NAME:		( )		CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

DATE :	**7 AUGUST 2018**	**DAY :**	**TUESDAY**
DURATION:	**2 h**	**MARKS:**	**80**

ADDITIONAL MATERIALS

Writing Paper x 6
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of **5** printed pages and 1 blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n.$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots\cdots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

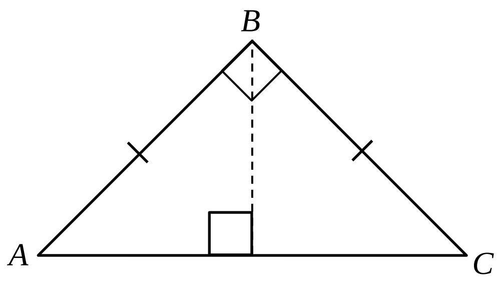
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1 Find the range of values of a for which $x^2 + ax + 2(a - 1)$ is always greater than 1. [4]

2 Find the distance between the points of intersection of the line $2x + 3y = 8$ and the curve $y = 2x^2$, leaving your answer in 2 significant figures. [5]

3 Express $\dfrac{x^2 - 2x - 6}{x(x^2 - x - 6)}$ as a sum of 3 partial fractions. [5]

4 Triangle ABC is an right angled isosceles triangle with angle ABC as the right angle. The height from point B to the base AC is $\dfrac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}$. **Without using a calculator**, express the area of the triangle ABC in the form $a + b\sqrt{2}$, where a and b are integers. [5]



5 **(i)** Given $\sin(A + B) + \sin(A - B) = k \sin A \cos B$, find k. [2]

(ii) Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sin 2x \cos x \, dx$. [4]

6 **(a)** State the values between which the principal value of $\tan^{-1}x$ must lie. [1]

(b) The function f is defined by $f(x) = 3\cos ax + 1$, where a is a positive integer and $-\pi \le x \le \pi$.

(i) State the amplitude and the minimum value of f. [2]

(ii) Given that $f(x) = 1$ when $x = \dfrac{\pi}{4}$, find the smallest possible value of a [1]

(iii) Using the value of a found in part **(ii)**, state the period of f and sketch the graph of $y = f(x)$. [4]

7 The function f is defined by $f(x) = 6x^3 - kx^2 + 3x + 10$, where k is a constant.

(i) Given that $2x + 1$ is a factor of $f(x)$, find the value of k. [2]

(ii) Using the value of k found in part **(i)**, solve the equation $f(x) = 0$. [4]

8 Solve the equation

(i) $3\log_3 x - \log_x 3 = 2$, [5]

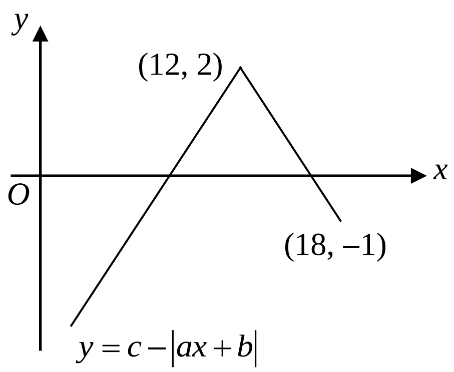
(ii) $2\log_2(1-2x) - \log_2(6-5x) = 0$. [4]

9 The equation of a curve is $y = \dfrac{2x^2}{x-1}$, $x > 1$.

(i) Find the coordinates of the stationary point of the curve. [4]

(ii) Use the second derivative test to determine the nature of the point. [3]

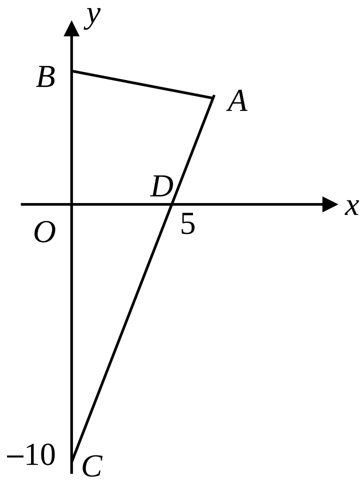
10 The diagram shows part of the graph $y = c - |ax + b|$ where $a > 0$. The graph has a maximum point (12, 2) and passes through the point (18, −1).



(i) Determine the value of each of a, b and c. [4]

(ii) State the set of value(s) of m for which the line $y = mx + 4$ cuts the graph $y = c - |ax + b|$ at exactly one point. [3]

11



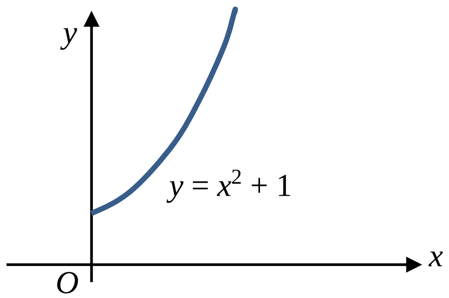
The diagram shows a triangle ABC in where points B and C are on the y-axis. The line AC cuts the x-axis at point D and the coordinates of point C and D are $(0, -10)$ and $(5, 0)$ respectively. $AD = \frac{2}{7}AC$ and points A, B and D are vertices of a rhombus $ABDE$.

(i) Show that the coordinates of A is $(7, 4)$. [1]

(ii) Find the coordinates of B and E. [5]

(iii) Calculate the area of the quadrilateral $ABOD$. [2]

12



The diagram above shows part of the curve $y = x^2 + 1$. P is the point on the curve where $x = p$, $p > 0$. The tangent at P cuts the x-axis at point Q and the foot of the perpendicular from P to x-axis is R.

(i) Show that the area A of the triangle PQR is given by $A = \frac{p^3}{4} + \frac{p}{2} + \frac{1}{4p}$. [5]

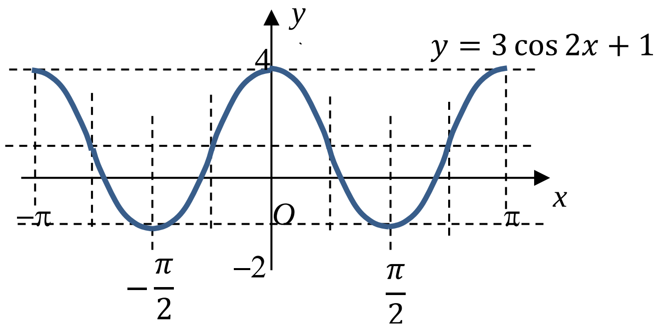
(ii) Obtain an expression for $\frac{dA}{dp}$. [1]

(iii) Find the least area of the triangle PQR, leaving your answer in 2 decimal places. [4]

End of Paper

YISHUN TOWN SECONDARY SCHOOL
2018 Preliminary Examination
Secondary Four Express / Five Normal
ADDITIONAL MATHEMATICS 4047/01

Answer Scheme

Qn	Answer
1	$2 < a < 6$
2	2.8 units
3	$\frac{x^2-2x-6}{x(x^2-x-6)} = \frac{1}{x} - \frac{1}{5(x-3)} + \frac{1}{5(x+2)}$
4	$18 - 12\sqrt{2}$
5(i)	$k = 2$ **5(ii)** $\frac{4-\sqrt{2}}{6}$
6(a)	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ or $-90^0 < \tan^{-1} x < 90^0$
6(b)(i)	Amplitude = 3, minimum value = – 2 **6(ii)** $a = 2$
(iii)	Period = π 
7(i)	$k = 31$ **7(ii)** $x = 5$ or $\frac{2}{3}$ or $-\frac{1}{2}$
8(i)	$x = 0.693$ or 3
(ii)	$x = -\frac{5}{4}$
9(i)	(2, 8)
(ii)	$\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3}$ Min point
10(i)	$a = \frac{1}{2}$, $b = -6$, $c = 2$
(ii)	$m = -\frac{1}{6}$ or $m > \frac{1}{2}$ or $m \le \frac{1}{2}$
11(i)	(7, 4)
(ii)	B (0, 5), E(12, – 1)
(iii)	27.5 units^2
12(ii)	$\frac{dA}{dp} = \frac{3}{4}p^2 + \frac{1}{2} - \frac{1}{4p^2}$ **12(ii)** 0.77 units^2

NAME:		( )		CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 2 (4047/02)

DATE :	**16 AUGUST 2018**	**DAY :**	**THURSDAY**
DURATION:	**2 h 30 min**	**MARKS:**	**100**

ADDITIONAL MATERIALS

Writing Paper x 8
Mathematics Cover Sheet x 1

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

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1. **(a)** Given that the roots of the equation $x^2 - 6x + k = 0$ differ by 2, find the value of *k*. [3]

(b) If α and β are the roots of the equation $x^2 + bx + 1 = 0$, where *b* is a non-zero constant, show that the equation with roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ is $x^2 - \left(b^2 - 2\right)x + 1 = 0$. [4]

2. If the first three terms in the expansion of $\left(1 - \dfrac{x}{2}\right)^n$ is $1 - 6x + ax^2$, find the value of *n* and of *a*. [4]

3. **(a)** Solve the equation $\sqrt{4 + \dfrac{3}{x}} = \dfrac{1}{\sqrt{x}} + 2$. [5]

(b) Given that $\dfrac{4}{n}(3x)^2\left(\dfrac{2}{9x^2}\right)^{n-2} \equiv \dfrac{m}{x^2}$, where $x \neq 0$, find the values of the constants *m* and *n*. [4]

4. A precious stone was purchased by a jeweler in the beginning of January 2003. The expected value, \$*V*, of the stone may be modelled by the equation $V = 6000\left(4^t\right) - 1000\left(16^t\right)$, where *t* is the number of years since the time of purchase. Find

(i) the expected value of the stone when $t = \dfrac{3}{4}$. [1]

(ii) the value(s) of *t* for which the expected value of the stone is \$8000. [3]

(iii) the range of values of *t* for which the expected value of the stone exceeds \$8000. [1]

5. The equation of a circle, *C*, is $x^2 + y^2 - 4ux + 2uy + 5\left(u^2 - 20\right) = 0$ where *u* is a positive constant.

(a) Given that $u = 6$, find the coordinates of the centre and the radius of the circle *C*. [3]

(b) Determine the value of *u* for which

(i) the circle, *C*, passes through the point $(-4, 4)$, [2]

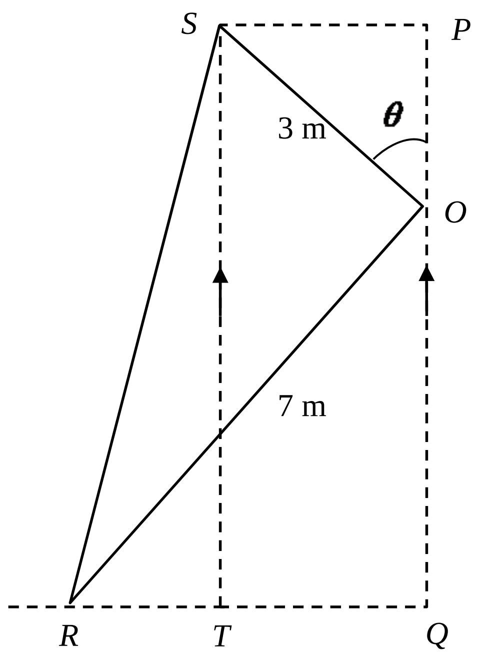
(ii) the line $x = 2$ is a tangent to the circle, *C*. [4]

6. The variables x and y are related by the equation $mx + ny - 3xy = 0$, where m and n are non-zero constants. When $\frac{1}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. Given that the line passes through the points $(1,0)$ and $(-5,9)$, find the values of m and of n. [6]

7. **(i)** Prove that $\sin^4\theta - \cos^4\theta \equiv 1 - 2\cos^2\theta$. [3]

(ii) Hence solve $\sin^4\theta - \cos^4\theta - 3\cos\theta = 2$ for $0 < \theta < 2\pi$. [4]

8. In the diagram, $OS = 3$ m, $OR = 7$ m and angle SOR = angle SPO = angle $RQO = 90^\circ$.
It is given that angle SOP is a variable angle θ where $0^\circ < \theta < 90^\circ$. The point T is on the line RQ such that ST is parallel to PQ.



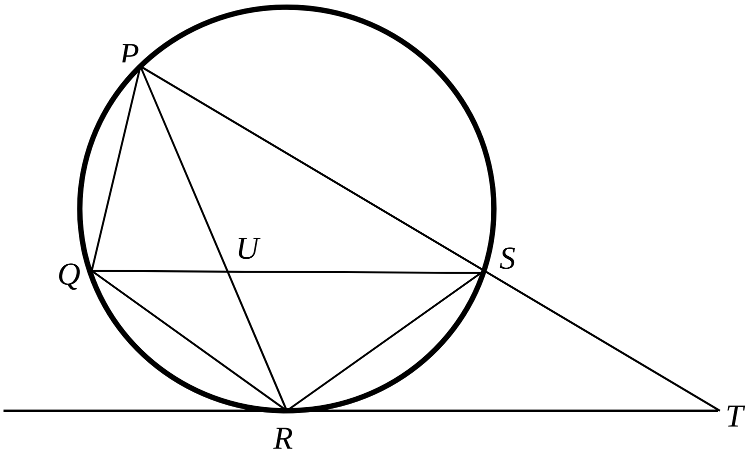
(i) Show that $PQ = 7\sin\theta + 3\cos\theta$. [1]

(ii) Show that the area of triangle RST is $\frac{21}{2}\cos 2\theta + 10\sin 2\theta$. [3]

(iii) Express the area of the triangle RST as $k\cos(2\theta - \alpha)$, where $k > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iv) Hence find the maximum area of triangle RST and the corresponding value of θ. [3]

9. The diagonals of a cyclic quadrilateral *PQRS* intersect at a point *U*. The circle's tangent at *R* meets the line *PS* produced at *T*.



If $QR = RS$, prove the following.

(i) *QS* is parallel to *RT*. [3]

(ii) Triangles *PUS* and *QUR* are similar. [3]

(iii) $PU^2 - QU^2 = (PU \times PR) - (QU \times QS)$. [3]

10. It is given that $y = xe^{-x} - 2e^{-2x}$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) If *x* and *y* can vary with time and *x* increases at the rate of 1.5 units per second at the instant when $x = \ln 2$, find the exact value of the rate of increase of *y* at this instant. [3]

11. A curve has the equation $y = \frac{\ln x}{x^2} - 2$.

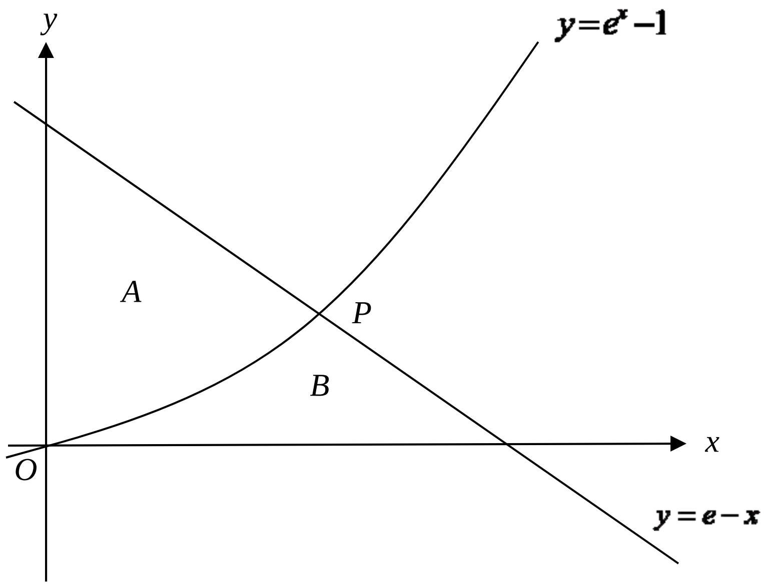
(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 2\ln x}{x^3}$. [2]

(ii) **(a)** The *x*-coordinate of a point *P* on the curve is 1. Find the equation of the tangent to the curve at *P*. [2]

(b) The tangent to the curve at the point *P* intersects the *x*-axis at *Q* and the *y*-axis at *R*. Calculate the shortest distance from the origin *O* to the line *QR*. [4]

(iii) Given that another curve $y = \mathrm{f}(x)$ passes through the point $(1, -0.25)$ and is such that $\mathrm{f}'(x) = \frac{\ln x}{x^3}$, find the function $\mathrm{f}(x)$. [3]

12. The diagram shows the graphs of $y = e^x - 1$ and $y = e - x$. P is the point of intersection of the two graphs.



(i) Show that $\alpha = 1$ is a root to the equation $e\left(1 - e^{\alpha - 1}\right) - \alpha + 1 = 0$. [1]

(ii) Hence, find the coordinates of P. [2]

(iii) Find the area of region A, which is enclosed by the two graphs and the y-axis. [4]

(iv) Find the exact value of $\dfrac{\text{area of region } A}{\text{area of region } B}$, given that the area of region B is enclosed by the two graphs and the x-axis. [2]

13. A particle moves pass a point A in a straight line with a displacement of -4 m from a fixed point O. Its acceleration, a m/s^2, is given by $a = \dfrac{t}{2}$, where t seconds is the time elapsed after passing through point A.

Given that the initial velocity is -1 m/s, find,

(i) the velocity when $t = 2$, [3]

(ii) the distance travelled by the particle in the first 5 seconds. [5]

END OF PAPER

YISHUN TOWN SECONDARY SCHOOL
2018 Preliminary Examination
Secondary Four Express / Five Normal
ADDITIONAL MATHEMATICS 4047/02

1(a)	$k=8$
1(b)	Show $x^2-\left(b^2-2\right)x+1=0$
2	$n=12$, $a=\frac{33}{2}$
3(a)	$x=\frac{1}{4}$
3(b)	$n=4$, $m=\frac{4}{9}$
4(i)	\$8970
4(ii)	$t=\frac{1}{2},1$
4(iii)	$\frac{1}{2}<t<1$
5(i)	Centre is $(12,-6)$ Radius = 10 units
5(ii)(a)	$u=2$
5(ii)(b)	$u=6$
6	$m=2$, $n=3$
7(i)	Show $\sin^4\theta-\cos^4\theta\equiv1-2\cos^2\theta$
7(ii)	$\theta=\frac{2\pi}{3},\pi,\frac{4\pi}{3}$
8(i)	Show $PQ=7\sin\theta+3\cos\theta$
8(ii)	Show Area = $\frac{21}{2}\cos2\theta+10\sin2\theta$
8(iii)	Area = $\frac{29}{2}\cos\left(2\theta-43.6^\circ\right)$
8(iv)	Max area of triangle $RST=\frac{29}{2}$ m^2, $\theta=21.8^\circ$
9(i)	Show QS is parallel to RT
9(ii)	Show Triangles PUS and QUR are similar
9(iii)	Show $PU^2-QU^2=\left(PU\times PR\right)-\left(QU\times QS\right.$
10(i)	$\frac{dy}{dx}=(1-x)e^{-x}+4e^{-2x}$
10(ii)	$\left.\frac{dy}{dt}\right\vert_{x=\ln2}=\frac{9}{4}-\frac{3}{4}\ln2$
11(i)	$\frac{dy}{dx}=\frac{1-2\ln x}{x^3}$
11(ii)(a)	$y=x-3$
11(ii)(b)	$h=\frac{3\sqrt{2}}{2}$ units
11(iii)	$\mathrm{f}(x)=-\frac{1+2\ln x}{4x^2}$
12(ii)	$P(1,e-1)$
12(iii)	Area of Region $A=\frac{3}{2}$ units^2
12(iv)	$\frac{\text{Area of Region } A}{\text{Area of Region } B}=\frac{3}{e^2-3}$
13(i)	Velocity = 0 m/s
13(ii)	Total distance travelled $=8\frac{1}{12}$ m