TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **Monday 21 August 2017**

2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The function f is defined, for all values of x, by

$$f(x) = -\frac{2}{3}x^3 - 3x^2 + 8x + 1.$$

Find the range of values of x such that f is a decreasing function. [3]

2 The graph of $y = -\log_2(x-a)$ has coordinates (7, -2) and (4, b).

(i) Determine the value of a and of b. [2]

(ii) Sketch the graph of $y = -\log_2(x-a)$. [2]

3 A fossil containing radioactive carbon–14 isotopes was recently found. The amount of carbon–14, N, can be expressed as $N = N_0e^{kt}$, where N_0 and k are constants and t refers to the time in years.

The time taken for half of the carbon–14 to decay is 5700 years.

(i) Show that $k = -0.0001216$. [2]

(ii) If the fossil now contains only 39% of its original amount of carbon–14, calculate to the nearest year, the age of the fossil. [3]

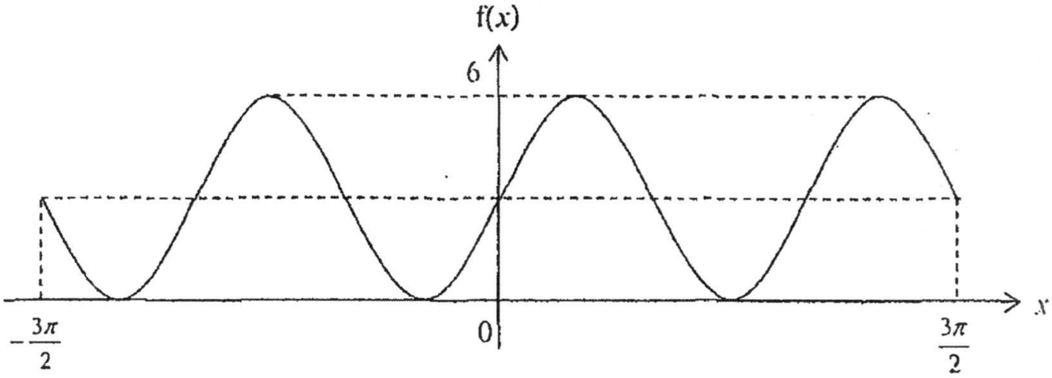
4 (i) Sketch the graph of $y = x^{\frac{2}{3}}$. [2]

(ii) The line $y = \frac{1}{2}x$ intersects the graph of $y = x^{\frac{2}{3}}$ at points A and B.
Find the equation of the perpendicular bisector of AB. [5]

5 (i) Given that $px^2 + qx + 2q$ is always negative, what conditions must apply to the constants p and q? [4]

(ii) Give an example of values of p and q which satisfy the conditions found in part (i). [1]

6



The figure shows part of the graph of $f(x) = a\sin bx + c$, where a, b and c are constants.

(i) State the period and amplitude of $f(x)$. [2]

(ii) State the value c. [1]

(iii) Show that $f(x)$ can be expressed as $3(\cos x + \sin x)^2$. [3]

7 (i) Prove that $\dfrac{1+\sec A}{\tan A + \sin A} = \operatorname{cosec} A$. [3]

(ii) Hence solve the equation $1 + \sec 2x = 5(\tan 2x + \sin 2x)$ for $-100° \leq x \leq 100°$. [4]

8 (i) Sketch the graph of $y = |5 - 3x| - 1$, showing all intercepts clearly. [3]

(ii) Find the coordinates of the point of intersection between the line $y = 5x$ and the graph of $y = |5 - 3x| - 1$. [2]

(iii) Determine the set of values of m for which the line $y = mx$ intersects the graph of $y = |5 - 3x| - 1$ at two points. [2]

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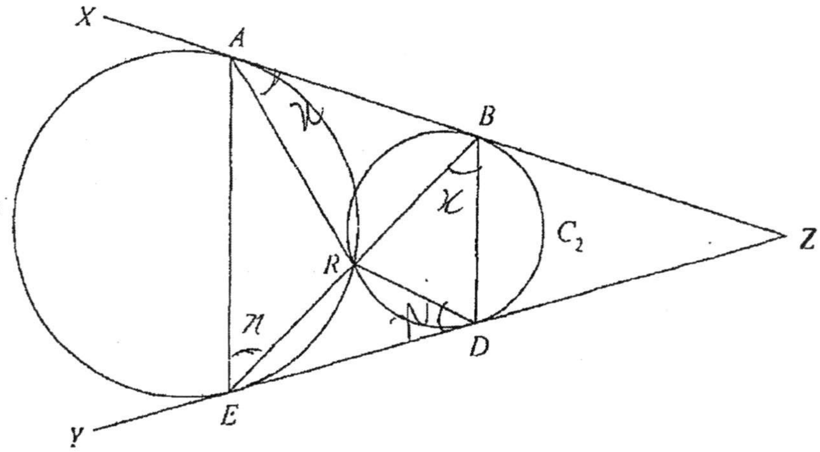

The diagram shows two intersecting circles, C_1 and C_2. The line XZ is tangent to C_1 and C_2 at A and B respectively. The line YZ is tangent to C_1 and C_2 at E and D respectively. Point B is such that $AB = BZ$. Point R lies on both circles and ERB is a straight line.

(i) Prove that AE and BD are parallel. [4]

(ii) Prove that angle BAR = angle RDE. [3]

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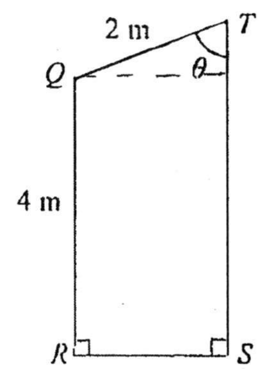


The diagram shows a trapezoidal glass window, $QRST$. The lengths of QR and QT are 4 m and 2 m respectively. Angle QTS is given as θ, where $0 < \theta < \frac{\pi}{2}$.

(i) Show that the area, A m^2, of the window is given by
$$A = 8\sin\theta + \sin 2\theta.$$ [4]

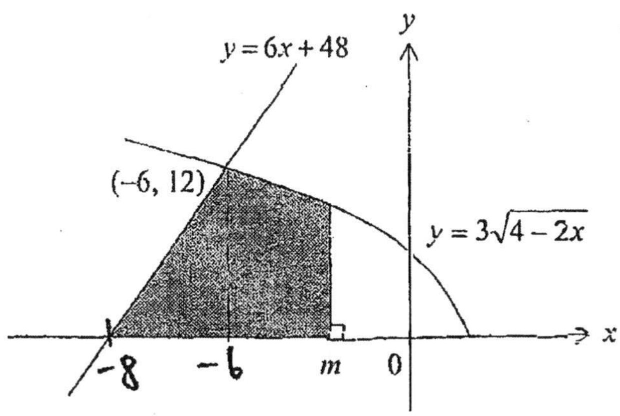
(ii) Given that θ can vary, find the value of θ which gives the greatest area of the window. [5]

11 The equation of a curve is $y = 3\sqrt{4-2x}$.

(i) A particle P moves along the curve in such a way that the y-coordinate of P decreases at a constant rate of 0.12 units per second.

Find the y-coordinate of P at the instant when the x-coordinate of the particle is increasing at 0.16 units per second. [5]

(ii) The diagram shows part of the curve $y = 3\sqrt{4-2x}$ and the line $y = 6x + 48$ which intersects the curve at (–6, 12).



Given that the area of the shaded region is 49 units^2, find the value of the constant m. [6]

12 A car is travelling along a straight road from Junction X to Junction Y. It passes X with a speed of 15 m/s. At the same instant, the driver spots a red traffic light at Y and applies the brakes immediately. During the journey from X to Y, the acceleration, a m/s^2, of the car, t seconds after passing X, is given by $a = -t - 3.5$.

(i) Show that the time taken for the car to come to an instantaneous rest after passing X is 3 seconds. [4]

(ii) Given that the distance between X and Y is 24 m, determine whether the car will be able to stop in time for the red light at Y. Justify your answer. [4]

(iii) State an assumption made. [1]

End of Paper

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS — 4047/02

Paper 2

Thursday 24 August 2017

2 hours 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

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Answer all questions.

1 A curve has equation $y = \frac{x}{e^{2x}}$.

(i) Find the gradient function for the curve. [3]

(ii) The equation of the normal to the curve at $x = 1$ cuts the axes at $(0, p)$ and $(q, 0)$. Find the value of p and of q, leaving your answers in terms of e. [6]

2 (i) Differentiate $3x \sin 2x$ with respect to x. [3]

(ii) Evaluate $\int_0^{\frac{\pi}{4}} x \cos 2x \, dx$. [6]

3 (a) Sketch the graph of the equation $y = e^{-2x}$. [2]

(b) (i) The gradient function of a curve is given by $\frac{dy}{dx} = e^{2x} + e^{-2x}$.

Explain why the function is an increasing function. [2]

(ii) Find the equation of the function. [3]

4 (a) (i) Expand and simplify $(1 + a)^8$, in ascending powers of a, up to the term containing a^3. [2]

(ii) Given that $a = x + x^2$, write down the expansion of $(1 + x + x^2)^8$ up to the term containing x^3. [3]

(iii) Using your expansion and a suitable value for x, find the value of 1.0101^8, giving your answer correct to 6 decimal places. [2]

(b) (i) Write down the general term in the binomial expansion of $\left(3x - \frac{2}{x^2}\right)^{12}$.

Write down the power of x in this general term. [2]

(ii) Hence, explain why there is no term in x^5 in the binomial expansion of $\left(3x - \frac{2}{x^2}\right)^{12}$. [2]

5 The roots of the equation $2x^2 - x + 6 = 0$ are p and q.

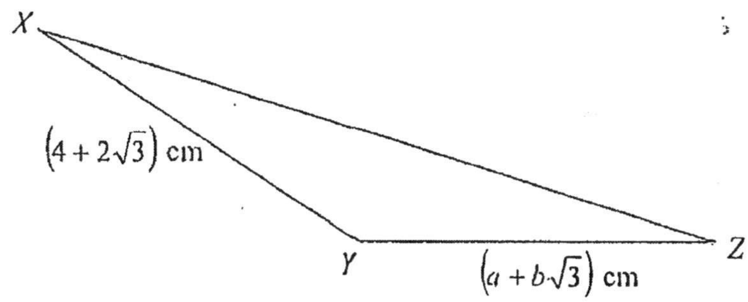
(i) Find the value of $p^2 + q^2$. [3]

(ii) Find the value of $p^3 + q^3$. [2]

(iii) Find a quadratic equation whose roots are $p^2 - q$ and $q^2 - p$. [5]

6 (i) Express $\dfrac{60+36\sqrt{3}}{4+2\sqrt{3}}$ in the form $r+s\sqrt{3}$, where r and s are integers. [3]

(ii) The diagram shows a triangle XYZ.



XY is $(4+2\sqrt{3})$ cm and YZ is $(a+b\sqrt{3})$ cm, where a and b are integers.
The included angle XYZ is 120°.
Given that the area of the triangle is $(15+9\sqrt{3})$ cm^2, find the value of a and of b. [5]

7 A point P has coordinates (8, 0). Another point Q is vertically above P such that $\tan \angle POQ = \dfrac{3}{4}$, where O is the origin.

(i) Find the coordinates of Q. [2]

(ii) Given that OQ is a diameter of a circle, find the equation of the circle. [3]

(iii) A point R is such that angle $ORP = 38°$.
Determine whether R lies within the circle, stating your reasons clearly. [3]

8 (i) Find the remainder when $3x^3 - x^2 + 12x$ is divided by $3x - 1$. [2]

(ii) Hence, state the remainder when $3x^3 - x^2 + 12x + c$ is divided by $3x - 1$. [1]

(iii) Factorise $3x^3 - x^2 + 12x - 4$ completely, showing your workings clearly. [3]

(iv) Express $\dfrac{11x^2+9x+4}{3x^3-x^2+12x-4}$ in partial fractions. [5]

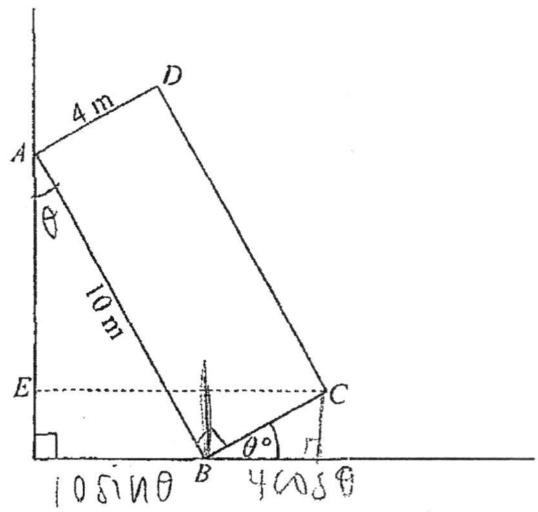
9 A curve has equation $y = (x-1)\ln 2x$, for all values of $x > 0$.

(i) Find $\dfrac{dy}{dx}$. [3]

(ii) Hence, explain why the curve has a stationary point for a value of x between 0.5 and 1. [2]

(iii) Determine the nature of the stationary point. [1]

10 The diagram shows a cuboid structure $ABCD$ that is placed tilted against a vertical wall.



AB is 10 metres, AD is 4 metres and the side BC makes an angle $\theta°$ with the floor. EC is horizontal distance of C from the wall.

(i) Show that EC can be expressed in the form $a \sin\theta + b\cos\theta$, where a and b are constants to be found. [2]

(ii) Express EC in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is an acute angle. [4]

The structure $ABCD$ will remain tilted against the wall provided EC lies between 7.5 metres and 9.5 metres.

(iii) Find the range of values of θ for the structure $ABCD$ to remain tilted. [4]

11 The amount of active ingredients in a lotion brand L, y units, is dependent on the time, t years after the lotion is manufactured.
The variables y and t are related by the equation $y = ae^{bt}$, where a and b are constants.
Some values of y and t are shown in the table below.

t (years)	1	2	4	5	6
y (units)	950.6	303.2	30.8	9.8	3.1

(i) Plot a graph of $\ln y$ against t. [2]

(ii) Using your graph, find the value of a and of b. [4]

(iii) Find the maximum amount of active ingredients after 3 years. [2]

The amount of active ingredients in another lotion brand M is given by the equation $y = \dfrac{1000}{e^{\frac{t}{2}}}$.

(iv) By adding a suitable straight line to your graph, find the time after which brand M contains a higher level of active ingredients. [3]

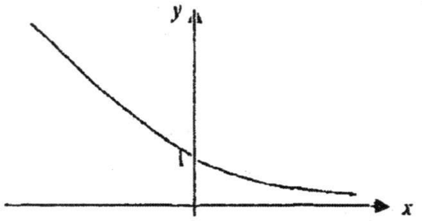
End of Paper

Answers:

1 (i) $\frac{dy}{dx} = \frac{1-2x}{e^{2x}}$ (ii) $p = \frac{1}{e^2} - e^2$ $q = 1 - \frac{1}{e^4}$

2 (i) $3 \sin 2x + 6x \cos 2x$. (ii) $\frac{1}{6}\left(\frac{3\pi}{4} - \frac{3}{2}\right)$

3 (a)



(b) (i) $e^{2x} > 0$ for all values of x

$\therefore \frac{dy}{dx} = e^{2x} + e - 2x > 0$

Hence, function is increasing

4 (a) (i) $1 + 8a + 28a^2 + 56a^3 + \ldots$ (ii) $1 + 8x + 36x^2 + 112x^3 + \ldots$

(iii) 1.083712

(b) (i) General term is $\binom{12}{r}(3x)^{12-r}\left(-\frac{2}{x^2}\right)^r$ Power of $x = 12 - 3r$

(ii) $r = \frac{7}{3}$, Since $\frac{7}{3}$ is not an integer, there is no term in x^5.

5 (i) $-\frac{23}{4}$ (ii) $-\frac{35}{8}$ (iii) $8x^2 + 50x + 131 = 0$

6 (i) $6 + 6\sqrt{3}$ (ii) $a = 6, b = 2$

7 (i) (8, 6) (ii) $(x-4)^2 + (y-3)^2 = 25$

(iii) $\angle ORP = 38° < 53.1$, $\therefore$ R lies outside the circle.

8 (i) 4 (ii) $4 + c$ (iii) $(3x-1)(x^2+4)$ (iv) $\frac{2}{(3x-1)} + \frac{3x+4}{(x^2+4)}$

9 (i) $\ln(2x) + 1 - \frac{1}{x}$ (ii) Since $\frac{dy}{dx} < 0$ for $x = 0.5$ and $\frac{dy}{dx} > 0$ for $x = 1$,

$\therefore \frac{dy}{dx} = 0$ for $0.5 < x < 1$

10 $EC = 10 \sin \theta + 4 \cos \theta$ (ii) $\sqrt{116} \sin(\theta + 21.80°)$ (iii) $22.3° < \theta < 40.1°$

11 (ii) $a = 2981$ (accept 2836 to 3134), $b = -1.14$ (accept 1.13 to 1.16)

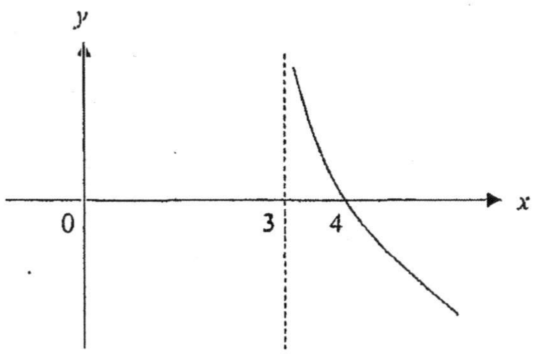
(iii) 94.6 units (accept 94.6 to 99.5) (iv) 1.7 years

Answers:

1 $x < -4$ or $x > 1$

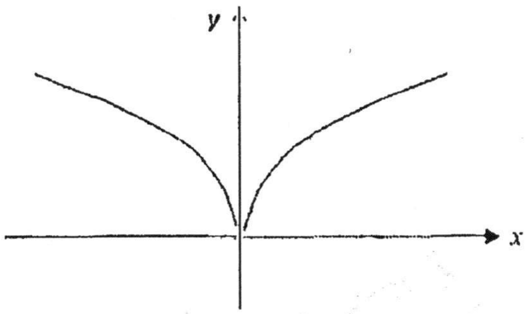
2(i) $a = 3, b = 0$

(ii)



3(ii) 7743 or 7744 years

4(i)



(ii) $y = -2x + 10$

5(i) $p < 0$ & $8p < q < 0$

(ii) Any answer as long as:

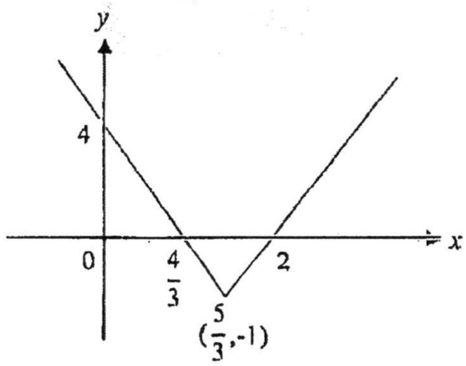
- p and q are negative, and
- $8p < q$

6(i) Period = π, Amplitude = 3

(ii) $c = 3$

7(ii) $x = -95.8°, 5.8°, 84.2°$

8(i)



(ii) Pt of intersection: $(\frac{1}{2}, \frac{5}{2})$

8(iii) $-\frac{3}{5} < m < 3$

10(ii) $\theta = 1.3441$

11(i) $x = -6, y = 12$

(ii) $m = -2.5$

12(ii) $s = 24.75$m, hence the car will not be able to stop in time.

(iii) The driver does not have any reaction time before stepping on the brakes / brakes are not faulty.