8

O Level Centre / Index Number	Class	Name
/		

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS — 4047/1

Paper 1

19 August 2016
2 hours

Additional Materials: Writing Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples. paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
/ 80

This document consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the following equations

(a) $5^{2+x} - 3(5^{1-x}) + 10 = 0$, [4]

(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$. [4]

2 (a) Find the greatest value of the integer k for which $-3x^2 + kx - 5$ is never positive for all values of x. [3]

(b) A curve has an equation $y = \frac{x^2}{2-3x}$, where $x \neq \frac{2}{3}$.

Find the range of values of x for which y is decreasing. [4]

3 (i) Prove the identity $1 + \frac{\sin^2 A}{1-\sec^2 A} + \frac{\cos^2 A}{1-\operatorname{cosec}^2 A} = 0$. [3]

(ii) Hence, solve the equation $\frac{\sin^2 A}{1-\sec^2 A} + \frac{\cos^2 A}{1-\operatorname{cosec}^2 A} = \tan(2A + 10°)$ for $-180° < A < 180°$. [4]

4 A curve has the equation $y = 4e^{\tan(\pi - \frac{x}{4})}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If x and y vary with time and y increases at the rate of e units per second when $x = \pi$ radian, find the exact value of the rate of decrease of x at this instant. [4]

5 (a) Sketch the graph of $f(x) = 2 - |5-3x|$ for $-1 \leq x \leq 6$.

Indicate clearly the vertex and the intercepts of the axes. [3]

(b) Solve the equation $2 - |5-3x| = x - 1$. [2]

(c) (i) State the range of the values of c if there is no solution for the equation $2 - |5-3x| = c$, [1]

(ii) State the range of the values of m if there are exactly two solutions for the equation $2 - |5-3x| = mx$. [1]

6 The amount of radioactive Sodium-24, M measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by $M = M_0e^{kt}$, where t is the time in hours, M_0 and k are a constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

(i) Given that there are only 20 grams of Sodium-24 left after 14.9 hours. Find the value of M_0 and of k. [3]

(ii) Find the amount of Sodium-24 remain after 60 hours. [1]

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7 (a) The function f is defined, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, by the equation $f(x) = 2\tan 3x$.

(i) State the period of f. [1]

(ii) Sketch the graph of $y = f(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

(b) On the same diagram drawn in part (a), sketch the graph of $g(x) = 1 - 2\sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

(c) State the number of solutions of the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [1]

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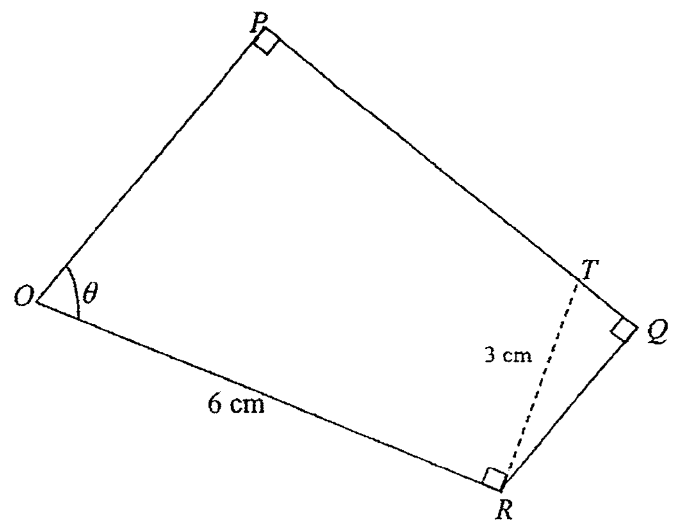
(i) State the value of k. [1]

(ii) Sketch the graph of $f(x) = -\ln x$ for $x > k$. [2]

(iii) Explain how another straight line drawn on your diagram in part (ii) can lead to the graphical solution of $xe^{3-2x} = 1$. Draw this straight line and hence state the number of solutions for $xe^{3-2x} = 1$. [3]

9 The diagram shows a quadrilateral $OPQR$ where $OR = 6$ cm, angle OPQ = angle $PQR = \frac{\pi}{2}$ radian and angle $ROP = \theta$ radian, θ is a variable and an acute angle. T is a point on PQ such that angle $ORT = \frac{\pi}{2}$ radian and $RT = 3$ cm.



(i) Show that the area, A cm^2 of the quadrilateral $OPQR$ is given by

$$A = 9\sin 2\theta + 18\sin^2\theta$$ [3]

(ii) Given that θ can vary, find maximum area of the quadrilateral $OPQR$.

[6]

10 A particle P moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v m/s is given by

$$v_P = 1 - \frac{9}{(3t+1)^2}.$$

(i) Calculate the initial acceleration of the particle P. [2]

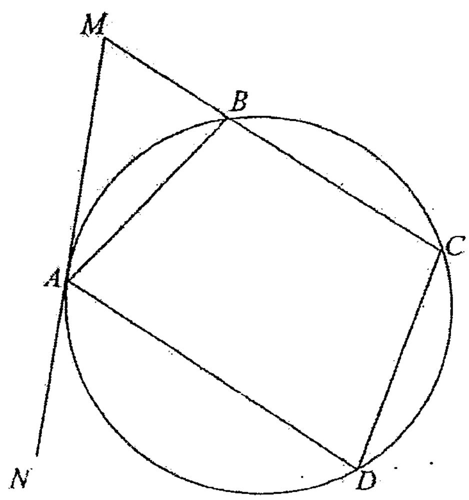
(ii) Show that the particle P is at instantaneously rest at $t = \frac{2}{3}$. [2]

(iii) Calculate the average speed of the particle P during the first 3 seconds after passing O. [4]

Another particle Q moves in a straight line and its displacement, S meter from O after t seconds is given by $S_Q = t - 1$.

(iv) Find the distance from the fixed point O when P first collides with Q. [2]

11 In the diagram, A, B, C and D are points on the circle. MN is a tangent to the circle at A. MBC is a straight line.



(a) Name a triangle which is similar to triangle CAM. [1]

Hence prove that $\left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$. [3]

(b) Given further that AD and BC are parallel, show that

(i) triangle ABM is similar to triangle ADC, [2]

(ii) $AD \times AM = AC \times CD$. [2]

~ End of Paper ~

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(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$ [4]

(a) $5^{2+x} - 3(5^{1-x}) + 10 = 0$

$25(5^x) - \frac{15}{5^x} + 10 = 0$

$5(5^x) - \frac{3}{5^x} + 2 = 0$ [M1]

Let $p = 5^x$

$5p - \frac{3}{p} + 2 = 0$

$5p^2 + 2p - 3 = 0$ [M1]

$(5p-3)(p+1) = 0$

$p = \frac{3}{5}$ or $p = -1$

$5^x = \frac{3}{5}$ or $5^x = -1$ (reject)

$\lg 5^x = \lg(\frac{3}{5})$ [M1] (p if never reject $5^x = -1$)

$x = \frac{\lg(\frac{3}{5})}{\lg 5}$

$x = -0.317$ [A1]

(b) $\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$

$$\log_9 \sqrt{3-3x} = \frac{1}{2} - \log_{81}(1-2x)$$

$$\log_9 \sqrt{3-3x} = \frac{1}{2} - \frac{\log_9(1-2x)}{\log_9 81}$$

$$\frac{1}{2}\log_9(3-3x) = \frac{1}{2} - \frac{\log_9(1-2x)}{2}$$ [M1 for changing base]

$$\log_9(3-3x) + \log_9(1-2x) = 1$$

$$\log_9(3-3x)(1-2x) = 1$$ [M1]

$$(3-3x)(1-2x) = 9^1$$

$$(1-x)(1-2x) = 3$$ [M1]

$$2x^2 - 3x - 2 = 0$$

$$(2x+1)(x-2) = 0$$

$x = -\frac{1}{2}$ or $x = 2$ (reject) (p if never reject $x = 2$)

$\therefore x = -\frac{1}{2}$ [A1]

2 **(a)** Find the greatest value of the integer k for which $-3x^2 + kx - 5$ is never positive for all values of x. [3]

(b) A curve has an equation $y = \frac{x^2}{2-3x}$, where $x \neq \frac{2}{3}$.

Find the range of values of x for which y is decreasing.

[4]

(a) For all values of x, $-3x^2 + kx - 5$ is never positive,

Discriminant ≤ 0

$k^2 - 4(-3)(-5) \leq 0$ [M1]

$k^2 - 60 \leq 0$

$(k - \sqrt{60})(k + \sqrt{60}) \leq 0$

$-\sqrt{60} \leq k \leq \sqrt{60}$ [A1]

OR $-2\sqrt{15} \leq k \leq 2\sqrt{15}$

OR $-7.7460 \leq k \leq 7.7460$

The greatest integer value of k is 7 [A1]

(b) $y=\dfrac{x^2}{2-3x}, x \neq \dfrac{2}{3}$

$\dfrac{dy}{dx}=\dfrac{2x(2-3x)+3x^2}{(2-3x)^2}$ [M1]

$=\dfrac{4x-3x^2}{(2-3x)^2}$

Since the curve is decreasing, $\dfrac{dy}{dx}<0$ and $x \neq \dfrac{2}{3}$

$\dfrac{4x-3x^2}{(2-3x)^2}<0$ [M1]

Since $(2-3x)^2>0$, $4x-3x^2<0$

$3x^2-4x>0$ [M1]

$x(3x-4)>0$

$x<0$ or $x>\dfrac{4}{3}$ [A1]

3 (i) Prove the identity $1+\dfrac{\sin^2 A}{1-\sec^2 A}+\dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}=0.$ [3]

(ii) Hence, solve the equation $\dfrac{\sin^2 A}{1-\sec^2 A}+\dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}=\tan(2A+10°)$ for $-180°<A<180°$. [4]

(i) To prove $1+\dfrac{\sin^2 A}{1-\sec^2 A}+\dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}=0.$

LHS $=1+\dfrac{\sin^2 A}{1-\sec^2 A}+\dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}$

$=1+\dfrac{\sin^2 A}{-\tan^2 A}+\dfrac{\cos^2 A}{-\cot^2 A}$ [B1]

$=1-\cos^2 A-\sin^2 A$ [B1]

$=1-1$ [B1]

$=0$

Hence $1+\dfrac{\sin^2 A}{1-\sec^2 A}+\dfrac{\cos^2 A}{1-\operatorname{cosec}^2 A}=0.$ (Proved)

(ii) Since $\frac{\sin^2 A}{1-\sec^2 A}+\frac{\cos^2 A}{1-\operatorname{cosec}^2 A}=\tan(2A+10°)$

$\tan(2A+10°)=-1$ [B1]

Basic angle $=45°$

$2A+10°=-45°,-225°,135°,315°$ [M1]

$A=-27.5°,-117.5°,62.5°,152.5°$

[A1 for both] [A1 for both]

4 A curve has the equation $y=4e^{\tan(\pi-\frac{x}{4})}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If x and y vary with time and y increases at the rate of e units per second when $x=\pi$ radian. Find the exact value of the rate of decrease of x at this instant. [4]

(i) $\frac{dy}{dx}=4(-\frac{1}{4})\sec^2(\pi-\frac{x}{4})e^{\tan(\pi-\frac{x}{4})}$ [M1]

$\frac{dy}{dx}=-\sec^2(\pi-\frac{x}{4})e^{\tan(\pi-\frac{x}{4})}$ [B1]

(ii) When $x=\pi$,

$\frac{dy}{dx}=-\sec^2(\frac{3\pi}{4})e^{\tan(\frac{3\pi}{4})}$ [M1]

$=-(-\sqrt{2})^2e^{-1}$

$=-\frac{2}{e}$ [A1]

$\frac{dy}{dt}=\frac{dx}{dt}\times\frac{dy}{dx}$

$e=\frac{dx}{dt}\times\left(-\frac{2}{e}\right)$ [M1]

$\frac{dx}{dt}=-\frac{e^2}{2}$

The exact rate of decrease of x is $\frac{e^2}{2}$ *units / s* [A1]

5 **(a)** Sketch the graph of $f(x) = 2 - |5 - 3x|$ for $-1 \le x \le 6$.

Indicate clearly the vertex and the intercepts of the axes. [3]

(b) Solve the equation $2 - |5 - 3x| = x - 1$ [2]

(c) **(i)** State the range of the values of c if there is no solution for the equation $2 - |5 - 3x| = c$, [1]

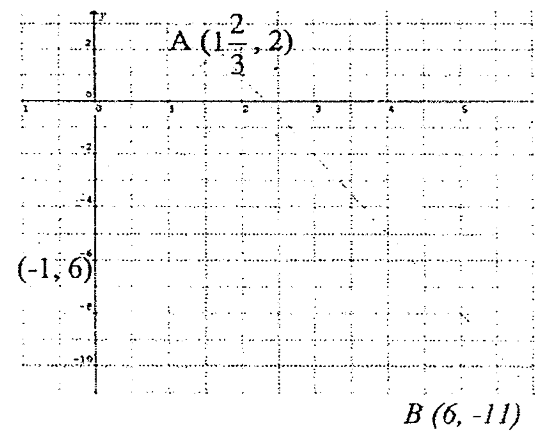
(ii) State the range of the values of m if there are exactly two solutions for the equation $2 - |5 - 3x| = mx$. [1]

(a) Turning Points = $(1\frac{2}{3}, 2)$ [B1]

Shape - inverted v-shape [B1]

intercepts : $(0, -3)$, $(1, 0)$, $(2\frac{1}{3}, 0)$

terminal points : (-1, -6) , (6, -11) [B1]



(b) $2 - |5 - 3x| = x - 1$

$|5 - 3x| = 3 - x$

$5 - 3x = 3 - x$ or $5 - 3x = -(3 - x)$ [M1]

$x = 1$ $x = 2$ [A1]

(c) (i) $c > 2$ [B1]

(ii) Gradient of OA = $\frac{6}{5}$

Gradient of AB = -3

The range of values of m : $-3 < m < \frac{6}{5}$ [B1]

6 The amount of radioactive Sodium-24, M measured in grams, used as a tracer to measure the rate of flow in an artery or vein can be modelled by

$M = M_0 e^{kt}$, where t is the time in hours, M_0 and k are a constants.

The hospital buys a 40-grams sample of Sodium-24 and will reorder when the sample is reduced to 3 grams.

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(ii) Find the amount of Sodium-24 remain after 60 hours. [1]

(iii) Calculate the time taken before the hospital reorders Sodium-24 . [2]

(i) When $t = 0, M = 40$

$M_0 = 40$ [B1]

When $t = 14.9$, $M = 20$

$$20 = 40e^{14.9k}$$

$$e^{14.9k} = \frac{1}{2}$$ [M1]

$$k = \frac{1}{14.9}\ln\frac{1}{2}$$

$$k = -\frac{1}{14.9}\ln 2$$

$$k = -0.046520$$

$$k = -0.0465 \quad (3s.f.)$$ [A1]

(ii) When $t = 60$,

$$M = 40e^{-(\frac{1}{14.9}\ln 2)(60)}$$

$$M = e^{-2.7912}$$

$$M = 0.0613\,g$$ [A1]

(iii) When $M = 3$,

$$3 = 40e^{-0.04652t}$$

$$\frac{3}{40} = e^{-0.04652t}$$ [M1]

$$\ln\left(\frac{3}{40}\right) = -0.04652t$$

$$t = -\frac{1}{0.04652}\ln\left(\frac{3}{40}\right)$$

$$t = 55.7 \text{ hours}$$ [A1]

7 **(a)** The function f is defined, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, by the equation $f(x) = 2\tan 3x$.

(i) State the period of f. [1]

(ii) Sketch the graph of $y = f(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

(b) On the same diagram drawn in part (a), sketch the graph of $g(x) = 1 - 2\sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

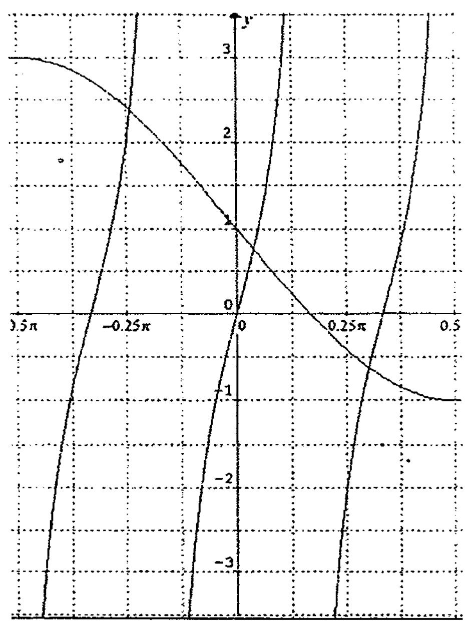
(c) State the number of solutions of the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [1]

(a) (i) Period $= \frac{\pi}{3}$ [B1]

(ii) Shape [B 1]

4 asymptotes [B 0.5]

x-intercept : $-\frac{\pi}{6}; 0; \frac{\pi}{6};$ [B 0.5]



(b) Shape [B1]

turning points $(-\frac{\pi}{2},3);(\frac{\pi}{2},-1);$[B 0.5]

intercepts : (0, 1), $(\frac{\pi}{6},0)$ [B 0.5]

(c)

$$\sin x + \tan 3x = \frac{1}{2}$$
$$2\sin x + 2\tan 3x = 1$$
$$2\tan 3x = 1 - 2\sin x$$

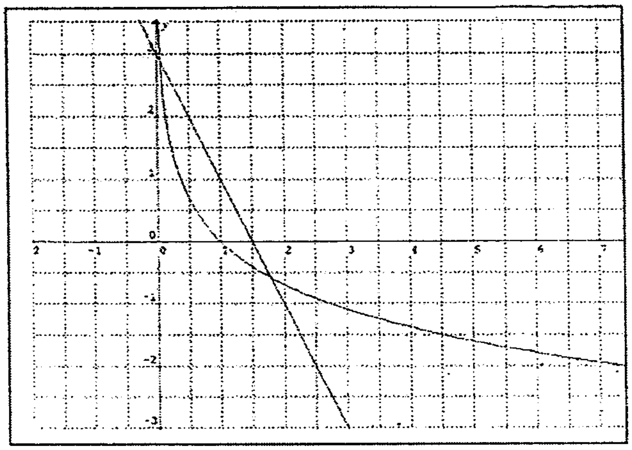
There are 3 solutions for the equation $\sin x + \tan 3x = \frac{1}{2}$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [A1]

8 The function $f(x) = -\ln x$ is defined for $x > k$.

(i) State the value of k. [1]

(ii) Sketch the graph of $f(x) = -\ln x$ for $x > k$. [2]

(iii) Explain how another straight line drawn on your diagram in part **(b)** can lead to the graphical solution of $xe^{2x-3} = 1$. Draw this straight line and state the number of solutions for $xe^{2x-3} = 1$ [3]



(i) $k = 0$ [B1]

(ii) Shape [B1]

Asymptote $x = 0$ [B 0.5]

x-intercept : $(1, 0)$ [B 0.5]

(iii) Since

$xe^{2x-3} = 1$

$\ln(xe^{2x-3}) = 0$

$\ln xe^{2x31} = 0$

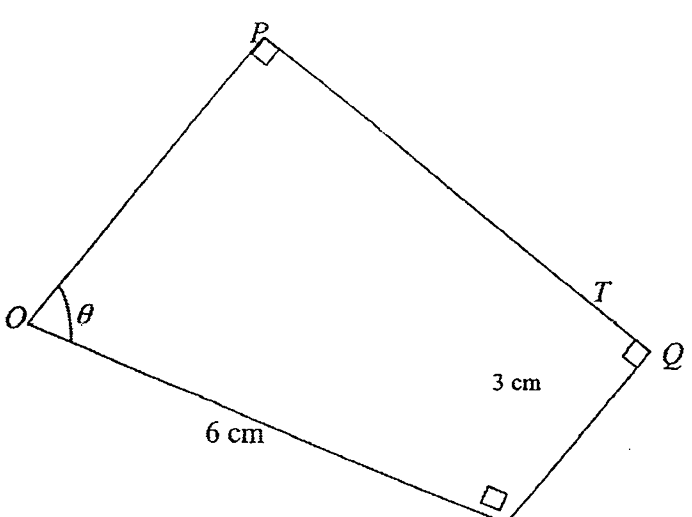
$\ln x + 2x - 3 = 0$

$y = 3 - 2x$ [B1]

Hence, by drawing the line $y = 3 - 2x$ on the diagram in part (b), the x-coordinates of the points of intersection would give the solutions for $xe^{2x-3} = 1$. [B1]

From the sketch, we can conclude that there are 2 solutions for $xe^{2x-3} = 1$. [A1]

9 The diagram shows a quadrilateral $OPQR$ where $OR = 6$ cm, angle OPQ = angle $PQR = \frac{\pi}{2}$ radian and angle $ROP = \theta$ radian, θ is a variable and an acute angle. T is a point on PQ such that angle $ORT = \frac{\pi}{2}$ radian and $RT = 3$ m.



(i) Show that the area, A cm^2 of the quadrilateral $OPQR$ is given by

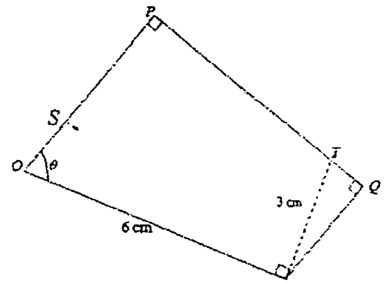
$$A = 9\sin 2\theta + 18\sin^2\theta \qquad [3]$$

(ii) Given that θ can vary, find maximum area of the quadrilateral $OPQR$.

[6]

$P\hat{S}R = \frac{\pi}{2}$ rad

$R\hat{T}Q = \theta$ (alt. $\angle$, $PQ//SR$)



$A = \frac{1}{2}(OS)(RS) + (RS)(RQ)$

$A = \frac{1}{2}(6\cos\theta)(6\sin\theta) + (6\sin\theta)(3\sin\theta)$ [M1][M1]

$A = 18\sin\theta\cos\theta + 18\sin^2\theta$ [A1]

$A = 9\sin 2\theta + 18\sin^2\theta$ (Shown)

$A = 9\sin 2\theta + 18\sin^2\theta$

$\frac{dA}{d\theta} = 18\cos 2\theta + 18(2)\sin\theta\cos\theta)$ [B1] [B1]

$= 18\cos 2\theta + 18\sin 2\theta$

For maximum area, $\frac{dA}{d\theta} = 0$.

$\frac{dA}{d\theta} = 18\cos 2\theta + 18\sin 2\theta = 0$ [B1]

$\cos 2\theta + \sin 2\theta = 0$

$1 + \tan 2\theta = 0$

$\tan 2\theta = -1$

Basic angle $= \frac{\pi}{4}$

$2\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

(N.A.)

$\theta = \frac{3\pi}{8}$ [A1]

$$\frac{d^2A}{d\theta^2} = -36\sin 2\theta + 36\cos 2\theta$$

[B1]

When $\theta = \frac{3\pi}{8}, \frac{d^2A}{d\theta^2} = -36\left(\frac{1}{\sqrt{2}}\right) + 36\left(-\frac{1}{\sqrt{2}}\right)$

$$= -36\sqrt{2} < 0$$

Therefore, maximum area

$$= 9\sin 2\left(\frac{3\pi}{8}\right) + 18\sin^2\left(\frac{3\pi}{8}\right)$$

$$= \frac{9}{\sqrt{2}} + 18\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 9(1 + \frac{\sqrt{2}}{2})$$

$= 15.4\ \text{cm}^2$ [A1]

10 A particle P moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v m/s is given by

$$v_P = 1 - \frac{9}{(3t+1)^2}.$$

(i) Calculate the initial acceleration of the particle P. [2]

(ii) Show that the particle P is at instantaneously rest at $t = \frac{2}{3}$. [2]

(iii) Calculate the average speed of the particle P during the first 3 seconds after passing O. [4]

Another particle Q moves in a straight line and its displacement, S m from O after t seconds is given by

$$S_Q = t - 1$$

(iv) Find the distance from the fixed point O when P first collides with Q.

[2]

(i) $v_P = 1 - \dfrac{9}{(3t+1)^2}$

acceleration, $a = \dfrac{dv}{dt}$

$$a = \frac{54}{(3t+1)^3} \qquad [M1]$$

Initial acceleration = 54 m/s^2 [A1]

(ii) At instantaneously rest, $v_P = 0$

$1 - \dfrac{9}{(3t+1)^2} = 0$

$(3t+1)^2 = 9$ [M1]

$3t + 1 = \pm 3$

$t = \dfrac{2}{3}$ or $-\dfrac{4}{3}$ (reject)

$\therefore t = \dfrac{2}{3}$ (Shown) [A1]

(iii) $S_P = \int [1 - \dfrac{9}{(3t+1)^2}]\, dx$

$S_p = t + \dfrac{3}{3t+1} + c$ [M1]

When $t = 0, S_p = 0,$

$0 = 3 + c$

$c = -3$

$\therefore S_p = t + \dfrac{3}{3t+1} - 3$ [A1]

When

$t = 0, S = 0m$

$t = \dfrac{2}{3}, S = -1\dfrac{1}{3}m$

$t = 3, S = \dfrac{3}{10}m$

average speed

$$= \frac{\frac{4}{3} \times 2 + \frac{3}{10}}{3} \quad [M1]$$

$$= \frac{89}{90}$$

$$= 0.989 \text{ m/s} \quad [A1]$$

(iv) When P collides with Q, $S_P = S_Q$,

$$t + \frac{3}{3t+1} - 3 = t - 1$$

$$\frac{3}{3t+1} = 2$$

$$3t + 1 = \frac{3}{2}$$

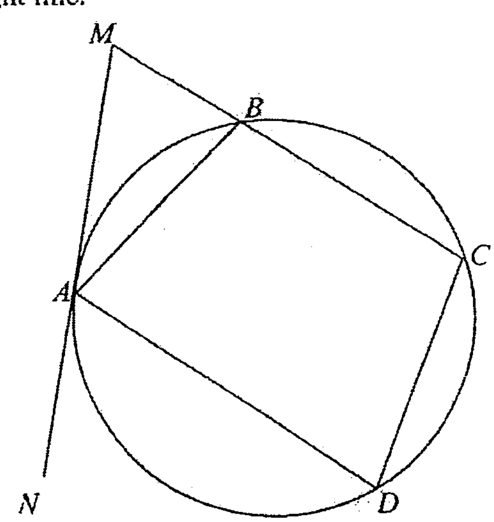
$$t = \frac{1}{6} \quad [M1]$$

When $t = \frac{1}{6}, S_Q = \frac{1}{6} - 1$

$$S_Q = -\frac{5}{6} \text{ m} \quad [A1]$$

Hence, the particles first collides at $\frac{5}{6}$ m from the fixed point O. [A1]

11 In the diagram,. A, B, C and D are on the circle. MN is a tangent to the circle at A. MBC is a straight line.



(a) Name a triangle which is similar to triangle CAM. [1]

Hence prove that $\left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$. [3]

(b) Given further that AD and BC are parallel, show that

(i) triangle ABM is similar to triangle ADC. [2]

(ii) $AD \times AM = AC \times CD$. [2]

(a)

$A\hat{M}B = C\hat{M}A$ (common angle)

$M\hat{A}B = M\hat{C}A$ (alternate segment theorem)

triangle CAM is similar to triangle ABM [B1]

$$\frac{AC}{BA} = \frac{AM}{BM} = \frac{MC}{MA}$$ [B1]

$$\left(\frac{AC}{BA}\right)^2 = \left(\frac{AM}{BM}\right)^2$$ [B1]

$$= \frac{BM \times MC}{BM^2}$$ $(AM^2 = MC \times BM)$ [B1]

$$= \frac{MC}{BM}$$

$$\therefore \left(\frac{AC}{BA}\right)^2 = \frac{CM}{BM}$$ (proved) [p if no conclusion]

(b)

$A\hat{B}M = A\hat{D}C$ (angle in opposite segment)

$M\hat{A}B = M\hat{C}A$ (alternate segment theorem)

$= C\hat{A}D$ (alternate angle, AD//BC)

triangle ABM is similar to triangle ADC [B 2,1,0]

$$\frac{AD}{AB} = \frac{CD}{MB}$$ [B1]

$$\frac{AD}{CD} = \frac{AB}{MB}$$

$$\frac{AD}{CD} = \frac{AC}{AM}$$ since $\frac{AB}{MB} = \frac{AC}{AM}$ (from part (a) [B1]

$AD \times AM = AC \times CD$ (Proved) [p if no conclusion]

~ End of Paper

O Level Centre/ Index Number	Class	Name
/		

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS
Paper 2

4047/2
18 August 2016
2 hours 30 minutes

Additional Materials: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $y = f(x)$ is such that $f'(x) = (k-2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for $f(x)$. [4]

2 (i) Differentiate $\ln(\sin x)$ with respect to x. [2]

(ii) Show that $\frac{d}{dx}(x\cot x) = \cot x - x\operatorname{cosec}^2 x$. [3]

(iii) Using the results from parts (i) and (ii), find $\int x\operatorname{cosec}^2 x \, dx$. [3]

3 The equation of a curve is $y = 6x^{\frac{2}{3}}$.

(i) Sketch the curve $y = 6x^{\frac{2}{3}}$. [2]

(ii) The point P lies on the curve such that the gradient of the normal to the curve is $-\frac{1}{2}$. The normal at P meets the x-axis at A and the y-axis at B. Find the ratio $AP:PB$. [6]

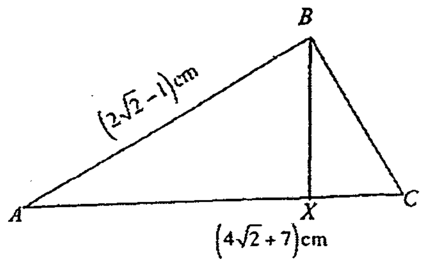
4 (i) Given that n is a positive integer, write down, without simplifying, the $(r+1)$th term in the binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^2}\right)^n$. [1]

(ii) The binomial expansion of $\left(\frac{x}{2} - \frac{k}{x^2}\right)^n$ has a constant term. Show that n is a multiple of 3. [1]

(iii) Given that $n = 9$ and that the constant term is $-\frac{2625}{2}$, find the value of k. [3]

(iv) Using the value of k found in part (iii), find the term independent of x in the expansion of $(2 + x^3)\left(\frac{x}{2} - \frac{k}{x^2}\right)^9$. [3]

5



The diagram shows a triangle ABC such that $AB = (2\sqrt{2}-1)$ cm and $AC = (4\sqrt{2}+7)$ cm. The point X lies on AC such that $\angle AXB = \angle ABC$.

(i) Show that $AX \times AC = AB^2$. [2]

(ii) Find an expression for AX in the form $\frac{1}{17}\left(a+b\sqrt{2}\right)$. [4]

(iii) Given that $BC^2 = 72+60\sqrt{2}$, show that $\angle AXB = 90°$. [3]

6 The equation of a curve is $y = \frac{(2x-5)^2}{x-1}$, where $x \neq 1$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and show that its can be expressed in the form $\frac{k}{(x-1)^3}$. Hence, or otherwise, determine the nature of these stationary points. [4]

7 The highest point on a circle C_1 is $(2,8)$. The line T, $3y = 42-4x$, is a tangent to C_1 at the point $(6,6)$.

(i) Find the coordinates of the centre of C_1. [4]

(ii) Find the equation of C_1. [2]

The circle C_2 is a reflection of C_1 in the line T.

(iii) Find the equation of C_2. [3]

8 **(i)** Show that $3x-1$ is a factor of $3x^3+11x^2+8x-4$ and hence factorise completely the cubic polynomial $3x^3+11x^2+8x-4$. [3]

(ii) Express $\dfrac{5x^2-2x+11}{3x^3+11x^2+8x-4}$ as the sum of 3 partial fractions. [4]

(iii) Hence find $\int \dfrac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx$. [3]

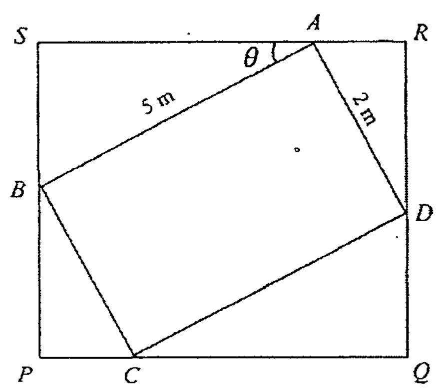
9 The roots of the quadratic equation $4x^2+3x+1=0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

(iii) Show that the value of $\alpha^3+\beta^3$ is 9. [2]

(iii) Find a quadratic equation whose roots are $\alpha^2+\beta$ and $\alpha+\beta^2$. [4]

10



The diagram shows a rug in the shape of a rectangle $ABCD$ such that $AB = 5$ m and $AD = 2$ m. The rug is placed inside a rectangular function room $PQRS$ such that each of the corners A, B, C and D touches the sides of the room SR, SP, PQ and QR respectively. The side of the rug AB makes an acute angle θ with the side of the room SR. The lengths of the room SR and SP are L m and W m respectively.

(a) **(i)** Find the values of the integers a and b for which
$$L = a\cos\theta + b\sin\theta.$$ [2]

(ii) Obtain a similar expression for W. [1]

(iii) Hence find the perimeter of the room $PQRS$ in exact form if $PQRS$ is a square. [3]

(b) Using the values of a and b found in (a) part (i),

(i) express L in the form $R\cos(\theta-\alpha)$, $R>0$ and $0° < \alpha < 90°$. [2]

(ii) find the value of θ if $L = 4$ and the area of the rectangular function room $PQRS$. [4]

11 The amount of expenditure, \$$y$, incurred by a textile company is related to \$$x$, the amount of sales generated. The variables x and y are related by the formula $y = 10^k x^a$, where a and k are constants. The following table shows corresponding values of x and y.

x (\$)	6	35	234	1995	6310
y (\$)	148	295	628	1480	2344

(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of k. [4]

(iii) Estimate the amount of expenditure incurred when the sales generated is \$4000. [2]

(iv) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven. [2]

- End of Paper -

O Level Centre/ Index Number	Class	Name
/		SOLUTIONS

PRELIMINARY EXAMINATION TWO
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/2**
Paper 2
18 August 2016
2 hours 30 minutes

Additional Materials: Answer Paper (7 sheets)

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$$\Delta = \frac{1}{2} bc \sin A$$

1 The curve $y = f(x)$ is such that $f'(x) = (k-2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

Solution:

For y is an increasing function of x,

$(k-2)e^{3x} > 0$. [M1]

Since $e^{3x} > 0$, $k-2 > 0$

$\therefore k > 2$. [A1]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for $f(x)$. [4]

Solution:

$f'(x) = (k-2)e^{3x}$

Subst $x = 0$ and $f'(x) = 4$,

$4 = k - 2$

$k = 6$ [A1]

$f(x) = \frac{(k-2)e^{3x}}{3} + c$ [M1]

Subst $x = 0$ and $f(x) = 3$,

$3 = \frac{4}{3} + c$

$c = 1\frac{2}{3}$ [A1]

$f(x) = \frac{4}{3}e^{3x} + \frac{5}{3}$ [A1]

2 (i) Differentiate $\ln(\sin x)$ with respect to x. [2]

Solution:

$$\frac{d}{dx}(\ln(\sin x)) = \frac{\cos x}{\sin x} \quad \text{[M1]}$$

$$= \cot x \quad \text{[A1]}$$

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}x} x\cot x = \cot x - x\cos ec^2 x$. [3]

Solution:

$$\frac{d}{dx} x\cot x = \frac{d}{dx}\frac{x}{\tan x}$$

$$= \frac{\tan x - x\sec^2 x}{\tan^2 x} \quad \text{[M1]}$$

$$= \cot x - x\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right) \quad \text{[M1]}$$

$$= \cot x - x\cos ec^2 x \quad \text{[A1]}$$

(iii) Using the results from parts (i) and (ii), find $\int x\cos ec^2 x \, \mathrm{d}x$. [3]

Solution:

$$\int \left(\cot x - x\cos ec^2 x\right) \mathrm{d}x = x\cot x + c \quad \text{[M1]}$$

$$\int \cot x \, \mathrm{d}x - \int x\cos ec^2 x \, \mathrm{d}x = x\cot x + c$$

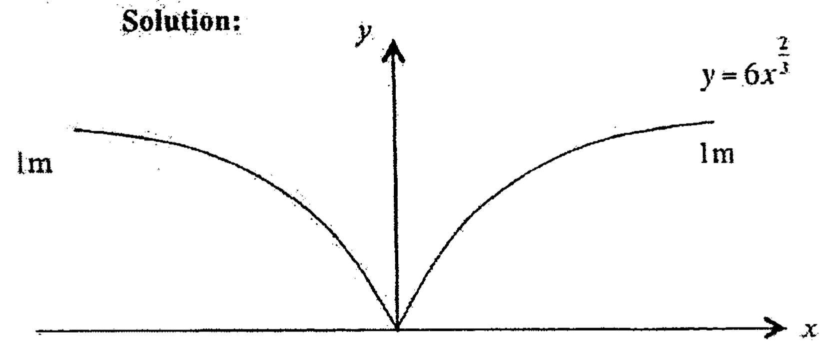
$$\left[\ln(\sin x) + c\right] - \int x\cos ec^2 x \, \mathrm{d}x = x\cot x + c \quad \text{[M1]}$$

$$\int x\cos ec^2 x \, \mathrm{d}x = \ln(\sin x) - x\cot x + c \quad \text{[A1]}$$

3 The equation of a curve is $y = 6x^{\frac{2}{3}}$

(i) Sketch the curve $y = 6x^{\frac{2}{3}}$. [2]

Solution:



(ii) The point P lies on the curve such that the gradient of the normal to the curve is $-\frac{1}{2}$. The normal at P meets the x-axis at A and the y-axis at B. Find the ratio $AP{:}PB$. [6]

Solution:

$y = 6x^{\frac{2}{3}}$

$\frac{dy}{dx} = 4x^{-\frac{1}{3}}$ [M1]

Gradient of tangent at P $= -1 \div \left(-\frac{1}{2}\right)$

$= 2$

When $\frac{dy}{dx} = 2$, $4x^{-\frac{1}{3}} = 2$ [M1]

$x^{-\frac{1}{3}} = \frac{1}{2}$

$x^{\frac{1}{3}} = 2$

$x = 8$ [A1]

$y = 6(8)^{\frac{2}{3}}$

$= 24$ [A1]

Equation of normal, $y - 24 = -\frac{1}{2}(x - 8)$

$y = -\frac{1}{2}x + 28$ [M1]

$A(56,0)$, $P(8,24)$, $B(0,28)$

$AP : PB = 24 - 0 : 28 - 24$

$= 24 : 4$

$= 6 : 1$ [A1]

4 **(i)** Given that n is a positive integer, write down, without simplifying, the $(r+1)$th term in the binomial expansion of $\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$. [1]

Solution:

$(r+1)\text{th term} = \binom{n}{r}\left(\frac{x}{2}\right)^{n-r}\left(-\frac{k}{x^2}\right)^r$ [B1]

(ii) The binomial expansion of $\left(\frac{x}{2}-\frac{k}{x^2}\right)^n$ has a constant term. Show that n is a multiple of 3. [1]

Solution:

For constant term, $n-r-2r=0$

$n=3r$

Since r is an integer and $n=3r$, n is a multiple of 3. [A1]

(iii) Given that $n=9$ and that the constant term is $-\frac{2625}{2}$, find the value of k. [3]

Solution:

Constant term $= -\frac{2625}{2}$

$\binom{9}{3}\left(\frac{1}{2}\right)^{9-3}(-k)^3 = -\frac{2625}{2}$ [M1]

$84\left(\frac{1}{64}\right)\left(-k^3\right) = -\frac{2625}{2}$

$k^3 = 1000$ [M1]

$k = 10$ [A1]

(iv) Using the value of k found in part (iii), find the term independent of x in the expansion of $(2+x^3)\left(\frac{x}{2}-\frac{k}{x^2}\right)^9$. [3]

Solution:

Let $9-3r=-3$

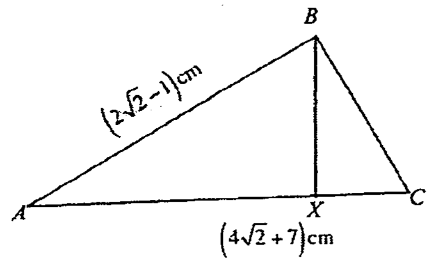
$r=4$

Constant term in the expansion of $\left(2+x^3\right)\left(\frac{x}{2}-\frac{10}{x^2}\right)^9$

$= 2\left(-\frac{2625}{2}\right)+x^3\binom{9}{4}\left(\frac{x}{2}\right)^5\left(-\frac{10}{x^2}\right)^4$ [M2]

$= 36750$ [A1]

5



The diagram shows a triangle ABC such that $AB = (2\sqrt{2} - 1)$ cm and $AC = (4\sqrt{2} + 7)$ cm. The point X lies on AC such that $\angle AXB = \angle ABC$.

(i) Show that $AX \times AC = AB^2$. [2]

Solution:

$\angle AXB = \angle ABC$ (given)

$\angle XAB = \angle BAC$ (common $\angle$)

ΔAXB is similar to ΔABC.

$\frac{AX}{AB} = \frac{AB}{AC}$ [M1]

$\therefore AX \times AC = AB^2$ [A1]

(ii) Find an expression for AX in the form $\frac{1}{17}\left(a + b\sqrt{2}\right)$. [4]

Solution:

$AX \times AC = AB^2$

$$AX = \frac{AB^2}{AC}$$

$$= \frac{\left[2\sqrt{2} - 1\right]^2}{7 + 4\sqrt{2}}$$ [M1]

$$= \frac{\left(2\sqrt{2}\right)^2 - 4\sqrt{2} + 1}{7 + 4\sqrt{2}}$$

$$= \frac{9 - 4\sqrt{2}}{7 + 4\sqrt{2}} \times \frac{7 - 4\sqrt{2}}{7 - 4\sqrt{2}}$$ [M1]

$$= \frac{63 - 36\sqrt{2} - 28\sqrt{2} + 32}{17}$$ [M1]

$$= \frac{1}{17}\left(95 - 64\sqrt{2}\right)$$ [A1]

(iii) Given that $BC^2 = 72 + 60\sqrt{2}$, show that $\angle AXB = 90°$. [3]

Solution:

$$AB^2 + BC^2 = \left[2\sqrt{2} - 1\right]^2 + 72 + 60\sqrt{2}$$

$$= 8 - 4\sqrt{2} + 1 + 72 + 60\sqrt{2}$$

$$= 81 + 56\sqrt{2}$$ [M1]

$AC^2 = \left[4\sqrt{2}+7\right]^2$

$= 32+56\sqrt{2}+49$

$= 81+56\sqrt{2}$ [M1]

Since $AC^2 = AB^2 + BC^2$, by Converse of Pythagoras' Theorem,
$\angle ACB = 90°$.
$\therefore \angle AXB = 90°$ (since $\angle AXB = \angle ACB$) [A1]

6 The equation of a curve is $y = \dfrac{(2x-5)^2}{x-1}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

Solution:

$\dfrac{dy}{dx} = \dfrac{(x-1)(2)(2x-5)(2)-(2x-5)^2(1)}{(x-1)^2}$ [M1]

$= \dfrac{(2x-5)(4x-4-2x+5)}{(x-1)^2}$

$= \dfrac{(2x-5)(2x+1)}{(x-1)^2}$ [M1]

When $\dfrac{dy}{dx} = 0$, $(2x-5)(2x+1) = 0$ [M1]

$x = 2.5$ or -0.5 [A1]

When $x = 2.5$, $y = 0$

When $x = -0.5$, $y = -24$

Stationary points are $(2.5, 0)$ and $(-0.5, -24)$ [A1]

(ii) Find an expression for $\dfrac{d^2y}{dx^2}$ and show that its can be expressed in the form $\dfrac{k}{(x-1)^3}$. Hence, or otherwise, determine the nature of these stationary points. [4]

Solution:

$\dfrac{d^2y}{dx^2} = \dfrac{(x-1)^2(8x-8)-(2x-5)(2x+1)(2)(x-1)}{(x-1)^4}$ [M1]

$= \dfrac{(x-1)\left(8x^2-16x+8-8x^2+16x+10\right)}{(x-1)^4}$

$= \dfrac{18}{(x-1)^3}$ [A1]

When $x = -0.5$, $\frac{d^2y}{dx^2} = \frac{18}{(-0.5-1)^3} < 0$

$(-0.5,-24)$ is a maximum point. [A1]

When $x = 2.5$, $\frac{d^2y}{dx^2} = \frac{18}{(2.5-1)^3} > 0$

$(2.5,0)$ is a minimum point. [A1]

7 The highest point on a circle C_1 is $(2,8)$. The line T, $3y = 42 - 4x$, is a tangent to C_1 at the point $(6,6)$.

(i) Find the coordinates of the centre of C_1. [4]

Solution:

Since the highest point on a circle C_1 is $(2,8)$, the centre is $(2, y)$. [M1]

Gradient of normal at (6, 6) $= 1 \div \left(-\frac{4}{3}\right)$ [M1]

Equation of the normal at $(6,6)$: $(y-6) = \frac{3}{4}(x-6)$

$(y-6) = \frac{3}{4}(x-6)$

$y = \frac{3}{4}x + \frac{3}{2}$ [A1]

When $x = 2$, $y = 3$

The centre of C_1 is (2, 3). [A1]

(ii) Find the equation of C_1. [2]

Solution:

Equation of C_1: $(x-2)^2 + (y-3)^2 = (8-3)^2$[M1]

$(x-2)^2 + (y-3)^2 = 25$ [A1]

The circle C_2 is a reflection of C_1 in the line T.

(iii) Find the equation of C_2. [3]

Solution:

The centre of C_2 is $(2 + 2(6-2), 3 + 2(6-3)) = (10, 9)$. [B2]

Equation of C_2: $(x-10)^2 + (y-9)^2 = 25$ [A1]

8 **(i)** Show that $3x-1$ is a factor of $3x^3+11x^2+8x-4$ and hence factorise completely the cubic polynomial $3x^3+11x^2+8x-4$. [3]

Solution:

Let $f(x)=3x^3+11x^2+8x-4$

$$f\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3+11\left(\frac{1}{3}\right)^2+8\left(\frac{1}{3}\right)-4 \quad \text{[M1]}$$

$$=0$$

Since $f\left(\frac{1}{3}\right)=0$, $(3x-1)$ is a factor.

$3x^3+11x^2+8x-4=(3x-1)(x^2+bx+4)$

Comparing x term, $12-b=8$

$b=4$

$3x^3+11x^2+8x-4=(3x-1)(x^2+4x+4)$ [M1]

$=(3x-1)(x+2)^2$ [A1]

(ii) Express $\dfrac{5x^2-2x+11}{3x^3+11x^2+8x-4}$ as the sum of 3 partial fractions. [4]

Solution:

$$\frac{5x^2-2x+11}{3x^3+11x^2+8x-4}=\frac{5x^2-2x+11}{(3x-1)(x+2)^2}$$

Let $\dfrac{5x^2-2x+11}{(3x-1)(x+2)^2}=\dfrac{A}{(3x-1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x+2)^2}$ [M1]

$5x^2-2x+11=A(x+2)^2+B(3x-1)(x+2)+C(3x-1)$

Let $x=-2$, $-7C=35$

$C=-5$ [A1]

Let $x=\frac{1}{3}$, $\frac{49}{9}A=\frac{98}{9}$

$A=2$ [A1]

Let $x=0$, $4A-2B-C=11$

$8-2B-(-5)=11$

$B=1$ [A1]

$$\frac{5x^2-2x+11}{3x^3+11x^2+8x-4}=\frac{2}{(3x-1)}+\frac{1}{(x+2)}-\frac{5}{(x+2)^2}$$

(iii) Hence find $\int \frac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx$. [3]

Solution:

$$\int \frac{5x^2-2x+11}{3x^3+11x^2+8x-4}\,dx = \int\left[\frac{2}{(3x-1)}+\frac{1}{(x+2)}-\frac{5}{(x+2)^2}\right]dx$$

$$=\frac{2}{3}\ln(3x-1)+\ln(x+2)-\frac{5}{(-1)}(x+2)^{-1}+c \quad \text{[M2]}$$

$$=\frac{2}{3}\ln(3x-1)+\ln(x+2)+\frac{5}{(x+2)}+c \quad \text{[A1]}$$

9 The roots of the quadratic equation $4x^2+3x+1=0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Find the value of $\alpha^2+\beta^2$. [4]

Solution:

Sum of roots: $\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{3}{4}$

$$\frac{\alpha+\beta}{\alpha\beta}=-\frac{3}{4}$$

Product of roots: $\frac{1}{\alpha\beta}=\frac{1}{4}$ [M1]

$$\alpha\beta=4$$

$$\alpha+\beta=\frac{\alpha+\beta}{\alpha\beta}\times\alpha\beta$$

$$=-\frac{3}{4}\times 4$$

$$=-3 \quad \text{[M1]}$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(-3)^2-2(4) \quad \text{[M1]}$$

$$=1 \quad [A1]$$

(iii) Show that the value of $\alpha^3+\beta^3$ is 9. [2]

Solution:

$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$

$$=(-3)(1-4) \quad \text{[M1]}$$

$$=9 \text{ (shown)} \quad \text{[A1]}$$

(iii) Find a quadratic equation whose roots are $\alpha^2+\beta$ and $\alpha+\beta^2$. [4]

Solution:

$$\alpha^2+\beta+\alpha+\beta^2=1+(-3)$$

$$=-2 \quad \text{[B1]}$$

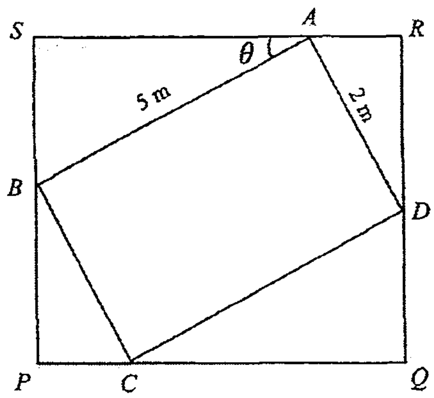
$$(\alpha^2+\beta)(\alpha+\beta^2)=\alpha^3+\alpha^2\beta^2+\alpha\beta+\beta^3$$

$$=9+(4)^2+4 \quad \text{[M1]}$$

$$=29 \quad \text{[A1]}$$

The new equation is $x^2+2x+29=0$ [A1]

10



The diagram shows a rug in the shape of a rectangle $ABCD$ such that $AB = 5$ m and $AD = 2$ m. The rug is placed inside a rectangular function room $PQRS$ such that each of the corners A, B, C and D touches the sides of the room SR, SP, PQ and QR respectively. The side of the rug AB makes an acute angle θ with the side of the room SR. The lengths of the room SR and SP are L m and W m respectively.

(a) **(i)** Find the values of the integers a and b for which

$$L = a\cos\theta + b\sin\theta.$$ [2]

Solution:

$L = SA + AR$

$= 5\cos\theta + 2\sin\theta$

$a = 5;\ b = 2$ [B2]

(ii) Obtain a similar expression for W. [1]

Solution:

$W = SB + BP$

$= 5\sin\theta + 2\cos\theta$ [B1]

(iii) Hence find the perimeter of the room $PQRS$ in exact form if $PQRS$ is a square. [3]

Solution:

$W = SB + BP$

$= 5\sin\theta + 2\cos\theta$ [B1]

If $PQRS$ is a square, $L = W$

$5\cos\theta + 2\sin\theta = 5\sin\theta + 2\cos\theta$ [M1]

$3\sin\theta = 3\cos\theta$

$\tan\theta = 1$

$\theta = 45°$ [A1]

Perimeter of $PQRS = 4(5\cos 45° + 2\sin 45°)$

$$= 4\left(\frac{5\sqrt{2}}{2} + \frac{2\sqrt{2}}{2}\right)$$

$$= 4\left(\frac{7\sqrt{2}}{2}\right)$$

$= 14\sqrt{2}$ m [A1]

(b) Using the values of a and b found in (a) part (i),

(i) express L in the form $R\cos(\theta - \alpha)$, $R > 0$ and $0° < \alpha < 90°$. [2]

Solution:

$L = 5\cos\theta + 2\sin\theta$

$$= \sqrt{5^2 + 2^2}\cos\left(\theta - \tan^{-1}\frac{2}{5}\right)$$

$$= \sqrt{29}\cos(\theta - 21.801°)$$

$= \sqrt{29}\cos(\theta - 21.8°)$ (1 dp) [B2]

(ii) find the value of θ if $L = 4$ and the area of the rectangular function room $PQRS$. [4]

Solution:

$$L = 4$$

$$\sqrt{29}\cos(\theta - 21.801°) = 4$$

$\cos(\theta - 21.801°) = \dfrac{4}{\sqrt{29}}$ [M1]

$$\theta - 21.801° = 42.031°$$

$$\theta = 63.832°$$

$= 63.8°$ (1 dp) [A1]

Area of room $PQRS = L \times W$

$= 4 \times (5\sin 63.832° + 2\cos 63.832°)$ [M1]

$$= 4 \times 5.3695$$

$= 21.5$ m^2 [A1]

11 The amount of expenditure, \$$y$, incurred by a textile company is related to \$$x$, the amount of sales generated. The variables x and y are related by the formula $y = 10^k x^a$, where incurs a and k are constants. The following table shows corresponding values of x and y.

x (\$)	6	35	234	1995	6310
y (\$)	148	295	628	1480	2344

(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

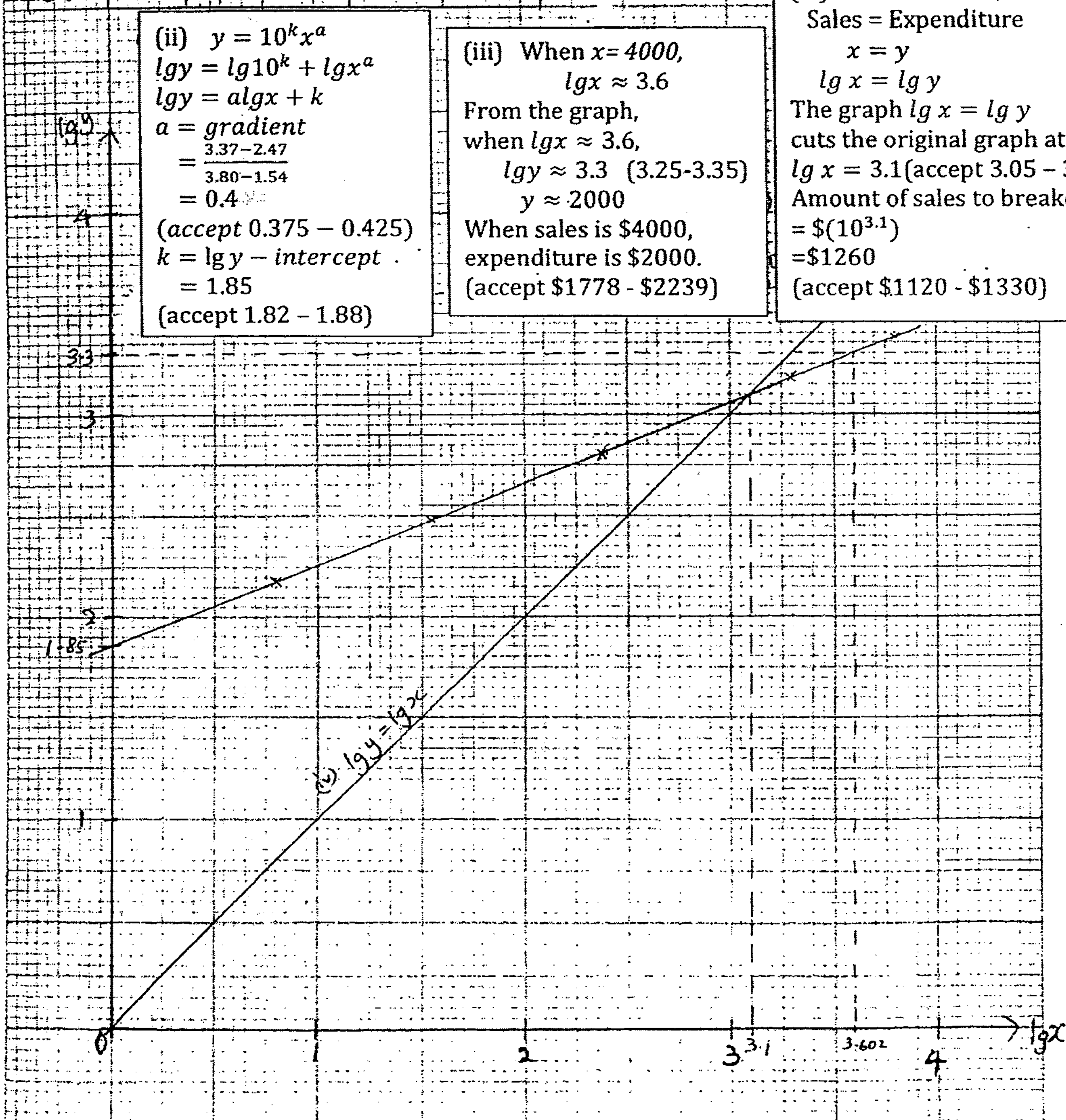
(ii) Use your graph to estimate the value of a and of k. [4]

(iii) Estimate the amount of expenditure incurred when the sales generated is \$4000. [2]

(iv) Draw a straight line on the same axes to estimate the amount of sales to be generated in order for the textile company to breakeven. [2]

Name:

$\lg x$	0.78	1.54	2.37	3.30	3.80
$\lg y$	2.17	2.47	2.80	3.17	3.37



(ii) $y = 10^k x^a$

$lgy = lg10^k + lgx^a$

$lgy = algx + k$

$a = gradient$

$= \frac{3.37-2.47}{3.80-1.54}$

$= 0.4$

($accept$ 0.375 – 0.425)

$k = \lg y - intercept$

$= 1.85$

(accept 1.82 – 1.88)

(iii) When $x = 4000$,

$lgx \approx 3.6$

From the graph,

when $lgx \approx 3.6$,

$lgy \approx 3.3$ (3.25-3.35)

$y \approx 2000$

When sales is \$4000,

expenditure is \$2000.

(accept \$1778 - \$2239)

(iv) To breakeven,

Sales = Expenditure

$x = y$

$lg\ x = lg\ y$

The graph $lg\ x = lg\ y$

cuts the original graph at

$lg\ x = 3.1$(accept 3.05 – 3.13)

Amount of sales to breakeven

= \$($10^{3.1}$)

=\$1260

(accept \$1120 - \$1330)