Answer all the questions.

1 A curve has the equation $y = 2x^3 \ln x$, where $x > 0$.

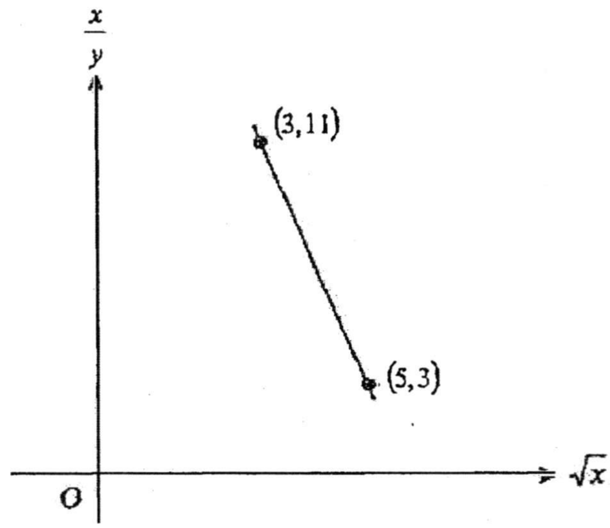
(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Show that x-coordinate of the turning point is $\dfrac{1}{\sqrt[3]{e}}$ and determine whether the turning point is a maximum ~~or a minimum.~~ [4]

2 (a) Find the range of values of p for which the expression $(p+6)x^2 - 8x + p$ is always positive for all real values of x. [4]

(b) Show that the line $y = \dfrac{x}{k} + \dfrac{k}{4}$ is a tangent to the curve $y^2 = x$ for all real values of k. [3]

3 The diagram shows part of a straight line graph drawn to represent the equation $y = \dfrac{x}{b\sqrt{x} - a}$, where a and b are constants.



Given that the line passes through $(3,11)$ and $(5,3)$, find the values of a and of b. [4]

4 (a) Using an appropriate substitution, or otherwise,

solve $\left(\sqrt{9}\right)^{2x} - 3^{x+2} = 6\left(3^x\right) - 54$. [6]

(b) **Without using a calculator**, find the value of 10^x, given

that $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$. [3]

5 (a) (i) State the values between which the principal value of $\cos^{-1} x$ must lie. [1]

(ii) Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ in radians. [1]

(b) It is given that $\cos A = \frac{4}{5}$ where $270° < A < 360°$

Without the use of calculator, find the exact value of each of the following.

(i) $\cot 2A$. [3]

(ii) $\sin \frac{1}{2} A$. [2]

6 (a) Evaluate $\int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}}\right) dx$. [3]

(b) The gradient of a curve is $\frac{12}{(4x-1)^2}$. Given that the curve passes through the

points $\left(\frac{1}{2}, 5\right)$ and $(-2, k)$, find the value of k. [4]

Questions 7 to 11 must be handed in separately from Questions 1 to 6.

Begin your answer to question 6 on a fresh sheet of paper.

7 It is given that $3x+4y=k$ and $(6x-20)^2+(4y+3)^2=200$.

(i) If $k=8$, find the solutions of these simultaneous equations. [4]

(ii) If $k=-10$, show that there are no solutions without solving the equations. [2]

(iii) Explain why there cannot be more than 2 solutions for all values of k. [2]

8 Express $\dfrac{2x^3-5x^2-11x+44}{x^2-x-6}$ in partial fractions. [5]

9 (a) Find the equation of the normal to the curve $y=\sin 4x-3\cos 2x$ at the point where $x=\dfrac{\pi}{12}$. [7]

(b) Singapore has two high tides and two low tides a day. The tidal movement on East Coast beach during a particular day can be modelled by the curve $y=\sin x$.
If the four tides occur at 5 am, 11 am, 5 pm and 11 pm respectively, at what time will the flow of the water onto East Coast beach be the fastest? [1]

10 (a) Given that the coefficient of x^2 in the expansion of $(1-3x)^2(1-kx)^8$ is 117, find the two possible values of the constant k. [5]

(b) Find the term independent of x in the expansion of $\left(\dfrac{1}{x}-\dfrac{x^3}{4}\right)^{24}$. [4]

11 A curve has the equation $y=\left(\frac{x}{2}+1\right)^2-4$.

(i) Explain why the lowest point on the curve has the coordinates $(-2,-4)$. [2]

(ii) Find the x–coordinates of the points at which the curve intersects the x–axis. [2]

(iii) Sketch the graph of $y=\left|\left(\frac{x}{2}+1\right)^2-4\right|$, indicating clearly the coordinates of the turning point and the points where the curve meets the axes. [3]

(iv) State the set of the values of k for which the line $y=k$ intersects the curve

(a) at 2 distinct points, [2]

(b) at 4 distinct points. [1]

End of Paper 1

Answer all the questions.

1 A particular species of fish living in a fish farm is being studied. After t years, its population P is given by $P = 300(2 + 5e^{-kt})$ where k is a constant.

(a) Find the initial population of the fish in the farm. [1]

The population of the fish in the farm after 3 years is predicted to be 2400.

(b) Find the value of k. [2]

The fish farm owner has to replenish the supply of fish in the farm when the population drops below 1000.

(c) Using the k value obtained in part (b), determine, with working, whether the fish farm owner needs to replenish the fish supply after 5 years. [2]

2 Given that $f(x) = 3x^4 + x^3 - mx^2 - nx + 36$,

(a) find the values of m and n when $(x^2 - 9)$ is a factor of $f(x)$, [4]

(b) hence solve the equation $f(x) = 0$. [3]

3 The quadratic equation $2x^2 + 4x - 7 = 0$ has roots α and β.

(a) State the values of $(\alpha + \beta)$ and $\alpha\beta$. [2]

(b) Find the value of $\alpha^2 + \beta^2$. [2]

(c) Hence, form a quadratic equation whose roots are $\frac{2}{\alpha^2}$ and $\frac{2}{\beta^2}$. [3]

4 (a) Solve the equation $\log_2(x-3) - 6\log_{x-3}(2) = 1$. [5]

(b) Given that $w = \log_5 a$, find, in terms of w,

(i) $\log_5 \frac{5}{a}$, [1]

(ii) $(\log_5 a)^4$ [1]

(iii) $\log_5 125a^2$ [2]

5 (a) (i) Prove that $\dfrac{1+\cos x}{1-\cos x} \equiv \cot^2 \dfrac{x}{2}$. [3]

(ii) Hence prove that $\cot^2 15^\circ = 7 + 4\sqrt{3}$. [3]

(b) Solve the equation $\dfrac{4\sec x}{1+\sec^2 x} = -1$ for $0 \le x \le 360^\circ$, giving your answers correct to 2 decimal places. [5]

6 The area of a triangle is $\left(3+\sqrt{15}\right)$ cm².

(a) In the case whereby the triangle is a right angle triangle with height $\left(\sqrt{5}-\sqrt{3}\right)$ cm, find, without using a calculator, the length of the base of this triangle in the form $\left(a\sqrt{5}+b\sqrt{3}\right)$ cm. [4]

(b) In the case whereby the triangle is an equilateral triangle with the length of each side w cm, find the value of w^2, giving your answer in the form $a\left(\sqrt{3}+\sqrt{5}\right)$. [4]

Begin Question 7 on a fresh sheet of Answer Paper.

7 A curve has the equation $y = x(3-x)^3$. Points A and B are the two stationary points on the curve.

(a) Find the coordinates of points A and B. [4]

(b) Determine the nature of these 2 stationary points on the curve. [4]

(c) (i) Find the values of x for which y is increasing. [1]

(ii) Find the values of x for which y is decreasing. [1]

8 **Answer the whole of this question on the graph paper provided.**

The table below shows the experimental values of x and y which are related by the equation $y = b^{a-x}$. One value of y has been recorded wrongly.

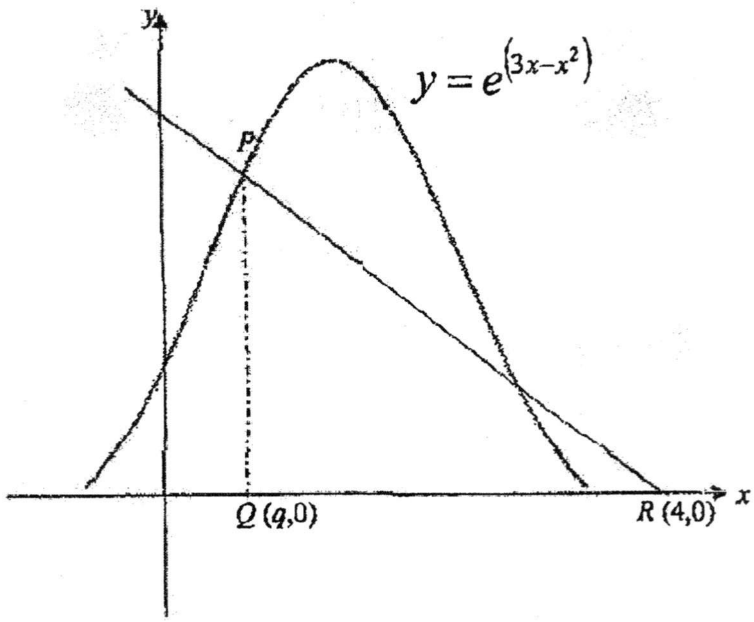
x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1

(a) Plot $\lg y$ against x and draw a straight line graph. [4]

(b) Use your graph to estimate the values of a and b. [4]

(c) Determine which value of y is incorrect and estimate the correct value of y. [2]

9 The diagram below shows the graph of $y = e^{(3x-x^2)}$. Points Q and R lie on the x-axis such that their coordinates are $(q, 0)$ and $(4, 0)$ respectively. P is a point on the curve such that PQ is parallel to the y-axis.

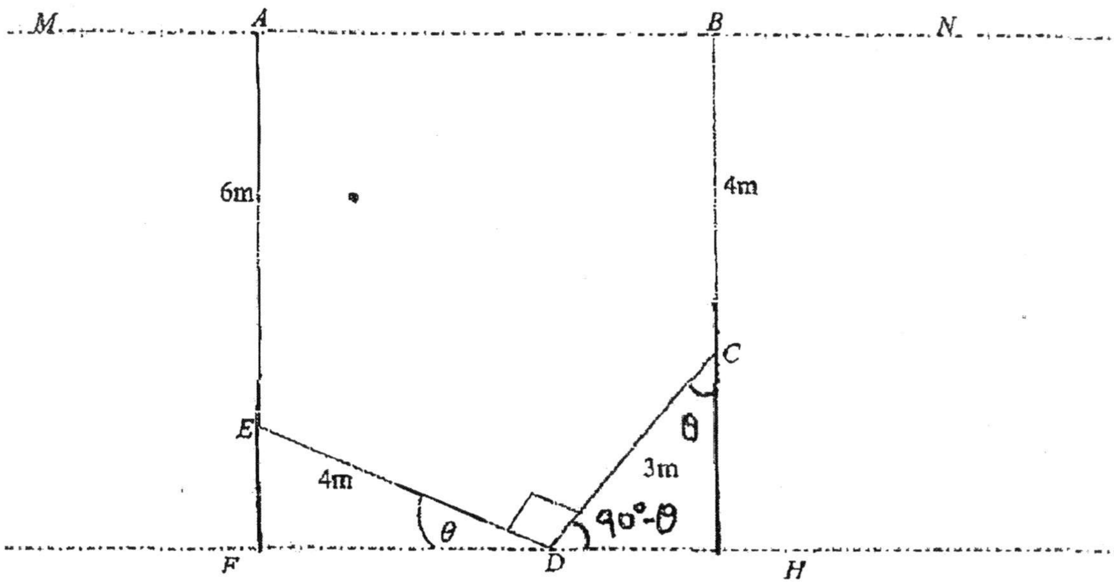


(a) Express the coordinates of P in terms of q. [1]

(b) Show that the area of triangle PQR, $A = \left(2 - \frac{q}{2}\right)\left(e^{3q-q^2}\right)$. [2]

(c) If q is decreasing at a rate of 2 units per second, find the rate at which the area of triangle PQR is changing at the instant when $q = 2$. [4]

10



In the diagram above, $AE = 6$ m, $BC = DE = 4$ m and $CD = 3$ m, $\angle CDE = 90^\circ$ $\angle EDF = \theta$, AE and BC are both perpendicular to the line MN. MN is parallel to FH.

(a) Show that $AB = 4\cos\theta + 3\sin\theta$. [2]

(b) Express AB in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$. [3]

(c) Find the maximum perimeter, P, of the figure and the corresponding value of θ. [4]

11 The function f is defined, for $x \geq 0$, by $f(x) = p\cos\left(\frac{x}{2}\right) - q$.

(a) State the period of $f(x)$. [1]

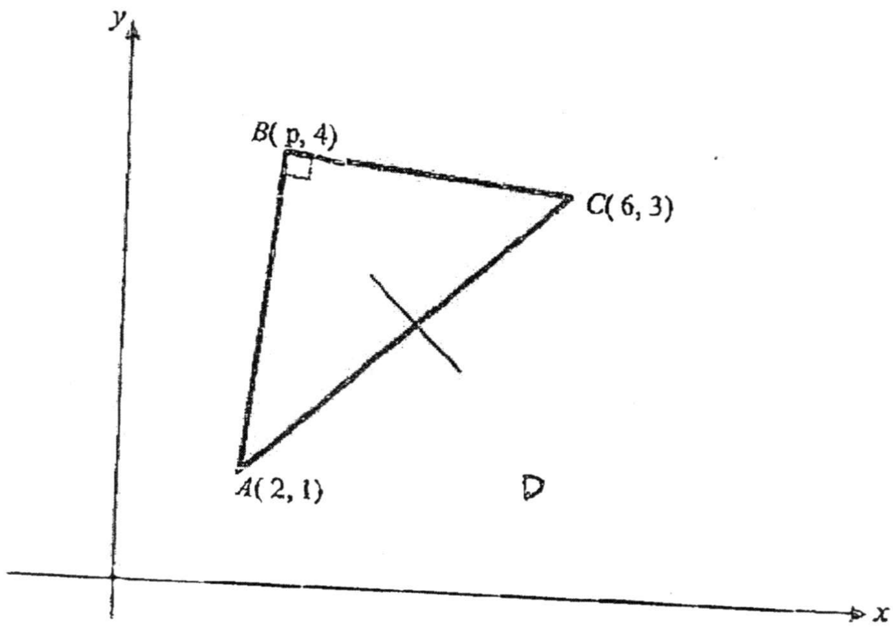
Given that the maximum and minimum values of $f(x)$ are 1 and -5 respectively, find

(b) the amplitude of f, [1]

(c) the values of p and q, [2]

(d) Using the values of p and q found in **part (iii)**, sketch the graph of $f(x)$ for $0 \leq x \leq 3\pi$. [3]

12 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a triangle ABC in which point A is (2, 1), point B is $(p, 4)$ and point C is (6, 3). The line AB is perpendicular to the line BC.

Find

(a) the value of p, where $p < 4$, [3]

(b) the equation of the perpendicular bisector of AC. [3]

The point D is such that $ABCD$ is a square. Find, using the value of p from part (i),

(c) the coordinates of D, [2]

(d) the area of $ABCD$. [2]

~ End of Paper 2 ~

Paper 1 Answer

Q1

SEC 4 EXP A-MATHS SA1 PAPER1 (2017)

Q1 (i) $y = 2x^3 \ln x, \ x > 0$

$\frac{dy}{dx} = 2x^3\left(\frac{1}{x}\right) + (\ln x)(6x^2)$ does not accept $\ln x \ \ 6x^2$

$= 2x^2 + 6x^2 \ln x$
[B1] [B1]

1(ii) when $\frac{dy}{dx} = 0$, $2x^2 + 6x^2 \ln x = 0$ [B1]

Does not know how to solve the equation by factorisation $2x^2 + 6x^2 \ln x = 0$

$2x^2(1 + 3\ln x) = 0$

Since $x \neq 0$, $\therefore 1 + 3\ln x = 0$

$\ln x = -\frac{1}{3}$

$\therefore x = e^{-\frac{1}{3}}$

$x = \frac{1}{\sqrt[3]{e}}$ (shown) [M1]

$\frac{d^2y}{dx^2} = 4x + 6x^2\left(\frac{1}{x}\right) + (\ln x)(12x)$

$= 10x + 12x \ln x$ [B1]

When $x = \frac{1}{\sqrt[3]{e}}$, $\frac{d^2y}{dx^2} = 10\left(\frac{1}{\sqrt[3]{e}}\right) + 12\left(\frac{1}{\sqrt[3]{e}}\right)\ln\left(\frac{1}{\sqrt[3]{e}}\right)$

$= 4.299 > 0$ (min). [B1]

$\therefore$ the turning pt is a Minimum point.

Q2

Q2(a): $b^2-4ac<0$ and $(p+6)>0$

$(-8)^2-4(p+6)(p)<0$ [M1] $p>-6$ [M1]

$64-4p^2-24p<0$

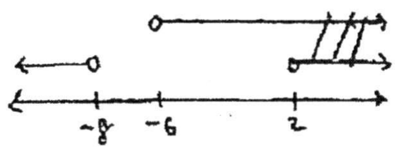
$4p^2+24p-64>0$

$p^2+6p-16>0$

$(p+8)(p-2)>0$

$p<-8$ or $p>2$ [M1]

Mistake made:
(1) Used $b^2-4ac>0$
(2) not indicate $p+6>0$
(3) Did not write down the final answer



Ans: $p>2$ [A1]

Q2(b): $\left(\frac{x}{k}+\frac{k}{4}\right)\left(\frac{x}{k}+\frac{k}{4}\right)=x$ [B1]

$\frac{x^2}{k^2}+\frac{x}{2}+\frac{k^2}{16}=x$

$\frac{x^2}{k^2}-\frac{x}{2}+\frac{k^2}{16}=0$

$16x^2-8k^2x+k^4=0$

$b^2-4ac=(-8k^2)^2-4(16)(k^4)$ [B1]

$=64k^4-64k^4$

$=0$

Since $b^2-4ac=0$ for all values of k, the line $y=\frac{x}{k}+\frac{k}{4}$ is a tangent to the curve $y^2=x$ (shown) [M1]

Alternate solution:

$y=\frac{y^2}{k}+\frac{k}{4}$

$4ky=4y^2+k^2$

$4y^2-4ky+k^2=0$

$(2y-k)^2=0$

$y=\frac{k}{2}$

there is only one solution

$\therefore y=\frac{x}{k}+\frac{k}{4}$ is a tangent to the curve

Students did not present the working correctly.

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Q3

Q3:

$$y = \frac{x}{b\sqrt{x} - a}$$

$$b\sqrt{x} - a = \frac{x}{y}$$

$$\frac{x}{y} = b\sqrt{x} - a \qquad [M1]$$

Let $Y = \frac{x}{y}$, $X = \sqrt{x}$.

then grad = b and vertical intercept = $-a$.

$$\text{Grad} = b = \frac{11-3}{3-5} = -4 \qquad [A1]$$

$$Y = -4X + c$$

When $X=5$, $Y=3$, we have,

$$3 = -4(5) + c$$

$$c = 23. \qquad [M1]$$

$$\therefore -a = 23$$

$$\therefore a = -23 \qquad [A1]$$

OR $(3, 11)$ $X = 3$, $Y = 11$

$$11 = -4(3) + c$$

$$c = 23$$

Q 4

Q. 4(a) $(\sqrt{9})^{2x} - 3^{x+2} = 6(3^x) - 54$

$(3^x)^2 - 9(3^x) = 6(3^x) - 54$

$(3^x)^2 - 15(3^x) + 54 = 0$ [M1]

Let $u = 3^x$.

$\therefore u^2 - 15u + 54 = 0$

$(u-6)(u-9) = 0$ [M1]

$\therefore u = 6$ or $u = 9$ [B1]

$3^x = 6$ $\qquad$ $3^x = 9$

$x \lg 3 = \lg 6$ $\qquad$ $3^x = 3^2$

$\therefore x = \frac{\lg 6}{\lg 3}$ [M1] $\qquad$ $\Rightarrow x = 2$ # [A1]

$= 1.63$ # (3s.f) [A1]

(1) Students gave answer $u=6$ or $u=9$. Did not solve the equations to find values of x.

(2) Did not know how to solve $3^x = 6$

4(b) $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$

$32(2^{2x}) \times \frac{5^x}{25} = 5^{2x} \times 8(2^{3x})$ [M1]

$\frac{32}{8 \times 25} = \frac{5^{2x} \times 2^{3x}}{5^x \times 2^{2x}}$ [M1]

$5^x \times 2^x = \frac{4}{25}$

$\Rightarrow 10^x = \frac{4}{25}$ #. [A1]

Students did not change 8^{x+1} to 2^{3x+3}

Q5

5(a)(i) Let the principal value of $\cos^{-1}x$ be θ.

$\therefore 0 \leq \theta \leq \pi$ *. [B1]

Most students did not know what principal value is.

5(a)(ii) Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \theta$, $0 \leq \theta \leq \pi$

$\therefore \cos\theta = -\frac{1}{\sqrt{2}}$

basic $\measuredangle \alpha = \frac{\pi}{4}$

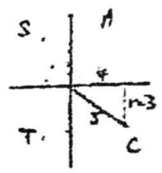
$\therefore \theta = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$ * [B1]

Did not know that $\cos^{-1}x$ is an angle

5(b)(i) $\cos A = \frac{4}{5}$, $270° < A < 360°$.

$\tan A = -\frac{3}{4}$ [B1]



$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= \frac{2(-\frac{3}{4})}{1-(-\frac{3}{4})^2}$ [M1]

$= -\frac{24}{7}$

$\therefore \cot 2A = -\frac{7}{24}$ * [A1]

Did not know that
$\sin A = -\frac{3}{5}$
$\tan A = -\frac{3}{4}$

5(b)(ii) $\cos 2A = 1 - 2\sin^2 A$

$\cos A = 1 - 2\sin^2\frac{A}{2}$

$\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$

$= \frac{1-\frac{4}{5}}{2}$ [M1]

$= \frac{1}{10}$

$\therefore \sin\frac{1}{2}A = \frac{1}{\sqrt{10}}$ * [A1] or $-\frac{1}{\sqrt{10}}$ (N.A.)

Most students tried to use $\sin 2A = 2\sin A\cos A$ formula

Q6

Q.6(a) $\int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}}\right) dx$

$= \int_1^9 \left(x^{\frac{1}{2}} + 2 - 4x^{-\frac{1}{2}}\right) dx$

$= \left[\underbrace{\frac{2x^{\frac{3}{2}}}{3}}_{[B1]} + \underbrace{2x - 8x^{\frac{1}{2}}}_{[B1]}\right]_1^9$

$= \left[\frac{2(9)^{\frac{3}{2}}}{3} + 2(9) - 8(9)^{\frac{1}{2}}\right] - \left[\frac{2}{3} + 2 - 8\right]$

$= 12 - \left(-5\frac{1}{3}\right)$

$= 17\frac{1}{3}$ ※. [B1]

6(b) $\frac{dy}{dx} = \frac{12}{(4x-1)^2} = 12(4x-1)^{-2}$

$\therefore y = \frac{12(4x-1)^{-1}}{(-1)(4)} + c = \underbrace{\frac{-3}{4x-1}}_{[B1]} + c$

When $x = \frac{1}{2}$, $y = 5$,

$5 = \frac{-3}{4(\frac{1}{2})-1} + c$

$c = 8$ [B1]

$\therefore y = \frac{-3}{4x-1} + 8$

When $x = -2$,

$k = \frac{-3}{4(-2)-1} + 8$ [M1]

$k = 8\frac{1}{3}$ ※. [A1]

Some students tried to use

$m = \frac{y_1 - y_2}{x_1 - x_2}$

formula

Q7

$3x + 4y = 8$

$4y = 8 - 3x$ —— ① $(6x-20)^2 + (4y+3)^2 = 200$ —— ②

Subst. ① into ②, $(6x-20)^2 + (8-3x+3)^2 = 200$ [M1]

$36x^2 - 240x + 400 + (-3x+11)^2 = 200$

$36x^2 - 240x + 400 + 9x^2 - 66x + 121 - 200 = 0$

$45x^2 - 306x + 321 = 0$

$\therefore x = \dfrac{(306) \pm \sqrt{(-306)^2 - 4(45)(321)}}{2(45)}$ [M1]

$= \dfrac{306 \pm \sqrt{35856}}{90}$

$= 5.50$ (3s.f.) or 1.30 (3s.f.) [A1]

$\therefore y = -2.13$ (3s.f.) or 1.03 (3s.f.) [A1]

* many studs do not show working for how they solve for x or y in the quad eqⁿ.
* many studs forgot to solve for y.

7(ii) If $k = -10$, then $4y = -10 - 3x$.

$\therefore (6x-20)^2 + (-10-3x+3)^2 = 200$

$36x^2 - 240x + 400 + (-3x-7)^2 = 200$

$36x^2 - 240x + 400 + 9x^2 + 42x + 49 - 200 = 0$

$45x^2 - 198x + 249 = 0$ [B1]

$b^2 - 4ac = (-198)^2 - 4(45)(249)$

$= -5616 < 0$ [B1]

Since $b^2 - 4ac < 0 \Rightarrow$ no real roots $\Rightarrow$ no solⁿs.

7(iii) Since $4y = k - 3x$,

$(6x-20)^2 + (k-3x+3)^2 = 200$

Addition of two quadratic expressions in this eqⁿ will result in a quadratic equation. [B1]

And the max. no. of roots in a quad eqⁿ is 2. [B1]

* many students thought that $(6x-20)^2 + (4y+3)^2 = 200$ is a quadratic curve.

Q8

Q 8 :

$$\begin{array}{r} 2x-3 \\ x^2-x-6 \overline{\smash{)}\,2x^3-5x^2-11x+44} \\ -(2x^3-2x^2-12x) \\ \hline -3x^2+x+44 \\ -(-3x^2+3x+18) \\ \hline -2x+26 \end{array}$$

* Many students did not find the quotient. They went straight into express the fract^n as partial fractions.

[B1]

$$\therefore \frac{2x^3-5x^2-11x+44}{x^2-x-6} = 2x-3 + \frac{-2x+26}{x^2-x-6}$$

$$\frac{-2x+26}{x^2-x-6} = \frac{-2x+26}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \quad [B1]$$

$$-2x+26 = A(x+2)+B(x-3)$$

When $x=-2$, $-2(-2)+26 = B(-2-3)$

$$-5B = 30$$

$$B = -6 \quad [B1]$$

When $x=3$, $-2(3)+26 = A(3+2)$

$$5A = 20$$

$$A = 4 \quad [B1]$$

$$\therefore \frac{2x^3-5x^2-11x+44}{x^2-x-6} = 2x-3+\frac{4}{x-3}-\frac{6}{x+2} \quad [A1]$$

Q9

Q² 9(a) $y = \sin 4x - 3\cos 2x$

$$\frac{dy}{dx} = \underbrace{4\cos 4x}_{[B1]} + \underbrace{6\sin 2x}_{[B1]}$$

When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 4\cos 4\left(\frac{\pi}{12}\right) + 6\sin 2\left(\frac{\pi}{12}\right)$

$$= 2 + 6\left(\frac{1}{2}\right)$$

$$= 5 \quad [B1]$$

$\therefore$ Grad of normal $= -\frac{1}{5}$. [B1]

*Many students cannot differentiate the trigo terms.

* Students were found working on degree mode (calculator) when this is in radian.

* Students should learn to give answer in exact value.

When $x = \frac{\pi}{12}$, $y = \sin 4\left(\frac{\pi}{12}\right) - 3\cos 2\left(\frac{\pi}{12}\right)$

$$= \frac{\sqrt{3}}{2} - 3\left(\frac{\sqrt{3}}{2}\right)$$

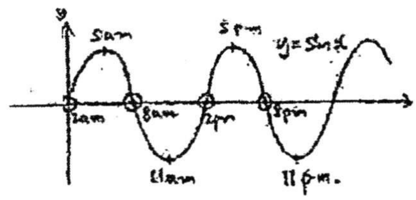
$$= \frac{\sqrt{3} - 3\sqrt{3}}{2} = -\sqrt{3} \quad [B1]$$

$\therefore y = -\frac{1}{5}x + c$ $\qquad -\sqrt{3} = -\frac{\pi}{60} + c$

When $x = \frac{\pi}{12}$, $y = -\sqrt{3}$, $c = \frac{\pi}{60} - \sqrt{3}$ [B1]

$\therefore$ eqⁿ of normal is $y = -\frac{1}{5}x + \frac{\pi}{60} - \sqrt{3}$ * [B1]

9(b)



From the graph, the timings of the fastest water flow are

2 am, 8 am, 2 pm, 8 pm *. [B1]

Q10

Q. 10 (a) $(1-3x)^2(1-kx)^8$

$= (1-6x+9x^2)(1-8kx+28k^2x^2+\cdots)$ [B1]

$\Rightarrow 28k^2+48k+9=117$ [B1]

$28k^2+48k-108=0$

$7k^2+12k-27=0$

$(7k-9)(k+3)=0$ [M1]

$\therefore k=\frac{9}{7}$ * [A1] or $k=-3$ * [A1]

10(b) $T_{r+1} = {}^{24}C_r\left(\frac{1}{x}\right)^{24-r}\left(-\frac{x^3}{4}\right)^r$ [M1]

$= {}^{24}C_r\left(-\frac{1}{4}\right)^r x^{-24+r+3r}$

$= {}^{24}C_r\left(-\frac{1}{4}\right)^r x^{4r-24}$

$\Rightarrow 4r-24=0$ [M1]

$4r=24$

$r=6$ [B1]

$\therefore T_7 = {}^{24}C_6\left(-\frac{1}{4}\right)^6 x^{4(6)-24}$

$= \frac{33649}{1024}$ * [A1]

* After finding $r=6$, Many students claimed that the 7^{th} term was the answer.

* Many students thought $\left(\frac{1}{x}-\frac{x^3}{4}\right)^{24} = {}^{24}C_r\left(\frac{1}{x}\right)^{24-r}\left(\frac{x^3}{4}\right)^r$

Q11

11 (i) $y = \left(\frac{x}{2}+1\right)^2 - 4$

When $\frac{x}{2}+1=0$

$\frac{x}{2}=-1$

$x=-2$ [B1]

When $x=-2$, $y=\left(\frac{-2}{2}+1\right)^2-4 = \underline{-4}$

Since the coeff of x^2 is +ve, $\therefore$ the lowest pt. of the graph is $(-2,-4)$ \#. [B1]

11 (ii) When $y=0$, $\left(\frac{x}{2}+1\right)^2-4=0$

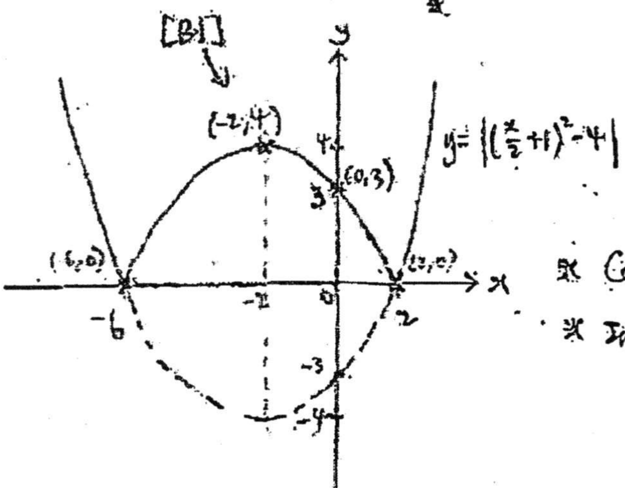
$\left(\frac{x}{2}+1\right)^2=4$

$\frac{x}{2}+1=2$ or -2. [M1]

$\frac{x}{2}=1$ or -3

$x=2$ \# or -6 \# [A1]

11 (iii)



When $x=0$, $y=\left(\frac{0}{2}+1\right)^2-4$

$y=-3$.

※ Correct shape. [B1]

※ Intercepts clearly indicated. [B1]

11 (iv)(a) $k=0$ [B1] and $k>4$ \#. [B1]

11 (iv)(b) $0<k<4$ \#. [B1]

Q1

Marking Scheme

(1) $P = 300(2+5e^{-kt})$

(a) When $t=0$: $P = 300[2+5] = 2100$ [B1]

(b) $2400 = 300(2+5e^{-3k})$

$\frac{2400}{300} = 2+5e^{-3k}$

$8 = 2+5e^{-3k}$

$6 = 5e^{-3k}$ [M1]

$e^{-3k} = \frac{6}{5}$

$-3k = \ln(\frac{6}{5})$

$k \doteq -0.0608$ [A1]

(c) $P = 300(2+5e^{0.061t})$

When $t=5$: $P = 300[2+5e^{0.061(5)}]$ [M1]

$= 2634.94 > 1000$

hence no need to replenish. [B1]

Q2

2(b) Hence $f(x) = 3x^4 + x^3 - 31x^2 - 9x + 36$

$$
\begin{array}{r}
3x^2 + x - 4 \\
x^2 - 9 \,\big)\, 3x^4 + x^3 - 31x^2 - 9x + 36 \\
-(3x^4 \quad - 27x^2) \\
\hline
x^3 - 4x^2 - 9x \\
-(x^3 \quad - 9x) \\
\hline
-4x^2 + 36 \\
-(-4x^2 + 36) \\
\hline
0
\end{array}
$$

Hence $(x^2-9)(3x^2+x-4) = 0$ [M1]

$x^2 - 9 = 0$ or $3x^2 + x - 4 = 0$

$x = \pm 3$ [A1] $\quad (3x+4)(x-1) = 0$

$\therefore x = -\frac{4}{3}$ or $x = 1$ [A1], [A1].

---

3) $\alpha + \beta = -\frac{4}{2} = -2$ [B1] $\quad \alpha\beta = -\frac{7}{2}$ [B1]

(b) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ [M1]

$= (-2)^2 - 2\left(-\frac{7}{2}\right) = 4 + 7 = 11$ [A1]

(c) $\frac{2}{\alpha^2} + \frac{2}{\beta^2} = \frac{2\beta^2 + 2\alpha^2}{\alpha^2\beta^2}$

$= \frac{2(\alpha^2+\beta^2)}{(\alpha\beta)^2}$

$= \frac{2(11)}{\left(-\frac{7}{2}\right)^2} = \cancel{\frac{22}{\frac{49}{4}}} = -\frac{88}{49}$ [B1]

wrong!

$\left(\frac{2}{\alpha^2}\right)\left(\frac{2}{\beta^2}\right) = \frac{4}{(\alpha\beta)^2}$

$= \frac{4}{\left(-\frac{7}{2}\right)^2}$

$= \frac{4}{\frac{49}{4}}$

$= \frac{16}{49}$ [B1].

Hence $x^2 - \left(-\frac{88}{49}\right)x + \frac{16}{49} = 0$

$49x^2 + 88x + 16 = 0.$ [B1]

er

Q4

(a) $\log_2(x-3) - 6\left[\frac{\log_2 2}{\log_2(x-3)}\right] = 1$ [M1] (change of base)

$\log_2(x-3) - \frac{6}{\log_2(x-3)} = 1$

Let $X = \log_2(x-3)$: $X - \frac{6}{X} = 1$ [M1].

$X^2 - 6 = X$

$X^2 - X - 6 = 0$

$(X-3)(X+2) = 0$

$X = 3$ or $X = -2$

$\log_2(x-3) = 3$ $\quad$ $\log_2(x-3) = -2$

$x - 3 = 2^3$ $\quad$ $x - 3 = 2^{-2}$ [M1]

$x = 5$ [B1] or $\quad$ $x = 3\frac{1}{4}$. [B1]

(b)(i) $\log_5\left(\frac{5}{a}\right) = \log_5 5 - \log_5 a$

$= 1 - w$ [B1]

(ii) $(\log_5 a)^4 = w^4$ [B1]

(iii) $\log_5 125a^2 = \log_5 125 + \log_5 a^2$ [M1]

$= \log_5 (5)^3 + 2\log_5 a$

$= 3 + 2w$ [A1].

Q5

5(a)(i) LHS: $\dfrac{1+\cos X}{1-\cos X} = \dfrac{1+\left[2\cos^2\frac{X}{2}-1\right]}{1-\left[1-2\sin^2\frac{X}{2}\right]}$ [B1] [B1]

$= \dfrac{2\cos^2\frac{X}{2}}{2\sin^2\frac{X}{2}} = \cot^2\left(\frac{X}{2}\right)$ (proven). [B1].

(ii) Let $X = 30°$: $\dfrac{1+\cos 30°}{1-\cos 30°} = \cot^2(15°)$
[B1].

$\cot^2 15° = \dfrac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}$

$= \dfrac{2+\sqrt{3}}{2} \div \dfrac{2-\sqrt{3}}{2}$

$= \dfrac{2+\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$ [M1]

$= \dfrac{4+4\sqrt{3}+3}{1} = 7+4\sqrt{3}$. [A1].

(b) $4\sec X = -1-\sec^2 X$

$\sec^2 X + 4\sec X + 1 = 0$

$\sec X = \dfrac{-4 \pm \sqrt{16-4}}{2}$ [M1].

$= \dfrac{-4 \pm \sqrt{12}}{2}$

$\sec X = -0.26795$ or $\sec X = -3.7321$ [M1].

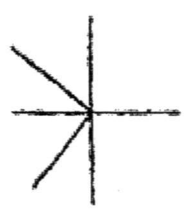
$\cos X = \dfrac{1}{-0.26795}$

$= -3.73$ (NA)
[A1]

$\cos X = \dfrac{1}{-3.7321}$

$\cos X = -0.26795$

Basic $\angle = 74.46°$



$\therefore X = 105.54°$ or $254.46°$

Q6

(6)(i) $\frac{1}{2}(b)(\sqrt{5}-\sqrt{3}) = (3+\sqrt{15})$ [B1].

$b = \frac{2(3+\sqrt{15})}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ [M1].

$= \frac{(6+2\sqrt{15})(\sqrt{5}+\sqrt{3})}{5-3}$

$= \frac{6\sqrt{5}+6\sqrt{3}+2\sqrt{75}+2\sqrt{45}}{2}$ [M1]

$= 3\sqrt{5}+3\sqrt{3}+\sqrt{75}+\sqrt{45}$

$= 3\sqrt{5}+3\sqrt{3}+5\sqrt{3}+3\sqrt{5}$

$= 6\sqrt{5}+8\sqrt{3}$ [A1]

(ii) $\frac{1}{2}w^2(\sin 60°) = 3+\sqrt{15}$ [B1].

$\frac{1}{2}w^2\left(\frac{\sqrt{3}}{2}\right) = 3+\sqrt{15}$

$w^2\left(\frac{\sqrt{3}}{2}\right) = 6+2\sqrt{15}$

$w^2 = (6+2\sqrt{15})\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{12+4\sqrt{15}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ [M1].

$= \frac{12\sqrt{3}+4\sqrt{45}}{3}$

$= \frac{12\sqrt{3}+4(3\sqrt{5})}{3}$ [M1].

$= \frac{12\sqrt{3}+12\sqrt{5}}{3}$

$= 4(\sqrt{3}+\sqrt{5})$. [A1].

Q7

(7) (i) $y = x(3-x)^3$

$$\frac{dy}{dx} = (x)[3(3-x)^2(-1)] + (3-x)^3 \quad [B1].$$

At Stat. pts: $(-3x)(3-x)^2 + (3-x)^3 = 0$

$$(3-x)^2[-3x + 3 - x] = 0 \quad [M1].$$

$(3-x)^2 = 0$ or $-4x + 3 = 0$

$\therefore x = 3$ or $x = \frac{3}{4}$

$y = 0$ [A1] $\quad y = \frac{2187}{256}$ or $y = 8.54$ [A1]

(ii) Using the first derivative test,

x	3^-	3	3^+
$\frac{dy}{dx}$	-ve	0	-ve
	\	—	\

[B1]

x	$\frac{3}{4}^-$	$\frac{3}{4}$	$\frac{3}{4}^+$
$\frac{dy}{dx}$	+ve	0	-ve
	/	—	\

[B1].

$(3, 0)$ is a pt of inflexion [B1] $\quad (0.75, 8.54)$ is a max point [B1].

(iii)(a) For y increasing, $x < 0.75$ [B1]

For y decreasing, $x > 0.75$. [B1].

<u>Alt. Solution</u>

(ii) $$\frac{d^2y}{dx^2} = (3-x)^2[-4] + (3-4x)[2(3-x)(-1)]$$

$$= -4(3-x)^2 - 2(3-4x)(3-x)$$

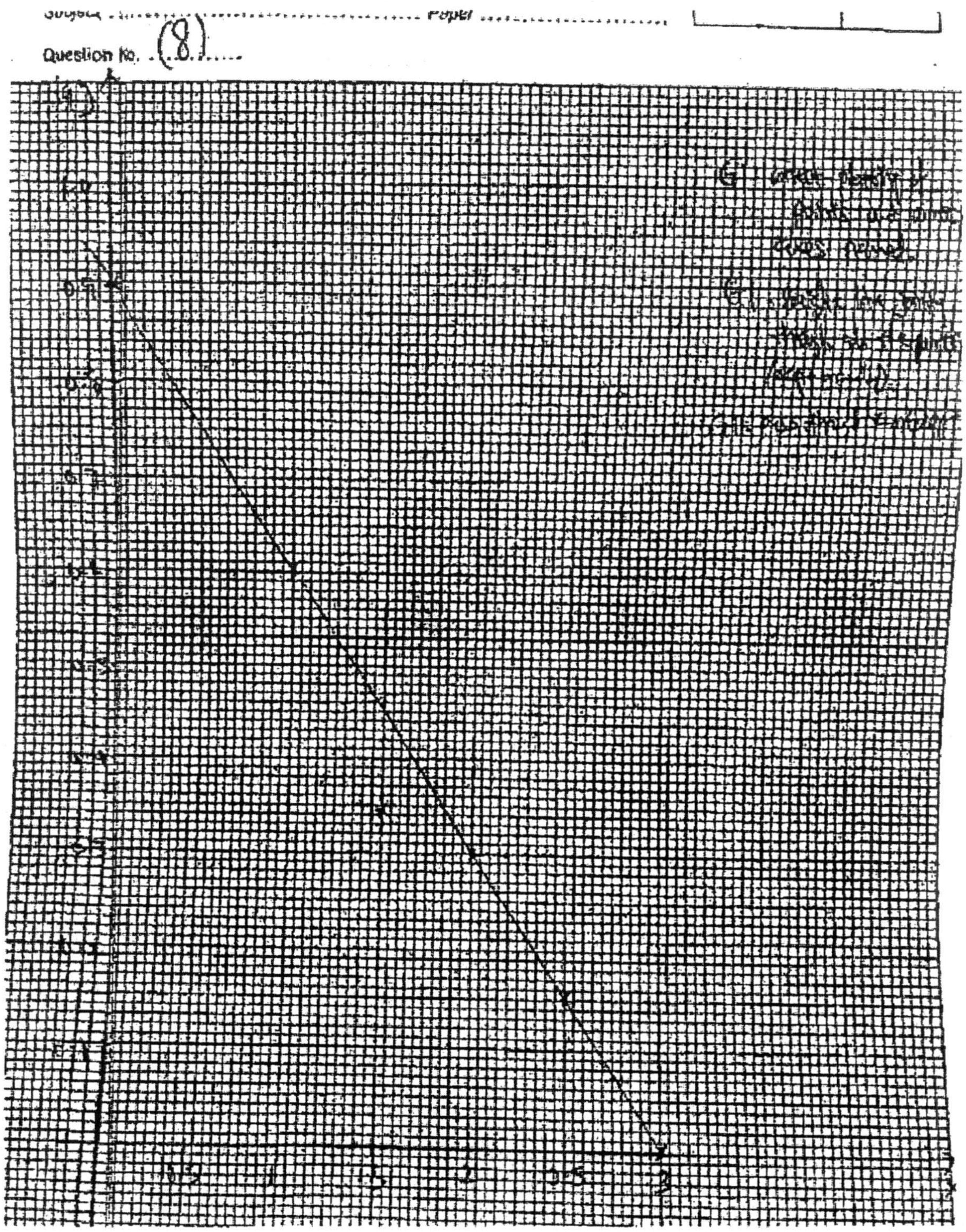
When $x = 3$, $\frac{d^2y}{dx^2} = 0$ [B1] (pt of inflexion) [B1]

$x = \frac{3}{4}$, $\frac{d^2y}{dx^2} < 0$ [B1] (max pt) [B1].

$\therefore (3, 0)$ is a pt of inflexion

(0.75 $\quad$ pt.

Q8



Q9

9(i) Co-ord of $P = (q, e^{3q-q^2})$ [B1]

(ii) $A = \frac{1}{2}(4-q)(e^{3q-q^2})$ [B1, B1].

$= (2 - \frac{q}{2})(e^{3q-q^2})$.

(iii) $\frac{dq}{dt} = -2$ units/sec

$\frac{dA}{dt} = (\frac{dA}{dq})(\frac{dq}{dt})$

$= (-\frac{3}{2}e^2)(-2)$ [M1]

$= 3e^2 = 22.2$ units^2/sec.
[A1]

$\frac{dA}{dq} = (2-\frac{q}{2})(e^{3q-q^2})(3-2q) +$
$(e^{3q-q^2})(-\frac{1}{2})$ [B1]

When $q=2$: $\frac{dA}{dq} = (2-\frac{2}{2})(e^{6-4})(3-4) + (e^2)(-\frac{1}{2})$

$= -e^2 - \frac{1}{2}e^2$

$= -\frac{3}{2}e^2$ [B1]
(-11.08358)

Q10

10(a) $\measuredangle CDH = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$

$DF = 4\cos\theta$ [B1]

$DH = 3\cos(90^\circ - \theta)$

$= 3\sin\theta$ [B1].

$\therefore AB = 4\cos\theta + 3\sin\theta$

(b) $R = \sqrt{3^2 + 4^2} = 5$ [B1], $\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$ [B1]

$\therefore AB = 5\cos(\theta - 36.9^\circ)$ [B1].

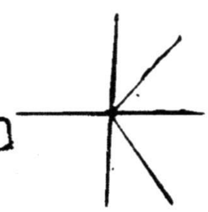
(c) Max AB is when $\cos(\theta - 36.9^\circ) = 1$ [B1]

$\therefore$ max. perimeter $= 6 + 4 + 3 + 4 + 5$

$= 22$m. [B1].

When $\cos(\theta - 36.9^\circ) = 1$

$\theta - 36.9^\circ = 0^\circ, 360^\circ$ [M1]



$\therefore \theta = 36.9^\circ, 396.9^\circ$

[A1]. (NA).

Q11

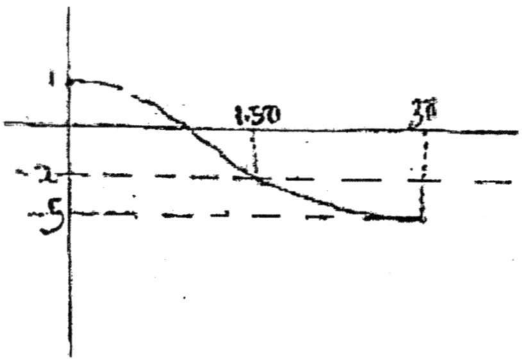
(11) $f(x) = p\cos(\frac{x}{3}) - q$

(i) Period $= \frac{2\pi}{\frac{1}{3}} = 6\pi$ [B1].

(ii) Amplitude $= 3$ [B1].

(iii) $p = 3$, $q = 2$. [B1 ; B1].

(iv)



$f(x) = 3\cos(\frac{x}{3}) - 2$.

G1 : correct shape

G1 : correct y-intercept

G1 : correct values of y at $x = 1.5\pi$ and $x = 3\pi$.

$y = b^{a-x}$

$\lg y = \lg b^{a-x}$

$= (a-x)\lg b$

$= a\lg b - x\lg b$

$\therefore \lg y = (-\lg b)x + a\lg b$

x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1
lg y	0.602	0.344	0.301	0.149	0

[B1]

Grad of graph $= \frac{0.95-0.65}{0.5-1.5} = -0.3$ [A1]

Hence $-\lg b = -0.3$ and $a\lg b = 0.9$

$b = 1.995 \approx 2.0$ [B1]

$a = \frac{0.9}{\lg b} = 3$ [B1].

Incorrect value of y is $y = 2.21$ [B1]

Correct value of $\lg y = 0.45$

hence correct $y = 2.82$. [B1].

Q12

Grad of $AB = \frac{4-1}{p-2} = \frac{3}{p-2}$

Since AB ⊥ BC: $\frac{1}{p-6} = \frac{p-2}{-3}$ [M1]

$-3 = (p-6)(p-2)$

$-3 = p^2 - 8p + 12$

$p^2 - 8p + 15 = 0$

$(p-3)(p-5) = 0$ [M1]

$\therefore p = 3$ or $p = 5$ (NA) [A1].

(ii) Midpt $AC = \left(\frac{6+2}{2}, \frac{1+3}{2}\right)$

$= (4, 2)$

Grad of $AC = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$ [B1]

Grad of ⊥ bisector of $AC = -2$

$\therefore y = -2x + c$

at $(4,2) \Rightarrow c = y + 2x$ [M1]

$= 2 + 2(4)$

$= 10$, hence $y = -2x + 10$ [A1].

(iii) Midpt of $AC = \left(\frac{6+2}{2}, \frac{3+1}{2}\right) = (4, 2)$

Let D have a coord of (x, y).

$\frac{3+x}{2} = 4$ and $\frac{4+y}{2} = 2$ [M1]

$x = 5$ $\quad$ $y = 0$

(iv) Area $= \frac{1}{2}\left|\begin{matrix} 3 & 6 & 5 & 2 & 3 \\ 4 & 3 & 0 & 1 & 4 \end{matrix}\right|$

[M1]

$= \frac{1}{2}|22 - 42|$

$= \frac{1}{2}(20) = 10$ units^2

[A1].