O

CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination in preparation for
the General Certificate of Education Ordinary Level 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

3 September 2019

Candidates answer on the Question Paper.
No additional materials are required.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **17** printed pages and **3** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

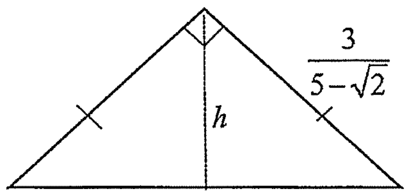
$$\Delta = \frac{1}{2} ab \sin C$$

1 The line $y=\frac{x}{k}+\frac{k}{8}$, where k is a constant, is tangent to the curve $8y=2x^2$ at the point Q.
Find

(i) the value of k, [4]

(ii) the coordinates of Q. [2]

2 The length of one side of an isosceles right-angled triangle is given as $\frac{3}{5-\sqrt{2}}$ units. Find the exact length of its height, h units, in the form $a+b\sqrt{2}$ where a and b are rational numbers. [4]



3 **(i)** Differentiate xe^{3x} with respect to x. [2]

(ii) Hence evaluate $\int_0^1 xe^{3x}\,dx$ leaving your answer in terms of e. [4]

4 (i) Express $\sin\left(2x+\dfrac{\pi}{2}\right)$ as $a\cos 2x$. [2]

(ii) Hence or otherwise, find the value of $\displaystyle\int_{1}^{\frac{\pi}{3}}\left(\frac{3}{4x}+\sin\left(2x+\frac{\pi}{2}\right)\right)\mathrm{d}x$. [5]

5 The equation of a curve is $y = \cos 2x + 2\sqrt{3}\sin x$ for $0 < x < \frac{\pi}{2}$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point. [2]

6 Express $\frac{x^3+2x^2+1}{x^2-4}$ in partial fractions. [5]

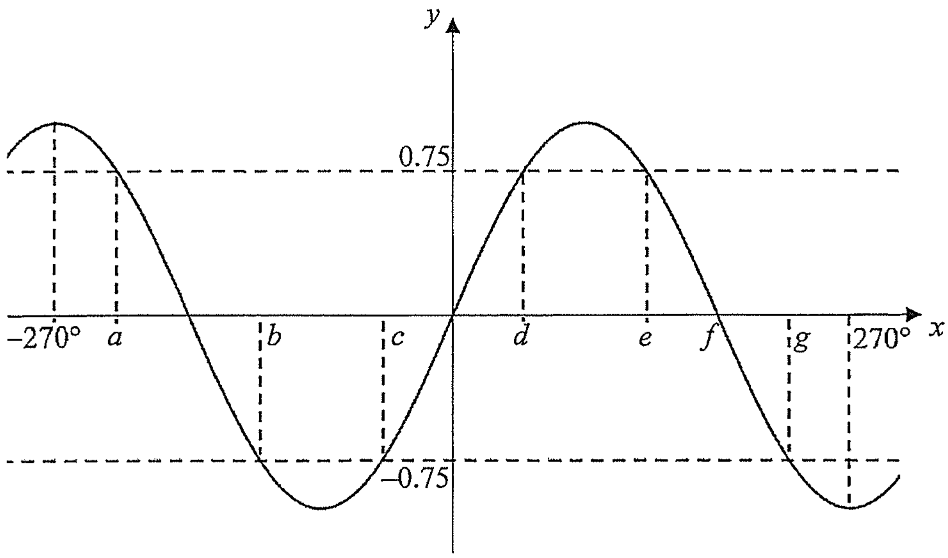
7 **(i)** On the same axes, sketch the curves $y = 9x^{\frac{1}{3}}$ and $y = \frac{1}{x}$ for $x > 0$. [2]

(ii) Find the x-coordinate of the point of intersection of the two curves. [2]

8 Show that $\frac{d}{dx}\left(\frac{7\sqrt{x}}{2x-5}\right)$ can be written in the form $\frac{k(2x+5)}{\sqrt{x}(2x-5)^2}$ and state the value of k.

[4]

9 The diagram shows the graph of $y = \sin x$ for $-270° \le x \le 270°$. The angles a, b, c, d, e, f and g are marked on the diagram on the x-axis.



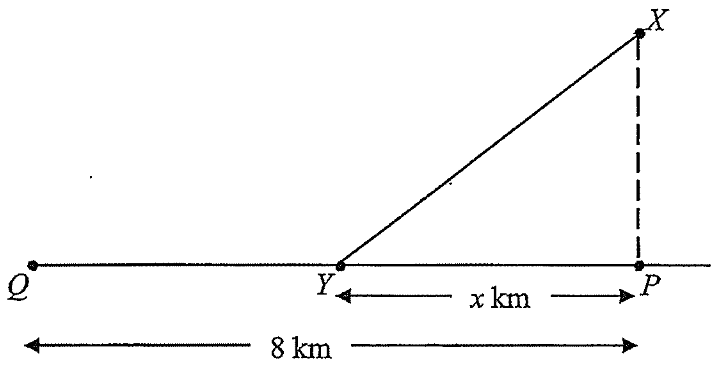
Name the angle(s) which represent the following statements.

(i) The angles for which $|\sin x| = 0.75$. [2]

(ii) The principal value of $\sin^{-1}(-0.75)$. [1]

(iii) The angles which satisfy the equation $2\sin x = -\frac{3}{2}$. [1]

10 A straight road running through P to a village Q, is 8 km long. John is camped at a point X which is 1 km from the nearest point P. A point Y is on PQ such that PY is x km.



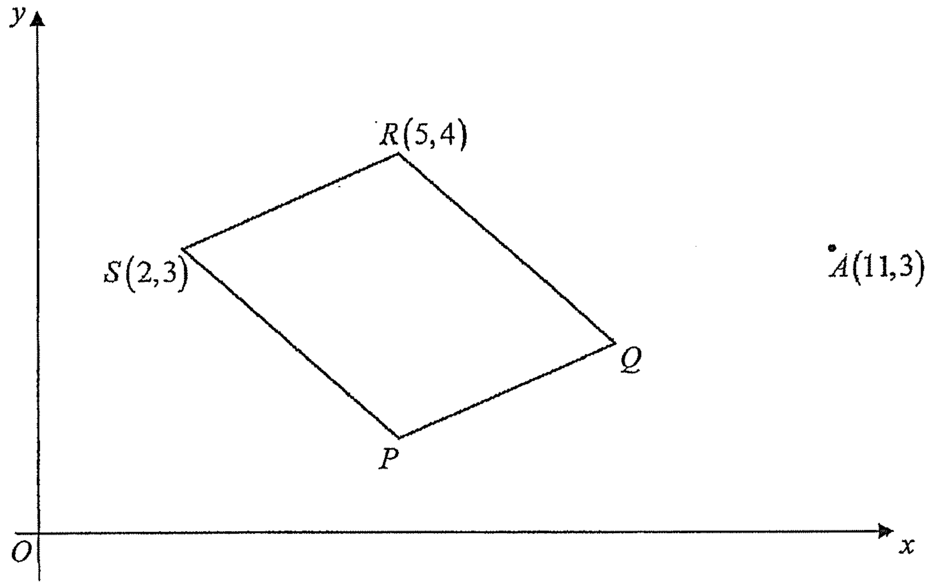
John walks in a straight line from X to Y at a speed of 5 km/h and then from Y to Q at 6 km/h.

(i) Show that the total time taken to reach the village Q is given by $T = \frac{1}{5}\sqrt{x^2+1} + \frac{8-x}{6}$.

[3]

(ii) Find the value of x if John is to make the journey from X to Q in as short a time as possible and hence find this shortest time. [5]

11



The diagram shows a parallelogram *PQRS* with points $R(5,4)$ and $S(2,3)$. The points *P*, *Q* and $A(11,3)$ are collinear. Find

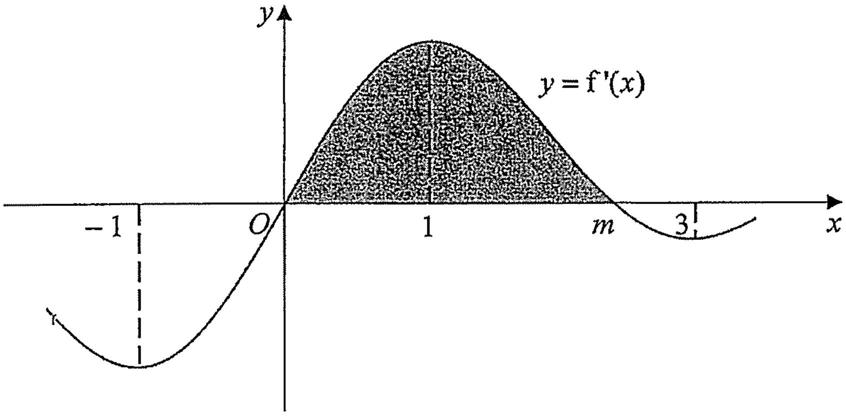
(i) the equation of the line *PQ*, [3]

(ii) the *y*-coordinate of *Q* given that its *x*-coordinate is 8, [1]

(iii) the coordinates of P, [2]

(iv) the area of triangle AQR. [2]

12 The figure shows a sketch of the derived function curve $y = f'(x)$ for $-1 \le x \le 3$.



The curve $y = f'(x)$ cuts the x axis at the origin and at $(m, 0)$ where $0 < m < 3$. The area of the shaded region is 7 sq units .

(i) With explanation, write down the x coordinate of the maximum and minimum points of the curve $y = f(x)$ for $-1 < x < 3$.

[3]

Given that $f(0) = 2$,

(ii) find the value of $f(m)$ by considering $\int_0^m f'(x)\,dx$. [3]

13 **(i)** Express $\sin 2x + 8\cos^2 x$ in the form $a + R\sin(2x + \alpha)$ where α and R are constants, $R > 0$ and $0° < x < 90°$. [6]

(ii) Hence find the minimum value of $\sin 2x + 8\cos^2 x$ and the value of x between $0°$ and $180°$ at which this minimum value occurs. [3]

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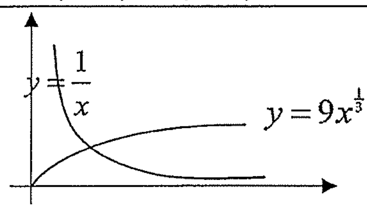
Additional Math

Answers to Sec 4/5 Prelim 2019 Paper 1

1(i)	$k = -2$	**(ii)**	$Q\left(-1, \frac{1}{4}\right)$
2	$\frac{3}{23} + \frac{15}{46}\sqrt{2}$		
3(i)	$e^{3x} + 3xe^{3x}$	**(ii)**	$\frac{2}{9}e^3 + \frac{1}{9}$
4(i)	$\cos 2x$	**(ii)**	0.0130
5(i)	$\frac{dy}{dx} = -2\sin 2x + 2\sqrt{3}\cos x$ $\frac{d^2y}{dx^2} = -4\cos 2x - 2\sqrt{3}\sin x$	**(ii)** **(iii)**	$\left(\frac{\pi}{3}, 2\frac{1}{2}\right)$ Maximum point
6	$x + 2 + \frac{17}{4(x-2)} - \frac{1}{4(x+2)}$		
7(i)		**(ii)**	0.192 or $\frac{\sqrt{3}}{9}$
8	$k = -\frac{7}{2}$		
9(i)	*a,b,c,d,e,g*	**(ii)**	*c* (iii) *b,c,g*
10		**(ii)**	$x = 1.5075$ Shortest time = 1.44h
11(i)	$3y = x - 2$	**(ii)**	$y = 2$
(iii)	$P(5 , 1)$	**(iv)**	4.5 sq units
12(i)	$x = 0$ is a minimum point since f''(x) > 0 $x = m$ is a maximum point since f''(x)<0	**(ii)**	f(m) = 9
13(i)	$4 + \sqrt{17}\sin(2x + 76.0°)$	**(ii)**	Min value= -0.123 when x = 97.0°

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CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS

4047/02

Paper 2

4 September 2019

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours and 30 minutes

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

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Quadratic Equation

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Binomial expansion

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

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Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Write down and simplify the first three terms of the expansion $(1-kx)^6$ in ascending powers of x. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(2-3x)^2(1-kx)^6$ is 393, find the possible values of k. [4]

2 **(a)** The variables x and y are related in such a way that when $\ln y$ is plotted against x, a straight line is obtained which passes through the points (3, 4) and (5, 6).

(i) Express y in terms of x. [3]

(ii) Hence, sketch the graph of y against x, indicating the coordinates at which the graph cuts the vertical axis. [2]

(b) Variables x and y are related by the equation $y = \frac{2a}{x^2 - 3b}$, where a and b are constants.

When the graph of $\frac{1}{y}$ against x^2 is plotted, a straight line with a gradient of $-\frac{1}{4}$ is obtained. Given that the intercept on the vertical axis is 3, calculate the value of a and of b. [3]

3 **(a)** Using $\sin 3x = \sin(x + 2x)$, show that $\sin 3x$ may be expressed as $3\sin x - 4\sin^3 x$. [4]

(b) By differentiating both sides of the identity $\sin 3x = 3\sin x - 4\sin^3 x$, show that $\cos 3x = 4\cos^3 x - 3\cos x$. [4]

4 The roots of the quadratic equation $x^2 - 4x + 7 = 0$ are α and β.

(i) Find the numerical value of

(a) $\alpha^2 + \beta^2$ [2]

(b) $\alpha^3 + \beta^3$ [2]

(ii) Find a quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta^2 + 1}$ and $\frac{\beta}{\alpha^2 + 1}$. [5]

5 The radius, r cm, of a spherical ball at time t seconds is given by the equation $r = 6 + \frac{1}{1+t}$, $t > 0$.

Find, when $t = 2$,

(i) the radius of the ball when t =2 seconds, [1]

(ii) the rate of change of the radius, [2]

(iii) the rate of change of the volume of the ball. [4]

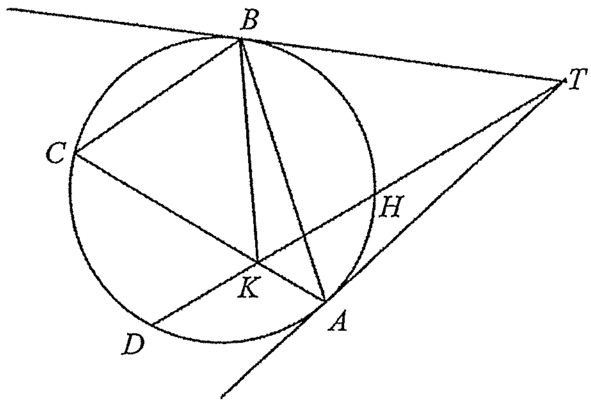
(iv) Explain why the surface area of the ball can never excced 144π cm^2 when it is left for a few days.

[2]

6 **(a)** Solve the equation $e^x - 1 - 12e^{-x} = 0$. [5]

(b) Explain why the equation $5^{2x} - 5^{x+1} = 5^x + k$ has no real solutions if $k < -9$. [3]

7

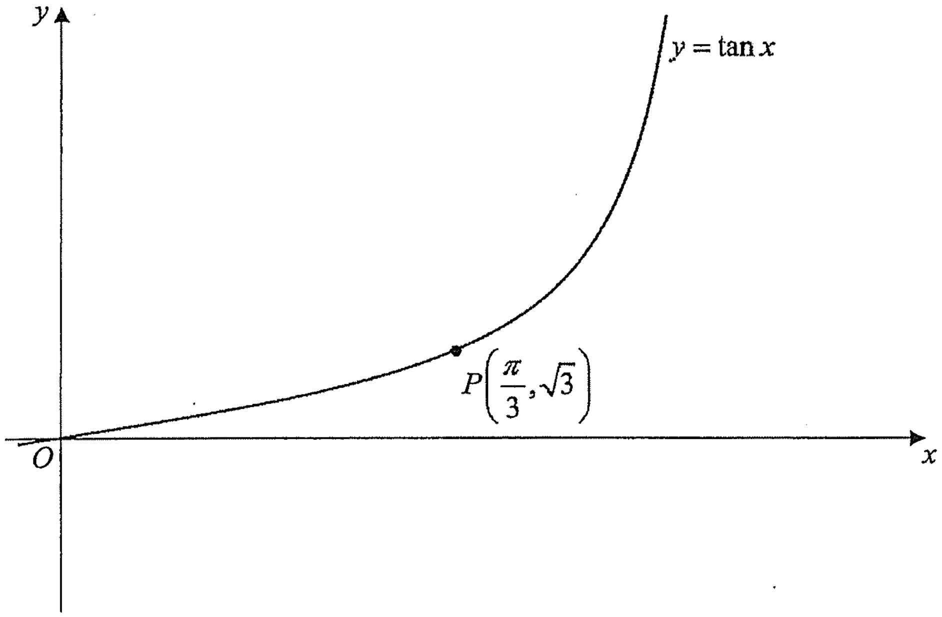


TA and *TB* are the tangents to the circle at *A* and *B*. A line through *T* cuts the circle at *H* and *D*. *AC* cuts the line *TD* at *K* and angle *TAB* = angle *AKT*, prove that

(i) *BC* is parallel to *TD*, [4]

(ii) a circle can be drawn to pass through A, K, B and T. [3]

8 The diagram shows part of the graph of $y = \tan x$.



(i) Find the gradient of the curve at the point $P(\frac{\pi}{3}, \sqrt{3})$. [2]

(ii) Find the equation of the normal to the curve at P. [2]

(iii) **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\ln \cos x) = -\tan x$. [1]

(b) The normal at P cuts the x-axis at the point Q. Find the coordinates of Q and hence find the area enclosed by the curve $y = \tan x$, the x- axis and the normal to the curve at P. Give your answer correct to two decimal places. [6]

9 A particle P moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v=12e^{-t}-18e^{-2t}$.

Find

(i) the velocity and the acceleration of the particle at O, [3]

(ii) the acceleration of the particle when it is at rest, [4]

(iii) the total distance travelled by the particle from $t = 0$ to $t = 3$. [5]

10 The equation of a circle C_1 is $x^2+y^2=4x+6y+12$.

(i) Find the coordinates of the centre and the radius of the circle. [3]

(ii) $P(t,t)$ is a point on the circle C_1, where t is a positive integer. Find the value of t and hence find the equation of the tangent to the circle at P. [4]

(iii) The tangent to the circle at P cuts the circle C_2, with equation $(x-5)^2+(y-7)^2=16$ at the points H and K. Find the x-coordinates of H and K, give the answers correct to 2 decimal places. [3]

(iv) Determine whether the point $Q(7,4)$ lies inside C_1 or C_2 or both C_1 and C_2. [2]

11 **(a)** It is given that $f(x)=2x^3-x^2-19x+12$. Solve the equation $f(x)=0$, expressing non-integer roots in the form $\dfrac{h\pm\sqrt{k}}{4}$, where h and k are integers. [5]

(b) The expression $f(x) = 3x^3 + ax^2 + bx + c$ leaves the same remainder when it is divided by $x - 1$ and by $x + 2$.

(i) Express a in terms of b. [3]

(ii) Given that f(x) leaves a remainder of 8 when divided by $(x + 1)$, find the value of c. [2]

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Solutions for CHIJ Sec TP 2019 AM Prelim P2

1. **(i)** $1-6kx+15k^2x^2$ **(ii)** $k=-\frac{16}{5}$ or $k=2$

2. **(a)(i)** $y=e^{x+1}$ **(b)** $a=-2$ $,b=4$

3. **(a)**
$$\sin 3x=\sin(x+2x)=\sin x\cos 2x+\cos x\sin 2x$$
$$=\sin x(1-2\sin^2 x)+\cos x(2\sin x\cos x)$$
$$=\sin x-2\sin^3 x+2\sin x(1-\sin^2 x)$$
$$=3\sin x-4\sin^3 x$$

(b)
$$\frac{d}{dx}(\sin 3x)=\frac{d}{dx}(3\sin x-4\sin^3 x)$$
$$3\cos 3x=3\cos x-12\sin^2 x\cos x$$
$$3\cos 3x=3\cos x-12\cos x(1-\cos^2 x)$$
$$3\cos 3x=3\cos x-12\cos x+12\cos^3 x$$
$$\cos 3x=4\cos^3 x-3\cos x$$

4. **(i)(a)** 2 (b) -20 **(ii)** $x^2-x(-\frac{16}{52})+\frac{7}{52}=0$ i.e. $52x^2+16x+7=0$

5. $(i)6\frac{1}{3}$ $(ii)-\frac{1}{9}$ (iii) $-17\frac{67}{81}\pi$ or -56.0 **(iv)** After a few days t becomes large and the value of $\frac{1}{1+t}$ will tend to 0 and the radius of the ball will tend to 6. Therefore the surface area of the ball will approximate to $4\pi(6)^2$ i.e. 144π cm^2

6. **(a)** $x=\ln 4=1.39$ **(b)** $(5^x)^2-6(5^x)-k=0$ For no real roots, $(-6)^2-4(1)(-k)<0$

$36+4k<0 \Rightarrow k<-9$

7. **(i)** Let $\angle TAB=\angle AKT=\theta$

$\angle ACB=\theta$ ($\angle$ in alt. seg.)

$\therefore \angle KCB=\angle AKB$ and they form corr.$\angle$,

$\therefore$ BC //TD.

(ii) $\angle TAB=\angle TBA$ (tan from external pt are equal)

$\therefore \angle AKT=\angle ABT$ (and they form $\angle$ in the same segment)

a circle can be drawn to pass through *A,K,B* and *T,*

8. **(i)** 4 **(ii)** $4y+x=4\sqrt{3}+\frac{\pi}{3}$ **(iii) (a)** $\frac{d}{dx}(\ln\cos x)=\frac{-\sin x}{\cos x}=-\tan x$ **(b)** 6.69

9. **(i)** -6, 24 **(ii)** 8 m/s^2 **(iii)** 4.42 m

10. **(i)** Centre (2, 3), radius= 5units **(ii)** t=6 , $y=14-\frac{4}{3}x$ **(iii)** 2.76 or 7.56 **(iv)** Q is outside C_1, and inside C_2

11. **(a)** $x=3$ or $x=\frac{-5\pm\sqrt{57}}{4}$ **(b)(i)** $a=b+9$ (ii) $c=2$