\varepsilon = \{x : x \text{ is a factor of } 72\}$

$A = \{x : x \text{ is a multiple of } 4\}$

$B = \{x : x \text{ is divisible by } 3\}$

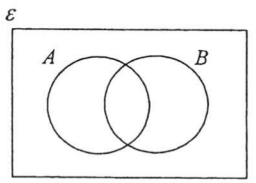
(a) List the elements in ε.

Answer (a) .....[1]

(b) Find $A \cap B'$.

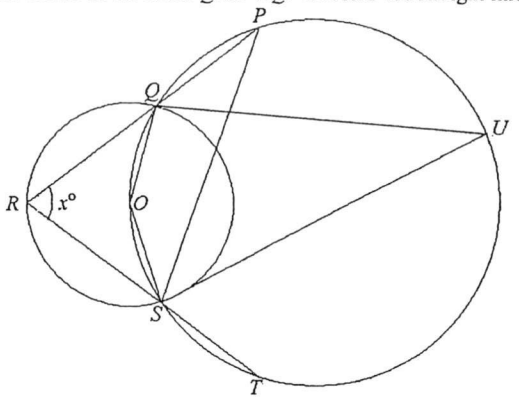
Answer (b) .....[1]

(c) In the Venn diagram below, shade $A' \cup B'$



[1]

10 In the diagram, circle QRS and circle PUT meet at points Q and S. O lies on circle PUT and is the centre of the circle QRS. RQP and RST are straight lines.



Given that $\angle QRS = x°$, showing all reasons clearly,

(a) find angle QUS in terms of x,

Answer (a) .....°[2]

(b) determine if ΔRPS is an isosceles triangle.

Answer (b)

.....

.....

.....[3]

11 **(a)** Express 630 and 495 each as a product of its prime factors.

Answer (a) 630 = ...

495 = ...[2]

(b) Find the highest common factor of 630 and 495.

Answer (b) .....[1]

(c) Find the smallest positive integer k, such that $630k$ is a perfect cube.

Answer (c) k = ...[1]

12 An estate of area 2.25 km^2 is represented on a map by an area of 36 cm^2.

(a) Express the scale of the map in the form 1: n.

Answer (a) .....[2]

(b) Find the area of the same piece of land, in cm^2, on another map of scale 5 : 900000.

Answer (b) .....cm^2 [2]

13 (a) The ratio of the distance from Town A to Town B and from Town B to Town C is 4 : 5. Sally drives at a constant speed of 65 km/h from Town A and she took 25 minutes to reach Town B. Find the distance between Town A and Town C.

Answer (a) ..... km [2]

(b) The price of petrol is \$1.35 per litre. Sally uses 0.27 l of petrol per 2.3 km of distance travelled. How much will it cost her to travel from Town A to Town C?

Answer (b) \$.....[2]

[Turn Over

14 $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \overrightarrow{QR} = \begin{pmatrix} -6 \\ -7 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} k^2 \\ -3 \end{pmatrix}$.

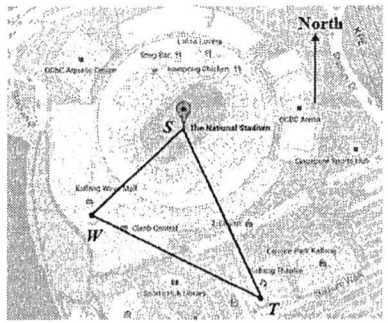
(a) Given that $\overrightarrow{QS}$ is parallel to $\overrightarrow{PQ}$, find the possible values of k.

Answer (a) k =.....[2]

(b) Hence, find $\left|\overrightarrow{PS}\right|$.

Answer (b) .....units [2]

15 The diagram below shows the map of the Singapore Sports Hub. S, T and W are positions of the National Stadium, Kallang Theatre and Kallang Wave Mall respectively. $SW = 550$ m, $WT = 675$ m, angle $SWT = 83°$ and the bearing of T from S is $161°$.



(a) Calculate the length of ST.

Answer (a) .....m [2]

(b) Calculate the bearing of W from T.

Answer (b) .....° [3]

16 A bag contains a total of 65 chocolates wrapped with silver, red and blue wrappers. There are 18 chocolates with red wrappers in the bag.

(a) If a chocolate is picked randomly, the probability of picking a chocolate with silver wrapper is $\frac{4}{13}$. How many chocolates with blue wrappers are there?

Answer (a) .....chocolates [1]

(b) If two chocolates are picked randomly, what is the probability of picking at least a chocolate with red wrapper?

Answer (b) .....[2]

(c) When x chocolates with blue wrappers are added to the 65 chocolates, the probability of picking a chocolate with silver wrapper becomes $\frac{5}{19}$. Find the value of x.

Answer (c) x =.....[2]

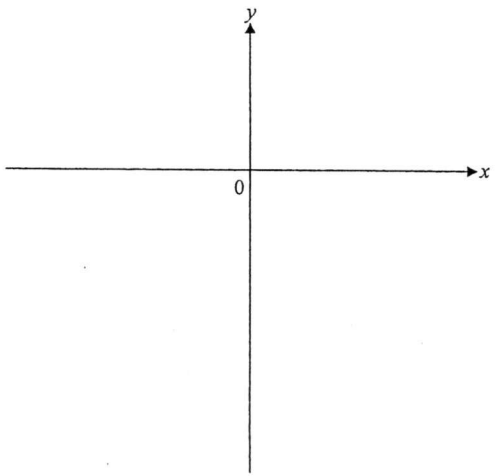
17 **(a)** Express $y = x^2 + 5x - 3$ in the form of $y = (x + h)^2 - k$.

Answer (a) .....[1]

(b) State the coordinates of the turning point.

Answer (b) (..... , ...) [1]

(c) Sketch the graph of $y = x^2 + 5x - 3$ on the axes provided, showing clearly the intercepts and turning point.



[2]

18 **(a)** Points A, B and C have coordinates $(0, 7)$, $(-3, k)$ and $(8, k)$ respectively. Given that the length of AB is $\sqrt{109}$ units and $k < 15$, find the value of k.

Answer (a) k = ...[2]

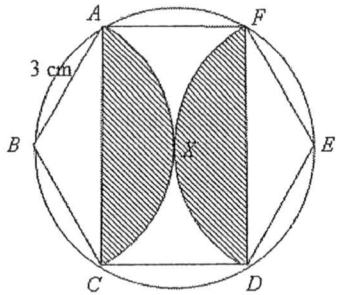
(b) Point D is a point such that $ABCD$ forms a trapezium with BC parallel to AD and the gradient of CD is $1\frac{3}{7}$. Find the coordinates of D.

Answer (b) D (..... , ...) [2]

(c) Calculate the area of trapezium $ABCD$.

Answer (c) .....square units [1]

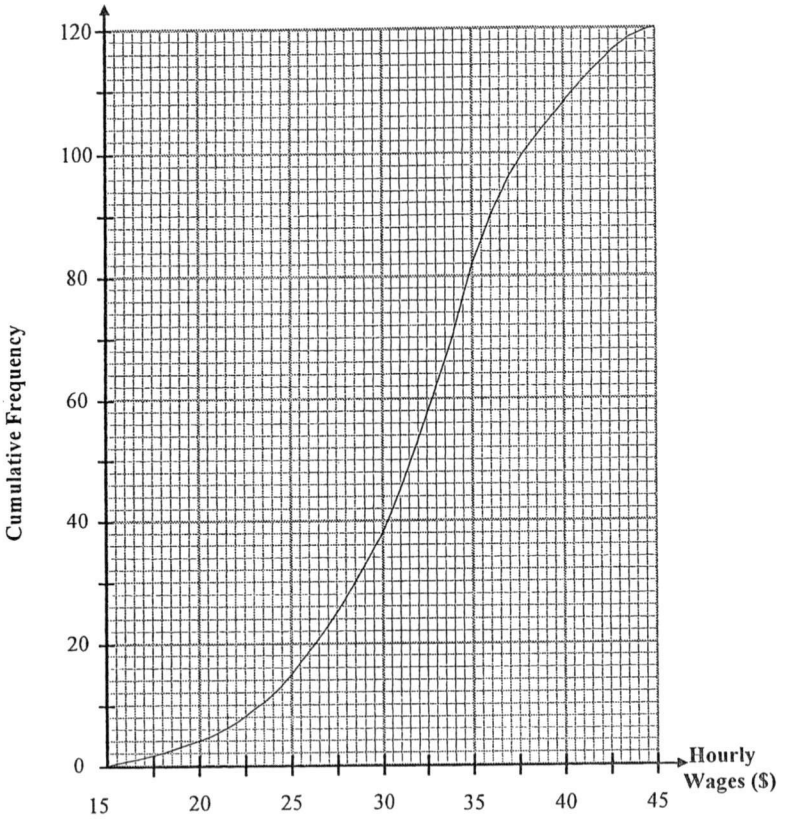
19 In the diagram below, *ABCDEF* is a regular hexagon inscribed in a circle with centre *X*. The length of each side of the hexagon is 3 cm, *AXC* and *DXF* are two identical circular arcs centered at *B* and *E* respectively and touch each other at the point *X*.



What percentage of the area of the hexagon is **not** shaded?

Answer .....% [5]

20 The cumulative frequency curve shows the distribution of the hourly wages of 120 workers in Sunshine Company.



(a) Use the graph to estimate the

(i) median hourly wage,

Answer (a)(i) $.....[1]

(ii) interquartile range.

Answer (a)(ii) $.....[1]

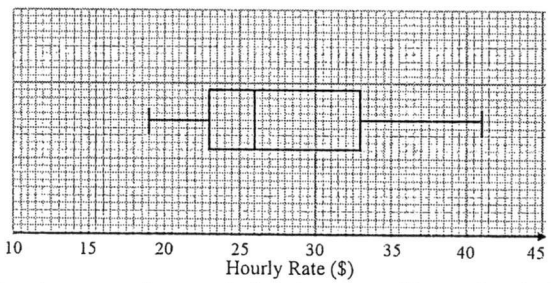
(b) Given that 10% of the workers hold managerial positions or higher, use the graph to estimate the minimum hourly wage of a middle manager.

Answer (b) $.....[1]

(c) Find the probability that a worker, chosen at random, is paid $30 or less per hour.

Answer (c) .....[1]

(d) The box-and-whisker plot below shows the hourly wages of 120 workers in Brightlight Company.



Write down two comparisons on the hourly wages paid to workers in Sunshine Company and Brightlight Company.

Answer (c)

.....

.....

.....

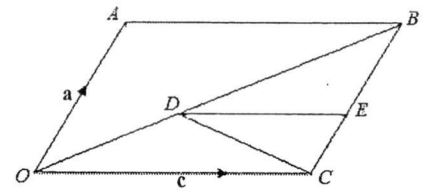
.....

.....

.....[2]

[Turn Over

21 In the diagram, $OABC$ is a parallelogram where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. D is the point on OB such that $5\overrightarrow{OD} = 3\overrightarrow{DB}$.



(a) Express, as simply as possible, in terms of **a** and/or **c**,

(i) $\overrightarrow{OB}$,

Answer (a) (i).....[1]

(ii) $\overrightarrow{CD}$.

Answer (a) (ii).....[2]

(b) Given that $\frac{3}{8}BC = EC$, express $\overrightarrow{DE}$ in terms of **a** and/or **c**.

Answer (b) .....[2]

(c) Find, in the simplest form, the ratio of

(i) $\dfrac{\text{area of } \Delta BDE}{\text{area of } \Delta BOC}$,

Answer (c) (i)... .....[1]

(ii) $\dfrac{\text{area of } \Delta OCD}{\text{area of parallelogram } OABC}$.

Answer (c) (ii)... .....[1]

End of Paper

Q1a) $2y-3(x-y+1)$
$= 2y-3x+3y-3$
$= 5y-3x-3$ B1

1b) $3p^2-pq-4q^2$
$= (3p-4q)(p+q)$ B1

Q2)

3	33	12	25
2	11	4	25
2	11	2	25
5	11	1	25
5	11	1	5
11	11	1	1
	1	1	1

M1

LCM of 33, 12 and 25 $= 3\times2^2\times5^2\times11$
$= 3300$
∴ Smallest possible length $= 3300$cm A1

Q3) Let the radius of circle be r and length of square be l.

$\pi r^2 = 1386$
$r^2 = \frac{1386}{\frac{22}{7}}$ M1
$r = 21$ cm
$l^2 + l^2 = (2\times21)^2$
$2l^2 = 1764$
$l^2 = 882$
Area of square $= 882$ cm^2 A1

Q4) Let the length of WQ be x.

$\left(\frac{7}{15}\right)^3 = \frac{7}{5}$ M1
$x = 15\times\sqrt[3]{\frac{7}{5}}$
$= 11.052\,0945$ cm
$\simeq 11.1$ cm (3 sf) A1

Q5) Readers may misinterpret the area of house as the average housing price of 4-room HDB flat B1
hence overestimating the average housing price of a 4-room HDB flat. B1

Q6a) Difference in temperature $= 7°C - (-11°C)$
$= 18°C$ B1

b) Rate of increase of temperature $= \frac{18°C}{8\text{ hours}}$
$= 2\frac{1}{4}°C/hr$

Duration taken for temperature to reach 1.375°C $= \frac{1.375°C-(-11°C)}{2\frac{1}{4}°C/hr}$ M1
$= 5\frac{1}{2}$ hr

Time when temperature was 1.375°C is 1130 am. A1

Q7) $\frac{2m-1}{21-3m} + \frac{3m+5}{5m-35}$
$= \frac{2m-1}{3(7-m)} + \frac{3m+5}{5(m-7)}$
$= \frac{2m-1}{3(7-m)} - \frac{3m+5}{5(7-m)}$ M1
$= \frac{5(2m-1)-3(3m+5)}{15(7-m)}$ M1
$= \frac{10m-5-9m-15}{15(7-m)}$
$= \frac{m-20}{15(7-m)}$ A1

OR

$\frac{2m-1}{21-3m} + \frac{3m+5}{5m-35}$
$= \frac{(2m-1)(5m-35)+(3m+5)(21-3m)}{(21-3m)(5m-35)}$ M1
$= \frac{10m^2-70m-5m+35+63m-9m^2+105-15m}{105m-735-15m^2+105m}$
$= \frac{m^2-27m+140}{-15m^2+210m-735}$
$= \frac{(m-20)(m-7)}{-15(m-7)^2}$ M1
$= \frac{m-20}{-15(m-7)}$
$= \frac{m-20}{15(7-m)}$ A1

Q8a) Total amount repayable $= \$125\,000\left(1+\frac{2.35}{100}\right)^{14}$ M1
$= \$147209.2147$
$\simeq \$147209.21$ (nearest cents) A1

b) Monthly instalment $= \frac{\$147209.2147}{12\times7}$
$= \$1752.490651$
$\simeq \$1752.49$ (nearest cents) B1

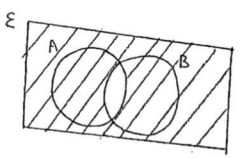
Q9a) $\xi = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$ B1

b) $A = \{4, 8, 12, 24, 36, 72\}$

$B = \{3, 6, 9, 12, 18, 24, 36, 72\}$

$A \cap B = \{4, 8\}$ B1

c)



B1

Q10a) $\angle QOS = 2 \times \angle QRS$ ($\angle$ at centre = 2$\angle$ at circumference)

$= 2x°$ M1

$\angle QUS = 180° - \angle QOS$ ($\angle$s in opp. segment)

$= 180° - 2x°$ A1

b) $\angle RPS = \angle QUS$

$= 180° - 2x°$ ($\angle$s in same segment) M1

$\angle PSR = 180° - x° - (180° - 2x°)$ ($\angle$ sum of $\triangle$)

$= x°$ M1

$\because \angle PSR = \angle RPS = x°$, $\triangle RPS$ is an isosceles triangle. A1

Q11a) $630 = 2 \times 3^2 \times 5 \times 7$ B1

$495 = 3^2 \times 5 \times 11$ B1

b) HCF $= 3^2 \times 5$

$= 45$ B1

c) Smallest positive $k = 2^2 \times 3 \times 5^2 \times 7^2$

$= 14700$ B1

Q12a) $36\text{ cm}^2 : 2.25\text{ km}^2$

$6\text{ cm} : 1.5\text{ km}$

$1\text{ cm} : \frac{1.5 \times 1000 \times 100}{6}\text{ cm}$ M1

$1 : 25000$ A1

b) $5 : 900\,000$

$5\text{ cm} : 9\text{ km}$

$1\text{ cm} : 1.8\text{ km}$

$1\text{ cm}^2 : 3.24\text{ km}^2$

Area of land on map $= \frac{2.25}{3.24}$ M1

$= \frac{25}{36}\text{ cm}^2$ A1

Q13a) Town A to Town B : Town B to Town C

$4 : 5$

Distance from Town A to Town B $= 65 \times \frac{25}{60}$ M1

$= 27\frac{1}{12}\text{ km}$

Distance from Town A to Town C $= \frac{27\frac{1}{12}\text{ km}}{4} \times 9$

$= 60\frac{15}{16}\text{ km}$ A1

b) Total cost $= \frac{60\frac{15}{16}\text{ km}}{2.3\text{ km}} \times 0.27 \times \$1.35$ M1

$= \$9.657269022$

$\simeq \$9.66$ (nearest cent) A1

Q14a) $\vec{QS} = \vec{QR} + \vec{RS}$

$= \begin{pmatrix} -6 \\ -7 \end{pmatrix} + \begin{pmatrix} k^2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -6+k^2 \\ -10 \end{pmatrix}$

$\because \vec{QS} \parallel \vec{PQ}$, $\vec{QS} = m\vec{PQ}$ where m is a constant

$\begin{pmatrix} -6+k^2 \\ -10 \end{pmatrix} = m \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ M1

Comparing, $-10 = 5m$

$m = -2$

Sub $m = -2$, $-6 + k^2 = -2(1)$

$-6 + k^2 = -2$

$k^2 = 4$

$k = \pm 2$ A1

14b) $\vec{PS} = \vec{PQ} + \vec{QR} + \vec{RS}$

$= \begin{pmatrix}1\\5\end{pmatrix} + \begin{pmatrix}-6\\-7\end{pmatrix} + \begin{pmatrix}4\\-3\end{pmatrix}$

$= \begin{pmatrix}-1\\-5\end{pmatrix}$ M1

$|\vec{PS}| = \sqrt{(-1)^2+(-5)^2}$ units

$= \sqrt{26}$ units

$= 5.099019514$ units

$\simeq 5.10$ units (3sf) A1

Q15a) $S_T = \sqrt{550^2+675^2-2(550)(675)\cos 83^\circ}$ M1

$= 817.0905779$ m

$\simeq 817$ m (3sf) A1

b) $\frac{\sin \angle SWT}{817.0905779} = \frac{\sin \angle WTS}{550}$ M1

$\sin \angle WTS = \frac{550 \times \sin 83^\circ}{817.0905779}$

$\angle WTS = 41.92079507^\circ$

Bearing of W from T $= 360^\circ - (180^\circ - 161^\circ) - (41.92079507^\circ)$ M1

$= 299.0792049^\circ$ OR $180^\circ + 161^\circ - 41.92079507^\circ$

$\simeq 299.1^\circ$ (1dp) A1 $= 299.1^\circ$ (1dp)

Q16a) P(silver wrapper) $= \frac{4}{13}$

$= \frac{20}{65}$

Number of chocolates with blue wrapper $= 65 - 18 - 20$

$= 27$ B1

b) P(picking at least 1 red wrapper) $= 1 - P(\text{both blue}) - P(\text{both silver}) - P(\text{silver, blue}) - P(\text{blue, silver})$

$= 1 - \frac{27}{65}\times\frac{26}{64} - \frac{20}{65}\times\frac{19}{64} - \frac{20}{65}\times\frac{27}{64} - \frac{27}{65}\times\frac{20}{64}$ M1

$= \frac{999}{2080}$ A1

OR P(red,red) + P(blue,red) + P(silver,red)

$= \frac{18}{65} + \frac{4}{13}\times\frac{18}{64} + \frac{27}{65}\times\frac{18}{64}$

c) P(silver wrapper) $= \frac{5}{19}$

$\frac{20}{65+x} = \frac{5}{19}$ M1

$380 = 325 + 5x$

$5x = 55$

$x = 11$ A1

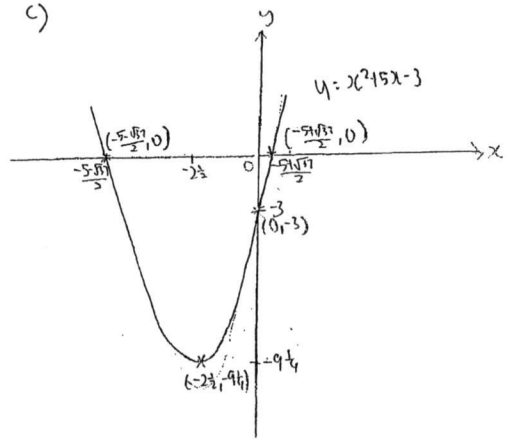
Q17a) x^2+5x-3

$= (x+\frac{5}{2})^2 - (\frac{5}{2})^2 - 3$

$= (x+\frac{5}{2})^2 - 9\frac{1}{4}$ B1

b) Turning point $= (-2\frac{1}{2}, -9\frac{1}{4})$ B1

c)



When $x=0$, $y=-3$

When $y=0$, $x^2+5x-3=0$

$x = \frac{-5\pm\sqrt{(5)^2-4(1)(-3)}}{2(1)}$

$= \frac{-5\pm\sqrt{37}}{2}$

$\simeq 0.541$ (3sf) or -5.54 (3sf)

B1 - shape, axis

B1 - Intercepts, turning point

Q18a) Length of AB $= \sqrt{109}$

$\sqrt{(0-(-3))^2+(7-k)^2} = \sqrt{109}$

$9 + (7-k)^2 = 109$

$9 + 49 - 14k + k^2 = 109$

$k^2 - 14k - 51 = 0$ M1

$(k-17)(k+3) = 0$

$k = 17$ or $k = -3$

(rejected)

$\therefore k = -3$ A1

18b) $\because BC \parallel AD$, gradient of AD = gradient of BC

$= 0$

$\therefore$ D must lie on $y = 7$

Let coordinates of D be $(x, 7)$

gradient of CD $= \frac{3}{7}$

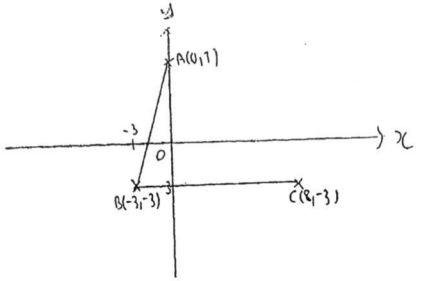
$\frac{7-(-3)}{x-8} = \frac{10}{7}$ M1

$\frac{10}{x-8} = \frac{10}{7}$

$x - 8 = \frac{7 \times 10}{10}$

$x = 15$

$\therefore D(15, 7)$ A1



c) Area of trapezium $= \frac{1}{2} \times (15+11) \times 10$

$= 130 \text{ unit}^2$ B1

Q19) Interior angle of hexagon $= \frac{180^\circ(6-2)}{6}$

$= 120^\circ$

$= \frac{2\pi}{3}$ rad M1

Area of segment $= \frac{1}{2} \times 3^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 3^2 \times \sin\frac{2\pi}{3}$ M1

$= 5.527663644 \text{ units}^2$

$\angle ABX = \frac{120^\circ}{2}$

$= 60^\circ$

$\therefore \angle BAX = 180^\circ - 2(60^\circ)$ ($\angle$ sum of isos $\triangle$)

$= 60^\circ$

Area of hexagon $= 6 \times \frac{1}{2} \times 3^2 \times \sin\frac{\pi}{3}$ M1

$= 23.3826859 \text{ units}^2$

Percentage of hexagon that is unshaded $= \frac{23.3826859 - 2(5.527663644)}{23.3826859} \times 100\%$ M1

$= 52.72002825\%$

$\simeq 52.7\%$ (3sf) A1

Q20ai) Median hourly wage $\simeq \$32.75$ B1

ii) Interquartile range $\simeq \$36.00 - \$28.50$

$= \$7.50$ B1

b) Number of workers with managerial positions or higher $= \frac{10}{100} \times 120$

$= 12$

Minimum hourly wage of middle manager $\simeq \$40.00$ B1

c) $P(\leq \$30 / \text{hr}) = \frac{38}{120}$

$= \frac{19}{60}$ B1

d) The hourly wages of workers in Sunshine Company is generally higher than the hourly wages of workers in Brightlight Company as the median hourly wage is higher for Sunshine company ($32.75) as compared to Brightlight Company ($26.00). B1

The hourly wages of workers in Sunshine Company is generally more consistent than that of Brightlight Company as the interquartile range is lower for Sunshine Company ($7.50) as compared to Brightlight Company ($10.00). B1

Q21ai) $\vec{OB} = \vec{OA} + \vec{AB}$

$= \vec{OA} + \vec{OC}$ ($\because$ OABC is a parallelogram)

$= \underset{\sim}{a} + \underset{\sim}{c}$ B1

ii) $\because 5\vec{OD} = 3\vec{DB}$

$\vec{OD} = \frac{3}{5}\vec{DB}$

$= \frac{3}{8}\vec{OB}$

$= \frac{3}{8}(\underset{\sim}{a} + \underset{\sim}{c})$

$\vec{CD} = \vec{CO} + \vec{OD}$

$= -\vec{OC} + \vec{OD}$

$= -\underset{\sim}{c} + \frac{3}{8}(\underset{\sim}{a} + \underset{\sim}{c})$ M1

$= \frac{1}{8}(3\underset{\sim}{a} - 5\underset{\sim}{c})$ A1

21 b) $\frac{3}{8}BC = EC$

$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$= \frac{5}{8}\overrightarrow{DB} + \frac{5}{8}\overrightarrow{BC}$

$= \frac{5}{8}(\underset{\sim}{a}+\underset{\sim}{c}) + \frac{5}{8}(-\underset{\sim}{a})$ M1

$= \frac{5}{8}\underset{\sim}{c}$ A1

ci) $\frac{\text{Area of } \triangle BDE}{\text{Area of } \triangle BDC} = \left(\frac{BE}{BC}\right)^2$

$= \left(\frac{5}{8}\right)^2$

$= \frac{25}{64}$ B1

ii) $\triangle ODC$ and $\triangle OBC$ has a common height, h cm.

$\frac{\text{Area of } \triangle ODC}{\text{Area of } \triangle OBC} = \frac{\frac{1}{2} \times OD \times h}{\frac{1}{2} \times DB \times h}$

$= \frac{OD}{DB}$

$= \frac{3}{8}$

$\frac{\text{Area of } \triangle OCD}{\text{Area of parallelogram } OABC} = \frac{\text{Area of } \triangle OCD}{\text{Area of } \triangle DBC} \times \frac{\text{Area of } \triangle OBC}{\text{Area of parallelogram } OABC}$

$= \frac{3}{8} \times \frac{1}{2}$

$= \frac{3}{16}$ B1

1 (a) Make t the subject of the formula $r = u + \sqrt{\frac{su}{t} + s^2}$. [2]

(b) Given that $x - \frac{1}{x} = 5$, find the value of $x^2 + \frac{1}{x^2}$. [2]

(c) Solve these simultaneous equations.

$$\frac{x-3y}{2} = \frac{2x-3y}{5}$$

$$7x - 6y = 19$$

[3]

(d) Express $\left(\frac{b}{a} + \frac{a}{b} + 2\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ as a single fraction in its simplest form. [3]

2 Tickets to a concert were sold and the seats were divided into 4 categories. The number of tickets sold for Saturday and Sunday are summarized in the table below.

Categories	CAT 1	CAT 2	CAT 3	CAT 4
Saturday	64	85	110	87
Sunday	50	65	128	90

(a) Write down a 2×4 matrix **T** to represent the number of tickets sold for both days. [1]

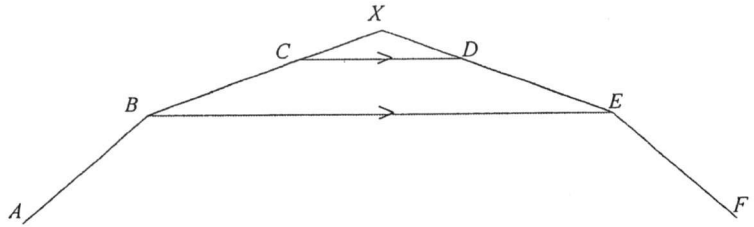
(b) The price per ticket is \$268 for Category 1, \$168 for Category 2, \$128 for Category 3 and \$78 for Category 4.
Represent the ticket prices in a column matrix **P**. [1]

(c) Evaluate the matrix **A** = **TP** [1]

(d) State what the elements of **A** represent. [1]

(e) A 10% discount was given for Matinee show tickets. 80 Category 1, 130 Category 2, 150 Category 3 and 185 Category 4 tickets were sold. Using matrix multiplication, calculate the total sales earned from the Matinee show. [3]

3



The diagram shows part of a regular n-sided polygon $ABCDEF\ldots$.
BCX and EDX are straight lines and CD is parallel to BE.

(a) Explain why triangles XCD and XBE are similar triangles. [2]

(b) It is given that $\angle BCD = 150°$. Find

(i) the value of n, [2]

(ii) angle CXD, [2]

(iii) angle ABE. [1]

4 The first 3 terms of a sequence of numbers T_1, T_2 and T_3 are given below:

$T_1 = 6(1)^2 + 30 = 36$
$T_2 = 6(2)^2 + 48 = 72$
$T_3 = 6(3)^2 + 66 = 120$

(a) (i) Find T_4. [1]

(ii) Find an expression, in terms of n, for T_n. Hence, or otherwise, explain why value of T_n must be even for all values of n. [3]

(iii) Evaluate T_{30}. [1]

(b) The first four terms in a different sequence are 4, 6, 8, 10.
Find an expression, in terms of n, for the nth term, P_n, of this sequence. [1]

(c) Find an expression, in terms of n, for the nth term, $\frac{T_n}{P_n}$. Hence, explain why the term $\frac{T_n}{P_n}$ is a multiple of 3. [3]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{3}x(5 - x^2).$$

x	-3	-2	-1	0	1	2	3	4
y	4	-0.7	p	0	1.3	0.7	-4	-14.7

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \le x \le 5$.
Using a scale of 2 cm to 2 units, draw a vertical y-axis for $-16 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Determine the number of solutions to the equation $x(5 - x^2) = 3$.
Explain how this can be seen from your graph. [2]

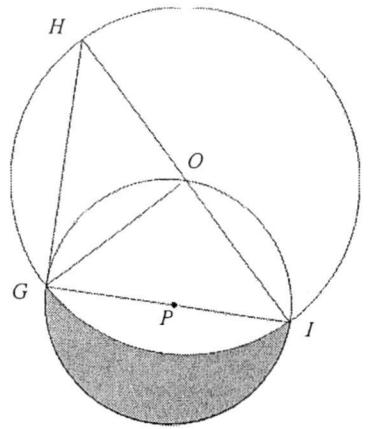
(d) By drawing a tangent, find the gradient of the curve at $(3,-4)$. [2]

(e) **(i)** On the same axes, draw a line with gradient 2 and passes through the point (0, 2). [2]

(ii) Write down the x-coordinate(s) of the point where this line intersects the curve. [1]

(iii) Use your graph to find the range of values of x for which $x(5 - x^2) > 6x + 6$. [2]

6 The diagram shows two circles C_1 and C_2 with centre O and P respectively. HI and GI are diameters of C_1 and C_2 respectively.



(a) Show that triangles HGI and GOI are similar.
Give a reason for each statement you make. [2]

(b) Given that $HI = 10$ cm and $GI = 5\sqrt{2}$ cm.
Find the ratio area of triangle HGI : area of triangle GOI. [2]

(c) Find the shaded area. [4]

7 Queenie has allocated a budget of \$49 on buying ingredients for baking a cake. The budget is divided between flour, sugar and saffron in the ratio of 3 : 2 : 5.

(a) Find the amount Queenie spent on buying saffron. [2]

(b) **(i)** Given that the unit price of saffron is \$$x$ per gram, write down an expression for the amount of saffron bought. [1]

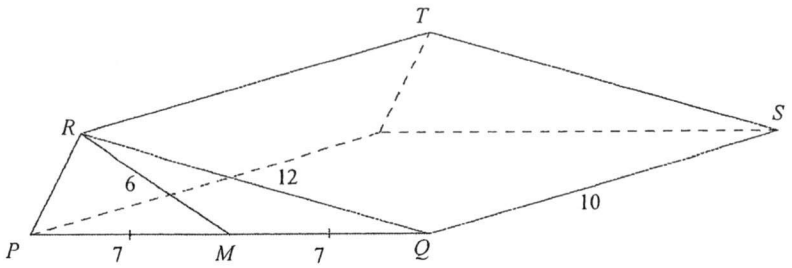
(ii) During a sale, the unit price of saffron is reduced by \$4. Write down an expression for the amount of saffron that can be bought for the same amount of money. [1]

(iii) If 4 more grams of saffron can be bought at the reduced price, form an equation in x and show that it reduces to $2x^2 - 8x - 49 = 0$. [3]

(iv) Solve the equation $2x^2 - 8x - 49 = 0$, giving your answers correct to 2 decimal places. [3]

(v) Janice who has \$50 wants to buy as much saffron as possible at the discounted price. Find the amount of saffron she can buy. [2]

8



The diagram shows a solid triangular prism with three rectangular faces.
$PM = QM = 7$ cm, $RM = 6$ cm, $QR = 12$ cm and $QS = 10$ cm.

(a) Show that $\angle MRQ = 24.53°$ corrected to 2 decimal places. [3]

(b) Calculate the vertical distance of R above PQ. [3]

(c) Given that $PR = \sqrt{26}$ cm, calculate the surface area of the prism. [3]

(d) Calculate the angle of elevation of T from Q. [3]

9 **(a)** The results of a Mathematics Test for a class of 20 students were recorded.
The results are shown in the stem-and-leaf diagram.

0	3 9
1	1 5
2	4 6 7
3	0 0 2 3 7 8
4	0 4 5 6 7 7 8

Key 1 | 3 means 13 marks

(i) Find the percentage of students who failed if the passing mark is 25. [1]

(ii) Find the mean mark. [1]

(iii) Find the standard deviation of the marks. [2]

(iv) It was discovered that the results had been recorded incorrectly.
The correct results were all 4 more than those recorded.

Explain how the median and standard deviation of the results have been affected by this error. [2]

(b) A bag contains 10 identical coloured balls. There are 4 pink and 6 blue balls.
Two balls are picked out at random, without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that one ball is pink and the other is blue. [2]

(iii) A third ball is now picked out at random. Calculate the probability that none of the three balls is blue. [2]

10 Sally is intending to set up a fruit punch stall during a charity bazaar to raise funds. Each cup will contain 200 ml of fruit punch and will be served in a plastic cup with a straw. Sally made a trip to the supermarket to find out the prices of the ingredients she needs. Information that Sally needs is provided in the tables below.

Sally's Fruit Punch Recipe

The recipe below makes 1 liter of punch:

1 lime

1 lemon

150ml pineapple juice

500ml lemonade

300ml orange juice

Ingredients Prices		
Item	**Description**	**Cost**
Lemon	Individual Pack of three	60¢ $1.50
Limes	Individual Pack of 5	75¢ $3.50
Lemonade	2L bottle	$1.85
Orange juice	1L carton	$1.25
Pineapple juice	500ml carton 2L carton	$2.50 $7.50
Plastic Cups	Pack of 20	$1.35
Straws	Pack of 100	$1.30

(a) Find the amount of orange juice needed to prepare 500ml of fruit punch. [1]

(b) Sally estimates that around 200 cups of fruit punch will be sold.

(i) What is the minimum number of bottles of lemonade that Sally should buy? [2]

Sally needs to decide how much she should charge for 1 cup of fruit punch. She must make sure that she covers all of her costs.

(ii) Suggest a sensible amount for her to charge 1 cup of fruit punch. Justify the decision you make and show your calculations clearly. [7]

- End of Paper -

4E5N Emath MYE 2019

Q1(a) $r = u + \sqrt{\frac{su}{t} + s^2}$

$r - u = \sqrt{\frac{su}{t} + s^2}$

$(r-u)^2 = \frac{su}{t} + s^2$ —— [M1]

$t = \frac{su}{(r-u)^2 - s^2}$ —— [A1]

(b) Given: $x - \frac{1}{x} = 5$

$(x - \frac{1}{x})^2 = 25$ —— [M1]

$x^2 - 2 + \frac{1}{x^2} = 25$

$x^2 + \frac{1}{x^2} = 27$ —— [A1]

(c) $\frac{x-3y}{2} = \frac{2x-3y}{5}$ —— ①

$7x - 6y = 19$ —— ②

From ②, $x = \frac{19+6y}{7}$; sub into ①

$\frac{\frac{19+6y}{7} - 3y}{2} = \frac{2(\frac{19+6y}{7}) - 3y}{5}$ —— [M1]

$\frac{5(19+6y)}{7} - 15y = 4(\frac{19+6y}{7}) - 6y$

$5(19+6y) - 105y = 4(19+6y) - 42y$

$51y = 19$

$y = \frac{19}{57} = \frac{1}{3}$ —— [A1]

$x = 3$ —— [A1]

$\therefore x = 3,\ y = \frac{1}{3}$

<17>

Q1(d) $\left(\frac{b}{a} + \frac{a}{b} + 2\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$

$= \left(\frac{b^2 + a^2 + 2ab}{ab}\right) \div \frac{b^2 - a^2}{(ab)^2}$ —— [M1]

$= \frac{(a+b)^2}{ab} \times \frac{(ab)^2}{(b-a)(b+a)}$ —— [M1]

$= \frac{ab(a+b)}{b-a}$ —— [A1]

Q2(a) $T = \begin{pmatrix} 64 & 85 & 110 & 87 \\ 50 & 65 & 128 & 90 \end{pmatrix}$ —— [B1]

(b) $P = \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$ —— [B1]

(c) $A = \begin{pmatrix} 64 & 85 & 110 & 87 \\ 50 & 65 & 128 & 90 \end{pmatrix} \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$

$= \begin{pmatrix} 52298 \\ 47724 \end{pmatrix}$ —— [B1]

(d) The elements of A represent the total sales of tickets on Saturday & Sunday respectively. —— [B1]

(e) Total sales $= 0.9 \begin{pmatrix} 80 & 130 & 150 & 185 \end{pmatrix} \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$ —— [M1]

$= \begin{pmatrix} 69219 \end{pmatrix}$ —— [M1]

∴ Total sales = $ 69219 —— [A1]

Q3 (a) $\angle BXE = \angle CXD$ (common $\angle$)

$\angle XCD = \angle XBE$ (corr. $\angle$s, $CD // BE$)

$\angle XDC = \angle XEB$ (corr. $\angle$s, $CD // BE$) } [M1]

∴ $\triangle XCD$ is similar to $\triangle XBE$ —— [A1]

(b) (i) $(n-2) \times 180 = 150 \times n$ —— [M1]

$n = 12$ —— [A1]

(ii) $\angle BCD = \frac{360}{12}$

$= 30°$ (ext. $\angle$ of polygon) —— [M1]

$\angle CXD = 180° - 30° - 30°$

$= 120°$ (base $\angle$ of isos $\triangle$) —— [A1]

(iii) $\angle ABE = 150° - 30°$

$= 120°$ —— [B1]

4(a)(i) $T_4 = 6(4)^2 + 84$

$= 180$ —— [B1]

(ii) $T_n = 6n^2 + 18n + 12$ —— [M1]

$= 6(n+1)(n+2)$

$= 2[3(n+1)(n+2)]$ —— [A1]

Since T_n is divisible by 2, it must be even $\forall$ values of n. —— [A1]

(iii) $T_{30} = 6(30+1)(30+2)$

$= 5952$ —— [B1]

(b) $P_n = 2n + 2$ —— [B1]

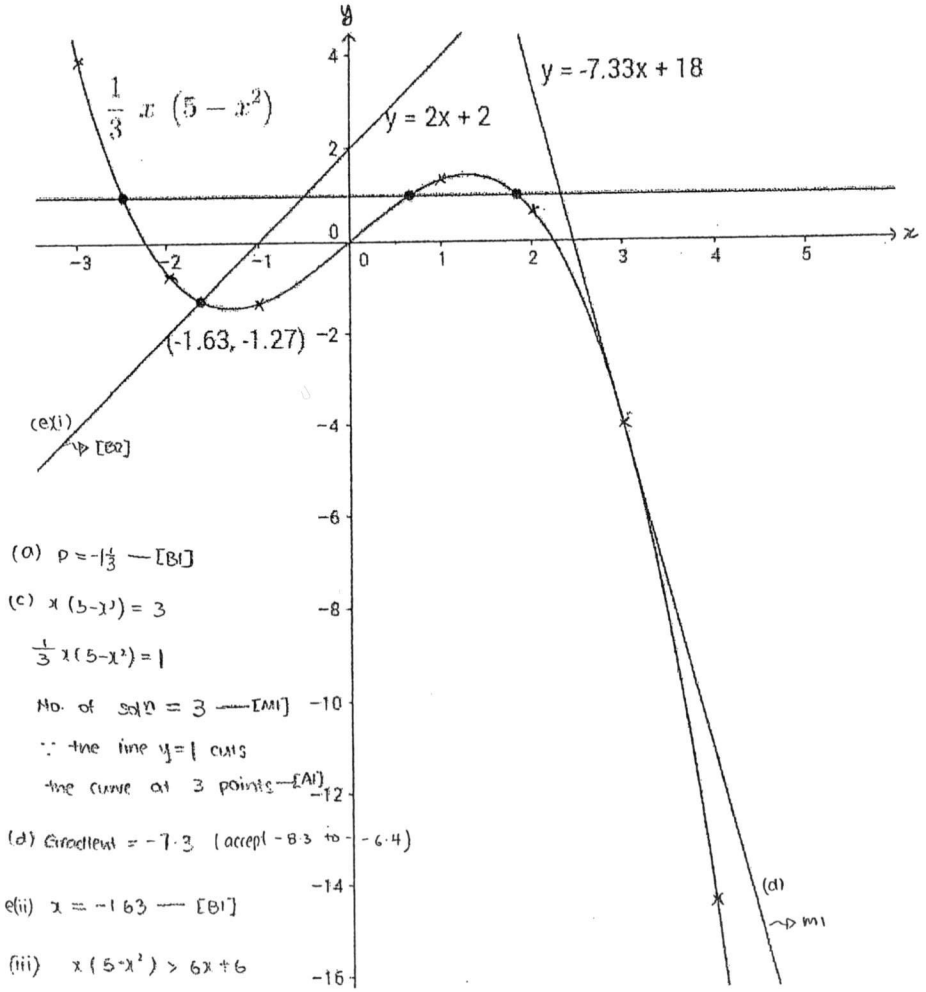
(c) $\frac{T_n}{P_n} = \frac{6(n+1)(n+2)}{2n+2}$ —— [M1]

$= \frac{6(n+1)(n+2)}{2(n+1)}$

$= 3(n+2)$ —— [A1]

Since $\frac{T_n}{P_n}$ is divisible by 3, it must be a multiple of 3. —— [A1]

Q5.



(a) $p = -1\frac{1}{3}$ —[B1]

(c) $x(5-x^2) = 3$

$\frac{1}{3}x(5-x^2) = 1$

No. of soln = 3 —[M1]

∵ the line $y = 1$ cuts the curve at 3 points —[A1]

(d) Gradient = −7.3 (accept −8.3 to −6.4)

e(ii) $x = -1.63$ — [B1]

(iii) $x(5-x^2) > 6x + 6$

$x(5-x^2) > 3(2x+2)$

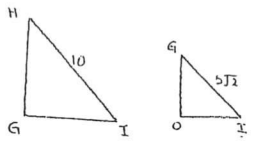
$\frac{1}{3}x(5-x^2) > 2x + 2$ —— [M1]

Range of x is $-3 \leq x < -1.63$

Q6 (a) $\angle HGI = \angle GOI = 90^\circ$ ($\angle$s in semi circle)
$\angle HIG = \angle GIO$ (common $\angle$) } — [M1]

$\therefore \triangle HGI$ is similar to $\triangle GOI$ — [A1]

(b)



Area of $\triangle GHI$: Area of $\triangle GOI$

$= 10^2 : \left(5\sqrt{2}\right)^2$ — [M1]

$= 2 : 1$ — [A1]

(c) Area of quadrant GOI $= \frac{1}{4} \times \pi (5^2)$

$= \frac{25}{4}\pi$ — [M1]

Area of segment GPI $= \frac{25}{4}\pi - \frac{1}{2} \times 5 \times 5$

$= \frac{25}{4}\pi - \frac{25}{2}$ — [M1]

Area of shaded area $= \frac{1}{2} \times \pi \left(\frac{5\sqrt{5}}{2}\right)^2 - \left(\frac{25}{4}\pi - \frac{25}{2}\right)$ — [M1]

$= 12.5\ cm^2$ # — [A1]

Q7(a) Amount spent on saffron $= \frac{49}{10} \times 5$ — [M1]

$= \$24.50$ — [A1]

(b)(i) Amount of saffron bought $= \frac{24.5}{x}$ g — [B1]

(ii) Amount of saffron bought $= \frac{24.5}{x-4}$ g — [B1]

(iii) $\frac{24.5}{x-4} - \frac{24.5}{x} = 4$ — [M1]

$24.5x - 24.5x + 98 = 4x(x-4)$ — [M1]

$4x^2 - 16x - 98 = 0$

$2x^2 - 8x - 49 = 0$ — [A1]

(iv) $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-49)}}{2(2)}$ — [M1]

$x = 7.3385\,39126$ or $x = -3.338539126$

≈ 7.34 (2dp) # — [A1] ≈ -3.34 (2dp) # — [A1]

(v) Amount bought $= \frac{\$50}{7.338539126 - 4}$ — [M1]

$= 14.97661046$

≈ 14.9 g (rounded down to 3sf) (accepted 15.0 g)

Q8(a) By Cosine Rule,

$6^2 + 12^2 - 2(6)(12)\cos \angle MRQ = 7^2$ —— [M1]

$\cos \angle MRQ = \dfrac{7^2 - 6^2 - 12^2}{-2(6)(12)}$ —— [M1]

$\angle MRQ = 24.533007^\circ$

$\approx 24.53^\circ$ (2 dp) —— [A1]

(b) Area of $\triangle RMQ = \frac{1}{2} \times 6 \times 12 \sin 24.53300712^\circ$ —— [M1]

$= 14.94782593$

Vertical dist. of R above PQ $= \dfrac{14.94782593}{\frac{1}{2} \times 7}$ —— [M1]

$= 4.270807409$

≈ 4.27 cm (3sf) —— [A1]

(c) Surface area $= \left(\frac{1}{2} \times 14 \times 4.270807409\right) \times 2 + (\sqrt{56} + 14 + 12) \times 10$ —— [M2]

$= 370.7814989$

$\approx 371\ cm^2$ (3sf) —— [A1]

(d) By Pythagoras Theorem,

$TQ = \sqrt{12^2 + 10^2}$

$= \sqrt{244}$ —— [M1]

Let $\angle$ of elevation be θ

$\sin\theta = \dfrac{4.270807409}{\sqrt{244}}$ —— [M1]

$\theta = 15.86730841^\circ$

$\approx 15.9^\circ$ (1 dp) —— [A1]

Q9(a)(i) % of failures $= \frac{5}{20} \times 100\%$

$= 25\%$ —— [B1]

(ii) Mean $= \dfrac{632}{20}$

$= 31.6$ marks —— [B1]

(iii) Standard Deviation $= \sqrt{\dfrac{23542}{20} - 31.6^2}$ —— [M1]

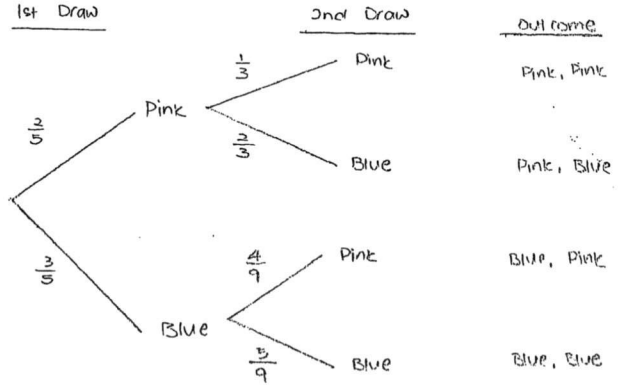
$= 13.36188609$

≈ 13.4 —— [A1]

(iv) Median would increase by 4 marks. —— [B1]

Standard deviation would remain the same —— [B1]

(b)(i)



1 branch 1 mark

(ii) $P(1\ pink, 1\ blue) = \left(\frac{3}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{4}{9}\right)$ —— [M1]

$= \dfrac{8}{15}$ —— [A1]

(iii) $P(\text{no blue balls}) = \left(\frac{2}{5}\right)\left(\frac{1}{3}\right)\left(\frac{3}{8}\right)$ —— [M1]

$= \dfrac{1}{30}$ —— [A1]

Q10 (a) Amount of orange juice $= \frac{1}{5} \times 300ml$

$= 150ml$ —— [A1]

(b) Total amount of fruit punch $= 200 \times 200ml$

$= 40\,000ml$

$= 40L$ —— [M1]

min. bottles of lemonade $= \frac{40 \times 0.5L}{2L}$

$= 10$ bottles —— [A1]

(c)

Amount of limes needed $= 40$

Amount of lemons needed $= 40$

Amount of pineapple juice $= 40 \times 0.15L$

$= 6L$

Amount of lemonade $= 40 \times 0.5L$

$= 20L$

Amount of orange juice $= 40 \times 0.3L$

$= 12L$

Accurate calculate of lime + lemonade — M1

" " " lemon — M1

" " " pineapple + orange — M1

" " " plastic + straw — M1

cost of lime $= \frac{40}{5} \times \$3.50$

$= \$28$

cost of lemon $= \frac{39}{3} \times \$1.50 + 60¢$

$= \$20.10$

cost of pineapple juice $= \frac{6}{2} \times \$7.50$

$= \$22.50$

cost of lemonade $= 10 \times \$1.85$

$= \$18.50$

cost of orange juice $= 12 \times \$1.25$

$= \$15$

cost of plastic cups $= 10 \times \$1.35$

$= \$13.50$

cost of straws $= \$1.30 \times 2 = \$2.60$

total cost $= \$120.20$ —— [A1]

min. cost $= \frac{\$120.20}{200}$ —— [M1]

$= \$0.601$

Since its a fund raising, Sally should consider her labour cost, rental of venue and also the amount she aims to raise for the event. ∴ sensible amount to charge will be more than $0.601 # —— [M1]