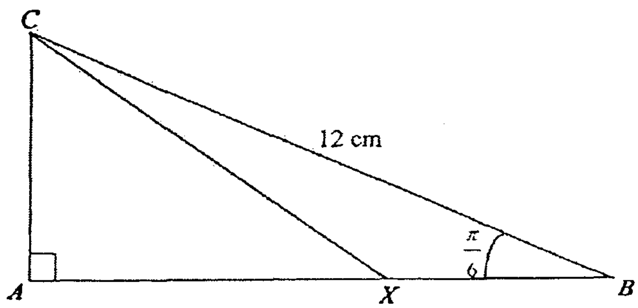
1



In the diagram, the right-angle triangle ABC is such that $BC = 12$ cm, $\angle ABC = \frac{\pi}{6}$ and $AX = \frac{2}{3}AB$.

Show that $\cos\angle BXC = -\frac{2\sqrt{7}}{7}$. [4]

2 Solve the equation $6\cos x = 4\sec x - \tan x$ for $0 < x < 5$. [5]

3 Air leaks from a spherical balloon at a constant rate of 25π cm^3 per second. Given that the initial volume is 5000π cm^3,

(i) calculate the radius of the balloon after 20 seconds, [3]

(ii) find the rate of change of radius at this instant. [2]

4 A curve is such that $\frac{d^2y}{dx^2} = 6x - 6$. The gradient of the curve at the point $(2, -1)$ is 4.

(i) Show that y is an increasing function for all real values of x. [4]

(ii) Find the equation of the curve. [2]

[Turn over...

5 Given the cubic expression $f(x) = x^3 + px^2 + qx + 4$ has a factor $(x+2)$ and leaves a remainder of 6 when divided by $(x+1)$,

(i) find the value of p and of q, [4]

(ii) factorize $f(x)$ completely. [2]

6 (a) Simplify the expression $\dfrac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$. [3]

(b) Solve the equation $\log_2 8x = 4\log_x 2$. [4]

7 Given that the roots of the equation $2x^2 - 2x + 5 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = -4$. [2]

(ii) Find the value of $\alpha^3 + \beta^3$. [2]

(iii) Find a quadratic equation whose roots are $\dfrac{\alpha}{2\beta^2}$ and $\dfrac{\beta}{2\alpha^2}$. [4]

8 The equation of the curve is given by $y = 3\cos 3x - 2$ for $0 \le x \le \pi$.

(i) Write down the amplitude and period of y. [2]

(ii) Find the coordinates of the maximum and minimum points for $0 < x < \pi$. [2]

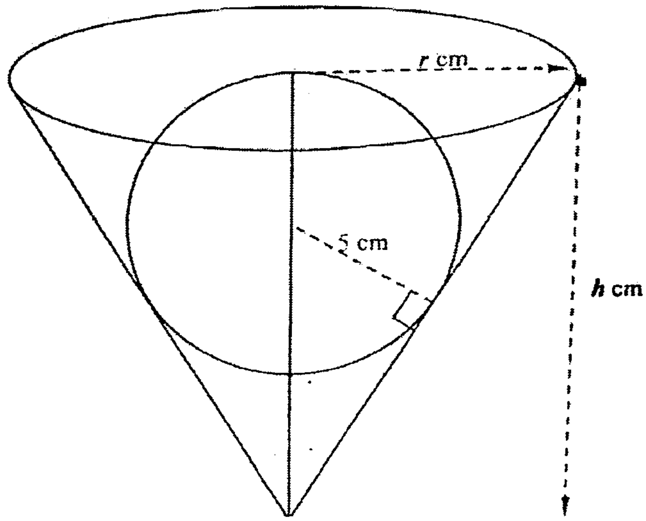
(iii) Calculate the values of x for which the curve cuts the x-axis. [2]

(iv) Sketch the curve $y = 3\cos 3x - 2$ for $0 \le x \le \pi$. [2]

(v) State the range of values of x for which y is decreasing between 0 and π. [2]

[Turn over...

9 A solid spherical ball is dropped into a cone of height h cm and radius r cm.



Given that the radius of the spherical ball is 5 cm,

(i) show that the volume of the cone, V is given by $V = \dfrac{25\pi h^2}{3(h-10)}$. [3]

(ii) Given that h can vary, find the value of h for which V has a stationary value. [3]

(iii) Calculate this stationary value of V and determine if the volume is a maximum or minimum value. [3]

10 (i) Express $\dfrac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(iii) Hence evaluate $\displaystyle\int_1^2 \frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}\,dx$. [4]

[Turn over...

11 The table show experimental values of two variables x and y.

x	2	3	4	6	10
y	3.24	5.79	9	17.05	38.43

It is known that x and y are related by the equation $\frac{y-b}{x} = a\sqrt{x} - 1$ for $x > 0$ where a and b are constants.

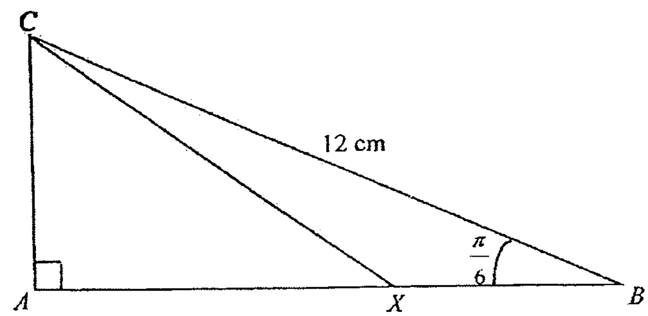
(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of $x + y$ against $x\sqrt{x}$. [3]

(ii) Use your graph to estimate, to 2 decimal places, the value of a and of b [4]

(iii) On the same diagram, draw a straight line representing the equation $y + x + 2x\sqrt{x} = 36$.
Hence find the value of x that satisfies the equation $(a+2)x\sqrt{x} = 36 - b$. [3]

~ End of Paper ~

1



In the diagram, the right-angle triangle ABC is such that $BC = 12$ cm, $\angle ABC = \frac{\pi}{6}$ and $AX = \frac{2}{3}AB$.

Show that $\cos \angle BXC = -\frac{2\sqrt{7}}{7}$. [4]

[soln] $\cos \angle BXC = -\cos \angle AXC$

$\sin \frac{\pi}{6} = \frac{AC}{12} \quad \Rightarrow AC = 6$

$AB = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3}$

$AX = 4\sqrt{3}$

$CX = \sqrt{36 + 48} = \sqrt{84} = 2\sqrt{21}$

$\cos \angle BXC = -\cos \angle AXC = -\frac{4\sqrt{3}}{2\sqrt{21}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7}$

2 Solve the equation $6\cos x = 4\sec x - \tan x$ for $0 < x < 5$. [5]

[soln]

$6\cos x = \frac{4}{\cos x} - \tan x$

$6\cos^2 x = 4 - \sin x$

$6\left(1 - \sin^2 x\right) = 4 - \sin x$

$6\sin^2 x - \sin x - 2 = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$

$\sin x = \frac{2}{3}$ or $\sin x = -\frac{1}{2}$

Basic angle = 0.7297   Basic angle = 0.5236

$x = 0.730, 2.41$ $x = 2.62, 5.76$ (NA)

3 Air leaks from a spherical balloon at a constant rate of 25π cm^3 per second. Given that the initial volume is 5000π cm^3,

(i) calculate the radius of the balloon after 20 seconds, [3]

(ii) find the rate of change of radius at this instant. [2]

[soln]

$$\frac{dV}{dt}-25\pi$$

After 20s, volume $= 5000\pi - 25\pi \times 20 = 4500\pi$

$$\frac{4}{3}\pi r^3 = 4500\pi$$

$$r^3 = 3375$$

$$r = 15$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$-25\pi = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{25}{4\times 225} = -\frac{25}{900} = -\frac{1}{36} \text{ cm/s}$$

4 A curve is such that $\frac{d^2y}{dx^2} = 6x - 6$. The gradient of the curve at the point $(2, -1)$ is 4.

(i) Show that y is an increasing function for all real values of x. [4]

(ii) Find the equation of the curve. [2]

[soln]

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\frac{dy}{dx} = 3x^2 - 6x + c$$

At $(2,-1)$, $\frac{dy}{dx} = 4$

$$12 - 12 + c = 4$$

$$c = 4$$

$$\frac{dy}{dx} = 3x^2 - 6x + 4$$

$$\frac{dy}{dx} = 3\left(x^2 - 2x\right) + 4$$

$$\frac{dy}{dx} = 3(x-1)^2 + 1$$

For all values of x, $\frac{dy}{dx} > 0$. y is increasing.

$$y = x^3 - 3x^2 + 4x + d$$

$$8 - 12 + 8 + d = -1$$
$$d = -5$$
$$y = x^3 - 3x^2 + 4x - 5$$

5 Given the cubic expression $f(x) = x^3 + px^2 + qx + 4$ has a factor $(x+2)$ and leaves a remainder of 6 when divided by $(x+1)$,

(i) find the value of p and of q, [4]

(ii) factorize $f(x)$ completely. [2]

[soln]

$$-8 + 4p - 2q + 4 = 0$$
$$2p - q = 2$$

$$-1 + p - q + 4 = 6$$
$$p - q = 3$$
$$p = -1;\ q = -4$$

$$f(x) = x^3 - x^2 - 4x + 4$$

$$f(x) = (x+2)(x^2 - 3x + 2)$$

$$f(x) = (x+2)(x-2)(x-1)$$

6 **(a)** Simplify the expression $\dfrac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$. [3]

(b) Solve the equation $\log_2 8x = 4\log_x 2$. [4]

[soln]

(a)

$$\frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} = \frac{3^n\left(\frac{1}{9} - 3\right)}{3^n\left(9 - \frac{1}{3}\right)} = -\frac{1}{3}$$

(b)

$$\log_2 8x = 4\log_x 2$$
$$\log_2 8 + \log_2 x = \frac{4\log_2 2}{\log_2 x}$$

$$3+\log_2 x=\frac{4}{\log_2 x}$$

Let $y=\log_2 x$ $\quad$ $y^2+3y-4=0$

$(y+4)(y-1)=0$

$\log_2 x=-4$ or $\log_2 x=1$

$x=\frac{1}{16}$ or $x=2$

7 Given that the roots of the equation $2x^2-2x+5=0$ are α and β.

(i) Show that $\alpha^2+\beta^2=-4$. [2]

(ii) Find the value of $\alpha^3+\beta^3$. [2]

(iii) Find a quadratic equation whose roots are $\frac{\alpha}{2\beta^2}$ and $\frac{\beta}{2\alpha^2}$. [4]

[soln] $\alpha+\beta=1$ and $\alpha\beta=\frac{5}{2}$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=1-2\times\frac{5}{2}=-4$$

$$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$

$$\alpha^3+\beta^3=1-3\times\frac{5}{2}=-\frac{13}{2}$$

$$\frac{\alpha}{2\beta^2}+\frac{\beta}{2\alpha^2}=\frac{\alpha^3+\beta^3}{2(\alpha\beta)^2}=\left(-\frac{13}{2}\right)\div\frac{25}{2}=-\frac{13}{25}$$

$$\frac{\alpha}{2\beta^2}\times\frac{\beta}{2\alpha^2}=\frac{1}{4\alpha\beta}=\frac{1}{10}$$

Quadratic equation is $x^2+\frac{13}{25}x+\frac{1}{10}=0$ or $50x^2+26x+5=0$

8 The equation of the curve is given by $y = 3\cos 3x - 2$ for $0 \le x \le \pi$.

(i) Write down the amplitude and period of y. [2]

(ii) Find the coordinates of the maximum and minimum points for $0 < x < \pi$. [2]

(iii) Calculate the values of x for which the curve cuts the x-axis. [2]

(iv) Sketch the curve $y = 3\cos 3x - 2$ for $0 \le x \le \pi$. [2]

(v) State the range of values of x for which y is decreasing between 0 and π. [2]

[soln] amplitude = 3, period = $\dfrac{2\pi}{3}$

Minimum point is $\left(\dfrac{\pi}{3}, -5\right)$ and Maximum point is $\left(\dfrac{2\pi}{3}, 1\right)$

$\cos 3x = \dfrac{2}{3}$

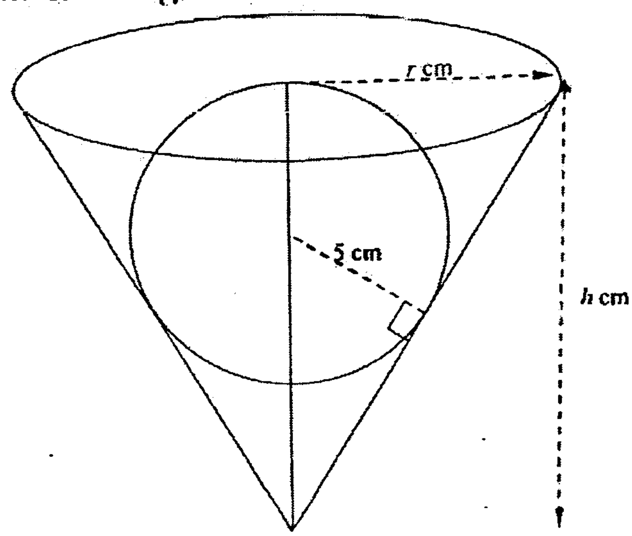
Basic angle = 0.841

$3x = 0.841, 5.4421, 7.124$

$x = 0.280, 1.81, 2.37$

y is decreasing for $0 < x < \dfrac{\pi}{3}$ and $\dfrac{2\pi}{3} < x < \pi$

9 A solid spherical ball is dropped into a cone of height h cm and radius r cm.



Given that the radius of the spherical ball is 5 cm,

(i) show that the volume of the cone, V is given by $V = \frac{25\pi h^2}{3(h-10)}$. [3]

(ii) Given that h can vary, find the value of h for which V has a stationary value. [3]

(iii) Calculate this stationary value of V and determine if the volume is a maximum or minimum value. [3]

[soln]

$$\frac{r}{\sqrt{h^2+r^2}} = \frac{5}{h-5}$$

$$\frac{r^2}{h^2+r^2} = \frac{25}{h^2-10h+25}$$

$$r^2h^2 - 10r^2h + 25r^2 = 25h^2 + 25r^2$$

$$r^2 = \frac{25h^2}{h^2-10h} = \frac{25h}{h-10}$$

$$V = \frac{1}{3}\pi h \times \frac{25h}{h-10} = \frac{25\pi h^2}{3(h-10)}$$

$$\frac{dV}{dh} = \frac{25\pi}{3}\left[\frac{(h-10)\times 2h - h^2}{(h-10)^2}\right]$$

$$\frac{dV}{dh}=\frac{25\pi}{3}\left[\frac{h^2-20h}{(h-10)^2}\right]$$

For stationary value,

$$\frac{dV}{dh}=0 \quad \Rightarrow h=20$$

$$V=\frac{25\pi\times 400}{3\times 10}=\frac{1000\pi}{3}=1047.20 \text{ (minimum volume)}$$

x	<20	20	>20
$\frac{dV}{dh}$	negative	0	positive

10 (i) Express $\dfrac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}$ in partial fractions. [5]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [1]

(iii) Hence evaluate $\displaystyle\int_1^2 \frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}\,dx$. [4]

[soln]

$$\frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}=2+\frac{9x^2-4x+2}{(2x-1)(x^2+2)}$$

$$\frac{9x^2-4x+2}{(2x-1)(x^2+2)}=\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$$

$$9x^2-4x+2=A(x^2+2)+(Bx+C)(2x-1)$$

Subst $x=\frac{1}{2}$, $\quad \frac{9}{4}A=\frac{9}{4} \quad A=1$

Coefficient of x^2: $\quad B=4$

Constant term: $\quad C=0$

$$\frac{9x^2-4x+2}{(2x-1)(x^2+2)}=\frac{1}{2x-1}+\frac{4x}{x^2+2}$$

$$\frac{d}{dx}\ln(x^2+2)=\frac{2x}{x^2+2}$$

$$\int_1^2 \frac{4x^3+7x^2+4x-2}{(2x-1)(x^2+2)}\,dx = \int_1^2 2+\frac{1}{2x-1}+\frac{4x}{x^2+2}\,dx$$

$$= \left[2x+\frac{1}{2}\ln(2x-1)+2\ln(x^2+2)\right]_1^2 = \left[4+\frac{1}{2}\ln 3+2\ln 6\right]-\left[2+\frac{1}{2}\ln 1+2\ln 3\right]$$

$$= 2-\frac{3}{2}\ln 3+2\ln 6$$

$$= 3.94$$

11 The table show experimental values of two variables x and y.

x	2	3	4	6	10
y	3.24	5.79	9	17.05	38.43

It is known that x and y are related by the equation $\frac{y-b}{x} = a\sqrt{x}-1$ for $x>0$ where a and b are constants.

(i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of $x+y$ against $x\sqrt{x}$. [3]

(ii) Use your graph to estimate, to 2 decimal places, the value of a and of b [4]

(iii) On the same diagram, draw a straight line representing the equation $y+x+2x\sqrt{x}=36$.
Hence find the value of x that satisfies the equation $(a+2)x\sqrt{x}=36-b$. [3]

[soln]

$$\frac{y-b}{x} = a\sqrt{x}-1$$

$$y-b = ax\sqrt{x}-x$$

$$x+y = ax\sqrt{x}+b$$

$x\sqrt{x}$	2.83	5.20	8	14.70	31.62
$x+y$	5.24	8.79	13	23.05	48.43

$a = 1.5$ and $b = 0.994$

$ax\sqrt{x}+2x\sqrt{x} = 36-b$

$ax\sqrt{x}+b = -2x\sqrt{x}+36$ (gradient = 2, intercept = 36)

~ *End of Paper* ~

1. (a) (i) Sketch the graph of the curve $y^2 = kx$, where k is a positive constant. [1]

(ii) Given that the line $y = 2x + 1$ meets the curve $y^2 = kx$, find the range of values of k. [4]

(b) Determine the conditions for p and q such that the curve $y = px^2 - 2x + 3q$ lies entirely above the x-axis, where p and q are constants. [3]

2. (i) Sketch the curve $y = 2\ln(x-3)$ for $x > 3$. [2]

(ii) The tangent to the curve $y = 2\ln(x-3)$ at the point P where $x = 5$ intersects the x-axis at A and the normal to the curve at P intersects the x-axis at B. Calculate the area of ΔAPB. [5]

3. (a) Write down and simplify the first three terms in the expansion of $(2-3x)^6$, in ascending powers of x. [2]

(b) Hence

(i) using a suitable value of x, find the estimated value of $(1.997)^6$, correct to 3 decimal places. [2]

(ii) determine the coefficient of x^2 in the expansion of $(2-3x)^7 - (2-3x)^6$. [3]

4. A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2+\cos x}{\sin x}$ for $-\pi \le x \le \pi$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the **exact** value of the x-coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

5. (a) (i) Show that $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(ii) Hence solve the equation $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos x$ for $0° < x < 360°$ [3]

(b) Without using a calculator, express $\sin 15°$ in the form $\frac{1}{k}(\sqrt{a} - \sqrt{b})$, where a, b and k are integers. [3]

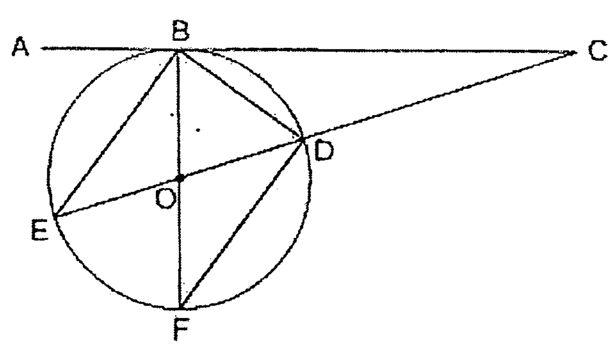
6. (i) Sketch the graph of $y = 1 - |x - 3|$. [3]

A line $y = mx + 1$ is drawn on the same axes with the graph $y = 1 - |x - 3|$.

(ii) In the case where $m = 2$, find the coordinates of the point of intersection of the line and the graph of $y = 1 - |x - 3|$. [2]

(iii) Determine the set of values of m for which the line does not intersect the graph of $y = 1 - |x - 3|$. [2]

7.



In the diagram, BF and DE are the diameters of the circle with centre O.

The tangent at B meets ED produced at C. Prove that

(i) $BE = DF$ [3]

(ii) $DF \times BC = BD \times CE$ [3]

(iii) $\angle BCE + 2\angle CBD = 90°$. [2]

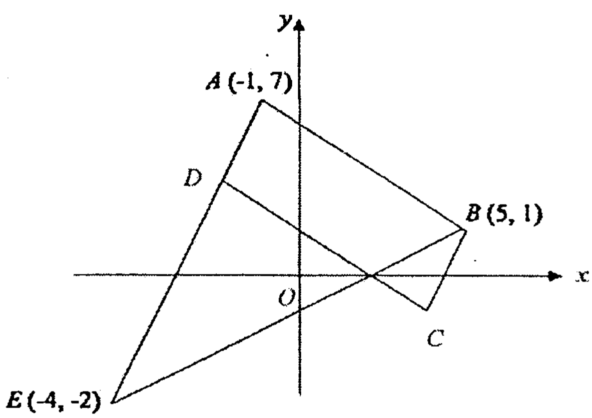
8. The equation of a circle C_1 is $x^2 + y^2 - 4x - 8y + 4 = 0$.

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is A.
State the coordinates of A. [1]

(c) Another circle, C_2, touches C_1 at the point A. Given that both circles do not overlap and the area of C_2 is four times that of the area of C_1, find the equation of C_2 in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of f, g and c. [4]

9. **Solutions to this question by accurate drawing will not be accepted.**



The diagram, not to scale, shows a parallelogram, $ABCD$. ADE and BE are straight lines. D divides AE such that $AD : DE$ is in the ratio $1 : 2$.

A, B and E have coordinates $(-1, 7)$, $(5, 1)$ and $(-4, -2)$ respectively.

(a) (i) Find the equation of the perpendicular bisector of AB and show that it passes through E. [3]

(ii) Hence deduce the geometrical property of triangle ABE. [1]

(b) Find the coordinates of D. [2]

(c) Find the area of the parallelogram $ABCD$. [2]

10. A particle starts from rest at 5 m from a fixed point O and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where t is the time in seconds after leaving from the initial rest position.

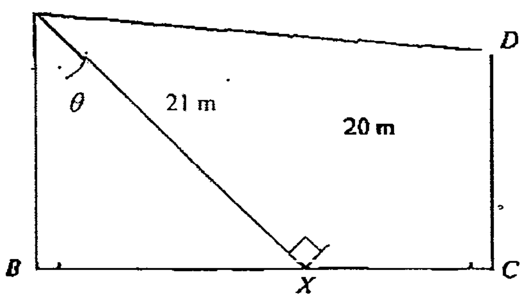
(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, s, from point O in terms of t. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

11.



The diagram shows a trapezium field $ABCD$. The point X lies on the side BC such that $AX = 21$ m, $DX = 20$ m, $\angle AXD = \angle ABX = \angle DCX = 90°$ and $\angle BAX = \theta$.

(i) Show that the length of fencing required for the perimeter of the field, L m, can be expressed in the form of $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be determined. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

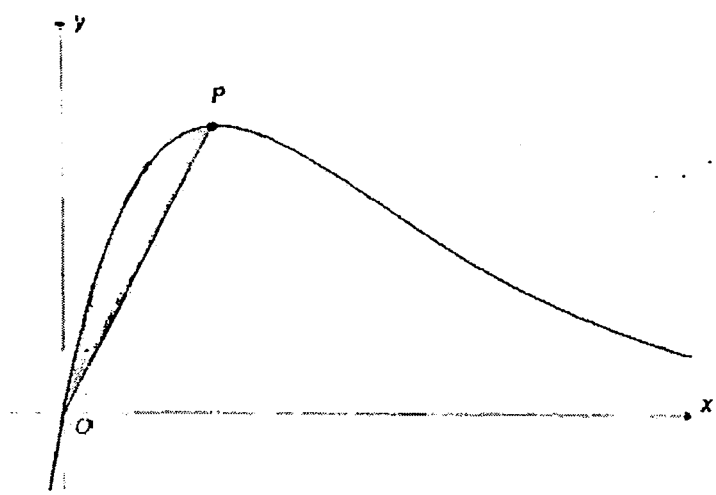
(iv) Given that the fencing used is 80 m, find the value(s) of θ. [3]

12. (a) (i) Given that $y = xe^{-2x}, x > 0$, show that $\frac{dy}{dx} = (1-2x)e^{-2x}$. [1]

(ii) Hence, find $\int xe^{-2x}dx$. [3]

(b) The diagram, which is not drawn to scale, shows part of the curve $y = xe^{-2x}$

A line drawn from the origin meets the curve at the maximum point P.



(i) Find the coordinates of P. [3]

(ii) Calculate the area of the region bounded by the curve and the line OP. [4]

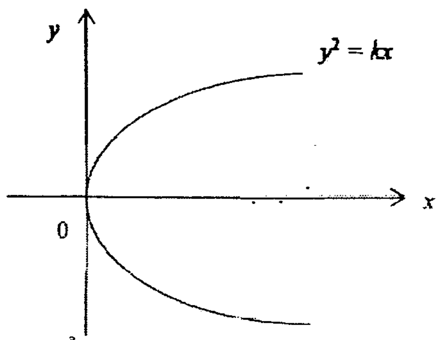
--- END OF PAPER ---

1. (a) (i) Sketch the graph of the curve $y^2 = kx$, where k is a positive constant. [1]

(ii) Given that the line $y = 2x + 1$ meets the curve $y^2 = kx$, find the range of values of k. [4]

(b) Determine the conditions for p and q such that the curve $y = px^2 - 2x + 3q$ lies entirely above the x-axis, where p and q are constants. [3]

(a)(i)



[D1]

(a)(ii) $y = 2x + 1$ .... (1)

$y^2 = kx$ .... (2)

(1) in (2) : $(2x+1)^2 = kx$

$4x^2 + (4-k)x + 1 = 0$ [A1]

For line meets the curve, $D \geq 0$.

$(4-k)^2 - 4(4)(1) \geq 0$ [M1]

$16 - 8k + k^2 - 16 \geq 0$

$k(k-8) \geq 0$ [M1A1]

$\therefore k \leq 0 (NA)$ *or* $k \geq 8$

(b) Curve lies entirely above line, $D < 0$ and $p > 0$.

$(-2)^2 - 4p(3q) < 0$

$4 - 12pq < 0$ [M1]

$pq > \frac{1}{3}$

$\therefore p > 0$ *and* $pq > \frac{1}{3}$ [A2]

2. (i) Sketch the curve $y = 2\ln(x-3)$ for $x > 3$. [2]

(ii) The tangent to the curve $y = 2\ln(x-3)$ at the point P where $x = 5$ intersects the x-axis at A and the normal to the curve at P intersects the x-axis at B.

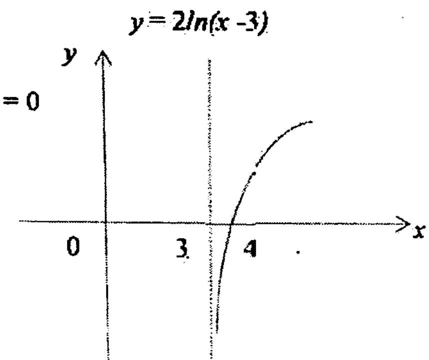
Calculate the area of ΔAPB. [5]

(i) Asymptote : $x = 3$

x-intercept : $2\ln(x-3) = 0$

$x - 3 = 1$

$x = 4$



[D1 : shape]

[D1 : Asymptote, x-int]

(ii) $\frac{dy}{dx} = \frac{2}{x-3}$ [A1]

When $x = 5$, gradient of tangent at P = 1

When $x = 5$, $y = 2\ln 2$

$P(1, 2\ln 2)$

Equation of tangent at P : $y - 2\ln 2 = x - 5$

$\therefore y = x - 5 + 2\ln 2$

At x-axis , y = 0 : $x = 5 - 2\ln 2$

$\therefore A(5 - 2\ln 2,\ 0)$ [A1]

Gradient of normal at P = -1 [M1]

Equation of normal at P : $y - 2\ln 2 = -1(x - 5)$

$\therefore y = -x + 5 + 2\ln 2$

At x-axis, $y = 0$: $x = 5 + 2\ln 2$

$\therefore B(5 + 2\ln 2,\ 0)$ [A1]

$\therefore Area\ of\ \Delta APB = \frac{1}{2}(5 + 2\ln 2 - 5 + 2\ln 2)(2\ln 2)$

$= 1.92$ units^2 [A1]

3. (a) Write down and simplify the first three terms in the expansion of $(2-3x)^6$, in ascending powers of x. [2]

(b) Hence

(i) using a suitable value of x, find the estimated value of $(1.997)^6$, correct to 3 decimal places. [2]

(ii) determine the coefficient of x^2 in the expansion of $(2-3x)^7-(2-3x)^6$. [3]

(a) $(2-3x)^6 = 2^6 + \binom{6}{1}2^5(-3x) + \binom{6}{2}2^4(-3x)^2 + ...$

$= 64 - 576x + 2160x^2 - ....$ (up to 1st 3 terms)[M1A1]

(b)(i) Put $2-3x = 1.997$

$x = 0.001$ [M1]

$(1.997)^6 = 64 - 576(0.001) + 2160(0.001)^2 + ...$

$= 63.42616 = 63.426$ (correct to 3dp) [A1]

(b)(ii) $(2-3x)^7 - (2-3x)^6 = (2-3x)^6[2-3x-1]$

$= (1-3x)(2-3x)^6$ [M1]

$= (1-3x)(64 - 576x + 2160x^2 - ....)$

Coefficient of x^2 = $1(2160) - 3(-576) = 3888$ [M1A1]

4. A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2+\cos x}{\sin x}$ for $-\pi \le x \le \pi$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the **exact** value of the x-coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

(i) $f'(x) = \dfrac{\sin x(-\sin x) - (2+\cos x)(\cos x)}{\sin^2 x}$ [M1]

$= \dfrac{-\sin^2 x - 2\cos x - \cos^2 x}{\sin^2 x}$

$= \dfrac{-1-2\cos x}{\sin^2 x}$ [A1]

(ii) For stationary points, $f'(x) = 0$.

$\frac{-1-2\cos x}{\sin^2 x}=0$

$-1-2\cos x=0$ [M1]

$\cos x=-\frac{1}{2}$

$x=\frac{2\pi}{3}$ *or* $\pi+\frac{2\pi}{3}-2\pi$

$\therefore x=\frac{2\pi}{3}$ *or* $-\frac{2\pi}{3}$ [A2]

x	-2.1	$-\frac{2\pi}{3}$	-2	2	$\frac{2\pi}{3}$	2.1
$f'(x)$	+ve	0	-ve	-ve	0	+ve
Tangent	/	—	\	\	—	/

[M1]

$\therefore x=-\frac{2\pi}{3}$ is a maximum point and $x=\frac{2\pi}{3}$ is a minimum point. [A2]

Alternate Mtd :

$f''(x)=\frac{\sin^2 x(2\sin x)-(-1-2\cos x)(2\sin x\cos x)}{\sin^4 x}$

$=\frac{2(\sin^2 x+\cos x+2\cos^2 x)}{\sin^3 x}$

$f''\left(-\frac{2\pi}{3}\right)=-2.31<0\Rightarrow \max\ point$

$f''\left(\frac{2\pi}{3}\right)=2.31>0\Rightarrow \min\ point$

5. (a) (i) Show that $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(ii) Hence solve the equation $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos x$ for $0° < x < 360°$. [3]

(b) Without using a calculator, express $\sin 15°$ in the form $\dfrac{1}{k}(\sqrt{a} - \sqrt{b})$, where a, b and k are integers. [3]

(a)(i) LHS : $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \dfrac{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}}$ [M1]

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$ [A1]

$= \cos 2x = RHS$ [A1]

(ii) $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos x$

$\cos 2x = \cos x$

$2\cos^2 x - \cos x - 1 = 0$ [M1]

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\dfrac{1}{2}$ *or* $\cos x = 1$

$\therefore x = 120°,\ 240°$ [A2]

(b) $\sin 15° = \sin(45° - 30°)$ Alt Mtd : $\sin 15° = \sin(60° - 45°)$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$ [M1]

$= \dfrac{\sqrt{2}}{2}\left(\dfrac{\sqrt{3}}{2}\right) - \dfrac{\sqrt{2}}{2}\left(\dfrac{1}{2}\right)$ [A1]

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$ [A1]

6. (i) Sketch the graph of $y = 1 - |x-3|$. [3]

A line $y = mx + 1$ is drawn on the same axes with the graph $y = 1 - |x-3|$.

(ii) In the case where $m = 2$, find the coordinates of the point of intersection of the line and the graph of $y = 1 - |x-3|$. [2]

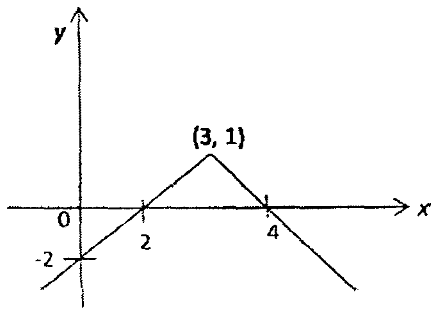
(iii) Determine the set of values of m for which the line does not intersect the graph of $y = 1 - |x-3|$. [2]

(i) y-int : Put $x = 0$: $y = -2$

x-int : $1 - |x-3| = 0$

$x = 4$ or $x = 2$

Max pt = (3, 1)



D1 : Correct shape

D1 : intercepts

D1 : max pt

(ii) $2x + 1 = 1 - |x-3|$

$|x-3| = -2x$

$x - 3 = -2x$ *or* $x - 3 = 2x$ [M1]

$x = 1$ (NA) *or* $x = -3$

When $x = -3$, $y = -5$

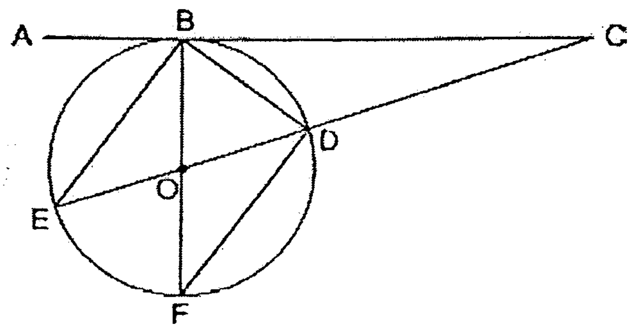
Pt of intersection is (-3, -5) [A1]

(iii) For line not to intersect graph of $y = 1 - |x-3|$, line must be parallel to the left arm.

Gradient of left arm = $\dfrac{1-(-2)}{3-0} = 1$

Set of values of m : $0 < m \le 1$ [B2]

7.



In the diagram, BF and DE are the diameters of the circle with centre O.

The tangent at B meets ED produced at C. Prove that

(i) $BE = DF$ [3]

(ii) $DF \times BC = BD \times CE$ [3]

(iii) $\angle BCE + 2\angle CBD = 90°$. [2]

(i) $\angle BED = \angle DFB$ (Angles in the same segment)
$\angle DBE = \angle BDF = 90°$ (right angle in a semi-circle)
$DE = BF$ (diameter) [M1]
$\therefore \Delta BDE \equiv \Delta DBF$ (AAS) [M1]

$\therefore BE = DF$ [A1]

Alt Mtd : Show $\Delta BOE \equiv \Delta DOF$

(ii) $\angle DBC = \angle BEC$ (Alternate segment theorem)

$\angle DCB = \angle BCE$ (Common angle)

$\therefore \Delta BEC$ is similar to ΔDBC (AA Similarity Test) [M1A1]

$$\frac{BE}{DB} = \frac{EC}{BC}$$

$BE \times BC = EC \times DB$ [M1]
$\therefore DF \times BC = BD \times CE$

(iii) $\angle BCE + \angle BEC + 90° + \angle CBD = 180°$ [M1]

$\angle BCE + 2\angle CBD = 180° - 90°$
$\therefore \angle BCE + 2\angle CBD = 90°$. [A1]

8. The equation of a circle C_1 is $x^2 + y^2 - 4x - 8y + 4 = 0$.

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is A.
State the coordinates of A. [1]

(c) Another circle, C_2 touches C_1 at the point A. Given that both circles do not overlap and the area of C_2 is four times that of the area of C_1, find the equation of C_2 in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$, stating the value of f, g and c. [4]

(a) $C_1: x^2 + y^2 - 4x - 8y + 4 = 0$.

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 + y^2 - 8y + \left(-\frac{8}{2}\right)^2 = -4 + \left(-\frac{4}{2}\right)^2 + \left(-\frac{8}{2}\right)^2 \quad \text{[M1]}$$

$(x-2)^2 + (y-4)^2 = 16$

Centre = (2, 4) and radius = 4 units [A2]

(b) x-coordinate of $A = 2$ (radius $\perp$ tangent)

$\therefore A = (2,\ 4+4) = (2,\ 8)$ [A1]

(c) Radius of $C_2 = 8$ [B1]

Centre of $C_2 = (2, 8+8) = (2, 16)$

Equation of C_2: $(x-2)^2 + (y-16)^2 = 8^2$ [M1]

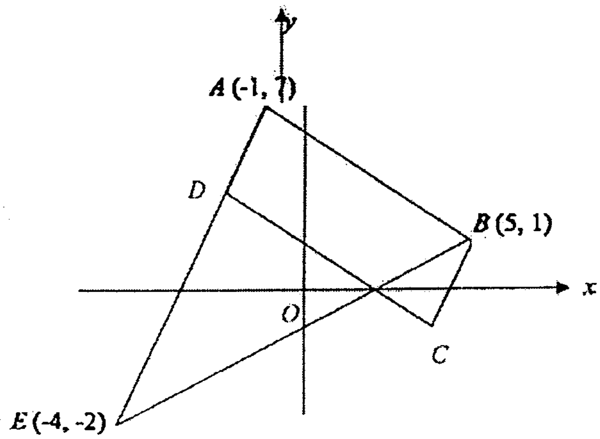
$x^2 - 4x + 4 + y^2 - 32y + 256 = 0$

$x^2 + y^2 - 4x - 32y + 196 = 0$ [A1]

$2g = -4$, $2f = -32$ and $c = -196$

$\therefore g = -2,\ f = 16,\ c = 196$ [A1]

9. **Solutions to this question by accurate drawing will not be accepted.**



The diagram, not to scale, shows a parallelogram, $ABCD$. ADE and BE are straight lines. D divides AE such that $AD : DE$ is in the ratio 1 : 2.

A, B and E have coordinates $(-1, 7)$, $(5, 1)$ and $(-4, -2)$ respectively.

(a) **(i)** Find the equation of the perpendicular bisector of AB and show that it passes through E. [3]

(ii) Hence deduce the geometrical property of triangle ABE. [1]

(b) Find the coordinates of D. [2]

(c) Find the area of the parallelogram $ABCD$. [2]

(a)(i) Gradient of $AB = \dfrac{7-1}{-1-5} = -1$

Gradient of perpendicular bisector of $AB = 1$

Mid-point of $AB = \left(\dfrac{-1+5}{2},\ \dfrac{7+1}{2}\right) = (2,\ 4)$ [A1]

Equation of perpendicular bisector of AB: $y - 4 = x - 2$

$\therefore y = x + 2$ [A1]

When $x = -4$, $y = -4 + 2 = -2$.

$\therefore$ perpendicular bisector of AB passes through E. *(Shown)* [M1]

(ii) ΔABE is an isosceles triangle. [A1]

(b) $\overrightarrow{AD} = \dfrac{1}{3}\overrightarrow{AE} = \begin{pmatrix}-1\\-3\end{pmatrix}$

$D = (-1-1\ ,\ 7-3) = (-2\ ,4)$ [M1A1]

(c) Area of $\Delta ABD = \dfrac{1}{2}\begin{vmatrix} -1 & -2 & 5 & -1 \\ 7 & 4 & 1 & 7 \end{vmatrix} = 12$ units^2 [M1]

Area of parallelogram $ABCD = 12 \times 2 = 24$ units^2 [A1]

10. A particle starts from rest at 5 m from a fixed point O and moves in a straight line with a velocity, $v = 12t - 3t^2$ m/s where t is the time in seconds after leaving from the initial rest position.

(i) Calculate the acceleration when the particle is instantaneously at rest. [3]

(ii) Calculate the maximum velocity. [2]

(iii) Express the displacement, s, from point O in terms of t. [1]

(iv) Find the average speed of the particle during the first five seconds. [3]

(i) $a = \frac{dv}{dt} = 12 - 6t$ [A1]

When particle is instantaneously at rest, $v = 0$

$12t - 3t^2 = 0$

$3t(4 - t) = 0$

$t = 0$ (NA) or $t = 4$ [M1]

Acceleration $= 12 - 6(4) = -12$ m/s². [A1]

(ii) For max or min velocity, $a = 0$

$12 - 6t = 0$

$t = 2$ [M1]

$\frac{d^2v}{dt^2} = -6 < 0 \Rightarrow$ max velocity

Max velocity $= 12(2) - 3(4) = 12$ m/s [A1]

(iii) $S = \int (12t - 3t^2)dt$

$= 6t^2 - t^3 + C$ where C is an arbitrary constant.

Subst $t = 0, s = 5 : C = 5$.

$\therefore s = 6t^2 - t^3 + 5$ [A1]

(iv) When $t = 0$, $s = 5$ m

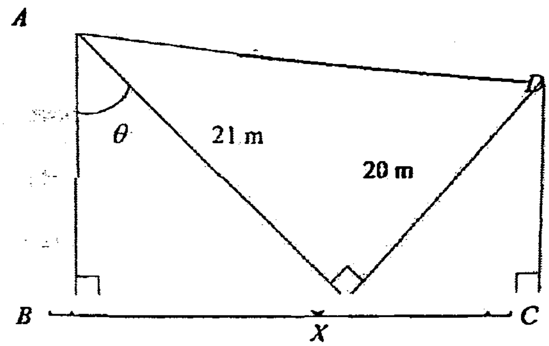
When $t = 4$, $s = 37$ m

When $t = 5$, $s = 30$ m [A1]

Total distance $= (37 - 5) + (37 - 30) = 39$ m [M1]

Average speed $= \frac{39}{5} = 7.8$ m/s [A1]

11.



The diagram shows a trapezium field $ABCD$. The point X lies on the side BC such that $AX = 21$ m, $DX = 20$ m, $\angle AXD = \angle ABX = \angle DCX = 90°$ and $\angle BAX = \theta$.

(i) Show that the length of fencing required for the perimeter of the field, L m, can be expressed in the form of $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be determined. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

(iv) Given that the fencing used is 80 m, find the value(s) of θ. [3]

(i) $AD = \sqrt{21^2 + 20^2} = 29m$

$\sin\theta = \frac{BX}{21}$

$BX = 21\sin\theta$

$\cos\theta = \frac{AB}{21}$

$AB = 21\cos\theta$

$\angle DXC = \theta$

$\sin\theta = \frac{DC}{20}$

$DC = 20\sin\theta$ [M1A1]

$\cos\theta = \frac{XC}{20}$

$XC = 20\cos\theta$

$L = AB + BC + CD + AD$

$= 21\cos\theta + 21\sin\theta + 20\cos\theta + 20\sin\theta + 29$

$\therefore L = 41\cos\theta + 41\sin\theta + 29$ [A1]

(ii) Let $41\cos\theta + 41\sin\theta = R\cos(\theta - \alpha)$

$R = \sqrt{41^2 + 41^2} = \sqrt{3362}$

$\tan\alpha = 1$

$\alpha = 45°$ [M1A1]

$\therefore L = 29 + \sqrt{3362}\cos(\theta - 45°)$

(iii) Max value of $L = 29 + \sqrt{3362} = 87.0m$ [A1]

$\cos(\theta - 45°) = 1$

$\theta - 45° = 0$ [A1]

$\therefore \theta = 45°$

(iv) $29 + \sqrt{3362}\cos(\theta - 45°) = 80$

$\cos(\theta - 45°) = \dfrac{51}{\sqrt{3362}}$

$\theta - 45° = 28.4°, 331.6°(NA), -28.4°$ [M1A2]

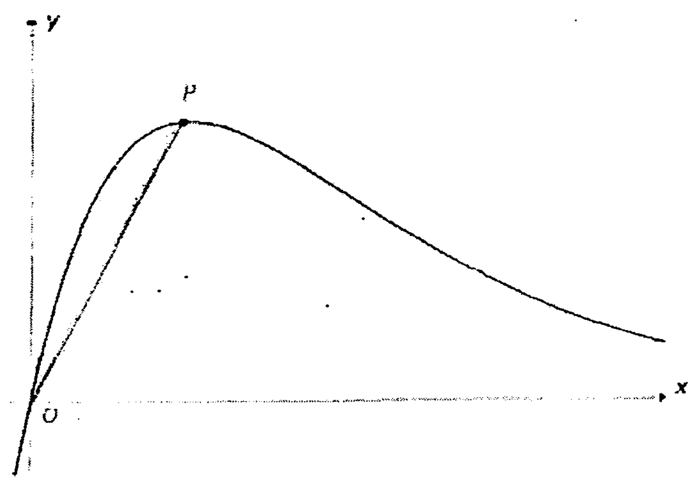
$\therefore \theta = 73.4°, 16.6°$

12. (a) (i) Given that $y = xe^{-2x}, x > 0$, show that $\frac{dy}{dx} = (1-2x)e^{-2x}$. [1]

(ii) Hence, find $\int xe^{-2x}dx$. [3]

(b) The diagram, which is not drawn to scale, shows part of the curve $y = xe^{-2x}$

A line drawn from the origin meets the curve at the maximum point P.



[3]

(ii) Calculate the area of the region bounded by the curve and the line OP. [4]

(a)(i) $y = xe^{-2x}$

$\frac{dy}{dx} = e^{-2x} - 2xe^{-2x}$ [M1]

$= (1-2x)e^{-2x}$

(ii) $\int e^{-2x}dx - 2\int xe^{-2x}dx = [xe^{-2x}]$ [M1]

$\int xe^{-2x}dx = \frac{1}{2}\int e^{-2x}dx - \frac{1}{2}xe^{-2x}$

[M1A1]

$\therefore \int xe^{-2x}dx = -\frac{1}{4}e^{-2x} - \frac{1}{2}xe^{-2x} + C$

(b)(i) For stationary points, $\frac{dy}{dx} = 0$

$(1-2x)e^{-2x} = 0$

$1 - 2x = 0$ [M1A1]

$x = \frac{1}{2}$

When $x=\frac{1}{2}, y=\frac{1}{2}e^{-1}=\frac{1}{2e}$

$\therefore P(\frac{1}{2},\frac{1}{2e})$ **[A1]**

(iii) Required area $= \int_0^{\frac{1}{2}} xe^{-2x}\,dx - \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2e}\right)$ **[M1]**

$= \left[-\frac{1}{4}e^{-2x} - \frac{1}{2}xe^{-2x}\right]_0^{\frac{1}{2}} - \frac{1}{8e}$ **[M1]**

$= \left[-\frac{1}{4}e^{-1} - \frac{1}{4}e\right] - (-\frac{1}{4}) - \frac{1}{8e}$ **[M1]**

$= \frac{5}{8}e^{-1} + \frac{1}{4}$ or 0.480 units² (3sf) **[A1]**