Register No.	Class

Name : ______________________________

DATE : **21 August 2017**
DURATION : **2 Hours**
TOTAL : **80 Marks**

ADDITIONAL MATERIALS
Cover Page (1 Sheet)
Answer Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
80

This document consists of **6** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2}bc\sin A.$$

Answer **ALL** questions. Omission of essential working will result in the loss of marks.
Write your answers clearly and neatly on foolscap paper.
Begin each question on a fresh page.

1. Solve the equation $\log_3 x^5 - \log_x 3 = 4$. [4]

---

2. (i) Find $\frac{d}{dx}(x^2 \ln 2x)$. [2]

(ii) Hence, find $\int x \ln 2x \, dx$. [3]

---

3. Points $A(36, 4)$, $B(q, -2)$ and $C(1, r)$ lies on the graph of $y = \log_p x^2$.

(i) Determine the value of p, of q and of r. [3]

(ii) Sketch the graph of $y = \log_p x^2$ [2]

---

4. It is given that $y = f(x)$ such that $f(x) = 3e^x - \frac{1}{4}e^{-2x} - \frac{3}{4}$.

(i) Explain why the curve $y = f(x)$ has no stationary point. [2]

(ii) Find the equation of the normal to the curve at the point $x = 0$. [3]

---

5. The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. Given that the quadratic factor of $f(x)$ is $x^2 - 4x + 2$ and $x = -1$ is a solution to the equation $f(x) = 0$, find

(i) an expression for $f(x)$ in descending power of x, [2]

(ii) the number of real roots of the equation $f(x) = 0$, justifying your answers, [2]

(iii) the remainder when $f(x)$ is divided by $x - 3$. [2]

---

6. The solution to the inequality $-ax^2+bx-1>0$, where a and b are constants is $\frac{1}{4}<x<1$

(i) Find the value of a and of b. [3]

(ii) Using the values of a and b found in part (i), find the set of values of x which the curve, $f(x)=-ax^2+bx-1$, lies completely below the line $y=1-4x$. [3]

---

7. (i) Express $\frac{3x^2+10x+15}{(2+x)^2(3-2x)}$ in partial fractions. [3]

(ii) Hence evaluate $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$ [3]

---

8. Without using a calculator, show that :

(a) $\tan 105° = -\left(2+\sqrt{3}\right)$ [4]

(b) $\sin^2 75° = \frac{1}{4}\left(2+\sqrt{3}\right)$ [3]

---

9. The roots of the quadratic equation $2x^2+px+1=0$, where p is a positive constant are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. The roots of the equation $2x^2-qx+10=0$, where q is a positive constant are $\frac{\alpha}{\beta}+2$ and $\frac{\beta}{\alpha}+2$.

(i) Find the value of p and of q. [6]

(ii) Show that the value of $\alpha^3+\beta^3$ is 4. [2]

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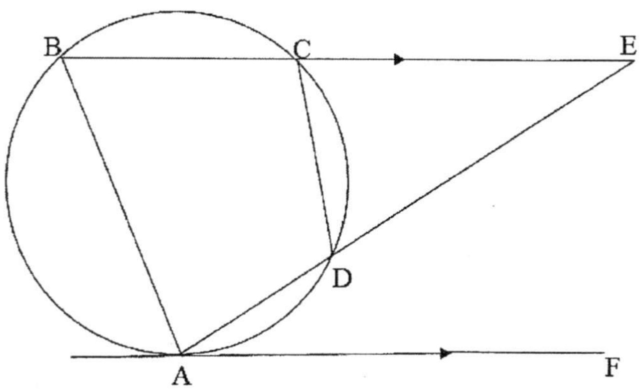
10. In the diagram, AF is a tangent to the circle at A. ADE and BCE are straight lines. AF is parallel to BE and $AB = CE$.



Prove that

(i) $\angle ABD = \angle CED$. [2]

(ii) ΔABD is congruent to ΔCED. [3]

(iii) $\frac{1}{2}\angle ABC = \angle DAF$ [3]

---

11. The equation of a curve is $y = f(x)$, where $f(x) = -\frac{49}{x} - x + 12$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the range of values of x for which $f(x)$ is an increasing function. [2]

(iii) Determine the nature of each of the stationary points of the curve. [2]

(iv) A particle moves alone the curve $y = -\frac{49}{x} - x + 12$. At the point $x = 4$, the y-coordinate is changing at a constant rate of 0.625 units per second. Find the rate of change of the x - coordinate. [2]

---

12. The function $f(x) = a\cos 2x + b$ is defined for $-\pi \le x \le \pi$, where a and b are positive constants.

(i) Given that the greatest and the least value of $f(x)$ are 8 and -2 respectively, find the value of a and of b. [2]

(ii) State the range of values between which the principal value of x must lie and find the principal value of x for which $f(x) = 0$. [3]

(iii) Sketch the graph of $y = a\cos 2x + b$ for $-\pi \le x \le \pi$. [3]

(iv) Hence, state the number of solutions to the equation $\frac{8}{\pi}x = a\cos 2x + b$. [2]

End of Paper

Register No.	Class

Name : ______________________________

DATE : 25 August 2017
DURATION : 2 Hours 30 Min
TOTAL : 100 Marks

ADDITIONAL MATERIALS
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Answer Paper (7 sheets)

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[Turn over

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Answer **ALL** questions. Omission of essential working will result in the loss of marks.
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Begin each question on a fresh page.

1. A point *P* lies on the curve $y = x^2 + 4x - 8$. The normal to curve is parallel to the line $2y - \frac{x}{3} = 1$. Find the coordinates of *P*. [3]

---

2. (a) Given that $2^{x-1} \times 3^{x+2} = 8^{x-1} \times 3^{2x}$, evaluate 12^x. [3]

(b) Solve the equation $e^y(5 - e^y) + 14 = 0$. [3]

---

3. Given that $\int_0^6 f(x)\,dx = 5$ and $\int_2^6 f(x)\,dx = 2$, find

(i) $\int_6^2 f(x)\,dx$. [1]

(ii) $\int_0^2 f(x)\,dx$. [2]

(iii) the value of k for which $\int_0^2 f(x) - kx\,dx = 15$. [3]

---

4. (a) Show that the binomial expansion $\left(x - \frac{1}{2x^3}\right)^{15}$ does not have an independent term. [3]

(b) In the binomial expansion of $(1 + kx)^n$, where $n \geq 3$ and k is a constant, the coefficient of x^3 and x^4 are equal. Express k in terms of n. [4]

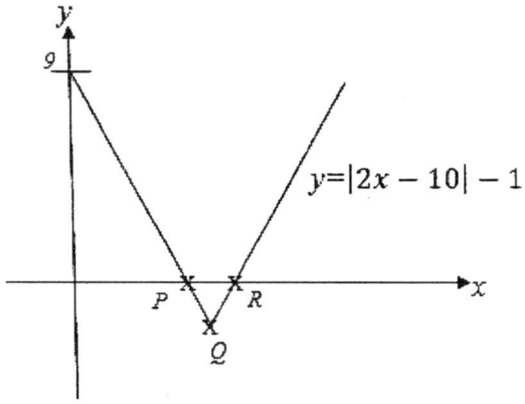
---

5. (i) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$. [4]

(ii) Hence, find in terms π, the solution to the equation $1 = 8\cos^3 A - 6\cos A$ for $0 < \theta < \pi$. [3]

---

6. The diagram shows part of the graph of $y = |2x - 10| - 1$.



(a) Find the coordinates of P, Q and R. [4]

(b) In the case when $mx + c = |2x - 10| - 1$, find

(i) the range of values of c when $m = -2$ where there is only **1** solution. [1]

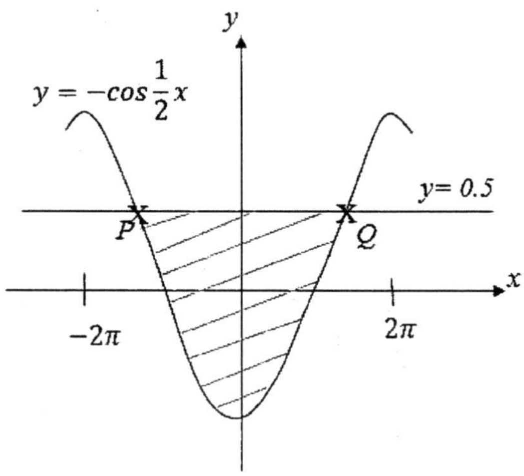
(ii) the range of values of m when $c = -1$ where there are **2** solutions. [2]

---

7. The diagram shows part of the graph $y = -\cos\frac{1}{2}x$ for $-2\pi \le x \le 2\pi$. The line $y = \frac{1}{2}$ intersects the curve at P and at Q.

(i) Find the coordinates of P and of Q. [2]

(ii) Find the area bounded by the curve and the line $y = \frac{1}{2}$. [5]



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8. The value, \$$V$, of a house is related to t, the number of years after it was built in year 2008. The variables are related by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of the house in year 2010, 2012, 2014 and 2016.

Year	2010	2012	2014	2016
t	2	4	6	8
V(\$)	517 600	595 400	684 800	787 800

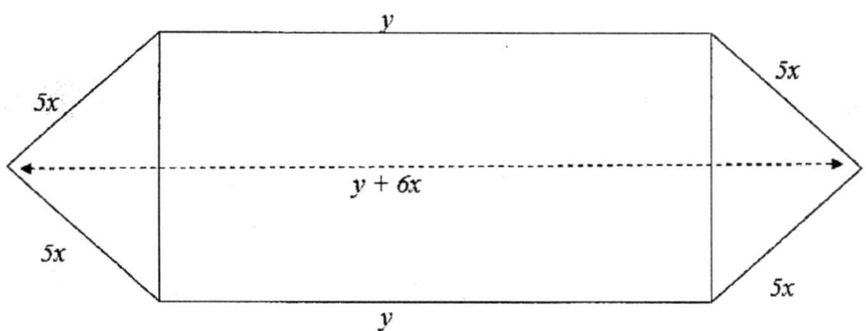
(i) On the graph paper, plot $\ln V$ against t and draw a suitable straight line. The vertical axis should start from 13.0 and have a scale of 2 cm to 0.1 [2]

(ii) Use the graph from part (i) to estimate the value of a and of k. [4]

(iii) Estimate the value of the house in 2015. [2]

---

9. An area is fenced up to enclose a landscape. The shape of the landscape is shown below. It is made up of a rectangle of length y cm and two isosceles triangles of sides $5x$ cm. The perimeter of the landscape is 420 cm and the length from one end to the other end is $y + 6x$ cm.



(i) Show that the area of the landscape, A cm^2, is given by $1680x - 56x^2$. [4]

(ii) Given that x can vary, find the stationary value of A. [2]

(iii) Determine whether this stationary value is a maximum or minimum. [2]

---

10. A particle travels in a straight line from a fixed point O where the distance S in meters is given by $S = \frac{4}{3}t^3 + kt^2 + qt$ where t is the time in seconds after passing O. k and q are constants. The velocity of the particle is 20 m/s when it passes O and at $t = 3s$, its acceleration is 0 m/s^2.

(i) Find the value of k and of q. [4]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii) Find the total distance travelled during the first 8 seconds. [3]

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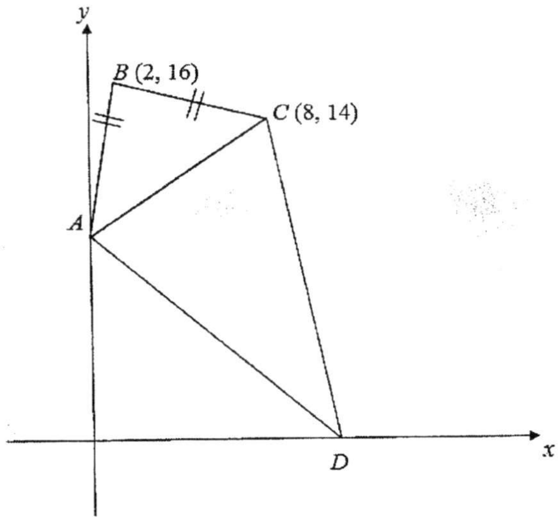
11. **[Solution to this question by accurate drawing will not be accepted]**



The diagram which is not drawn to scale shows a quadrilateral *ABCD*. The point *B* is (2, 16) and the point *C* is (8, 14). Triangle *ABC* is an isosceles triangle and point *A* and point *D* lies the y – axis and x – axis respectively.

(i) Find the coordinates of A. [3]

(ii) Given that the ratio of area of ΔABC : area of ΔACD is 1 : 3 , find the coordinates of D. [4]

(iii) Show that *ABCD* is a kite. [3]

---

12. The line $x = 17$ is a tangent to a circle and the points A (1, 9) and B (1, – 7) are on the circumference of the circle.

(i) Show that the radius of the circle is 10 units. [4]

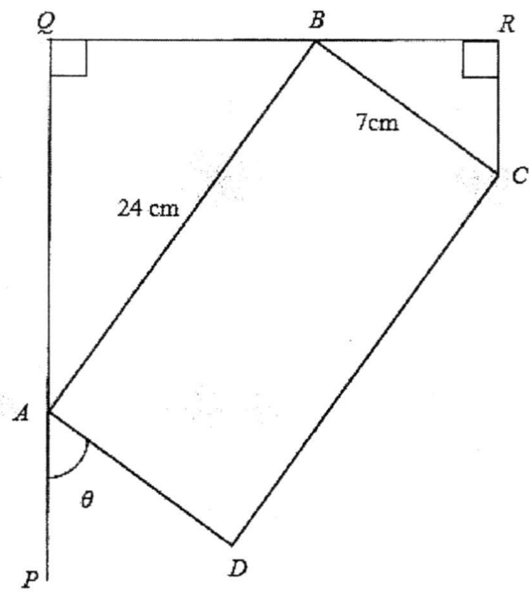
(ii) State the coordinates of the centre of the circle. [1]

(iii) Write down the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$ [2]

(iv) The circle is reflected along the line $y = -1$, show that the point (3, 10) does not lie on the reflected circle. [3]

---

13. In the diagram below, $ABCD$ is a rectangle. The line QR is perpendicular to the lines PQ and CR. Points A and B lie on the lines PQ and QR respectively and angle $PAD = \theta$. AB is 24 cm and BC is 7 cm.



(i) Show that the length of QR is $24\cos\theta + 7\sin\theta$. [4]

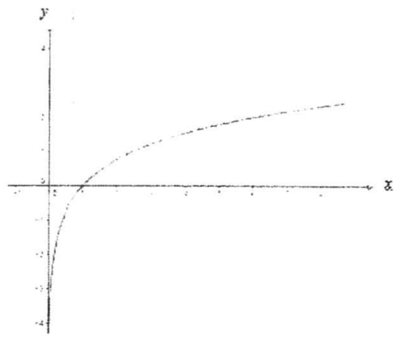
(ii) Express $24\cos\theta + 7\sin\theta$ in the form of $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

(iii) Find the value of θ when QR is 17 cm. [2]

(iv) Find the maximum length of QR and state the corresponding value of θ. [3]

End of Paper

ANSWER KEY

1	$x = 0.803$ or $x = 3$	2	(i) $\frac{d}{dx}(x^2 \ln 2x) = x + 2x \ln 2x$ (ii) $\int x \ln 2x \, dx = \frac{2x^2 \ln 2x - x^2}{4}$
3	(i) $p = 6$ $q = \frac{1}{6}$ $r = 0$ (ii) 	4	(ii) $y = -\frac{2}{7}x + 2$
5	(i) $f(x) = 2x^3 - 6x^2 - 4x + 4$ (ii) 3 real roots (iii) -8	6	(i) $a = 4, b = 5$ (ii) $x < \frac{1}{4}$ or $x > 2$
7	(i) $\frac{3x^2 + 10x + 15}{(2+x)^2(3-2x)} = \frac{1}{(2+x)^2} + \frac{3}{3-2x}$ (ii) 1.81	8	
9	$p = 2$, $q = 8$	10	
11	(i) $\frac{dy}{dx} = \frac{49}{x^2} - 1$ $\frac{d^2y}{dx^2} = -\frac{98}{x^3}$ (ii) $-7 < x < 7$ (iii) $x = 7$ is a maximum point $x = -7$ is a minimum point (iv) $\frac{dx}{dt} = \frac{10}{33}$	12	$a = 5$ $b = 3$ $x = 1.11$ 3 solutions

Solution

1. Solve the equation $\log_3 x^5 - \log_x 3 = 4$. [4]

$$5\log_3 x - \frac{\log_3 3}{\log_3 x} = 4 \quad \text{-M1}$$

Let $y = \log_3 x$

$$5y - \frac{1}{y} = 4$$

$$5y^2 - 4y - 1 = 0 \quad \text{-M1}$$

$$(5y+1)(y-1) = 0$$

$$\log_3 x = -\frac{1}{5} \quad \text{or} \quad \log_3 x = 1 \quad \text{- M1}$$

$$x = 3^{-\frac{1}{5}} \quad \text{or} \quad x = 3$$

$$x = 0.803 \quad \text{-A1}$$

---

2. (i) Find $\frac{d}{dx}(x^2 \ln 2x)$. [2]

(ii) Hence, find $\int x \ln 2x \, dx$. [3]

(i) $$\frac{d}{dx} x^2 \ln 2x = x^2 \frac{1}{x} + 2x \ln 2x \quad \text{- M1}$$

$$= x + 2x \ln 2x \quad \text{-A1}$$

(ii) $$\int x + 2x \ln 2x \, dx = x^2 \ln 2x$$

$$\int 2x \ln 2x \, dx = x^2 \ln 2x - \int x dx \quad \text{-M1}$$

$$\int 2x \ln 2x \, dx = x^2 \ln 2x - \frac{x^2}{2} \quad \text{- M1}$$

$$\int x \ln 2x \, dx = \frac{x^2 \ln 2x}{2} - \frac{x^2}{4} \quad \text{or} \quad \int x \ln 2x \, dx = \frac{2x^2 \ln 2x - x^2}{4} \quad \text{- A1}$$

3 Points $A(36, 4)$, $B(q, -2)$ and $C(1, r)$ lies on the graph of $y = \log_p x^2$.

(i) Determine the value of p, of q and of r. [3]

(ii) Sketch the graph of $y = \log_p x^2$ [2]

$4 = \log_p 36^2$

$2 = \log_p 36$

$p^2 = 36$

$p = 6$ - B1

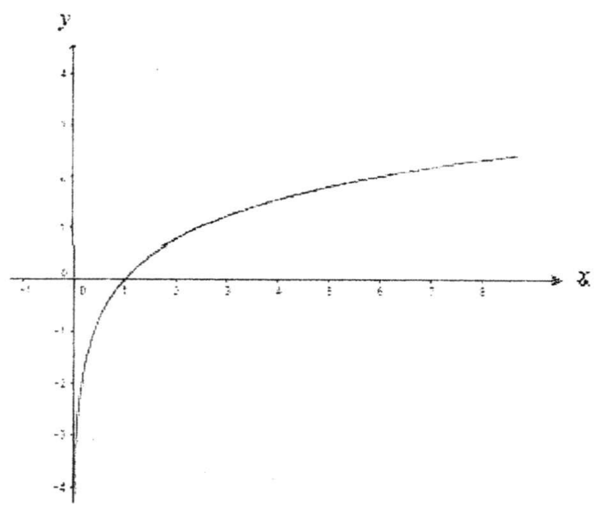
$6^{\frac{-2}{2}} = q$

$q = \frac{1}{6}$ - B1

$6^{\frac{r}{2}} = 1$

$\frac{r}{2} = 0$

$r = 0$ -B1



Shape – M1

Showing that it cuts at (1,0) – A1

4. It is given that $y = f(x)$ such that $f(x) = 3e^x - \frac{1}{4}e^{-2x} - \frac{3}{4}$.

(i) Explain why the curve $y = f(x)$ has no stationary point. [2]

(ii) Find the equation of the normal to the curve at the point $x = 0$. [3]

(i) $f'(x) = 3e^x + \frac{1}{2}e^{-2x}$

If $3e^x + \frac{1}{2}e^{-2x} = 0$ - M1

$3e^x = -\frac{1}{2}e^{-2x}$

$e^{3x} = -\frac{1}{6}$ (N.A)

Since $f'(x) = 0$ has no solution, $f(x)$ has no stationary point - A1

(ii) When $x = 0$, $y = 3 - \frac{1}{4} - \frac{3}{4}$

$= 2$

$f'(x) = 3 + \frac{1}{2}$

$= \frac{7}{2}$ -M1

$y - 2 = -\frac{2}{7}(x - 0)$ -M1

$y = -\frac{2}{7}x + 2$ - A1

5. The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. Given that the quadratic factor of $f(x)$ is x^2-4x+2 and that $x=-1$ is a solution to the equation $f(x)=0$, find

(i) an expression for $f(x)$ in descending power of x, [2]

(ii) the number of real roots of the equation $f(x)=0$, justifying your answers, [2]

(iii) the remainder when $f(x)$ is divided by $x-3$. [2]

(i) $2(x+1)(x^2-4x+2)=2x^3-8x^2+4x+2x^2-8x+4$ - M1

$f(x)=2x^3-6x^2-4x+4$ - A1

(ii) For x^2-4x+2, $(-4)^2-4(1)(2)=8$

>0 - M1

Therefore $f(x)$ has 3 real roots - A1

(iii) $f(3)=2x^3-6x^2-4x+4$ -M1

$=-8$ - A1

---

6. The solution to the inequality $-ax^2+bx-1>0$, where a and b are constants is $\frac{1}{4}<x<1$

(i) Find the value of a and of b. [3]

(ii) Using the values of a and b found in part (i), find the set of values of x which the curve, $f(x)=-ax^2+bx-1$, lies completely below the line $y=1-4x$. [3]

(i) $(x-\frac{1}{4})(x-1)=0$ - M1

$x^2-x-\frac{1}{4}x+\frac{1}{4}=0$

$x^2-\frac{5}{4}x+\frac{1}{4}=0$ - M1

$-4x^2+5x-1=0$

$a=4, b=5$ - A1

(ii) $-4x^2+5x-1<1-4x$ - M1

$t-1$

$(4x-1)(x-2)>0$ - M1

$x<\frac{1}{4}$ or $x>2$ - A1

---

7. (i) Express $\frac{3x^2+10x+15}{(2+x)^2(3-2x)}$ in partial fractions. [3]

(ii) Hence evaluate $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$ [3]

$$\frac{3x^2+10x+15}{(2+x)^2(3-2x)}=\frac{A}{(2+x)^2}+\frac{B}{2+x}+\frac{C}{3-2x}$$

$3x^2+10x+15=A(3-2x)+B(2+x)(3-2x)+C(2+x)^2$ - M1

Let $x=-2$

$7=7A$

$A=1$ - M1

$3=-2B+C$

$C=3-2B$

Compare coeff x

$10=-2A-B+4C$

$10=-2-B+4C$

Subst $C=3-2B$ into $10=-2-B+4C$

$10=-2-B+12-8B$

$B=0$ - M1

$C=3$ - M1

$$\frac{3x^2+10x+15}{(2+x)^2(3-2x)}=\frac{1}{(2+x)^2}+\frac{3}{3-2x}$$ - A1

(ii) $\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,dx$

$$\int_0^1 \frac{3x^2+10x+15}{(2+x)^2(3-2x)}\,\mathrm{dx}=\int_0^1 \frac{1}{(2+x)^2}+\frac{3}{3-2x}\,dx$$

$$=\left[\frac{-1}{(2+x)}\right]_0^1-\frac{3}{2}\left[\ln(3-2x)\right]_0^1$$

$$=\frac{1}{6}-\left(\frac{3}{2}(\ln 1-\ln 3)\right)$$

$$=1.81$$

8. Without using a calculator, show that : [3]

(a) $\tan 105° = -(2+\sqrt{3})$

[3]

(b) $\sin^2 75° = \frac{1}{4}(2+\sqrt{3})$

$$\tan 105° = \frac{\tan 60° + \tan 45°}{1-\tan 60° \tan 45°}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \quad \text{-M1}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad \text{-M1}$$

$$= \frac{(\sqrt{3}+1)^2}{1-3}$$

$$= \frac{3+2\sqrt{3}+1}{1-\sqrt{3}}$$

$$= -[2+\sqrt{3}] \quad \text{-A1}$$

$$\text{(ii)} \sin^2 75° = \frac{1-\cos 2(75°)}{2}$$

$$= \frac{1-\cos 150°}{2}$$

$$= \frac{1-(-\frac{\sqrt{3}}{2})}{2} \quad \text{- M1}$$

$$= \frac{2+\sqrt{3}}{2} \times \frac{1}{2} \quad \text{-M1}$$

$$= \frac{1}{4}[2+\sqrt{3}] \quad \text{- A1}$$

---

9. The roots of the quadratic equation $2x^2 + px + 1 = 0$, where p is a positive

const 1 1 ots of the equation $2x^2 - qx + 10 = 0$, where

q is a positive constant are $\frac{\alpha}{\beta}+2$ and $\frac{\beta}{\alpha}+2$.

[6]

(i) Find the value of p and of q.

(iii) Show that the value of $\alpha^3+\beta^3$ is 4. [2]

$\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{p}{2}$

$\frac{\beta+\alpha}{\alpha\beta}=-\frac{p}{2}$

$\beta+\alpha=-p$ - M1

$\frac{1}{\alpha\beta}=\frac{1}{2}$

$\alpha\beta=2$ - M1

$\frac{\alpha}{\beta}+2+\frac{\beta}{\alpha}+2=\frac{q}{2}$ - eqn 1

$\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)=\frac{10}{2}$

$\frac{\alpha\beta}{\beta\alpha}+2\frac{\alpha}{\beta}+2\frac{\beta}{\alpha}+4=5$

$1+2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+4=5$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=0$ - eqn 2 -M1

Subst eqn 1 into eqn 2

$4=\frac{q}{2}$

$q=8$ - A1

Fm eqn 2 : $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=0$

$\frac{\alpha^2+\beta^2}{\alpha\beta}=0$

$(\alpha+\beta)^2-2\alpha\beta=0$ -M1

$(\alpha+\beta)^2=2(2)$

$(\alpha+\beta)=2$ (NA)or -2 (p is positive)

$p=2$ - A1

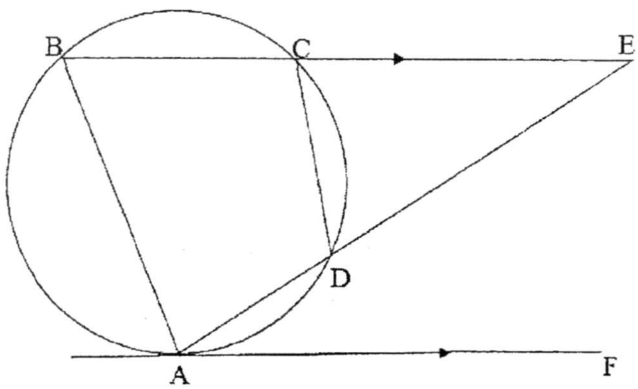
$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$\alpha^3+\beta^3=(\alpha+\beta)((\alpha+\beta)^2-2\alpha\beta-\alpha\beta)$ - M1

$=(-2)[(-2)^2-3(2)]$
$=4$ - A1

---

10. In the diagram, AF is a tangent to the circle at A. ADE and BCE are straight lines. AF is parallel to BE and $AB=CE$.



Prove that

(i) $\angle ABD=\angle CED$. [2]

(ii) ΔABD is congruent to ΔCED. [3]

(iii) $\frac{1}{2}\angle ABC=\angle DAF$ [3]

(i) $\angle ABD=\angle DAF$ (alt seg. Thm) - M1
$\angle CED=\angle DAF$ (alt $\angle$, BE // AF)
$\angle ABD=\angle CED$ (Shown) - A1

(ii) Let $\angle BAD=a$
$\angle BCD=180°-a$ ($\angle$ in opp segment)
$\angle ECD=a$ (adj $\angle$ on a st line)
$\angle BAD=\angle ECD$ - M1
$AB=CE$ (given)
$\angle ABD=\angle CED$ (part (i) - M1
$\Delta ABD\equiv\Delta CED$ (ASA test) - A1

(i ΔCED)
ase $\angle$ of isos Δ) - M1

$\angle CED = \angle DBE$ $(\angle BED = \angle CED)$

$\angle ABC = \angle ABD + \angle DBE$ - M1

$\angle ABC = \angle CED + \angle CED$ ($\angle ABD = \angle CED$ fm part (i))

$$\frac{1}{2}\angle ABC = \angle CED$$ - A1

---

11. The equation of a curve is $y = f(x)$, where $f(x) = -\frac{49}{x} - x + 12$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the range of values of x for which $f(x)$ is an increasing function. [2]

(iii) Determine the nature of each of the stationary points of the curve. [2]

(iv) A particle moves alone the curve $y = -\frac{49}{x} - x + 12$. At the point $x = 4$, y-coordinate is changing at a constant rate of 0.625 units per second. Find the rate of change of the x-coordinate. [2]

(i) $y = -49x^{-1} - x + 12$

$\frac{dy}{dx} = \frac{49}{x^2} - 1$

$\frac{d^2y}{dx^2} = (-2)49x^{-3}$

$= \frac{-98}{x^3}$

(ii) $\frac{49}{x^2} - 1 > 0$

$(7 + x)(7 - x) > 0$

The solution is $-7 < x < 7$

(iii) Stationary points are $x = -7$ or $x = 7$.

At $x = -7$

$\frac{d^2y}{dx^2} = -\frac{98}{(-3)^3} > 0$

um point

At $x = 7$

$$\frac{d^2y}{dx^2} = -\frac{98}{(7)^3} < 0$$

$\therefore x = 7$ is a maximum point

(iv) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{49}{4^2} - 1 = 0.625 \times \frac{dt}{dx}$$

$$\frac{33}{16} = 0.625 \times \frac{dt}{dx}$$

$$\frac{dx}{dt} = \frac{10}{33}$$

---

12. The function $f(x) = a\cos 2x + b$ is defined for $-\pi \leq x \leq \pi$, where a and b are positive constants.

(i) Given that the greatest and the least value of $f(x)$ are 8 and -2 respectively, find the value of a and of b. [2]

(ii) State the range of values between which the principal value of x must lie and find the principal value of x for which $f(x) = 0$. [3]

(iii) Sketch the graph of $y = a\cos 2x + b$. [3]

(v) Hence , state the number of solutions to the equation $\frac{8}{\pi}x = a\cos 2x + b$. [2]

(i) $a = 5$ - B1

$b = 3$ - B1

(c) Range of values = $0 \leq x \leq \frac{\pi}{2}$ - B1

$\cos 2x = -\frac{3}{5}$ - M1

$2x = 2.21$

$x = 1.11$ - A1

ANSWER KEY

1	P = (− 5, − 3)	**2**	$12^x = 36$ $y = 1.95$
3	$\int_6^2 f(x)\,dx = -2$ $\int_0^2 f(x)\,dx = 3$ $k = -6$	**4**	$k = \frac{4}{n-3}$
5	$A = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$	**6**	$R = \left(5\frac{1}{2}, 0\right), Q = (5, -1), P = \left(4\frac{1}{2}, 0\right)$ c > 9 $0 < m < 2$
7	$P = (-\frac{4}{3}\pi, \frac{1}{2})$, $Q = (\frac{4}{3}\pi, \frac{1}{2})$ 7.65 units^2	**8**	
9	$A = 12600cm^2$ A is a maximum value	**10**	(i) $k = -12$, $q = 20$ (ii) $t = 1 \text{ or } 5$ (iii) 160 m
11	$A(0,10)$ $D(10,0)$	**12**	Centre of circle = (7,1)
13	$24\cos\theta + 7\sin\theta = 25\cos(\theta - 16.3°)$ $\theta = 63.5°$ Max length $QR = 25$ $\theta = 16.3°$		

Solution

1. A point P lies on the curve $y = x^2 + 4x - 8$. The normal to curve is parallel to the line $2y - \frac{x}{3} = 1$. Find the coordinates of P. [3]

$\frac{dy}{dx} = 2x + 4$

$\frac{-1}{2x+4} = \frac{1}{6}$ - M1

$-6 = 2x + 4$

$x = -5$ - M1

$y = -3$

$P = (-5, -3)$ - A1

2. (a) Given that $2^{x-1} \times 3^{x+2} = 8^{x-1} \times 3^{2x}$, evaluate 12^x. [3]

(b) Solve the equation $e^y(5 - e^y) + 14 = 0$. [3]

(a) $2^x \times \frac{1}{2} \times 3^x \times 9 = 2^{3x} \times \frac{1}{8} \times 3^{2x}$ - M1

$\frac{1}{2} \times 9 \times 8 = \frac{2^{3x} \times 3^x}{2^x \times 3^x}$ - M1

$36 = (2^2)^x \times 3^x$

$12^x = 36$ - A1

(b) $-e^{2y} + 5e^y + 14 = 0$ - M1

$e^{2y} - 5e^y - 14 = 0$

$(e^y - 7)(e^y + 2) = 0$ - M1

$e^y = 7$ or $e^y = -2$ (N.A)

$y = \ln 7$

$y = 1.95$ - A1

3. Given that $\int_0^6 f(x)\,dx = 5$ and $\int_2^6 f(x)\,dx = 2$, find

(i) $\int_6^2 f(x)\,dx$. [1]

(ii) $\int_0^2 f(x)\,dx$. [2]

(iii) the value of k for which $\int_0^2 f(x) - kx\,dx = 15$. [3]

(i) $\int_6^2 f(x)\,dx = -2$ - B1

(ii) $\int_0^2 f(x)\,dx = \int_0^6 f(x)dx - \int_2^6 f(x)dx$ - M1

$= 5 - 2$

$= 3$ - A1

(iii) $\int_0^2 f(x) - kx\,dx = 15$

$\int_0^2 f(x)\,\mathrm{dx} - \int_0^2 kx\,dx = 15$ - M1

$3 - 15 = k\left[\frac{x^2}{2}\right]_0^2$ - M1

$3 - 15 = k[2 - 0]$

$k = -6$ - A1

4.

(a) Show that the binomial expansion $\left(x - \frac{1}{2x^3}\right)^{15}$ does not have an independent term. [3]

(b) In the binomial expansion of $(1 + kx)^n$, where $n \geq 3$ and k is a constant, the coefficient of x^3 and x^4 are equal. Express k in terms of n. [4]

(a) $T_{r+1} = 15C_r (x)^{15-r}\left(-\frac{1}{2x^3}\right)^r$

$= 15C_r(-\frac{1}{2})^r (x)^{15-r-3r}$ - M1

$15 - 4r = 0$

r - M1

Since r $\neq$ integer, the binomial expansion does not have an independent term. –A1

(b) Coeff of $x^3 = \dfrac{n(n-1)(n-2)}{6}k^3$ - M1

Coeff of $x^4 = \dfrac{n(n-1)(n-2)(n-3)}{24}k^4$ -M1

$\dfrac{n(n-1)(n-2)}{6}k^3 = \dfrac{n(n-1)(n-2)(n-3)}{24}k^4$ - M1

$4 = (n-3)k$

$k = \dfrac{4}{n-3}$ - A1

---

5. (i) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$. [4]

(ii) Hence , find in terms π, the solution to the equation $1 = 8\cos^3 A - 6\cos A$ for $0 < \theta < \pi$. [3]

(i) $\cos 3A = \dfrac{5}{2}\cos^3 A - \dfrac{3}{2}\cos A$

$\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$ - M1

$= (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A$ -M1

$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$ - M1

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$ - A1

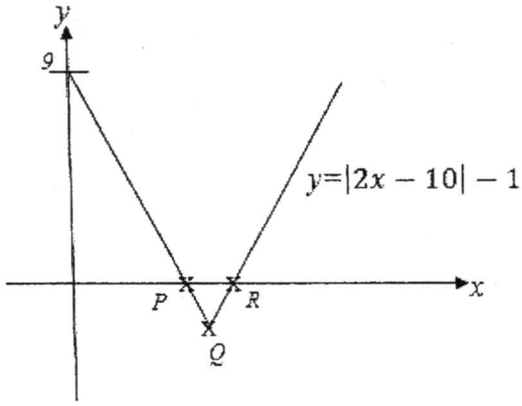
(ii) $1 = 8\cos^3 A - 6\cos A$

$\dfrac{1}{2} = \cos 3A$ - M1

$3A = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}$ -M1

$A = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$ -A1

6. The diagram shows part of the graph of $y = |2x - 10| - 1$.



(a) Find the coordinates of P , Q and R . [4]

(b) In the case when $mx + c = |2x - 10| - 1$, find

(i) the range of values of c when $m = -2$ where there is only **1** solution. [1]

(ii) the range of values of m when $c = -1$ where there are **2** solutions. [2]

(a) $|2x - 10| - 1 = 0$

$2x - 10 = 1$ or $2x - 10 = -1$

$x = 5\frac{1}{2}$ or $x = 4\frac{1}{2}$

$P = \left(4\frac{1}{2}, 0\right)$ or $R = \left(5\frac{1}{2}, 0\right)$ - B2

x- coordinates of Q = $\frac{4.5 + 5.5}{2} = 5$ - M1

y- coordinates of Q = -1

$Q = (5, -1)$ - A1

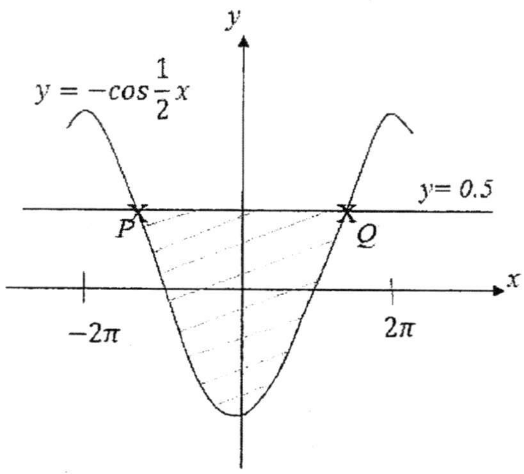
b(i) $c > 9$ - A1

(iii) $0 < m < 2$ - A2

7. The diagram shows part of the graph $y = -\cos\frac{1}{2}x$ for $-2\pi \le x \le 2\pi$. The line $y = \frac{1}{2}$ intersects the curve at P and at Q.

(i) Find the coordinates of P and of Q. [2]

(ii) Find the area bounded by the curve and the line $y = \frac{1}{2}$. [5]



(i) $-\cos\frac{1}{2}x = \frac{1}{2}$

$\cos\frac{1}{2}x = -\frac{1}{2}$

$\frac{1}{2}x = \frac{2}{3}\pi, -\frac{2}{3}\pi$

$x = \frac{4}{3}\pi, -\frac{4}{3}\pi$ -M1

$P = (-\frac{4}{3}\pi, \frac{1}{2}),\ Q = (\frac{4}{3}\pi, \frac{1}{2})$ - A2

(ii) $\int_{-\frac{4}{3}\pi}^{\frac{4}{3}\pi} \left| -\cos\frac{1}{2}x - \frac{1}{2} \right| dx$. - M1

$= \left| \left[-2\sin\frac{1}{2}x - \frac{1}{2}x \right]_{-\frac{4}{3}\pi}^{\frac{4}{3}\pi} \right|$ - M1

$= \left| \left[2\sin\frac{2}{3}\pi - \frac{2}{3}\pi \right] - \left[-2\sin -\frac{2}{3}\pi + \frac{2}{3}\pi \right] \right|$ -M1

$= \left| \left[\quad 2\sqrt{3} \quad 2 \quad \right] \left[2\sqrt{3} \quad \frac{2}{3}\pi \right] \right|$ - M1

$= 7.65 \text{ units}^2$ – A1

8. The value , $V, of a house is related to t, the number of years after it was built in year 2008.The variables are related by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of the house in year 2010, 2012, 2014 and 2016.

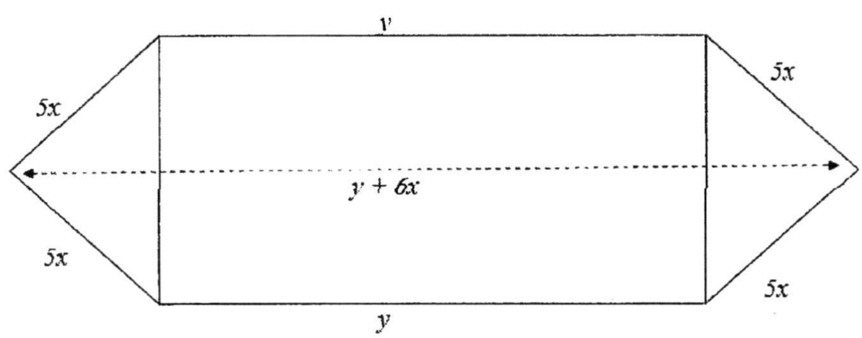
Year	2010	2012	2014	2016
t	2	4	6	8
V($)	517 600	595 400	684 800	787 800

(iv) On the graph paper, plot $\ln V$ against t and draw a suitable straight line. The vertical axis should start from 13.0 and have a scale of 2 cm to 0.1. [2]

(v) Use the graph from part (i) to estimate the value of a and of k. [3]

(vi) Estimate the value of the house in 2015. [3]

9. An area is fenced up to enclose a landscape. The shape of the landscape is as shown below. The shape is made up of a rectangle of length y cm and two isosceles triangles of sides
$5x$ cm. The perimeter of the landscape is 420 cm and the length from one end to the other end is $y + 6x$ cm.



(i) Show that the area of the landscape , A cm^2 , is given by $1680x - 56x^2$. [4]

(ii) Given that x can vary, find the stationary value of A. [2]

(iii) stationary value is a maximum or minimum. [2]

(i)

$y = \frac{420-20x}{2}$

$y = 210-10x$ - M1

Length of rect = $8x$ - M1

Area of isosceles triangle = $24x^2$

$A = y(8x)+24x^2$

$= (210-10x)8x+24x^2$ - M1

$= 1680x-56x^2$ - A1

(ii) $\frac{dA}{dx} = 1680-112x$

$1680-112x = 0$ - M1

$x = 15$ - M1

$A = 12600cm^2$ - A1

(iii) $\frac{d^2A}{dx^2} = -112$ - M1

$\frac{d^2A}{dx^2} < 0$, A is a maximum value. – A1

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10. A particle travels in a straight line from a fixed point O where the distance S in meters is given by $S = \frac{4}{3}t^3 + kt^2 + qt$ where t is the time in seconds after passing O . k and q are constants. The velocity of the particle is 20 m/s when it passes O and at $t = 3s$, its acceleration is 0 m/s².

(i) Find the value of k and of Q. [4]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii) Find the total distance travelled during the first 8 seconds. [3]

$V = 4t^2 + 2kt + q$ - M1

$20 = 4(0)^2 + 2k(0) + q$

$q = 20$ - A1

$a = 8t + 2k$ - M1

$0 = 8(t) + 2k$

$k = -12$

(ii) $V = 4t^2 - 24t + 20$

$4t^2 - 24t + 20 = 0$

$4(t-5)(t-1) = 0$ - M1

$t = 1 \text{ or } 5$ -A1

(iii) $S = \frac{4}{3}t^3 - 12t^2 + 20t$

When $t = 1$

$S = 9\frac{1}{3}m$

When $t = 5$

$S = -33\frac{1}{3}m$ - M1

When $t = 8$

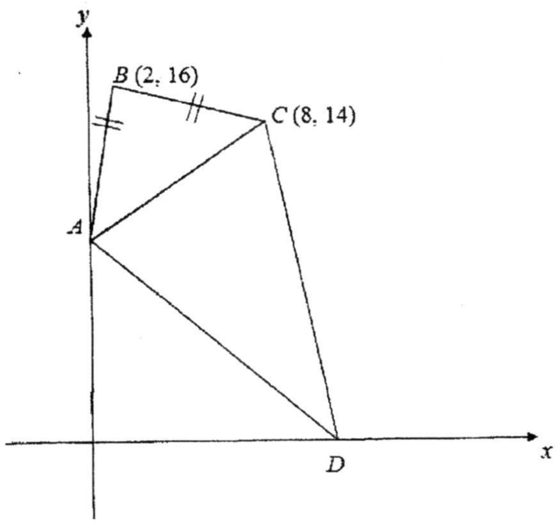
$S = 74\frac{2}{3}m$ -M1

Total distance travelled during the 1st 8 s

$S = 9\frac{1}{3} + 9\frac{1}{3} + 33\frac{1}{3} + 33\frac{1}{3} + 74\frac{2}{3}m$

$= 160$ m -A1

11. [Solution to this question by accurate drawing will not be accepted]



The diagram which is not drawn to scale shows a quadrilateral *ABCD*. The point *B* is (2, 16) and the point *C* is (8, 14). Triangle *ABC* is an isosceles triangle and point *A* and point *D* lies the y – axis and x – axis respectively.

(i) Find the coordinates of A. [3]

(i) Given tl |C : area of ΔACD is $1 : 3$, find the coordinates of *D*. [4]

(ii) Show that $ABCD$ is a kite. [3]

(i) Let A be (0,y)

$\sqrt{2^2+(16-y)^2}=\sqrt{36+4}$

$\sqrt{4+256-32y+y^2}=\sqrt{40}$ - M1

$y^2-32y+220=0$

$(y-10)(y-22)=0$ -M1

$y=10$ or $y=22$ (reject)

$A(0,10)$ - A1

(ii) Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 2 & 0 & 8 & 2 \\ 16 & 10 & 14 & 16 \end{vmatrix}$ - M1

$= 20$ units

Area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & x & 8 & 0 \\ 10 & 0 & 14 & 10 \end{vmatrix}$ - M1

$120 = 14x + 80 - 10x$

$x = 10$ -M1

$D(10,0)$ - A1

(iii) $AD = \sqrt{10^2+10^2}$

$AD = \sqrt{200}$

$CD = \sqrt{14^2+2^2}$

$CD = \sqrt{200}$ - M1

Grad of $AC = \frac{4}{8}$

$= \frac{1}{2}$

Grad of $BD = -\frac{16}{8}$

$= -2$ - M1

Since grad. of AC x grad. $BD = -1$ and length of AD = length of CD, $ABCD$ is a kite. – A1

12. The line $x = 17$ is a tangent to a circle and the points A (1, 9) and B (1, – 7) are on the circumference of the circle.

(i) Show that the radius of the circle is 10 units. [4]

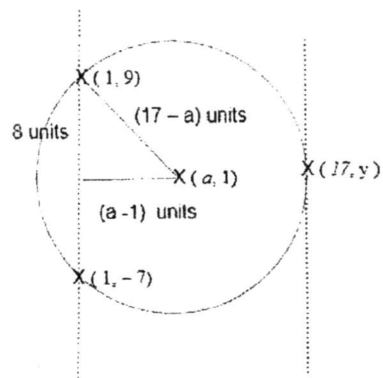
(ii) State the coordinates of the centre of the circle. [1]

(iii) Write down the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$ [2]

(iv) The circle is reflected along the line $y = -1$, show that the point (3, 10) does not lie on the reflected circle. [3]

(i) y– coordinates of the centre of the circle

$= \dfrac{9+(-7)}{2} = 1$ -M1



Let the x coordinates of the centre of the circle be a

$(17-a)^2 = (a-1)^2 + 8^2$ - M1

$289 - 34a + a^2 = a^2 - 2a + 1 + 64$

$224 = 32a$

$a = 7$ -M1

Radius $= 17 - 7 = 10$ units (Shown) - A1

(ii) Centre of circle = (7,1) - B1

(iii) $(x-7)^2 + (y-1)^2 = 100$ - M1

$x^2 + y^2 - 14x - 2y - 50 = 0$ - A1

(iv) Center of the reflected circle is (7,–3) - M1

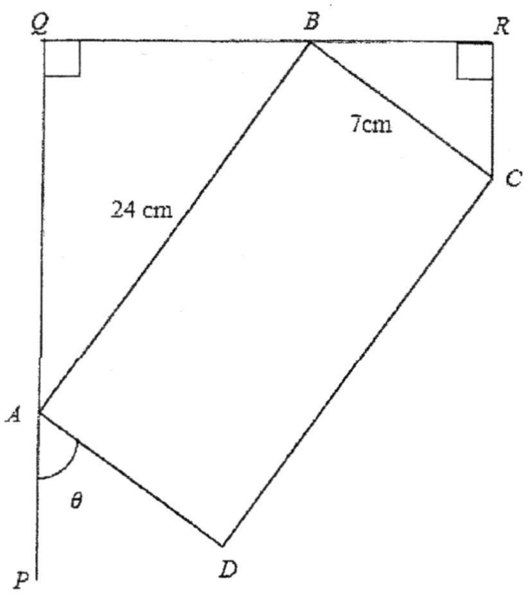
Distance $= \sqrt{(3-7)^2 + (10+3)^2}$

$= \sqrt{16+169}$

$= \sqrt{185}$ -M1

$= 13.6 > 10$ -A1

13. In the diagram below, $ABCD$ is a rectangle. The line QR is perpendicular to the lines PQ and CR. Points A and B lie on the lines PQ and QR respectively and angle $PAD = \theta$. AB is 24 cm and BC is 7 cm.



(i) Show that the length of QR is $24\cos\theta + 7\sin\theta$. [4]

(ii) Express $24\cos\theta + 7\sin\theta$ in the form of $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

(iii) Find the value of θ when QR is 17 cm. [2]

(iv) Find the maximum length of QR and state the corresponding value of θ [3]

(i) $\angle QAB = 90° - \theta$

$\angle QBA = \theta$ - M1

$QB = 24\cos\theta$ - M1

$\angle RBC = 90° - \theta$

$\angle BCR = \theta$

$BR = 7\sin\theta$ - M1

$QR = QB + BR$

$QR = 24\cos\theta + 7\sin\theta$ - A1

(ii) $24\cos\theta + 7\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$24 = R\cos\alpha$

$7 = R\sin\alpha$

$R = 25$ - M1

$\tan\alpha = \dfrac{7}{24}$

$\alpha =$ -M1

$24\cos\theta + 7\sin\theta = 25\cos(\theta - 16.3°)$ - A1

(iii) $17 = 5\cos(\theta - 16.3°)$

$\cos(\theta - 16.3°) = \frac{17}{25}$ -M1

$\theta = 63.5^0$ -A1

(iv) $QR = 25\cos(\theta - 16.3°)$

Max length of $QR = 25$ -A1

$\cos(\theta - 16.3°) = 1$ -M1

$\theta = 16.3°$ -A1
