Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **11 September 2018, Tuesday**

Additional Materials : Writing Papers (7 sheets)
Graph Paper (1 sheet)
2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Ms Renuka Ramakrishnan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1 **(i)** On the same axes, sketch the curves $y = \frac{2}{x^2}$ and $y^2 = 128x$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [2]

2 **(i)** Factorise completely the cubic polynomial $2x^3 - 11x^2 + 12x + 9$. [3]

(ii) Hence, express $\frac{6x^3-33x^2+35x+51}{2x^3-11x^2+12x+9}$ in partial fractions. [5]

3 A quadratic curve passes through $(0, -1)$ and $(2, 7)$. The gradient of the curve at $x = -2$ is -8. Find the equation of the curve. [5]

4 **(i)** Show that $\cos 3\theta - \cos\theta = -2\sin 2\theta \sin\theta$. [3]

(ii) Hence find the values of θ between $0°$ and $360°$ for which $\cos 3\theta - \cos\theta = \sin 2\theta$. [3]

5 The volume of a right square pyramid of length $(3 + \sqrt{2})$ cm is $\frac{1}{3}(29 - 2\sqrt{2})$ cm^3. Without using a calculator, find the height of the pyramid in the form $(a + b\sqrt{2})$ cm, where a and b are integers. [5]

6 The roots of the quadratic equation $6x^2 - 5x + 2 = 0$ are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

(i) Find the value of $\frac{\alpha}{\alpha+2} + \frac{\beta}{\beta+2}$. [5]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha+2}$ and $\frac{\beta}{\beta+2}$. [2]

7 A particle moves in a straight line such that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = t^2 - 5t + 4$.

(i) Find the acceleration of the particle when it first comes to an instantaneous rest. [3]

(ii) Find the average speed of the particle for the first 5 seconds. [4]

8 The following table shows the experimental values of two variables, x and y, which are related by the equation $y = ab^{x+1}$, where a and b are constants.

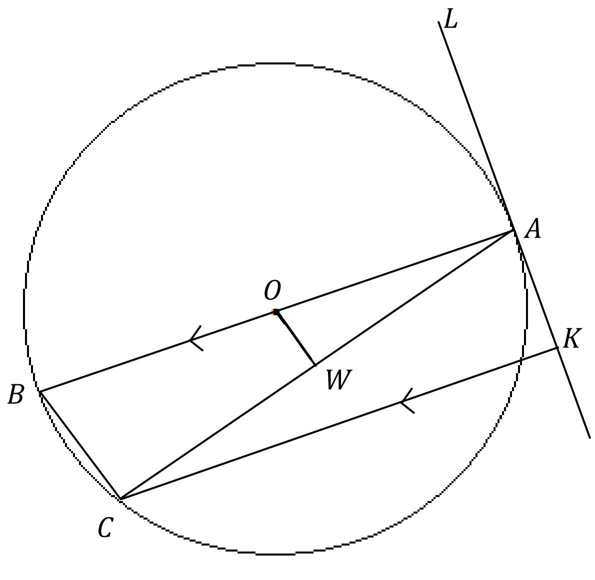
x	1	2	3	4
y	10.12	10.23	10.35	10.47

(i) On graph paper, plot $\lg y$ against x and draw a straight line graph. The vertical $\lg y$ – axis should start from 0.995 and have a scale of 4 cm to 0.005. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Explain how the value of a and of b will change if a graph of $\ln y$ against x was plotted instead. [1]

9



In the diagram, A, B and C are three points on the circle such that AB is the diameter of the circle and W is the midpoint of AC. AB and CK are parallel to each other and KL is a tangent to the circle at A.

(i) Prove that OW is parallel to BC. [2]

(ii) Prove that Angle AWO = Angle AKC. [3]

10 Diagram I shows a right angled ΔABC, with hypotenuse AB of length 4 m. This triangle is revolved around BC to generate a right circular cone as shown in Diagram II.

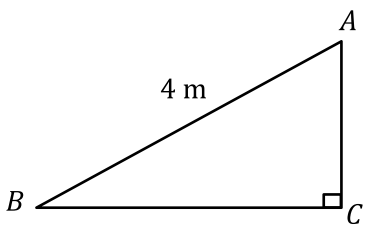


Diagram I

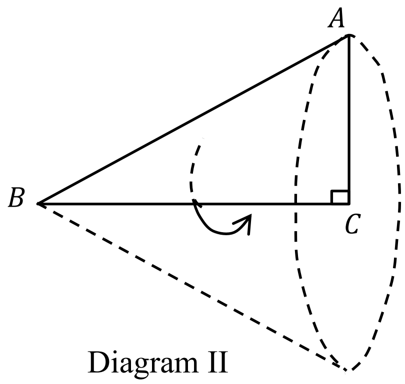


Diagram II

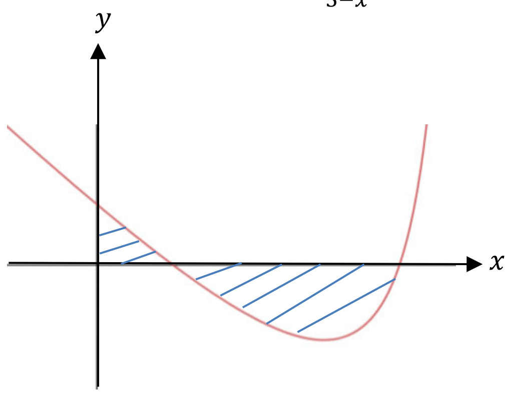
(i) Find the **exact** height that gives the maximum volume of the cone. [6]

(ii) Show that this maximum volume is obtained when $BC : CA = 1 : \sqrt{2}$. [2]

11 The equation of a curve is $y = \frac{4-5x+x^2}{5-x}$, $x \neq 5$.

(i) Find the set of values of x for which y is an increasing function of x. [3]

(ii) The diagram below shows part of the curve $y = \frac{4-5x+x^2}{5-x}$, $x \neq 5$.



By expressing $\frac{4-5x+x^2}{5-x}$ in the form $ax + \frac{b}{5-x}$, where a and b are constants, find the total area of the shaded regions. [5]

12 A circle C_1, with centre C, passes through four points A, B, F and G. The coordinates of A and B are (0, 4) and (8, 0) respectively. The equation of the normal to the circle at F is $y = -\frac{4}{3}x + 4$.

(i) Show that the coordinates of C is (3, 0). [5]

(ii) Hence find the equation of the circle. [2]

Another circle C_2 passes through the points C, F and G.

(iii) Given that GF is the diameter of the circle, calculate the radius of C_2. [2]

- End of Paper -

Answer Key

1ii) $\left(\frac{1}{2}, 8\right)$

2i) $(x-3)^2(2x+1)$

2ii) $3 + \frac{2}{2x+1} - \frac{1}{x-3} + \frac{3}{(x-3)^2}$

3) $y = 2x^2 - 1$

4ii) $\theta = 90°, 180°, 210°, 270°, 330°$

5) $(7 - 4\sqrt{2})cm$

6i) $\frac{17}{13}$

6ii) $x^2 - \frac{17}{13}x + \frac{6}{13} = 0$

7i) $-\frac{3m}{s^2}$

7ii) 1.63 m/s

8ii) $b = 1.01, a = 9.89$

8iii) remain unchanged

10i) $\frac{4\sqrt{3}}{3}$ cm

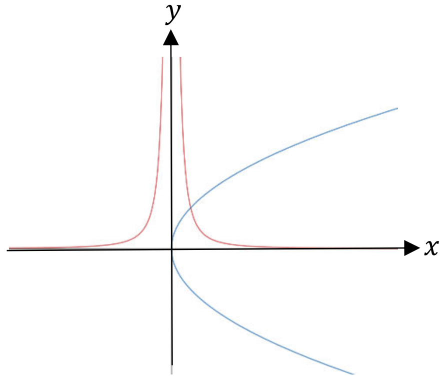
11i) $3 < x < 7, x \neq 5$

11ii) 2.35 $units^2$

12ii) $(x-3)^2 + y^2 = 25$

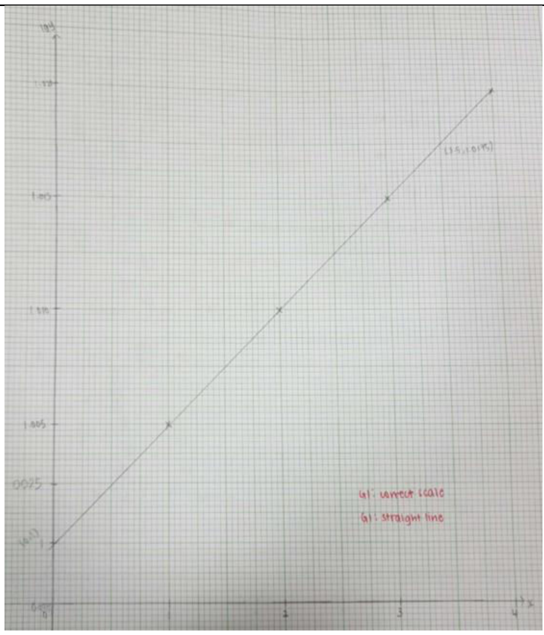
12iii) 3.54 $units$

NCHS 2018 Prelim 2 AM Paper 1 Solutions

1i		G1: graph of $y = \frac{2}{x^2}$ G1: graph of $y^2 = 128x$
1ii	$\left(\frac{2}{x^2}\right)^2 = 128x$ $x^5 = \frac{4}{128}$ $x = \frac{1}{2}$ $\rightarrow y = 8$ $\left(\frac{1}{2}, 8\right)$	M1: Equating both functions A1: award only if written as coordinates
2i	Let $f(x) = 2x^3 - 11x^2 + 12x + 9$ $f(3) = 0$ $\therefore$ $(x-3)$ is a factor of $f(x)$. $f(x) = (x-3)(2x^2 - 5x - 3)$ $= (x-3)^2(2x+1)$	M1: show 1st factor using factor theorem M1: Find quadratic factor (by long division/synthetic method) A1
2ii	$\frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} = 3 + \frac{-x+24}{(x-3)^2(2x+1)}$ $\frac{-x+24}{(x-3)^2(2x+1)} = \frac{A}{2x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ $-x + 24 = A(x-3)^2 + B(x-3)(2x+1) + C(2x+1)$ $\rightarrow$ Using substitution/comparing coefficient $A = 2, B = -1, C = 3$ $\therefore \frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9}$ $= 3 + \frac{2}{2x+1} - \frac{1}{x-3} + \frac{3}{(x-3)^2}$	M1: Change to proper fraction M1: Split correctly to respective partial fractions M2: Using substitution/comparing coefficient to find A, B and C A1

3	$y = ax^2 + bx + c$ When $x = 0, y = -1 \rightarrow c = -1$ $\frac{dy}{dx} = 2ax + b$ $When\ x = -2, \quad \frac{dy}{dx} = -8$ $-4a + b = -8$ $b = 4a - b$ --- (1) Sub $y = 7$ and $x = 2$ into $y = ax^2 + bx - 1$ $7 = 4a + 2b - 1$ $b = 4 - 2a$ --- (2) From (1) and (2), $4a - 8 = 4 - 2a$ $a = 2$ $\rightarrow b = 0$ $\therefore$ equation of curve:$y = 2x^2 - 1$	B1: writing quad eqn in a general form A1: Solving for c M1: differentiate quad function M1: forming 2 simultaneous equations and solving it A1
4i	$\cos 3\theta - \cos\theta$ $= \cos(2\theta + \theta) - \cos\theta$ $= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta - \cos\theta$ $= \cos\theta\,(\cos 2\theta - 1) - \sin 2\theta \sin\theta$ $= \cos 2\theta\,(-2\sin^2\theta) - \sin 2\theta \sin\theta$ $= -2\sin^2\theta \cos 2\theta - \sin 2\theta \sin\theta$ $= (-2\sin\theta\cos\theta)\sin\theta - \sin 2\theta \sin\theta$ $= -\sin 2\theta \sin\theta - \sin 2\theta \sin\theta$ $= -2\sin 2\theta \sin\theta$	M1: applying addition formula to $\cos(2\theta + \theta)$ M1: changing $\cos 2\theta - 1 = -2\sin^2\theta$ A1: changing $-2\sin^2\theta\cos 2\theta = -\sin 2\theta \sin\theta$
4ii	$\cos 3\theta - \cos\theta = \sin 2\theta$ $\sin 2\theta + 2\sin 2\theta \sin\theta = 0$ $\sin 2\theta(1 + 2\sin\theta) = 0$ $\sin 2\theta = 0 \quad or \quad \sin\theta = -0.5$ $2\theta = 180, 360, 540 \quad or \quad \theta = 210, 330$ $\theta = 90, 180, 270$ $\therefore \theta = 90°, 180°, 210°, 270°, 330°$	M1 apply hence + factorization A1 solve $\sin 2\theta = 0$ correctly A1 solve $\sin\theta = -0.5$ correctly
5	$\frac{1}{3}(3 + \sqrt{2})^2 h = \frac{1}{3}(29 - 2\sqrt{2})$ $h = \frac{29 - 2\sqrt{2}}{11 + 6\sqrt{2}}$ $= \frac{(29 - 2\sqrt{2})(11 - 6\sqrt{2})}{49}$ $= \frac{319 - 174\sqrt{2} - 22\sqrt{2} + 24}{49}$ $= \frac{343 - 196\sqrt{2}}{49}$ $= (7 - 4\sqrt{2})cm$	M1: forming an equation M1: Calculating $(3 + \sqrt{2})^2$ M1: Rationalising denominator M1: Simplifying after expansion A1

6i	$\frac{2}{\alpha}+\frac{2}{\beta}=\frac{5}{6}$ $\frac{2(\alpha+\beta)}{\alpha\beta}=\frac{5}{6}$ --- (1) $\left(\frac{2}{\alpha}\right)\left(\frac{2}{\beta}\right)=\frac{1}{3}$ $\alpha\beta=12$ --- (2) Sub (2) into (1) $\alpha+\beta=5$ $\frac{\alpha}{\alpha+2}+\frac{\beta}{\beta+2}=\frac{\alpha(\beta+2)+\beta(\alpha+2)}{(\alpha+2)(\beta+2)}$ $=\frac{2\alpha\beta+2(\alpha+\beta)}{\alpha\beta+2(\alpha+\beta)+4}$ $=\frac{17}{13}$	M1: applying concept of sum and product of roots A1: $\alpha\beta$ A1: $\alpha+\beta$ M1 A1
6ii	$\left(\frac{\alpha}{\alpha+2}\right)\left(\frac{\beta}{\beta+2}\right)=\frac{\alpha\beta}{(\alpha+2)(\beta+2)}$ $=\frac{6}{13}$ $\therefore$ Equation : $x^2-\frac{17}{13}x+\frac{6}{13}=0$ $or\ 13x^2-17x+6=0$	M1 A1
7i	When v = 0, $t^2-5t+4=0$ $(t-4)(t-1)=0$ $t=1\ \ or\ \ t=4$ $a=\frac{dv}{dt}$ $=2t-5$ When $t=1, a=-3m/s^2$	M1 A1: Differentiate correctly A1
7ii	$s=\int t^2-5t+4\,dt$ $=\frac{1}{3}t^3-\frac{5}{2}t^2+4t+c$ When $t=0, s=0\rightarrow c=0$ $\therefore s=\frac{1}{3}t^3-\frac{5}{2}t^2+4t$ When $t=1, s=\frac{11}{6}m$ $t=4, s=-\frac{8}{3}m$ $t=5, s=\ -\frac{5}{6}m$	A1: integrate correctly (look out for +c, unless definite integral)

	$\therefore Average\ Speed = \dfrac{\frac{11}{6} + \left(\frac{11}{6} + \frac{8}{3}\right) + \left(\frac{8}{3} - \frac{5}{6}\right)}{5}$ $= 1\frac{19}{30}\ m/s$ or $1.63\ m/s$	M1: Calculating distance in 5th sec M1: formula for average speed A1
8	 $y = ab^{x+1}$ $lgy = (x+1)lgb + lga$ $lgy = xlgb + (lgb + lga)$ x: 1, 2, 3, 4 lg y: 1.005, 1.010, 1.015, 1.020	G1: correct scale G1: straight line M1: table of values
8ii	$\lg b = \dfrac{1.0175 - 1}{3.5}$ $= 0.005$ $b = 1.01\ (3sf)$ $\lg b + \lg a = 1$ $\lg a = 1 - 0.005$ $a = 9.89\ (3sf)$	M1 A1 M1 A1
8iii	The values of a and b will remain unchanged.	B1
9i	O is the midpoint of AB and W is the midpoint of AC, By Midpoint Theorem, BC parallel to OW.	M1 A1
9ii	$Angle\ AOW = Angle\ ABC\ (corr\ angles, OW//BC)$ $Angle\ ABC = Angle\ CAK (alt\ segment\ theorem)$ $\rightarrow Angle\ AOW = Angle\ CAK$ $Angle\ BAC = Angle\ ACK\ (alt\ angles, AB//CK)$ $\therefore Angle\ AWO$ $= 180° - Angle\ BAC - Angle\ AOW\ (Angle\ sum\ of\ \Delta)$ $= 180° - Angle\ ACK - Angle\ CAK$ $= Angle\ AKC$ (shown)	M1 M1 A1

x	1	2	3	4
lg y	1.005	1.010	1.015	1.020

10i	Let $AC = r$ and $BC = h$ $r^2 = 16 - h^2$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(16 - h^2)h$ $= \frac{16}{3}\pi h - \frac{1}{3}\pi h^3$ $\frac{dV}{dh} = \frac{16}{3}\pi - \pi h^2$ $When \frac{dV}{dh} = 0,$ $\frac{16}{3}\pi = \pi h^2$ $h = \frac{4}{\sqrt{3}} \left(rej\ h = -\frac{4}{\sqrt{3}}\ since\ h > 0\right)$ $\frac{d^2V}{dh^2} = -2\pi h$ $= -\frac{8}{\sqrt{3}}\pi\ (< 0)$ $\therefore h = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}\ cm$	M1: Finding r/s between h and r M1: finding V in terms of one variable M1: differentiation M1: Stationary point M1: Prove Max A1
10ii	$r^2 = 16 - \left(\frac{4}{\sqrt{3}}\right)^2$ $r = \frac{4\sqrt{2}}{\sqrt{3}}$ $\frac{h}{r} = \frac{\frac{4}{\sqrt{3}}}{\frac{4\sqrt{2}}{\sqrt{3}}}$ $\frac{h}{r} = \frac{1}{\sqrt{2}}$ $\therefore BC : CA = 1 : \sqrt{2}$	M1 A1
11i	$\frac{dy}{dx} = \frac{(2x-5)(5-x) - (4-5x+x^2)(-1)}{(5-x)^2}$ $= \frac{10x - 2x^2 + 5x - 25 + 4 - 5x + x^2}{(5-x)^2}$ $= \frac{-x^2 + 10x - 21}{(5-x)^2}$ Since $(5-x)^2 > 0$, for y to be an increasing function, $-x^2 + 10x - 21 > 0$ $x^2 - 10x + 21 < 0$ $(x-3)(x-7) < 0$ $3 < x < 7, x \neq 5$	M1: Applying quotient rule M1 A1

11ii	When $y = 0$ $4 - 5x + x^2 = 0$ $(x-4)(x-1) = 0$ $x = 4\ or\ x = 1$ $\dfrac{x^2 - 5x + 4}{5 - x} = \dfrac{-x(5-x)+4}{(5-x)}$ $= -x + \dfrac{4}{5-x}$ Area of shaded region $= \int_0^1 -x + \dfrac{4}{5-x}\ dx + \left\lvert \int_1^4 -x + \dfrac{4}{5-x}\ dx \right\rvert$ $= [-0.5x^2 - 4\ln(5-x)]_0^1 + \lvert[-0.5x^2 - 4\ln(5-x)]_1^4\rvert$ $= 0.39257 + \lvert -1.95482 \rvert$ $= 2.35\ units^2\ (3sf)$	M1 M1, M1 M1: correct integration A1
12i	Gradient of AB $= -\frac{1}{2}$ Midpoint of AB = (4,2) Eqn of perpendicular bisector of AB: $y - 2 = 2(x - 4)$ $y = 2x - 6$ Sub $y = 2x - 6$ into $y = -\dfrac{4}{3} + 4$, $2x - 6 = -\dfrac{4}{3} + 4$ $x = 3$ $\rightarrow y = 0$ $C\ (3, 0)$	M1: Midpoint M1: gradient =2 M1: forming equation M1: Solving simultaneous A1
12ii	Radius $= \sqrt{(3-0)^2 + (0-4)^2}$ $= 5\ units$ Equation of circle: $(x-3)^2 + y^2 = 25$ Or $x^2 + y^2 - 6x - 16 = 0$	M1: Finding radius A1
12iii	$Angle\ GCF = 90°\ (Angle\ in\ Semicircle)$ $GF^2 = 5^2 + 5^2$ $GF = \sqrt{50}$ $Radius\ of\ C_2 = \dfrac{1}{2}\sqrt{50}$ $= \dfrac{5}{2}\sqrt{2}\ units$ $or\ = 3.54\ units\ (3sf)$	M1 A1

END ☺

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **12 September 2018, Wednesday**

Additional Materials : Writing Paper (8 sheets) **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions.

1. **(i)** Given $\frac{3\lg 3x - 2\lg x}{4} = \lg 3$, find the value of x. [3]

(ii) Given $\log_{(x-2)} y = 2$ and $log_y(x+k) = \frac{1}{2}$, find the value of k if k is an integer. [3]

2. **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right] = -\frac{1}{\sin x}$. [4]

(ii) Hence evaluate $\int \sin^2 x + \frac{2}{\sin x}\,\mathrm{d}x$. [4]

3. It is given that $y_1 = -2\cos x + 1$ and $y_2 = \sin\frac{1}{2}x$.

For the interval $0 < x < 2\pi$,

(i) state the amplitude and period of y_1 and of y_2, [2]

(ii) sketch, on the same diagram, the graphs of of y_1 and y_2, [4]

(iii) find the x-coordinate of the points of intersection of the two graphs drawn in **(ii)**, [3]

(iv) hence, find the range of values of x for which $y_1 \leq y_2$. [2]

4. Some liquid is poured onto a flat surface and formed a circular patch. This circular patch is left to dry and its surface area decreases at a constant rate of 4 cm^2/s. The patch remains circular during the drying process. Find the rate of change of the circumference of the circular patch at the instant when the area of the patch is 400 cm^2. [4]

5. **(i)** In the expansion of $\left(2+\frac{4}{x^4}\right)\left(kx^3 - \frac{2}{x}\right)^{13}$ where k is a constant and $k \neq 0$, find the value of k if there is no coefficient of $\frac{1}{x}$. [5]

(ii) Given the coefficients of $\frac{1}{x}$ and $\frac{1}{x^2}$ in the expansion of $\left(1-\frac{c}{x}\right)^n$ are -80 and 3000 respectively. Find the value of c and of n where n is a positive integer greater than 2 and c is a constant. [5]

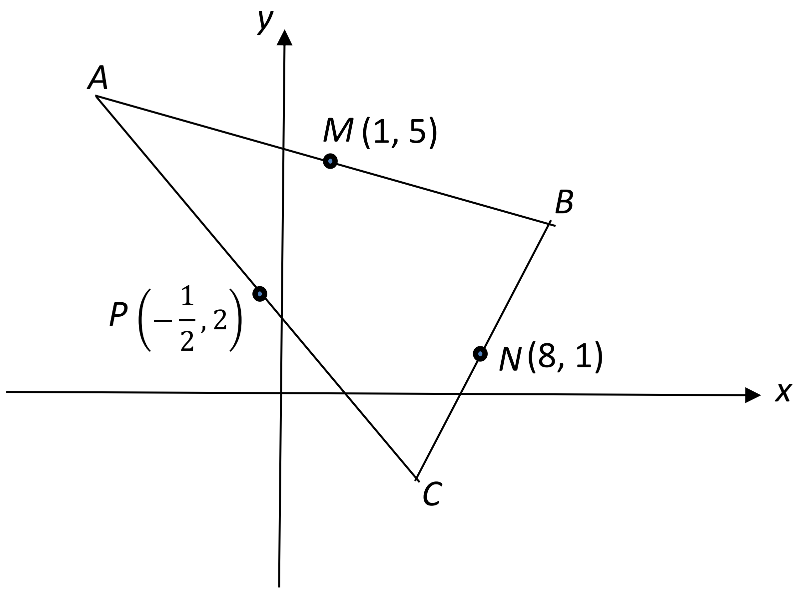
6. Curve A is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 27(2x-1)^2$ and curve B is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = -27(2x-1)^3$, and the y-coordinates of the stationary points for both curves are -4.

(i) Find the coordinates of the stationary points for curve A and B. [2]

(ii) Determine the nature of the stationary points for curve A and B. [4]

(iii) Find the equations of curve A and B. [4]

7. The diagram shows a triangle ABC. The mid-points of the sides of the triangle are $M(1,5)$, $N(8,1)$ and $P\left(-\frac{1}{2},2\right)$.



(i) State and explain which line is parallel to AB. [1]

(ii) Find the equation of the line AB. [3]

(iii) Find the equation of the line AC. [3]

(iv) Show the coordinates of A is $\left(-7\frac{1}{2},6\right)$. [3]

(v) Find the area of the quadrilateral $AMNP$. [2]

8. Diagram I and II show two types of isosceles triangular cards, ΔOAB with $\angle AOB = \theta$, $OA = OB = 3$ cm and ΔOMN with $OM = ON = 2$ cm. These two types of cards are connected as shown in diagram III where $\angle AON = 120^{\circ}$.

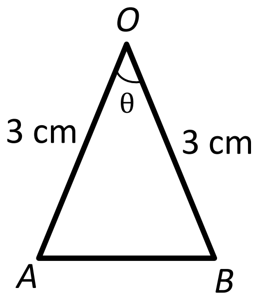


Diagram I

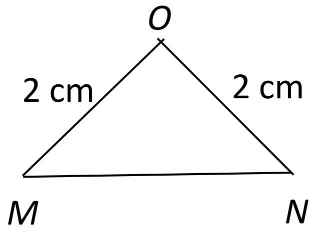


Diagram II

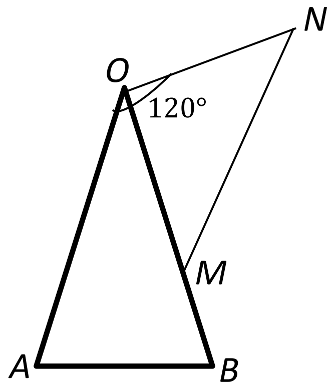


Diagram III

Three sets of cards from diagram III are connected as shown in diagram IV.

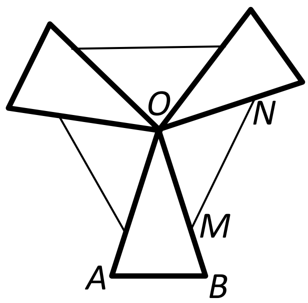


Diagram IV

(i) Show that the area of all the connected cards in diagram IV, A cm^2 is given by $A = \frac{33}{2}\sin\theta + 3\sqrt{3}\cos\theta$. [3]

(ii) Express A in the form $A = R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90^{\circ}$. [3]

(iii) Find the value of θ for which $A = 15$, where $0° < \theta < 90^{\circ}$. [3]

(iv) Find the maximum value of A and the corresponding value of θ. [2]

9. In an experiment to study the growth of a certain type of bacteria, the bacteria are injected into a mouse and the mouse's blood samples are collected at various time interval for testing. The blood test result shows that the population, P, of the bacteria is related to the time, t hours, after the injection, by the equation $P = 550 + 200e^{kt}$, where k is a constant. It takes **one day** for the population of bacteria to double.

(i) Find the population of the bacteria at the start of the experiment. [1]

(ii) Find the value of k. [2]

(iii) Find the percentage increase of the population of the bacteria when $t = 30$. [4]

(iv) The line $P = mt + c$ is a tangent to the curve $P = 550 + 200e^{kt}$ at the point where $t = 30$. Find the constant value of m and of c. [3]

(v) At $t = 50$, an antibiotics dosage is injected into the same mouse to stop the growth of bacteria. The dosage is able to kill the bacteria at a constant rate of 25 bacteria per hour. How much time needed for the dosage fully take its effectiveness? Hence sketch the graph of P against t for the whole experiment. [4]

10. A curve has the equation of $y = p(x-2)^2 - (x-3)(x+2)$ where p is a constant and $p \neq 1$.

(i) Find the range of values of p for which curve has a minimum point. [2]

Given that the curve touches the x-axis at point A.

(ii) Show that $p = \frac{25}{16}$. [3]

(iii) Find the coordinates of point A. [4]

(iv) Given that the line $y = mx + 2$ intersects the curve $y = p(x-2)^2 - (x-3)(x+2)$ at two distinct points where one of the points is at point A. Another line of the equation $y = mx + c$, is a tangent to the same curve at point B. Find the value of c where m and c are constants. [5]

End of Paper

Answers

$1i)\ x = 3,$

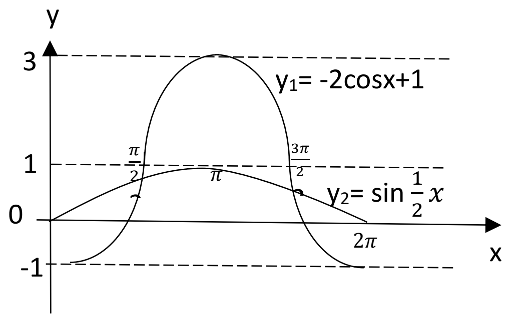
$1ii)\ \ k = -2$

2ii) $\frac{1}{2}x - \frac{1}{4}sin2x - 2ln\left(\frac{sinx}{1-cosx}\right) + c$

3ii)

Amplitude of y_1 = 2, Period of y_1 = 2π

Amplitude of y_2 = 1, Period of y_2 = 4π



3iii) x = 1.39, 4.89

3iv) $0 < x \leq 1.39$ Or $4.89 \leq x < 2\pi$

4) - k = $\frac{2}{5}$

5i) k = $\frac{2}{5}$

5ii) c = 5, n = 16

6i) $\left(\frac{1}{2}, -4\right)$

6ii) $\left(\frac{1}{2}, -4\right)$ *is point of inflexion* $-$ *Curve A*, $\left(\frac{1}{2}, -4\right)$ *is a* max *stationary point* $-$ *curve B*

6iii) $y_A = \frac{9}{2}(2x-1)^3 - 4$, $y_B = -\frac{27}{8}(2x-1)^4 - 4$

7i) Line PN is parallel to AB (Mid-Point Theorem)

7ii) $y = -\frac{2}{17}x + 5\frac{2}{17}$

7iii) $y = -\frac{4}{7}x + \frac{12}{7}$

7v) A = 27

8ii) $A = \frac{3\sqrt{133}}{2}\cos(\theta - 72.5°)$ $=17.3\cos(\theta - 72.5°)$

8iii) 42.6°

8iv) Max A = $\frac{3\sqrt{133}}{2}$ =17.3, $\theta = 72.5°$

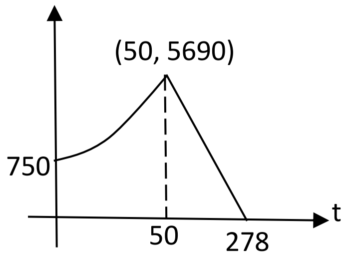
9i)750

9ii) k = $\frac{1}{24}\ln\frac{19}{4}$ = 0.0649

9iii) 160 %

9iv) m =91.1, c = -779

9v)



10i) p >1

10iii) $A\left(\frac{14}{3},0\right)$

10iv) c = $\frac{94}{49}$

2018 NCHS A-Math Prelim 2/ Paper 2 solution

(1i)

$$\frac{3\lg 3x - 2\lg x}{4} = \lg 3$$

3lg 3x -2lg x = 4lg3

$$\lg(3x)^3 - lgx^2 = lg3^4$$

$$\frac{(3x)^3}{x^2} = 3^4$$

$$27x = 81$$

x = 3

(1ii)

$$\log_{(x-2)}y = 2$$

$$y = (x-2)^2$$

$$y^{\frac{1}{2}} = x - 2$$

$$log_y(x+k) = \frac{1}{2}$$

$$x + k = y^{\frac{1}{2}}$$

x+ k = x-2

k = -2

$$\frac{1}{log_{(x-2)}y} = \frac{1}{2}$$

$$\log_y(x-2) = \frac{1}{2}$$

$$log_y(x+k) = \frac{1}{2}$$

k = -2

(2i)

$$\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right]$$

$$= lnsinx - ln(1 - cosx)$$

$$\frac{d}{dx}\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right] =$$

$$\frac{d}{dx}[lnsinx - ln(1-cosx)]$$

$$= \frac{cosx}{sinx} - \frac{-(-sinx)}{1-cosx}$$

$$= \frac{cosx}{sinx} - \frac{sinx}{1-cosx}$$

$$= \frac{cosx(1-cosx)-sin^2x}{sinx(1-cosx)}$$

$$= \frac{cosx-cos^2x-sin^2x}{sinx(1-cosx)}$$

$$= \frac{cosx-1}{sinx(1-cosx)}$$

$$= -\frac{1}{\sin x} \text{ (shown)}$$

(2ii)

$$\int \sin^2 x + \frac{2}{\sin x}dx$$

$$= \int \frac{1-cos2x}{2} + \frac{2}{sinx}dx$$

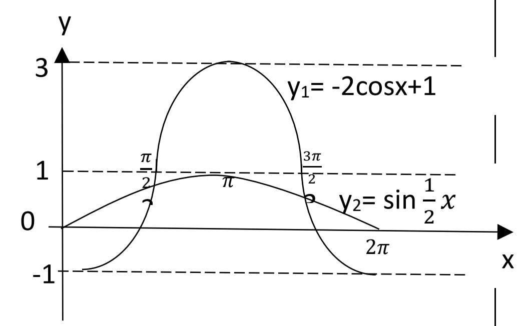
$$= \frac{1}{2}x - \frac{1}{4}sin2x - 2ln\left(\frac{sinx}{1-cosx}\right) + c$$

(3i)

Amplitude of y_1 = 2, Period of y_1 = 2π

Amplitude of y_2 = 1, Period of y_2 = 4π

(3ii)



(3iii)

$$-2\cos x + 1 = \sin\frac{1}{2}x$$

$$-2\left(1 - 2sin^2\frac{x}{2}\right) + 1 = sin\frac{1}{2}x$$

$$4sin^2\frac{x}{2} - sin\frac{1}{2}x - 1 = 0$$

$$sin\frac{1}{2}x = 0.6403882$$

$$\alpha = 0.69500$$

$$\frac{1}{2}x = 0.69500, \qquad \pi - 0.69500$$

= 0.695 or 2.4466

x = 1.39, 4.89

(3iv)

$0 < x \leq 1.39$ Or $4.89 \leq x < 2\pi$

(4)

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$C = 2\pi r$$

$$\frac{dC}{dr} = 2\pi$$

$$\pi r^2 = 400$$

$$r = \frac{20}{\sqrt{\pi}}$$

$$\frac{dA}{dr} \text{ x } \frac{dr}{dt} = \frac{dA}{dt}$$

$$2\pi r \text{ x } \frac{dr}{dt} = -4$$

(4)

$$\frac{dr}{dt} = \frac{-4}{2\pi r}$$

$$\frac{dC}{dt} = \frac{dC}{dr} \text{ x } \frac{dr}{dt}$$

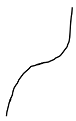
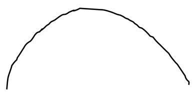
$$\frac{dC}{dt} = 2\pi \text{ x } \frac{-4}{2\pi r}$$

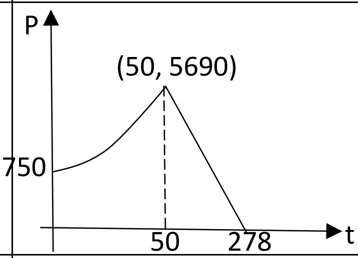
$$= \frac{-4}{r}$$

$$= \frac{-4}{\left(\frac{20}{\sqrt{\pi}}\right)}$$

$$= -\frac{\sqrt{\pi}}{5} \text{ cm/s}$$

= - 0.354 cm/s

(5i)	$\left(2+\frac{4}{x^4}\right)\left(kx^3-\frac{2}{x}\right)^{13}$ $=\left(2+\frac{4}{x^4}\right)\left(\frac{1}{x}, \quad x^3\right)$ $\left(kx^3-\frac{2}{x}\right)^{13}=\binom{13}{r}(kx^3)^{13-r}\left(-\frac{2}{x}\right)^r+\cdots$ $=\binom{13}{9}(kx^3)^4\left(-\frac{2}{x}\right)^9+\binom{13}{10}(kx^3)^3\left(-\frac{2}{x}\right)^{10}+\cdots$ $=715k^4x^{12}\left(-\frac{512}{x^9}\right)+286k^3x^9\left(\frac{1024}{x^{10}}\right)+\cdots$ $=-366080k^4x^3+\frac{292864}{x}k^3+\cdots$ $=\left(2+\frac{4}{x^4}\right)(-366080k^4x^3+\frac{292864}{x}k^3+\cdots)$ $=\frac{585728k^3}{x}-\frac{1464320}{x}k^4+\cdots$ $585728k^3-1464320k^4=0$ $k^3(585728-1464320k)=0$ k = $\frac{2}{5}$	(5ii)	$\left(1-\frac{c}{x}\right)^n=\binom{n}{1}\left(-\frac{c}{x}\right)+\binom{n}{2}\left(-\frac{c}{x}\right)^2+\cdots$ $=-\frac{nc}{x}+\frac{n(n-1)}{2}\cdot\frac{c^2}{x^2}+\cdots$ - nc = - 80 nc = 80 $\frac{n(n-1)c^2}{2}=3000$ $n^2c^2-nc^2=6000$ $80^2-80c=6000$ c = 5 , n =16
(6i)	Curve A --- $\frac{dy}{dx}=27(2x-1)^2$ $27(2x-1)^2=0$ $x=\frac{1}{2}$ $\left(\frac{1}{2},-4\right)$	(6i)	Curve B --- $\frac{dy}{dx}=-27(2x-1)^3$ $-27(2x-1)^3=0$ $x=\frac{1}{2}$ $\left(\frac{1}{2},-4\right)$
(6ii)	Curve A --- x = 0.4, x = 0.5, x = 0.6 $\frac{dy}{dx}>0 \quad \frac{dy}{dx}=0 \quad \frac{dy}{dx}>0$  $\left(\frac{1}{2},-4\right)$ *is point of inflexion*	(6ii)	Curve A --- x = 0.4, x = 0.5, x = 0.6 $\frac{dy}{dx}>0 \quad \frac{dy}{dx}=0 \quad \frac{dy}{dx}<0$  $\left(\frac{1}{2},-4\right)$ *is a maximum stationary point*
(6iii)	$y_A=\frac{27(2x-1)^3}{3(2)}+c$ c = -4 $y_A=\frac{9}{2}(2x-1)^3-4$	(6iii)	$y_B=\frac{-27(2x-1)^4}{4(2)}+c$ c = -4 $y_B=-\frac{27}{8}(2x-1)^4-4$
(7i)	Line PN is parallel to AB (Mid-Point Theorem)		
(7ii)	$m_{PN}=\frac{2-1}{-\frac{1}{2}-8}=-\frac{2}{17}$, $m_{AB}=-\frac{2}{17}$ $y=-\frac{2}{17}x+c_1$ $5=-\frac{2}{17}(1)+c_1$ $c_1=5\frac{2}{17}$ $y=-\frac{2}{17}x+5\frac{2}{17}$	(7iii)	$m_{MN}=\frac{5-1}{1-8}=-\frac{4}{7}$, $m_{AC}=-\frac{4}{7}$ $y=-\frac{4}{7}x+c_2$ $2=-\frac{4}{7}(-\frac{1}{2})+c_2$ $c_2=\frac{12}{7}$ $y=-\frac{4}{7}x+\frac{12}{7}$
(7iv)	$-\frac{2}{17}x+5\frac{2}{17}=-\frac{4}{7}x+\frac{12}{7}$ $x=-7\frac{1}{2}$ $y=-\frac{4}{7}\left(-\frac{15}{2}\right)+\frac{12}{7}$ $y=6$ $\left(-7\frac{1}{2},6\right)$ Shown	(7v)	A = $\frac{1}{2}\begin{vmatrix}-\frac{15}{2} & -\frac{1}{2} & 8 & 1 & -\frac{15}{2} \\ 6 & 2 & 1 & 5 & 6\end{vmatrix}$ $=\frac{1}{2}\left(\frac{61}{2}-\left(-\frac{47}{2}\right)\right)$ = 27

(8i)	$A = 3\left[\frac{1}{2}(3)(3)sin\theta + \frac{1}{2}(2)(2)\sin(120-\theta)\right]$ $=3\left[\frac{9}{2}sin\theta + (2)(sin120cos\theta - sin\theta cos120)\right]$ $=3\left[\frac{9}{2}sin\theta + 2(\frac{\sqrt{3}}{2}cos\theta - sin\theta(-\frac{1}{2}))\right]$ $=3\left[\frac{9}{2}sin\theta + \sqrt{3}cos\theta + sin\theta)\right]$ $=\frac{33}{2}\sin\theta + 3\sqrt{3}\cos\theta$ (shown)	(8ii)	R = $\sqrt{\left(\frac{33}{2}\right)^2 + (3\sqrt{3})^2} = \sqrt{\frac{1197}{4}} = \frac{3\sqrt{133}}{2}$ $\tan\alpha = \frac{\frac{33}{2}}{3\sqrt{3}}$ $\alpha = 72.5198$ $A = \frac{3\sqrt{133}}{2}\cos(\theta - 72.5°)$ $=17.3\cos(\theta - 72.5°)$
(8iii)	$\frac{3\sqrt{133}}{2}\cos(\theta - 72.5198°) = 15$ $\cos(\theta - 72.5198°) = 0.86711$ $\cos\alpha_1 = 0.86711$ $\alpha_1 = 29.8755$ $\theta - 72.5198 = 29.8755, -29.8755$ $\theta = 102.4\ (reject)\ \ or\ \ 42.6°$	(8iv)	Max A = $\frac{3\sqrt{133}}{2}$ =17.3 $\cos(\theta - 72.5198°) = 1$ $\cos\alpha_2 = 1$ $\alpha_2 = 0$ $\theta - 72.5198 = 0$ $\theta = 72.5°$
(9i)	$P = 550 + 200e^{kt}$ P = 550 + 200 = 750	(9ii)	$2(750) = 550 + 200e^{k(24)}$ $e^{24k} = \frac{19}{4}$ 24k = $\ln\frac{19}{4}$ k = $\frac{1}{24}\ln\frac{19}{4}$ = 0.0649
(9iii)	$P = 550 + 200e^{\left(\frac{1}{24}\ln\frac{19}{4}\right)(30)}$ $P = 550 + 200e^{\left(\frac{30}{24}\ln\frac{19}{4}\right)}$ = 1952.4811 $\frac{1952.4811-750}{750}$x100 = 160 %	(9iv)	$\frac{dP}{dt} = 200ke^{kt}$ $\frac{dP}{dt} = 200ke^{k(30)}$ $m = 200\left(\frac{1}{24}ln\frac{19}{4}\right)e^{\left(\frac{30}{24}\ln\frac{19}{4}\right)}$ m = 91.053 m =91.1 P = mt + c 1952.481 = 91.053(30) + c c = -779
(9v)		(9v)	$\frac{5688.169-0}{50-t}$ = -25 t = 277.56 t = 278 278 – 50 = 228 h
(10i)	$y = p(x-2)^2 - (x-3)(x+2)$ $\frac{dy}{dx} = 2p(x-2) - [(x-3)+(x+2)]$ $= 2p(x-2) - 2x + 1$ $\frac{d^2y}{dx^2} = 2p - 2 > 0$ $2p - 2 > 0$, p >1	(10i)	OR $y = (p-1)x^2 + x - 4px + 4p + 6$ (p - 1) > 0 (happy face since it is a min quadratic curve) p >1
(10ii)	$p(x-2)^2 - (x-3)(x+2)$ $p(x^2 - 4x + 4) - (x^2 - x - 6) = 0$ $px^2 - x^2 - 4px + x + 4p + 6 = 0$ $b^2 - 4ac = (-4p+1)^2 - 4(p-1)(4p+6)$ $=16p^2 - 8p + 1 - 16p^2 - 8p + 24$ = -16p+25 -16p + 25 =0 p = $\frac{25}{16}$ (shown)	(10iii)	y = $px^2 - x^2 - 4px + x + 4p + 6$ y = $px^2 - x^2 - 4px + x + 4p + 6$ $y = \frac{25}{16}x^2 - x^2 - 4\left(\frac{25}{16}\right)x + x + 4\left(\frac{25}{16}\right) + 6$ $y = \frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4}$ $\frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4} = 0$ $9x^2 - 84x + 196 = 0$ $(3x-14)^2 = 0$ $x = \frac{14}{3}$ $A\left(\frac{14}{3}, 0\right)$

(10iv)	$y = mx + 2$ $0 = m\left(\frac{14}{3}\right) + 2$ $m = -\frac{3}{7}$ $y = -\frac{3}{7}x + c$ $\frac{dy}{dx} = -\frac{3}{7}$ $2p(x-2) - 2x + 1 = -\frac{3}{7}$ $2\left(\frac{25}{16}\right)(x-2) - 2x + 1 = -\frac{3}{7}$ $x = \frac{30}{7}$ $y = \frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4}$ $y = \frac{9}{16}\left(\frac{30}{7}\right)^2 - \frac{21}{4}\left(\frac{30}{7}\right) + \frac{49}{4}$ $y = \frac{4}{49}$ $y = -\frac{3}{7}x + c$ $\frac{4}{49} = -\frac{3}{7}\left(\frac{30}{7}\right) + c$ $c = \frac{94}{49}$		