Name: Register Number: Class:

PRELIMINARY EXAMINATION (3) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **15 September 2016, Thursday**

Additional Materials : Writing Paper (8 sheets) **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1 Given that $y = \dfrac{x^4 - 2}{x}$, $x \neq 0$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Hence, show that y is an increasing function for all real values of x except zero. [1]

2 (a) Given that $\log_9 m = n$, express each of the following in terms of n.

(i) $\log_9 (9m^2)$ [2]

(ii) $\log_3 \dfrac{1}{m}$ [3]

(b) Solve the equation $2(\ln x)^2 + 3\ln\left(\dfrac{1}{x}\right) = 5$. [4]

3 On a university campus of 6 000 students, one student returned from vacation with a contagious flu virus. The spread of the virus through the student body is given by

$$f(t) = \frac{6000}{1 + 5999e^{-0.5t}}$$

where f(t) is the total number of students infected after t days. The university will cancel classes when 50% or more of the students are infected. Estimate,

(i) the number of students infected after 5 days, giving your answer to the nearest whole number, [1]

(ii) after how many days will the classes be cancelled. [3]

4 (a) Find the range of values of x for which $(x-2)(x+3) \geq 6$, [3]

(b) Find the range of values of k for which the line $y + kx = 8$ and the curve $x^2 + 4y = 16$ do not intersect. [4]

5 The function f is defined by $f(x) = 4x^2 - 4x - 15$ for $-3 \leq x \leq 4$.

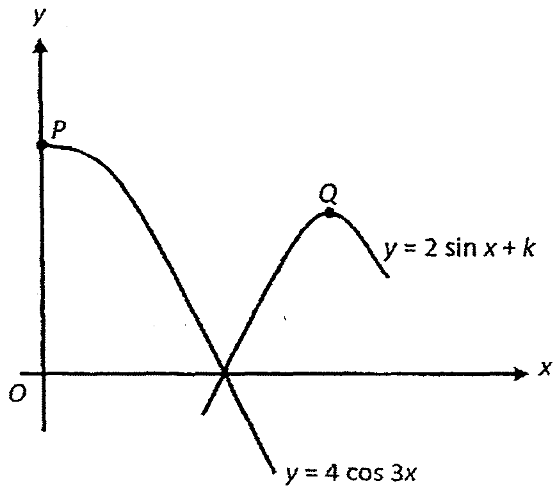
(i) Sketch the graph of $y = |f(x)|$, indicate clearly the x and y intercepts. [4]

(ii) Determine the set of values of m for which there are two or three distinct solutions for the equation $|f(x)| = m$. [2]

6 (a) Prove that $(\sec\theta + \tan\theta)^2 = \dfrac{1 + \sin\theta}{1 - \sin\theta}$. [4]

(b) Find all the values of t between 0 and 12 for which $\sin(\dfrac{\pi t}{5}) = \dfrac{\sqrt{3}}{2}$. [3]

7 The diagram, which is not drawn to scale, shows parts of the graphs of $y = 4\cos 3x$ and $y = 2\sin x + k$.



(i) State the amplitude of $y = 2\sin x + k$ and the period of $y = 4\cos 3x$. [2]

(ii) Points P and Q are the respective maximum points on these graphs. Given that the two graphs intersect at the x-axis, find the value of k and the coordinates of P and of Q. [6]

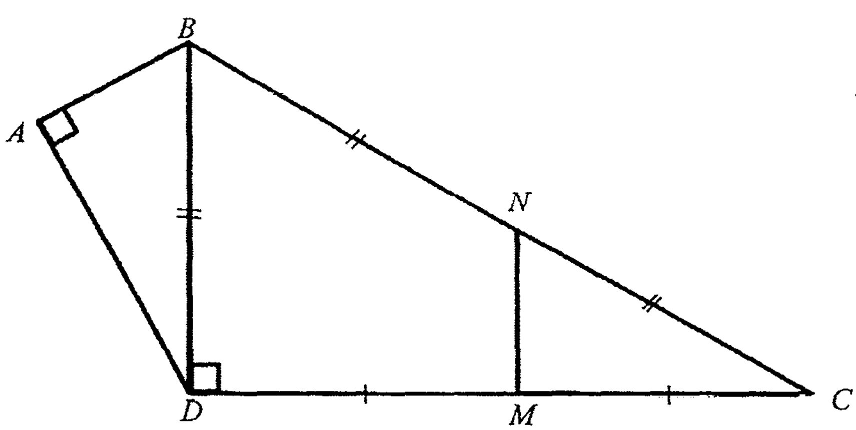
8 A particle P is traveling in a straight line with a velocity v ms^{-1}, given by $v = -2t^2 + 7t + 4$, where t is the number of seconds after passing a fixed point O. Calculate

(i) the value of t at which the particle comes to instantaneous rest, [2]

(ii) the maximum velocity achieved by the particle, [3]

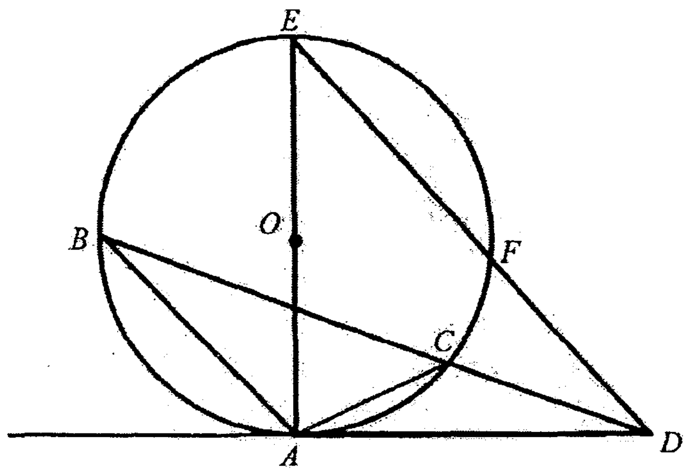
(iii) the total distance travelled by P from $t = 0$ to $t = 5$. [4]

9 (a)



In the diagram, M and N are mid-points of CD and BC respectively. DB bisects $\angle ABC$, $DB = CN$ and $\angle BAD = \angle BDC = 90°$. Prove that ΔABD is congruent to ΔMNC. [4]

(b)



In the diagram, triangle ABC is inscribed in the circle with centre O. The tangent at A meets the line EF and BC produced at D.

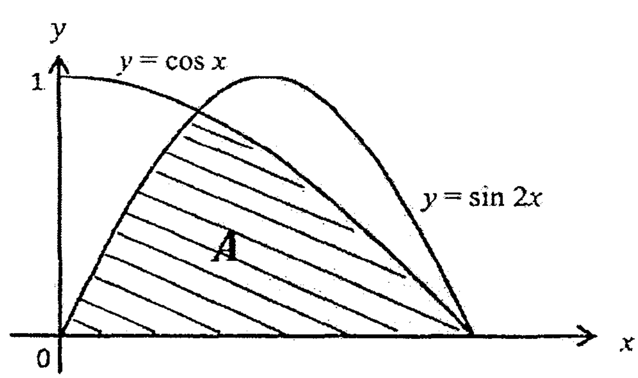
Prove that

(i) ΔADC and ΔBDA are similar. [2]

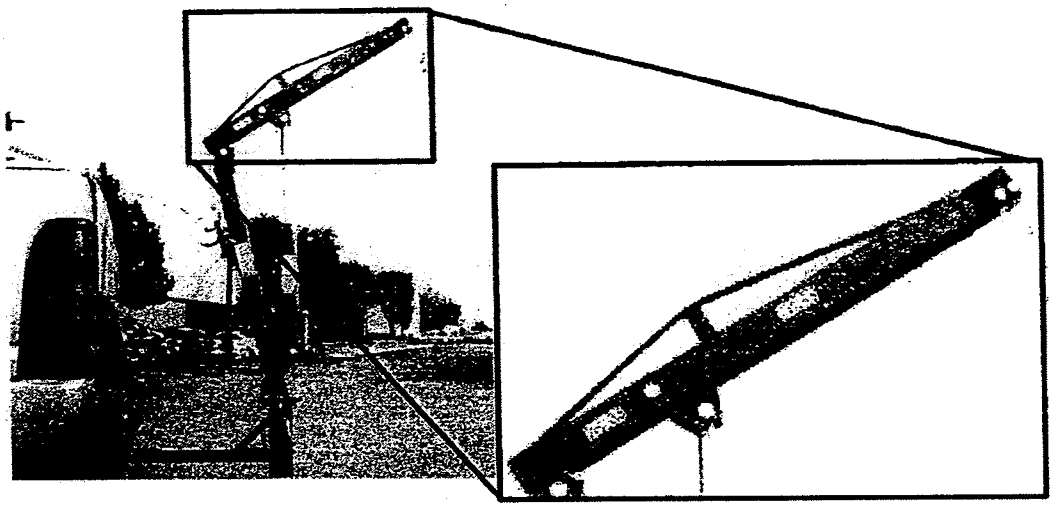
(ii) $BD \times CD = DE^2 - AE^2$ [3]

10 (a) It is given that $y = (x-2)\sqrt{2x-1}$. Find the exact value of x when the rate of decrease of y is three times the rate of increase of x. [5]

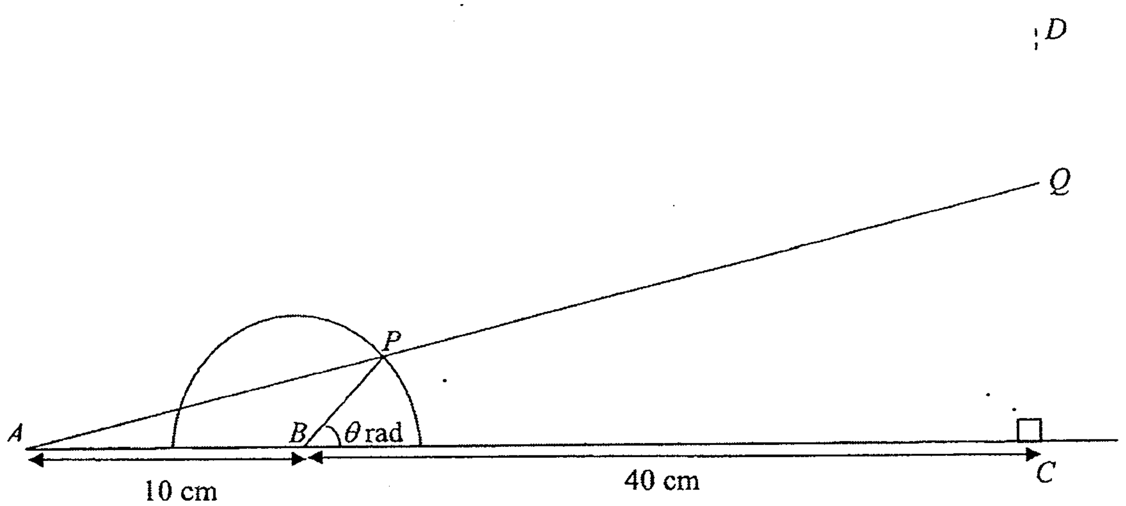
(b) The region A, shown in the diagram is bounded by the curves $y = \sin 2x$, $y = \cos x$ and the x-axis. Find its area. [5]



11 The pictures below show a load lifter and the close-up of its extensible arm.



The movement of the arm can be modelled with the diagram shown below.



(i) In the diagram, APQ is a straight line representing the arm. ABC is a straight line with $AB = 10$ cm and $BC = 40$ cm and CD is perpendicular to ABC. The arm is lifting an object vertically from point C. P is a variable point on the semicircle with centre B, radius 6 cm and $\angle CBP = \theta$. The length of the arm is adjusted so that the point Q lies along the vertical line CD during the lifting of the object.

Show that $CQ = \dfrac{150\sin\theta}{5 + 3\cos\theta}$. [3]

(ii) Find the value of θ for which CQ is a maximum. [5]

~~~~~ **End of Paper** ~~~~~
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Answers

1 (a) $\frac{dy}{dx} = 3x^2 + \frac{2}{x^2}$

(b) Since $3x^2 + \frac{2}{x^2} > 0$ thus $\frac{dy}{dx} > 0$ for all values of x, except $x = 0$

$\Rightarrow$ y is an increasing function (shown)

2 (a) (i) $1 + 2n$

(ii) $-2n$

(b) $x = e^{\frac{5}{2}}$ or $x = \frac{1}{e}$

$x = 12.2$ or $x = 0.368$ (to 3 s.f.)

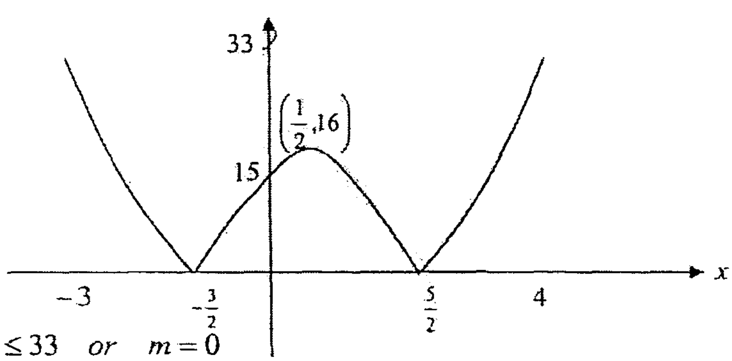
3 (i) 12 *student*

(ii) 18 *days*

4 (a) $x \leq -4$ or $x \geq 3$

(b) $-2 < k < 2$

5 (i)



(ii) $16 \leq m \leq 33$ or $m = 0$

6 (a) $LHS = (\sec\theta + \tan\theta)^2$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{2\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1 + 2\sin\theta + \sin^2\theta}{1 - \sin^2\theta}$

$= \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$$= \frac{1+\sin\theta}{1-\sin\theta} \text{ (proven)}$$

(b) $t = \frac{5}{3}, \frac{10}{3} \; or \; \frac{35}{3}$

7 **(i)** Amplitude $= 2$ and Period $= 120°$ or $\frac{2\pi}{3}$

(ii) $k = -1 \quad P(0, 4) \quad Q(\frac{\pi}{2}, 1)$ or $(90°, 1)$

8 **(i)** $t = 4$

(ii) max $velocity = 10\frac{1}{8} ms^{-1}$

(iii) $34.5\, m$

9 **(a)** Since M and N are mid-points of CD and BC

$MN \parallel DB$ (Mid-point Theorem)

$\Rightarrow \angle NMC = \angle BDC = 90°$ (Corr. $\angle s$ $MN \parallel DB$)

$\Rightarrow \angle MNC = \angle DBC$ (Corr. $\angle s$ $MN \parallel DB$)

Given DB bisects $\angle ABC$

$\Rightarrow \angle ABD = \angle DBC = \angle MNC$

$DB = CN$ (given)

$\Delta ABD \equiv \Delta MNC$ (AAS) (proven)

(b) (i) $\angle ADC = \angle BDA$ (common angle)
$\angle CAD = \angle ABD$ (alternate segment theorem)
$\therefore \Delta ADC$ *and* ΔBDA *are similar* (angle-angle similarity test)

(ii) $\frac{BD}{AD} = \frac{AD}{CD}$ (corr ratios of similar triangles)
$\Rightarrow BD \times CD = AD^2$
Since AD is tangent to circle
$\angle DAE = 90°$ (tangent $\perp$ radius)
$\therefore AD^2 = DE^2 - AE^2$ (pythagoras' theorem)
$\Rightarrow BD \times CD = DE^2 - AE^2$ (proven)

10 **(a)** $x = 2 - \sqrt{2}$

(b) $\frac{3}{4} units^2$

11 (a) From the diagram, PT is perpendicular to AC

ΔAPT *and* ΔAQC *are similar* (*angle* – *angle similarity test*)

$\dfrac{CQ}{50} = \dfrac{6\sin\theta}{10+6\cos\theta}$ (corr ratios of similar triangles)

$CQ = \dfrac{150\sin\theta}{5+3\cos\theta}$ (shown)

(b) $\theta = 2.21\, rad$ $(to\ 3\ s.f.)$

Prelim 3 Add Math P1
Answer Scheme.

1 (a) $y = x^3 - 2x^{-1}$ [M1]

$\frac{dy}{dx} = 3x^2 + \frac{2}{x^2}$ [A1]

(b) Since $3x^2 + \frac{2}{x^2} > 0$ thus $\frac{dy}{dx} > 0$ for all values of x, except $x = 0$ [B1]

$\Rightarrow$ y is an increasing function (shown)

2 (a) (i) $\log_9(9m^2) = \log_9 9 + 2\log_9 m$ [M1]

$= 1 + 2n$ [A1]

(ii) $\log_3 \frac{1}{m} = \log_3 1 - \log_3 m$ [M1]

$= 0 - \frac{\log_9 m}{\frac{1}{2}}$ [M1]

$= -2n$ [A1]

(b) $2(\ln x)^2 + 3\ln\left(\frac{1}{x}\right) - 5 = 0$

Let $y = \ln x$

$2y^2 - 3y - 5 = 0$ [M1]

$(2y - 5)(y + 1) = 0$ [M1]

$y = \frac{5}{2}$ *or* $y = -1$

$\ln x = \frac{5}{2}$ *or* $\ln x = -1$ [M1]

$x = e^{\frac{5}{2}}$ *or* $x = \frac{1}{e}$ [A1]

Accept $x = 12.2$ *or* $x = 0.368$ (*to* 3 *s.f.*)

3 (i) When t = 5

$$f(5) = \frac{6000}{1+5999e^{-0.5(5)}}$$

$$= \frac{6000}{1+5999e^{-0.5(5)}}$$

$= 12.159 \approx 12$ *student* [B1]

(ii) For classes to be cancelled, $f(t) \geq 3000$

$$\frac{6000}{1+5999e^{-0.5t}} \geq 3000$$ [M1]

$2 \geq 1+5999e^{-0.5t}$

$$e^{-0.5t} \leq \frac{1}{5999}$$ [M1]

$$t \geq -2\ln\left(\frac{1}{5999}\right) = 17.398$$

$\therefore$ *after* 18 *days* [A1]

4 (a) $x^2+x-12 \geq 0$ [M1]

$(x+4)(x-3) \geq 0$ [M1]

$x \leq -4$ *or* $x \geq 3$ [A1]

(b) $y = 8-kx$

$x^2+4(8-kx)=16$ [M1]

$x^2-4kx+16=0$

For no intersection, discriminant < 0

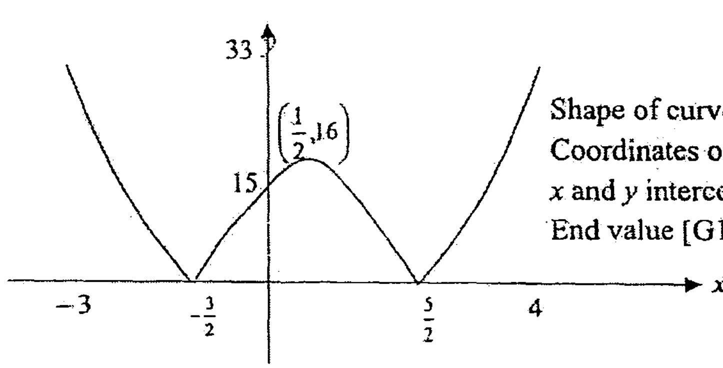
$16k^2-4(1)(16)<0$ [M1]

$k^2-4<0$

$(k-2)(k+2)<0$ [M1]

$-2<k<2$ [A1]

5 (i)



Shape of curve [G1]
Coordinates of max pt [G1]
x and y intercepts [G1]
End value [G1]

(ii) $16 \le m \le 33 \quad or \quad m = 0$ [B2]

6 (a) $LHS = (\sec\theta + \tan\theta)^2$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$ [M1]

$= \dfrac{1}{\cos^2\theta} + \dfrac{2\sin\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$ [M1]

$= \dfrac{1 + 2\sin\theta + \sin^2\theta}{1 - \sin^2\theta}$

$= \dfrac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$ [M1]

$= \dfrac{1+\sin\theta}{1-\sin\theta}$ (proven) [A1]

(b) $\sin\left(\dfrac{\pi t}{5}\right) = \dfrac{\sqrt{3}}{2} \qquad 0 < t < 12 \quad \Rightarrow 0 < \dfrac{\pi t}{5} < \dfrac{12\pi}{5}$

$\alpha = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$ [M1]

$\dfrac{\pi t}{5} = \dfrac{\pi}{3}, \dfrac{2\pi}{3} \; or \; \dfrac{7\pi}{3}$ [M1]

$t = \dfrac{5}{3}, \dfrac{10}{3} \; or \; \dfrac{35}{3}$ [A1]

7 (i) Amplitude = 2 and Period = 120° or $\dfrac{2\pi}{3}$ [B2]

(ii) Coordinates of P (0, 4) [B1]

Since the two curves intersect at the first x-intercept for $y = 4\cos 3x$,

$\Rightarrow x = \dfrac{\pi}{6}$ [M1]

When $x = \dfrac{\pi}{6}$, $y = 0$ [M1]

$0 = 2\sin\left(\dfrac{\pi}{6}\right) + k$

$\Rightarrow k = -1$ [A1]

For graph of $y = 2\sin x - 1$, first maximum is at $x = \dfrac{\pi}{2}$ [M1]

When $x = \dfrac{\pi}{2}$, $y = 1$

$\therefore$ coordinates of $Q\,(\frac{\pi}{2},1)$ or $(90°, 1)$ [A1]

8 **(i)** For particle at rest, v = 0

$-2t^2+7t+4=0$

$(-2t-1)(t-4)=0 \quad or \quad (2t+1)(t-4)=0$ [M1]

$t=-\frac{1}{2}\,(rejected) \quad or \quad t=4$ [A1]

(ii) For maximum velocity, $\frac{dv}{dt}=0$ [M1]

$-4t+7=0$

$t=\frac{7}{4}s$ [M1]

max $velocity=-2\left(\frac{7}{4}\right)^2+7\left(\frac{7}{4}\right)+4=\frac{81}{8}=10\frac{1}{8}\,ms^{-1}$ [A1]

(iii) $s=\int v\,dt=-\frac{2t^3}{3}+\frac{7t^2}{2}+4t+C$ [M1]

When t=0, s=0 $\Rightarrow C=0$

When t=4, s = $29\frac{1}{3}$ m [M1]

When t=5, s = 24.17m [M1]

$\therefore total\ distance=29\frac{1}{3}+\left(29\frac{1}{3}-24.17\right)=34.5\ m$ [A1]

9 **(a)** Since M and N are mid-points of CD and BC

$MN\ //\ DB$ (Mid-point Theorem) [M1]

$\Rightarrow \angle NMC=\angle BDC=90°$ (Corr. $\angle s$ $MN\ //\ DB$)

$\Rightarrow \angle MNC=\angle DBC$ (Corr. $\angle s$ $MN\ //\ DB$) [M1]

Given DB bisects $\angle ABC$

$\Rightarrow \angle ABD=\angle DBC=\angle MNC$ [M1]

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Since AD is tangent to circle

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$\Rightarrow BD\times CD = DE^2 - AE^2$ (proven) [A1]

10 **(a)** $y=(x-2)\sqrt{2x-1}$

$$\frac{dy}{dx}=\sqrt{2x-1}+(x-2)\left(\frac{1}{2\sqrt{2x-1}}\right)(2)$$

$$\frac{dy}{dx}=\frac{2x-1+x-2}{\sqrt{2x-1}}=\frac{3x-3}{\sqrt{2x-1}}$$ [M1]

$$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dx}$$

$$-3=\frac{3x-3}{\sqrt{2x-1}}$$ [M1]

$$\sqrt{2x-1}=1-x$$

$$2x-1=1-2x+x^2$$

$$x^2-4x+2=0$$ [M1]

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(2)}}{2}$$ [M1]

$$x=2\pm\sqrt{2}$$

Therefore, $x=2-\sqrt{2}$ since $\dfrac{dy}{dx}<0$ [A1]

(b) $\cos x=\sin 2x$ [M1]

$\cos x = 2\sin x\cos x$

$\cos x(2\sin x - 1) = 0$

$\Rightarrow x = \frac{\pi}{6}$ or $\frac{\pi}{2}$ [M1]

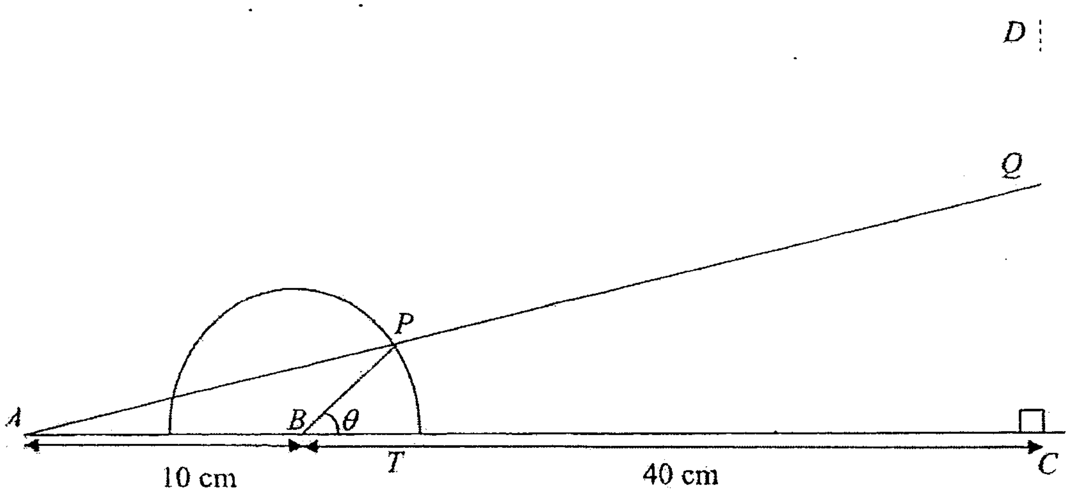
$Area = \int_0^{\frac{\pi}{6}} \sin 2x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x dx$ [M1]

$= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{6}} + [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ [M1]

$= \left[-\frac{1}{4} + \frac{1}{2}\right] + \left[1 - \frac{1}{2}\right]$

$= \frac{3}{4}\ units^2$ [A1]

11 (a)



From the diagram, PT is perpendicular to AC

ΔAPT *and* ΔAQC *are similar* (*angle – angle similarity test*) [M1]

$\frac{CQ}{50} = \frac{6\sin\theta}{10 + 6\cos\theta}$ (corr ratios of similar triangles) [M1]

$CQ = \frac{150\sin\theta}{5 + 3\cos\theta}$ (shown) [A1]

(b) $\frac{d}{d\theta}(CQ) = \frac{(5 + 3\cos\theta)(150\cos\theta) - (-3\sin\theta)(150\sin\theta)}{(5 + 3\cos\theta)^2}$ [M1]

$$= \frac{750\cos\theta + 450}{(5+3\cos\theta)^2}$$ [M1]

For maximum CQ,

$$\frac{d}{d\theta}(CQ) = \frac{750\cos\theta + 450}{(5+3\cos\theta)^2} = 0$$

$$750\cos\theta + 450 = 0$$

$$\cos\theta = -\frac{3}{5}$$ [M1]

$\theta = 2.21\, rad \quad (to\ 3\ s.f.)$ [A1]

θ	2.21^-	2.21	2.21^+
$\frac{d}{d\theta}(CQ)$	+	0	-

$\therefore$ *when* $\theta = 2.21\, rad$, CQ *is* max. [A1]

Name: **Register Number:** **Class:**

Preliminary Examination (3) 2016
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**
PAPER 2 **16 September 2016, Friday**

Additional Materials : Writing Papers (8 sheets) **2 hours 30 minutes**

Graph Paper (1 sheet)

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At the end of the examination, fasten all your work securely together. Tie your answer script into 2 separate bundles such as first bundle consists of question1 to 6 and second bundle consists of question 7 to 11.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1. The roots of the quadratic equation $3x^2 + \frac{27}{4} = 3x$ are α^2 and β^2.

   (i) Find the value of $\alpha + \beta$ and of $\alpha\beta$ where α and β are both negative. [5]

   (ii) Hence find the quadratic equation whose roots are α^3 and β^3. [4]

2. Given $\mathrm{f}(x) = 2 - 24\sin x\cos x$ and $\mathrm{g}(x) = 10(1 + \cos^2 x)$.

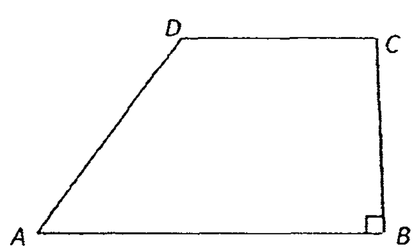
   (i) Express the sum of $\mathrm{f}(x)$ and $\mathrm{g}(x)$ in the form $R\cos(2x + \alpha) + q$ where R and q are constants and $R > 0$, $0 < \alpha < \frac{\pi}{2}$. [5]

   (ii) Hence find the minimum value of $\frac{2}{\mathrm{f}(x) + \mathrm{g}(x)}$ and the corresponding values of x for $0 < x < 2\pi$. [3]

3. **(i)** Show $\frac{d}{dx}\ln(\tan^2 3x) = 12\operatorname{cosec} 6x$. [4]

   (ii) Hence integrate $\frac{1}{\sin 6x} + \frac{1}{3e^{2-3x}}$ with respect to x. [4]

4. The diagram shows a right-angled trapezium $ABCD$ such that $2AB = 3CD$ and AB is parallel to DC. Given the height BC of the trapezium is $(3 - \sqrt{3})$cm and area of the trapezium is $(2 + 3\sqrt{3})\text{cm}^2$.



Find length CD in the form $(a + b\sqrt{3})$cm, where a and b are rational numbers. [5]

5. **(i)** The sum of the second and third term of the expansion of $(1 + kx)^n$ is $60x + 1740x^2$. Find the value of k and of n. [5]

   (ii) Hence write down the first 4 terms in the expansion of $(1 + kx)^n$ in ascending powers of x. [2]

   (iii) Hence determine the coefficient of a^3 in the expansion of $(1 + k(a - 2a^2))^n$. [3]

6. An experiment to find the constant acceleration, a m/s^2, of an electric toy car moving in one direction, requires students to measure the speed, v m/s from the speedometer when distance, s m varies. The table below shows the experimental values of v and s, which are connected by the equation $v=\sqrt{e^p+2as}$, where p is a constant.

s	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
v	3	5	6	10

(i) Plot v^2 against s and draw a straight line graph. Hence determine which value of v, in the table above, is the incorrect recording. Using your graph to estimate the correct v value. [4]

(ii) Use your graph to estimate the value of a and of p. [3]

(iii) Explain what does the value of e^p represents. [1]

(iv) By drawing a suitable straight line on your graph, solve $s=\left(\frac{120-2e^p}{4a+3}\right)$. [2]

Start on a fresh sheet of writing paper and tie answer script from question 7 to 11 together.

7. (i) Explain whether the curve $y=4-3e^{2x}$ has any stationary point. [2]

(ii) Sketch the graph $y=4-3e^{2x}$ indicating clearly the asymptote and x and y-intercepts. [3]

(iii) Hence solve $2x=\ln\left(1-\frac{4}{3}x\right)$ by inserting a straight line on the same graph in part (ii). [3]

8. (i) Factorise $8x^3+4x^2-2x-1$ completely. [3]

(ii) Hence express $\frac{2x+2}{(8x^3+4x^2-2x-1)}$ in partial fractions. [4]

(iii) The polynomial $8x^3+4x^2-2x-1$ leaves a remainder of $(px+q)$ when divided by (x^2-1). Find the value of p and of q. [4]

9. Given the curve $y = \frac{2}{3}x^{-\frac{1}{2}}$ and $y = \frac{8}{27}x^{\frac{3}{2}}$.

(i) Sketch the two graphs on the same diagram for $x > 0$ and label the graphs clearly. [2]

(ii) Calculate the coordinates of the point of intersection of the two graphs drawn in (i). [3]

10. The gradient function of a curve $y = f(x)$ is given by $m + n(3x-2)^3$. A point P lies on the curve and its x-coordinate is 2. The equation of the normal to the curve at P is given by $37y = 9x - 129$. The curve has a turning point at Q whose x-coordinate is $\frac{5}{3}$.

(i) Show that the value of m is 3 and n is $-\frac{1}{9}$. [3]

(ii) Find the equation of the curve. [4]

(iii) Find the area of triangle PQR where R is the point the curve intersect the y-axis. [4]

11. Given that a circle C_1 passes through the point $A(2, 0)$, $B(5, 1)$ and $C(6, 0)$.

(i) Show that the coordinates of centre D of the circle C_1 is $(4, -1)$ and hence find the radius of the circle. [6]

(ii) Find the equation of the circle C_1 in standard form. [1]

(iii) Given 2 tangents are drawn from a point E to touch the circle at point B and C. Find the coordinates of point E. [5]

(iv) Explain why a circle can be drawn to pass through the points B, C, D and E. Hence find the coordinate of the centre of this circle. [3]

End of Paper

Answers

1i) $\alpha\beta = \frac{3}{2}$ or $-\frac{3}{2}$ (rej)
$(\alpha+\beta) = 2$ (rej) or -2

1ii) $x^2 - x + \frac{27}{8} = 0$

2i) $f(x) + g(x) = 13\cos(2x + 1.18) + 17$

2ii) min $= \frac{1}{15}$, $x = 2.55,\ 5.70$

3ii) $\frac{1}{12}\ln(\tan^2 3x) + \frac{1}{9}e^{3x-2} + c$ OR
$\frac{1}{6}\ln(\tan 3x) + \frac{1}{9}e^{3x-2} + c$

4) $CD = 2 + \frac{22}{15}\sqrt{3}$

5i) $k = 2$
$n = 30$

5ii) $1 + 60x + 1740x^2 + 32480x^3 + \ldots$

5iii) coeff. of $a^3 = 25520$

6i)

S	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
V^2	9	25	36	100

6ii) incorrect $v = 6$ m/s
corrected $v = 7$

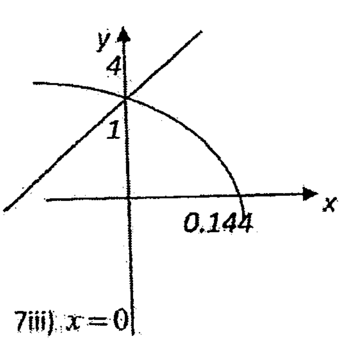
6ii) $p = \ln 4$ or 1.39
$a = 0.605$

6iii) e^p represents the square of initial speed or square of initial velocity

6iv) $s = 20.5$ or $21m$

7i) $\frac{dy}{dx} = -6e^{2x}$
$\frac{dy}{dx} < 0$, $\frac{dy}{dx} \neq 0$, no stationary point

7ii)



7iii) $x = 0$

8i) $(2x-1)(4x^2+4x+1)=(2x-1)(2x+1)^2$

8ii) $\dfrac{2x+2}{(8x^3+4x^2-2x-1)}=\dfrac{3}{4(2x-1)}-\dfrac{3}{4(2x+1)}-\dfrac{1}{2(2x+1)^2}$

8iii) $q=3$ and $p=6$

9ii) (1.5, 0.544) or $\left(\dfrac{3}{2}, \dfrac{2}{9}\sqrt{6}\right)$

10iii) $y=3x-\dfrac{1}{108}(3x-2)^4-\dfrac{179}{27}$ 10iii) $\dfrac{5}{4}$

$R=\sqrt{5}$

$(x-4)^2+(y+1)^2=5$

$E\left(\dfrac{17}{3}, \dfrac{2}{3}\right)$

11) Since $\angle DBE=\angle DCE=90^\circ$

(tangent perpendicular to radius).

$\therefore$ A circle with diameter DE ($\angle$ in semicircle).

Centre $\left(\dfrac{4+\frac{17}{3}}{2}, \dfrac{-1+\frac{2}{3}}{2}\right)=\left(\dfrac{29}{6}, -\dfrac{1}{6}\right)$

Prelim Exam (3) 2016 Additional Mathematics Paper 2 – Secondary 4 Express

Qn No	Suggested Solutions
1i	$3x^2+\frac{27}{4}=3x$
	$3x^2-3x+\frac{27}{4}=0$
	$x^2-x+\frac{9}{4}=0$
	$\alpha^2+\beta^2=1$ —— M1
	$(\alpha+\beta)^2-2\alpha\beta=1$
	$(\alpha\beta)^2=\frac{9}{4}$ —— M1
	$\alpha\beta=\frac{3}{2}$ or $-\frac{3}{2}$ (rej) —— A1
	$(\alpha+\beta)^2-2\left(\frac{3}{2}\right)=1$ —— M1
	$(\alpha+\beta)^2=4$
	$(\alpha+\beta)=2$ (rej) or -2 —— A1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$
1ii	$=(-2)^3-3\left(\frac{3}{2}\right)(-2)$ —— M1
	$=1$ —— M1
	$(\alpha\beta)^3=\left(\frac{3}{2}\right)^3$ —— M1
	$=\frac{27}{8}$
	$x^2-x+\frac{27}{8}=0$ —— A1
3ii	$\int\frac{1}{\sin 6x}+\frac{1}{3e^{2-3x}}dx=\int\frac{1}{\sin 6x}+\frac{1}{3}e^{3x-2}dx$
	$=\frac{1}{12}\ln\left(\tan^2 3x\right)+\frac{1}{3(3)}e^{3x-2}+c$ —— M2
	$=\frac{1}{12}\ln\left(\tan^2 3x\right)+\frac{1}{9}e^{3x-2}+c$ OR
	$=\frac{1}{6}\ln\left(\tan 3x\right)+\frac{1}{9}e^{3x-2}+c$ —— A2

Qn No	Suggested Solutions
2i	$f(x)+g(x)=2-24\sin x\cos x+10+10\cos^2 x$
	$=12-12\sin 2x+10\left(\frac{\cos 2x+1}{2}\right)$ —— M1
	$=12-12\sin 2x+5\cos 2x+5$
	$==17+5\cos 2x-12\sin 2x$ —— M1
	$R=\sqrt{5^2+12^2}=13$ —— M1
	$\tan\alpha=\frac{12}{5},\quad \alpha=1.176$ —— M1
	$f(x)+g(x)=13\cos(2x+1.18)+17$ —— A1
2ii	$\min\left(\frac{2}{f(x)+g(x)}\right)=\frac{2}{13\cos(2x+1.176)+17}$
	$=\frac{2}{13+17}=\frac{1}{15}$ —— M1
	$\cos(2x+1.176)=1$ —— M1
	basic angle $=0$
	$(2x+1.176)=0$ (rej), 2π, 4π
	$x=2.55,\quad 5.70$ —— A1
3i	$\frac{d}{dx}\ln\left(\tan^2 3x\right)=\frac{d}{dx}2\ln\left(\tan 3x\right)$
	$=\frac{2(3)\sec^2 3x}{\tan 3x}$ —— M1
	$=\frac{6\sec^2 3x}{\tan 3x}$
	$=\frac{6(\cos 3x)}{\cos^2 3x\sin 3x}$
	$=\frac{6}{\cos 3x\ \sin 3x}$ —— M1
	$=\frac{12}{\sin 6x}$ —— M1
	$=12\operatorname{cosec} 6x$ (*shown*) —— A1

4

Let AB = 3x and CD = 2x

$\frac{1}{2}(3x+2x)(3-\sqrt{3}) = 2+3\sqrt{3}$ — M1

$\frac{5x}{2} = \frac{2+3\sqrt{3}}{3-\sqrt{3}}$

$\frac{5x}{2} = \frac{2+3\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$ — M1

$\frac{5x}{2} = \frac{6+2\sqrt{3}+9\sqrt{3}+3(3)}{9-3}$

$\frac{5x}{2} = \frac{15+11\sqrt{3}}{6}$ — M1

$x = 1+\frac{11}{15}\sqrt{3}$ — M1

$CD = 2+\frac{22}{15}\sqrt{3}$ — A1

6i

B1

S	$4\frac{1}{6}$	$17\frac{1}{2}$	$37\frac{1}{2}$	80
V²	9	25	36	100

Straight line graph of correct axes — B1

incorrect v = 6 m/s — M1

$v^2 = 49$

corrected $v = 7$ — A1

6ii

gradient $= \frac{80\text{-}28}{63\text{-}20} = 1.209$ — M1

$v^2 = e^P + 2as$

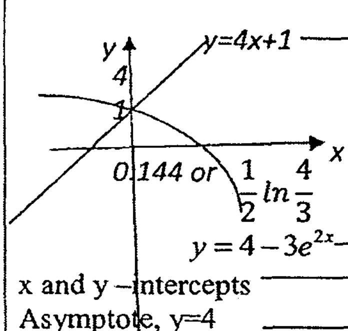
$e^P = 4$

$p = \ln 4$ or 1.39 — B1

$2a = 1.209$

$a = 0.605$ — A1

7ii



y=4x+1 — M1

$y = 4-3e^{2x}$ — B1

x and y – intercepts — B1

Asymptote, y=4 — B1

5i

$(1+kx)^n = \binom{n}{1}(1)(kx) + \binom{n}{2}(1)(kx)^2 + ..$ — M

$= nkx + \frac{n(n-1)}{2}k^2x^2 + ...$

$= 60x + 1740x^2 + ...$

$nk = 60$ — M1

$\frac{n(n-1)}{2}k^2 = 1740$ — M1

$n^2k^2 - nk^2 = 3480$

$60^2 - 60k = 3480$

$k = 2$ — A1

$n = 30$ — A1

5ii

$(1+2x)^{30} = 1 + 60x + 1740x^2 + \binom{30}{3}(1)(kx)^3 +$

$= 1 + 60x + 1740x^2 + 32480x^3 + ...$ — B2

5iii

$(1+k(a-2a^2))^n = 1 + 60(a-2a^2)$

$+ 1740(a-2a^2)^2 + 32480(a-2a^2)^3 + ...$ — M

$= 1740(2)(a)(-2a^2) + 32480a^3$

$= -6960a^3 + 32480a^3 + .$ — M1

$= 25520a^3 + ...$

coeff. of $a^3 = 25520$ — A1

6iii

e^P represents the square of initial speed or square of initial velocity — B1

6iv

$s = \frac{120\text{-}2e^P}{4a+3}$

$s(4a+3) = 120 - 2e^P$

$2e^P + 4as = 120 - 3s$

$e^P + 2as = 60 - 1.5s$

$v^2 = 60 - 1.5s$ Draw the line — M1

$s = 20.5$ or $21m$ — A1

7i

$\frac{dy}{dx} = -6e^{2x}$ — M1

$\frac{dy}{dx} < 0$, $\frac{dy}{dx} \neq 0$, no stationary point — A1

7iii

$2x = \ln\left(1-\frac{4}{3}x\right)$

$e^{2x} = 1-\frac{4}{3}x$

$3e^{2x} = 3-4x$

$4x = 3-3e^{2x}$

$4x+1 = 4-3e^{2x}$

$y = 4x+1$ (Draw this striaght line) — M1

$x = 0$ — A1

8i

$8x^3+4x^2-2x-1$

by trialand error, let $x = \frac{1}{2}$ — M1

$8\left(\frac{1}{2}\right)^3+4\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-1=0$

$\therefore (2x-1)$ is a factor

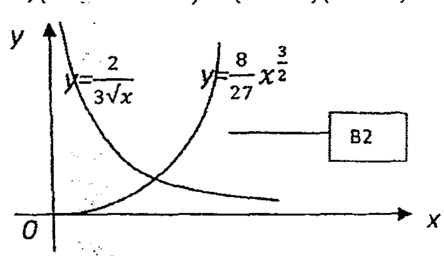
$$
\begin{array}{r l}
 & 4x^2+4x+1 \\
2x-1 \; \big) & 8x^3+4x^2-2x-1 \\
 & -\left(8x^3-4x^2\right) \\
 & 8x^2-2x-1 \\
 & -\left(8x^2-4x\right) \\
 & 2x-1 \\
 & -(2x-1) \\
 & 0
\end{array}
$$

M1

A1

$(2x-1)(4x^2+4x+1) = (2x-1)(2x+1)^2$

9i



B2

9ii

$\frac{2}{3}x^{-\frac{1}{2}} = \frac{8}{27}x^{\frac{3}{2}}$ — M1

$x^2 = \frac{9}{4}$ thus $x = \frac{3}{2}$ or $-\frac{3}{2}$ (rej) — M1

$(1.5,\ 0.544)$ or $\left(\frac{3}{2},\ \frac{2}{9}\sqrt{6}\right)$ — A1

8ii

$\frac{2x+2}{(8x^3+4x^2-2x-1)} = \frac{2x+2}{(2x-1)(2x+1)^2}$

Let $\frac{2x+2}{(2x-1)(2x+1)^2} = \frac{A}{2x-1}+\frac{B}{2x+1}+\frac{C}{(2x+1)^2}$ — M1

$2x+2 = A(2x+1)^2+B(2x-1)(2x+1)+C(2x-$

Let $x=\frac{1}{2}$, $2(\frac{1}{2})+2 = A(2(\frac{1}{2})+1)^2$

$A = \frac{3}{4}$ — M1

Let $x = -\frac{1}{2}$, $2\left(-\frac{1}{2}\right)+2 = C\left(2\left(-\frac{1}{2}\right)-1\right)$

$C = -\frac{1}{2}$

Let $x=0$, $2 = A-B-C$

$B = -\frac{3}{4}$ — M1

$\frac{2x+2}{(8x^3+4x^2-2x-1)} = \frac{3}{4(2x-1)}-\frac{3}{4(2x+1)}-\frac{1}{2(2x+1)^2}$ — A1

8iii

Let $x^2-1=0$, $x=1$ or $x=-1$

$8(1)^3+4(1)^2-2(1)-1 = p+q$

$p+q=9$ — M1

$8(-1)^3+4(-1)^2-2(-1)-1 = -p+q$

$q-p=-3$ — M1

$q=3$ and $p=6$ — A2

OR

$$
\begin{array}{r l}
 & 8x+4 \\
x^2-1 \; \big) & 8x^3+4x^2-2x-1 \\
 & -\left(8x^3-8x\right) \\
 & 4x^2+6x-1 \\
 & -\left(4x^2-4\right) \\
 & 6x+3
\end{array}
$$

M2

p = 6, q = 3 — A2

10i	$f'(x)=m+n(3x-2)^3$ $m+n\left(3\left(\frac{5}{3}\right)-2\right)^3=0$ $m+n(27)=0$ $m=-27n$ (eqn 1) M1 $y=\frac{9}{37}x-\frac{129}{37}$ gradient of tangent $=-\frac{37}{9}$ $m+n(3(2)-2)^3=-\frac{37}{9}$ $m+64n=-\frac{37}{9}$ (eqn 2) M1 $n=-\frac{1}{9},\ m=3$ (shown) A1
10ii	$f'(x)=3-\frac{1}{9}(3x-2)^3$ $y=3x-\frac{1}{9}\frac{(3x-2)^4}{4(3)}+c$ M1 $y=3x-\frac{1}{108}(3x-2)^4+c$ $37y=9(2)-129,\quad y=-3$ M1 $-3=3(2)-\frac{1}{108}(3(2)-2)^4+c$ $c=-\frac{179}{27}$ M1 $y=3x-\frac{1}{108}(3x-2)^4-\frac{179}{27}$ (eqn of curve) A1
10iii	$x=0,\ y=\frac{-(-2)^4}{108}-\frac{179}{27}=-\frac{61}{9}$ M1 $R\left(0,-\frac{61}{9}\right)$ $y=3\left(\frac{5}{3}\right)-\frac{1}{108}\left(3\left(\frac{5}{3}\right)-2\right)^4-\frac{179}{27}$ $=-\frac{257}{108}$ M1 $Q\left(\frac{5}{3},-\frac{257}{108}\right)$
10iii	$Area=\frac{1}{2}\begin{vmatrix} 2 & \frac{5}{3} & 0 & 2 \\ -3 & -\frac{257}{108} & -\frac{61}{9} & -3 \end{vmatrix}$ M1 $=\frac{1}{2}\left(-\frac{289}{18}-\left(-\frac{167}{9}\right)\right)$ $=\frac{5}{4}$ A1
11i	$x_D=\frac{6+2}{2}=4$ M1 $\sqrt{(y_D-0)^2+(4-2)^2}=\sqrt{(5-4)^2+(1-y_D)^2}$ M1 A1 $y_D=-1,\ D(4,-1)$ (shown) A1 $R=\sqrt{(5-4)^2+(1-(-1))^2}=\sqrt{5}$ A1 M1
11ii	$(x-4)^2+(y+1)^2=5$ B1
11iii	$M_{DB}=\frac{1-(-1)}{5-4}=2,\quad M_{BE}=-\frac{1}{2}$ M1 $y=-\frac{1}{2}x+C$ $1=-\frac{1}{2}(5)+C$ $C=3.5,\quad y=-\frac{1}{2}x+\frac{7}{2}$ (equation BE) M1 $M_{DC}=\frac{-1-0}{4-6}=\frac{1}{2},\quad M_{CE}=-2$ M1 $y=-2x+C$ $0=-2(6)+C$ $C=12,\quad y=-2x+12$ (equation CE) M1 $-\frac{1}{2}x+\frac{7}{2}=-2x+12,\quad E\left(\frac{17}{3},\frac{2}{3}\right)$ A1
11iv	Since $\angle DBE=\angle DCE=90°$ M1 (tangent perpendicular to radius). $\therefore$ A circle with diameter DE ($\angle$ in semicircle). A1 Centre $\left(\frac{4+\frac{17}{3}}{2},\frac{-1+\frac{2}{3}}{2}\right)=\left(\frac{29}{6},-\frac{1}{6}\right)$ B1

*** End of Paper ***