4

Name:	Class	Class Register Number/ Centre No./Index No.

PRELIMINARY EXAMINATION 2016
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1

3 August 2016

2 hours

Additional Materials: Answer Paper
Graph Paper (1 Sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

/ 80

This document consists of 5 printed pages 1 blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

1. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

1 The area of a triangle is $\left(1+\frac{5\sqrt{5}}{2}\right)$cm^2. If the length of the base of the triangle is $\left(3+2\sqrt{5}\right)$cm, find, without using a calculator, the height of the triangle in the form of $\left(a+b\sqrt{5}\right)$cm, where a and b are integers. [4]

2 Express $\frac{4x^2+6x+5}{2x^2+x-3}$ in partial fractions. [5]

3 The function $f(x)$ is such that $f(x)=2x^3+3x^2-x-4$,

(i) find a factor of $f(x)$. [2]

(ii) Hence, determine the number of solutions in the equation $f(x)=0$. [4]

4 The roots of the quadratic equation $3x^2-x+5=0$ are α and β.

(i) Evaluate $\alpha^2+\beta^2$. [2]

(ii) Find the quadratic equation whose roots are α^3-1 and β^3-1. [4]

5 The table shows experimental values of 2 variables, R and V, which are connected by an equation of the form $RV^n=k$ where n and k are constants.

R	33	19.95	5.07	2.38
V	2	2.9	8	14

(i) Plot lg R against lg V for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of n. [3]

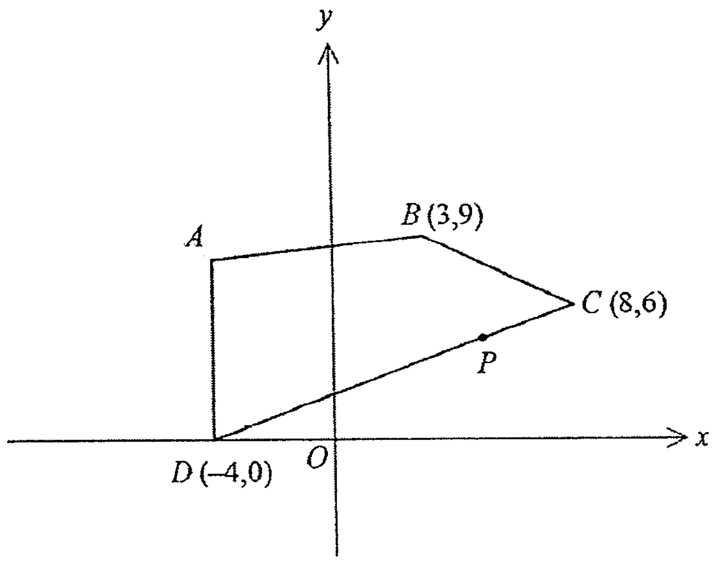
(iii) By drawing a suitable straight line on your graph in (i), find the value of V such that $\frac{R}{V^2}=1$. [3]

6 Given that $y=1-\frac{1}{2}\sin 3x$, $0^\circ \le x \le 240^\circ$.

(i) State the maximum and minimum values of y. [2]

(ii) Sketch the graph of $y=1-\frac{1}{2}\sin 3x$. [3]

7



A quadrilateral $ABCD$ passes through vertices B (3, 9), C (8, 6) and D (– 4, 0), line AD is parallel to the y – axis.

(i) Find the coordinates of A given that the length of AD is 8 units. [1]

(ii) A point P divides the line DC in the ratio of 2 : 1. Find the coordinates of P. [3]

(iii) Hence, find the area of the quadrilateral $ABPD$. [3]

8 (a) Sketch the graph $y^2 = 3x$. [2]

(b) Given that $f(x) = -2x^3 + 5x^2 + 4x + a$,

(i) find the coordinates of the turning points in terms of a. [4]

(ii) Determine the nature of each turning point. [3]

(iii) In the case where $a = 1$, explain why the part of the graph between the turning points lie above the x – axis. [1]

9 (i) Show that $\sec x + \tan x$ can be expressed as $\dfrac{1+\sin x}{\cos x}$. [1]

(ii) Differentiate $\ln(\sec x + \tan x)$ with respect to x. [3]

(iii) Hence, find $\int_{0.25}^{0.5} 2\sec x \, dx$. [3]

10 The points A and B lie on the circumference of a circle C_1 where A is the point (0, 8) and B is the point (4, 0). The line $y = 2x$ also passes through the centre of the circle C_1.

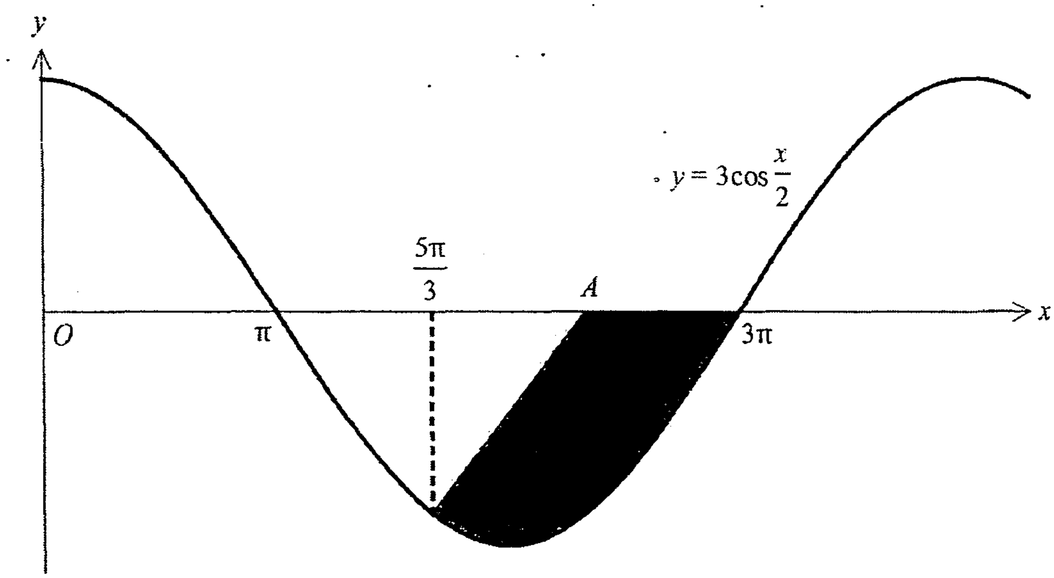
(i) Find the centre and radius of the circle C_1. [4]

(ii) Find the equation of the circle C_1 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

Another circle C_2 of radius $\sqrt{2}$ units has its centre inside C_1 and it cuts the circle C_1 at the origin and at the point where $x = 2$.

(iii) Find the centre of C_2. [5]

11



The diagram shows part of the curve $y = 3\cos\frac{x}{2}$ that cuts the x-axis at $x = \pi$ and $x = 3\pi$. The normal to the curve at $x = \frac{5\pi}{3}$ cuts the x-axis at A.

(i) Find the coordinates of A, leaving your answer in exact form. [6]

(ii) Hence, find the area of the shaded region. [4]

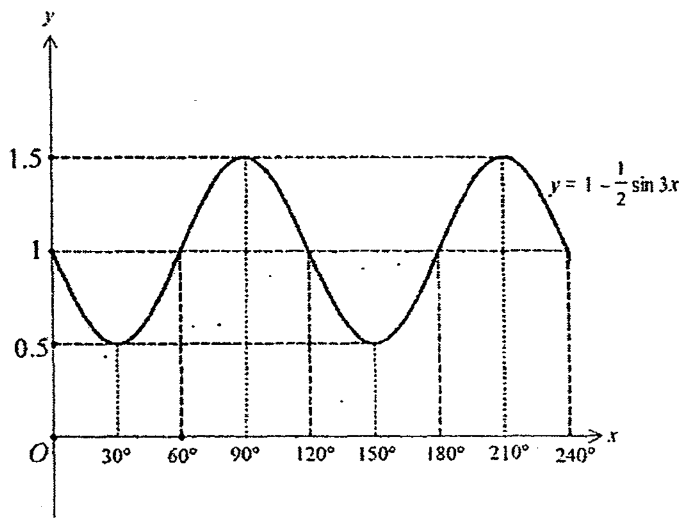
1. $4-\sqrt{5}$

2. $2-\frac{2}{2x+3}+\frac{3}{x-1}$

3. (ii) one solution

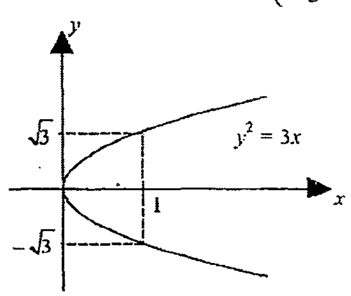
4. (i) $\frac{-29}{9}$ (ii) $27x^2+98x+196=0$

6. (i) Max y = 1.5; Min y = 0.5 (ii)



7. (i) (−4,8) (ii) $P(4,4)$ (iii) 50 units^2

8. (a) (b)(i). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ and $(2, 12+a)$ (b)(ii). $\left(-\frac{1}{3}, a-\frac{19}{27}\right)$ min ; $(2, 12+a)$ max



9. (ii) $\sec x$ (iii). 0.539

10. (i) Centre (2, 4), Radius $=2\sqrt{5}$ (ii) $x^2+y^2-4x-8y=0$ (iii) Centre of $C_2(1.22, 0.710)$

11. (i) $A\left(\frac{5\pi}{3}+\frac{9}{8}\sqrt{3}, 0\right)$ (ii) $6\frac{15}{32}$ / 6.47 units^2

	Workings
1	$1+\frac{5\sqrt{5}}{2}=\frac{1}{2}\left(3+2\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ $2+5\sqrt{5}=\left(3+2\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ $a+b\sqrt{5}=\frac{2+5\sqrt{5}}{3+2\sqrt{5}}$ $=\frac{2+5\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$ $=\frac{6-4\sqrt{5}+15\sqrt{5}-50}{9-4(5)}$ $=\frac{-44+11\sqrt{5}}{-11}$ $=4-\sqrt{5}$ The height of the triangle is ($4-\sqrt{5}$) cm
2	Given $\frac{4x^2+6x+5}{2x^2+x-3}$ As this is an improper fraction, **By long division,** $2x^2+x-3 \overline{\smash{)}4x^2+6x+5}$ quotient 2 $\underline{4x^2+2x-6}$ $4x+11$ $\frac{4x^2+6x+5}{2x^2+x-3}=2+\frac{4x+11}{(2x+3)(x-1)}$ Let $\frac{4x+11}{(2x+3)(x-1)}=\frac{A}{2x+3}+\frac{B}{x-1}$ $=\frac{A(x-1)+B(2x+3)}{(2x+3)(x-1)}$

$4x+11=A(x-1)+B(2x+3)$

Let $x = 1$,

$15=5B$

$B=3$

Let $x = 0$,

$11=-A+9$

$A=-2$

$$\frac{4x^2+6x=5}{(2x+3)(x-1)}=2-\frac{2}{2x+3}+\frac{3}{x-1}$$

3(i) Given $f(x)=2x^3+3x^2-x-4$

By trial and error,

Consider $(x-1)$

$f(1)=2(1)^3+3(1)^2-1-4$

$=0$

$\therefore (x-1)$ is a factor.

(ii) $f(x)=2x^3+3x^2-x-4$

By inspection,

$f(x)=(x-1)(2x^2+ax+4)$

By comparing coefficient of

$x^2: 3=a-2$

$\therefore a=5$

$f(x)=(x-1)(2x^2+5x+4)$

Applying discriminant for $2x^2+5x+4$,

$b^2-4ac=5^2-4(2)(4)$

$=25-32$

$=-7<0$

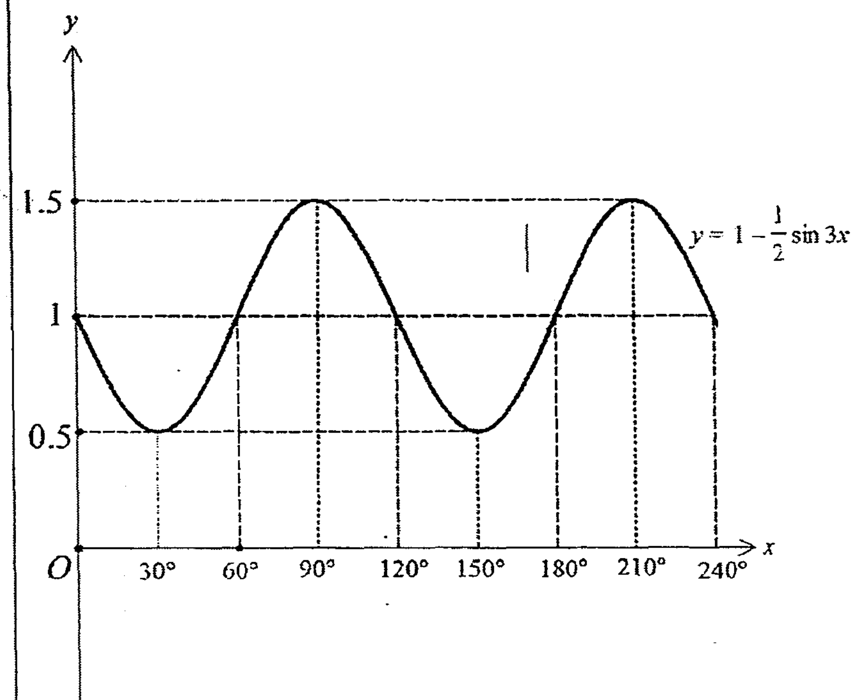
Thus $2x^2+5x+4$ has no real roots.

Therefore, there is only **one solution .**

4(i)	$3x^2-x+5=0$ $\alpha+\beta=\frac{1}{3}$ $\alpha\beta=\frac{5}{3}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{1}{3}\right)^2-2\left(\frac{5}{3}\right)$ $=\frac{1}{9}-\frac{10}{3}$ $=\frac{-29}{9}$
(ii)	New sum of roots = $\alpha^3-1+\beta^3-1$ $=\alpha^3+\beta^3-2$ $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)-2$ $=\left(\frac{1}{3}\right)(\alpha^2+\beta^2-\alpha\beta)-2$ $=\left(\frac{1}{3}\right)\left(\frac{-29}{9}-\frac{5}{3}\right)-2$ $=\frac{-98}{27}$ New product of roots = $(\alpha^3-1)(\beta^3-1)$ $=\alpha^3\beta^3-\beta^3-\alpha^3+1$ $=(\alpha\beta)^3-(\alpha^3+\beta^3)+1$ $=\left(\frac{5}{3}\right)^3-\left(\frac{-44}{27}\right)+1$ $=\frac{196}{27}$ Quadratic eqn: $x^2-\left(\frac{-98}{27}\right)x+\frac{196}{27}=0$ $27x^2+98x+196=0$

6(i) Max y = 1.5; Min y = 0.5

(ii)



7(i) Since line AD is parallel to $y-$axis,

Coordinates of $A = (-4, 0+8)$

$= (-4, 8)$

7(ii) Since P divides the line DC in ratio 2 : 1,

$P_x = \frac{8+4}{3} \times 2 + (-4); P_y = \frac{6}{3} \times 2 + 0$

$= 4 \qquad ; = 4$

$\therefore P(4,4)$

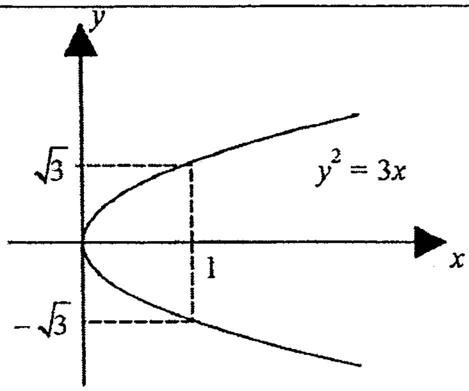
7(iii) Area of quadrilateral $ABPD = \frac{1}{2}\begin{vmatrix} -4 & 4 & 3 & -4 & -4 \\ 0 & 4 & 9 & 8 & 0 \end{vmatrix}$

$= \frac{1}{2}\left[(-16+36+24)-(12-36-32)\right]$

$= \frac{1}{2}[44+56]$

$= 50 unit^2$

8(a)



8(b)(i)

Given $\mathrm{f}(x) = -2x^3 + 5x^2 + 4x + a$

$\mathrm{f}'(x) = -6x^2 + 10x + 4$

For stationary point, $\mathrm{f}'(x) = 0$

$-6x^2 + 10x + 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$\therefore x = -\frac{1}{3}$ or $x = 2$

$$\mathrm{f}(x) = -2\left(-\frac{1}{3}\right)^3 + 5\left(-\frac{1}{3}\right)^2 + 4\left(-\frac{1}{3}\right)^1 + a$$

$$= -2\left(\frac{1}{27}\right) + \frac{5}{9} - \frac{4}{3} + a$$

$$= a - \frac{19}{27}$$

or

$$\mathrm{f}(x) = -2(2)^3 + 5(2)^2 + 4(2)^1 + a$$

$$= -16 + 20 + 8 + a$$

$$= a + 12$$

$\left(-\frac{1}{3}, a - \frac{19}{27}\right)$ and $(2, 12 + a)$ are turning points

8(b)(ii)

$\mathrm{f}''(x) = -12x + 10$

At $x = -\frac{1}{3}$, $\mathrm{f}''(x) = -12\left(-\frac{1}{3}\right) + 10$

$$= 14$$

$$> 0$$

$\therefore \left(-\frac{1}{3}, a - \frac{19}{27}\right)$ is a minimum turning point.

	At $x=2, f''(x)=-12(2)+10$ $=-14$ <0 $\therefore (2,12+a)$ is a maximum turning point.
8(b)(iii)	When $a=1$, min point $=\left(-\frac{1}{3},\frac{8}{27}\right)$ is above x - axis max point $=(2,13)$ is above x - axis Since graph has no other turning points, the part of the graph between the 2 turning points lie above x - axis.
9(i)	$\sec x+\tan x=\frac{1}{\cos x}+\frac{\sin x}{\cos x}$ $=\frac{1+\sin x}{\cos x}$
(ii)	$\frac{d}{dx}\ln(\sec x+\tan x)=\frac{d}{dx}\ln\left(\frac{1+\sin x}{\cos x}\right)$ $=\frac{d}{dx}\left[\ln(1+\sin x)-\ln(\cos x)\right]$ $=\frac{\cos x}{1+\sin x}-\frac{-\sin x}{\cos x}$ $=\frac{\cos x(\cos x)+\sin x(1+\sin x)}{(1+\sin x)\cos x}$ $=\frac{\cos^2 x+\sin^2 x+\sin x}{(1+\sin x)\cos x}$ $=\frac{1+\sin x}{(1+\sin x)\cos x}$ $=\frac{1}{\cos x}$ $=\sec x$
(iii)	$\int_{0.25}^{0.5} 2\sec x\, dx=2\int_{0.25}^{0.5}\sec x\, dx$ $=2\left[\ln\left(\frac{1+\sin x}{\cos x}\right)\right]_{0.25}^{0.5}$ $=2\left[\ln\left(\frac{1+\sin 0.5}{\cos 0.5}\right)-\ln\left(\frac{1+\sin 0.25}{\cos 0.25}\right)\right]$ $=0.539184$ $=0.539$ (3s.f)

10(i)	Midpoint of $AB=\left(\frac{0+4}{2},\frac{8+0}{2}\right)$ $=(2,4)$ Gradient of $AB=\frac{8-0}{0-4}$ $=-2$ Eqn of perpendicular bisector of AB: $y-8=\frac{1}{2}(x-0)$ $y=\frac{1}{2}x+3---(1)$ $y=2x---(2)$ *Equating*, $2x=\frac{1}{2}x+3$ $x=2$ $y=4$ $\therefore$ center of $C_1(2,4)$ Radius $=\sqrt{(2-4)^2+(4-0)^2}$ $=\sqrt{20}$ $=2\sqrt{5}units$
10(ii)	Thus eqn of C_1: $(x-2)^2+(y-4)^2=(2\sqrt{5})^2$ $x^2-4x+4+y^2-8y+16=20$ $x^2+y^2-4x-8y=0$
10(iii)	Since $C_1: x^2+y^2-4x-8y=0$ When $x=2$, $y^2-8y-4=0$ $y=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-4)}}{2(1)}$ $=4\pm2\sqrt{5}$

	Use $y=4-2\sqrt{5}$ (C_2 radius is only $\sqrt{2}unit$ and lies in C_1) Midpoint $=\left(1,2-\sqrt{5}\right)$ Gradient $=\dfrac{4-2\sqrt{5}-0}{2-0}$ $=2-\sqrt{5}$ Eqn of perpendicular bisector: $y-\left(2-\sqrt{5}\right)=\left(\dfrac{-1}{2-\sqrt{5}}\right)(x-1)$ $y=\dfrac{10-4\sqrt{5}-x}{2-\sqrt{5}}---(1)$ Since equation C_2 is of the form $(x-a)^2+(y-b)^2=2$ where center is $(a,\ b)$ Using $(0,0)$, $a^2+b^2=2---(2)$ By substituting (1) in (2), $a^2+\left(\dfrac{10-4\sqrt{5}-a}{2-\sqrt{5}}\right)^2=2$ $a^2+\dfrac{a^2+a(8\sqrt{5}-20)+180-80\sqrt{5}}{9-4\sqrt{5}}=2$ $\left(10-4\sqrt{5}\right)a^2+a(8\sqrt{5}-20)+162-72\sqrt{5}=0$ **Solving** $a=\dfrac{-(8\sqrt{5}-20)\pm\sqrt{(8\sqrt{5}-20)^2-4\left(10-4\sqrt{5}\right)\left(162-72\sqrt{5}\right)}}{2\left(10-4\sqrt{5}\right)}$ $=1.223$ or 0.7767 (rejected as it outside of C_1) Hence $b=0.7101$ Thus center of $C_2(1.22,0.710)$

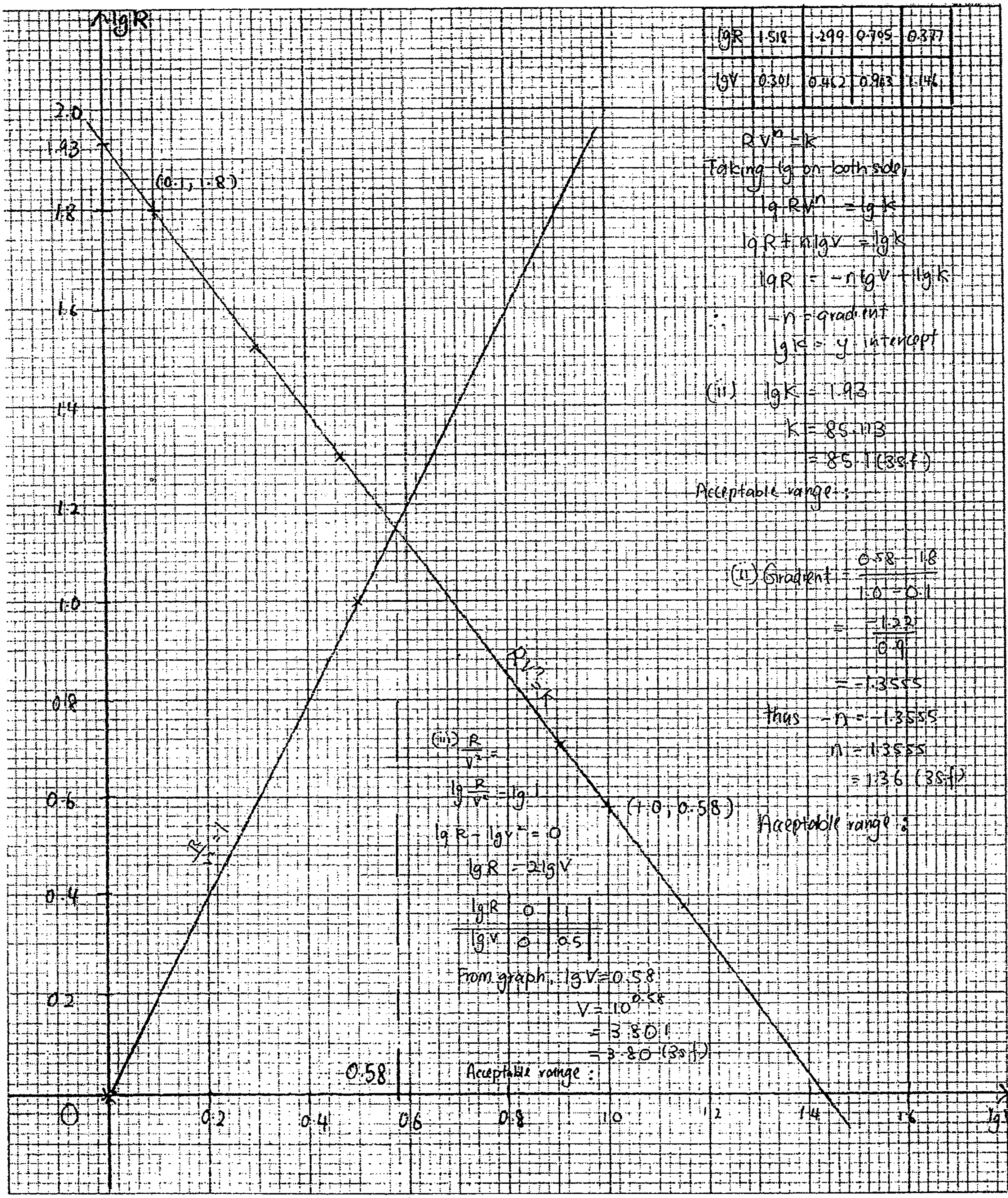
11(i)	Given $y = 3\cos\frac{x}{2}$ $\frac{dy}{dx} = -3\left(\frac{1}{2}\right)\sin\frac{x}{2}$ $= -\frac{3}{2}\sin\frac{x}{2}$ At $x = \frac{5\pi}{3}$, $\frac{dy}{dx} = -\frac{3}{2}\sin\frac{5\pi}{6}$ $= -\frac{3}{4}$ Gradient of normal $= \frac{4}{3}$ At $x = \frac{5\pi}{3}, y = -\frac{3\sqrt{3}}{2}$ Eqn of normal: $y + \frac{3\sqrt{3}}{2} = \frac{4}{3}\left(x - \frac{5\pi}{3}\right)$ $y = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}$ Since the normal cuts x - axis, $y = 0$ $0 = \frac{4}{3}x - \frac{20\pi}{9} - \frac{3\sqrt{3}}{2}$ $x = \frac{5\pi}{3} + \frac{9}{8}\sqrt{3}$ $\therefore A\left(\frac{5\pi}{3} + \frac{9}{8}\sqrt{3}, 0\right)$
11(ii)	Shaded area $= \left\lvert\int_{\frac{5\pi}{3}}^{3\pi} 3\cos\frac{x}{2}\ dx\right\rvert - \frac{1}{2}\times\frac{3\sqrt{3}}{2}\times\frac{9\sqrt{3}}{8}$ $= \left\lvert\left[6\sin\frac{x}{2}\right]_{\frac{5\pi}{3}}^{3\pi}\right\rvert - \frac{81}{32}$ $= \left\lvert 6\sin\frac{3\pi}{2} - 6\sin\frac{5\pi}{6}\right\rvert - \frac{81}{32}$ $= \lvert -6-3\rvert - \frac{81}{32}$ $= 6\frac{15}{32}\, unit^2 / 6.47 unit^2 (3sf)$

Candidate Name ....

Subject .... Paper 01

Centre Number | Index Number

Question No. 5



lgR	1.518	1.299	0.705	0.377
lgV	0.301	0.462	0.963	1.146

$RV^n = k$

Taking lg on both side,

$\lg RV^n = \lg k$

$\lg R + n\lg V = \lg k$

$\lg R = -n\lg V + \lg k$

$\therefore -n = \text{gradient}$

$\lg k = y\text{ intercept}$

(ii) $\lg k = 1.93$

$k = 85.113$

$= 85.1\ (3sf)$

Acceptable range :

(ii) Gradient $= \dfrac{0.58 - 1.8}{1.0 - 0.1}$

$= \dfrac{-1.22}{0.9}$

$= -1.3555$

thus $-n = -1.3555$

$n = 1.3555$

$= 1.36\ (3sf)$

Acceptable range :

(iii) $\dfrac{R}{V^2} =$

$\lg \dfrac{R}{V^2} = \lg 1$

$\lg R - \lg V^2 = 0$

$\lg R = 2\lg V$

lgR	0	1
lgV	0	0.5

From graph, $\lg V = 0.58$

$V = 10^{0.58}$

$= 3.801$

$= 3.80\ (3sf)$

Acceptable range :

Name:	Class	Class Register Number/ Centre No./Index No.

PRELIMINARY EXAMINATION 2016
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **5 August 2016**

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/ 100

This document consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (a) The equation of a curve is $y=2x^2+ax+(6+a)$, where a is a constant. Find the range of values of a for which the curve lies completely above the x-axis. [3]

(b) The equation of a curve is $y=3x^2+4x+6$.

(i) Find the set of values of x for which the curve is above the line $y=6$. [3]

(ii) Show that the line $y=-8x-6$ is a tangent to the curve. [2]

2 (a) Given that $\log_a 125-3\log_a b+\log_a c=3$, express a in terms of b and c. [3]

(b) Solve the equation

(i) $\lg 8x-\lg(x^2-3)=2\lg 2$, [3]

(ii) $2\log_5 x=3+7\log_x 5$. [4]

3 The equation of a curve is $y=x^2\sqrt{(5x-1)^3}$, for $x>0.2$. Given that x is changing at a constant rate of 0.25 units per second, find the rate of change of y when $x=2$. [4]

4 The graph of $y=\left|2x^2-ax-5\right|$ passes through the points with coordinates $(-1,\ 0)$ and $(0.75,\ b)$.

(i) Find the value of the constants a and b. [3]

(ii) Sketch the graph of $y=\left|2x^2-ax-5\right|$. [3]

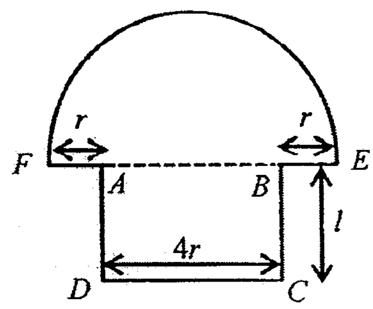
(iii) Determine the set of positive values of m for which the line $y=mx+2$ intersects the graph of $y=\left|2x^2-ax-5\right|$ at two points. [2]

5 In the binomial expansion of $\left(2x+\dfrac{k}{x}\right)^8$, where k is a positive constant, the coefficient of x^2 is 28.

(i) Show that $k=\dfrac{1}{4}$. [4]

(ii) Hence, determine the term in x in the expansion of $\left(6x-\dfrac{1}{x}\right)\left(2x+\dfrac{k}{x}\right)^8$. [4]

6



The diagram shows a design of a bookmark that includes a rectangle $ABCD$, where $BC = l$ cm, $CD = 4r$ cm, a semicircle with radius $3r$ cm, and $AF = BE = r$ cm. The area of the bookmark is 90 cm^2.

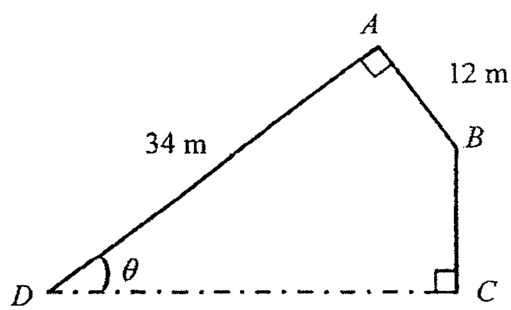
(i) Express l in terms of r. [2]

(ii) Given that the perimeter of the bookmark is P cm, show that

$$P = \left(6 + \frac{3\pi}{4}\right)r + \frac{45}{r}.$$ [2]

(iii) Given that r and l can vary, find the value of r for which P has a stationary value. Explain why this value of r gives the minimum perimeter. [5]

7



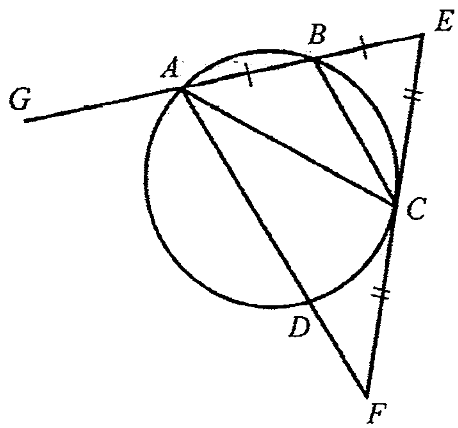
The diagram shows an animal exhibition area that is surrounded by glass panels at AB, BC and AD, where $AB = 12$ m, $AD = 34$ m, angle DAB = angle $BCD = 90°$ and the acute angle $ADC = \theta$ can vary.

(i) Show that L m, the length of the glass panels can be expressed as $L = 46 + 34\sin\theta - 12\cos\theta$. [2]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$, where p and $R > 0$ are constants and α is an acute angle. [4]

(iii) Given that the exact length of the glass panels is 62 m, find the value of θ. [3]

8



The diagram shows points A, B, C and D on a circle, line EF is tangent to the circle at C, lines ADF and $EBAG$ are straight lines, and points B and C are the midpoints of AE and EF.

Prove that

(i) $BC \times EC = AC \times BE$, [3]

(ii) $AF \times EC = AC \times AE$, [2]

(iii) angle GAD = angle ACF. [2]

9 **(a)** **(i)** Show that $\cot 2x = \dfrac{1 - \tan^2 x}{2 \tan x}$. [2]

(ii) Hence, solve the equation $8 \cot 2x \tan x = 1$, for $0° < x < 360°$. [4]

(b) The Ultraviolet Index (UVI) describes the level of solar radiation. The UVI measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours from the lowest value of the UVI, $0 \le t \le 10$, and q is a constant. It takes 10 hours for the UVI to reach its lowest value again.

(i) Explain why we are not able to measure a UVI of 12. [1]

(ii) Show that $q = \dfrac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the UVI is at least 3. Find the duration, in hours and minutes, that the building is supplied with power from the solar panels. [4]

10 **(a)** It is given that $y=\frac{2x^2}{4x-3}$, where $x>\frac{3}{4}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find the range of values of x for which $y=\frac{2x^2}{4x-3}$ is a decreasing function. [4]

(b) It is given that f(x) is such that $f'(x)=\frac{1}{2x-5}-\frac{4}{(2x-5)^2}$.

Given also that $f(3)=1.75$, show that $8f(x)-(2x-5)^2 f''(x)=\ln(2x-5)^4$. [7]

11 A particle moves in a straight line, so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v=2e^{0.1t}-10e^{0.1-0.3t}$. The particle comes to an instantaneous rest at the point A.

(i) Show that the particle reaches A when $t=\frac{5}{2}\ln 5+\frac{1}{4}$. [3]

(ii) Find the acceleration of the particle at A. [3]

(iii) Find the distance OA. [4]

(iv) Explain whether the particle is again at O at some instant during the eleventh second after first passing through O. [2]

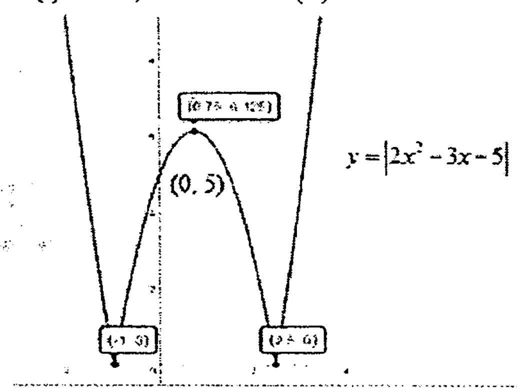
Answer Key

1. (a) $-4 < a < 12$ (b)(i) $x < -1\frac{1}{3}$ or $x > 0$

2. (a) $a = \frac{5\sqrt[3]{c}}{b}$ (b)(i) $x = 3$ (ii) $x = 85.7$ or $x = 0.130$

3. 49.5 units / s

4. (i) $a = 3$, $b = 6.125$ (ii) (iii) $m > 2$



5. (ii) $-1\frac{3}{4}x$

6. (i) $l = \frac{45}{2r} - \frac{9}{8}\pi r$ (iii) $r = 2.32$; min value

7. (ii) $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (iii) $45.8°$

9. (a)(ii) $x = 40.9°, 139.1°, 220.9°, 319.1°$ (b)(iii) 7 hrs and 3 mins

10. (a)(i) $\frac{4x(2x-3)}{(4x-3)^2}$ (ii) $\frac{3}{4} < x < \frac{3}{2}$

11. (ii) 1.23 m/s^2 (iii) 16.0 m (iv) passed through O

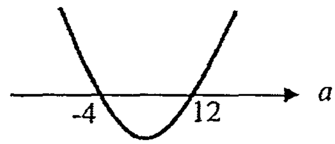
		Working
1	(a)	For $y=2x^2+ax+(6+a)$ to lie above the x-axis, discriminant $b^2-4ac<0$

$(a)^2-4(2)(6+a)<0$

$a^2-8a-48<0$

$(a-12)(a+4)<0$

$-4<a<12$



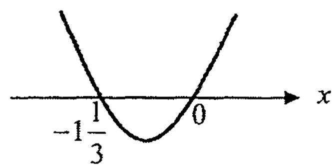
(b) (i)

$3x^2+4x+6>6$

$3x^2+4x>0$

$x(3x+4)>0$

$x<-1\frac{1}{3}$ or $x>0$



(ii)

$3x^2+4x+6=-8x-6$

$3x^2+12x+12=0$

$x^2+4x+4=0$

Discriminant $=(4)^2-4(1)(4)=0$

Since discriminant = 0, the line and curve intersects only at one point.
Line $y=-8x-6$ is tangent to the curve. (shown)

2 (a)

$\log_a 125-3\log_a b+\log_a c=3$

$\log_a 125-\log_a b^3+\log_a c=3$

$\log_a \frac{125c}{b^3}=3$

$a^3=\frac{125c}{b^3}$

$a=\frac{5\sqrt[3]{c}}{b}$

(b) (i)

$\lg 8x-\lg(x^2-3)=2\lg 2$

$\lg\left(\frac{8x}{x^2-3}\right)=\lg 2^2$

$\frac{8x}{x^2-3}=4$

	Working
	$4x^2-8x-12=0$
	$x^2-2x-3=0$
	$(x-3)(x+1)=0$
	$x=3$ or -1 (reject $x=-1$ as $\lg 8x$ is undefined)
	$x=3$
(b) (ii)	$2\log_5 x=3+7\log_x 5$
	$2\log_5 x=3+7\left(\frac{\log_5 5}{\log_5 x}\right)$
	$2(\log_5 x)^2-7-3\log_5 x=0$
	Let $u=\log_5 x$
	$2u^2-3u-7=0$
	$u=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-7)}}{2(2)}$
	$\log_5 x=\frac{3\pm\sqrt{65}}{4}$
	$x=5^{\frac{1}{4}(3+\sqrt{65})}$ or $x=5^{\frac{1}{4}(3-\sqrt{65})}$
	$x=85.7$ or $x=0.130$ (3 sig. fig.)
3	$y=x^2\sqrt{(5x-1)^3}$
	$\frac{dy}{dx}=x^2\left(\frac{3}{2}(5x-1)^{\frac{1}{2}}(5)\right)+2x\sqrt{(5x-1)^3}$
	$=(5x-1)^{\frac{1}{2}}\left(\frac{15x^2}{2}+2x(5x-1)\right)$
	$=(5x-1)^{\frac{1}{2}}\left(\frac{35x^2}{2}-2x\right)$
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$
	$=(5(2)-1)^{\frac{1}{2}}\left(\frac{35(2)^2}{2}-2(2)\right)\times 0.25$
	$=49.5$ units/s

		Working

4 (i)

$y=\left|2x^2-ax-5\right|$

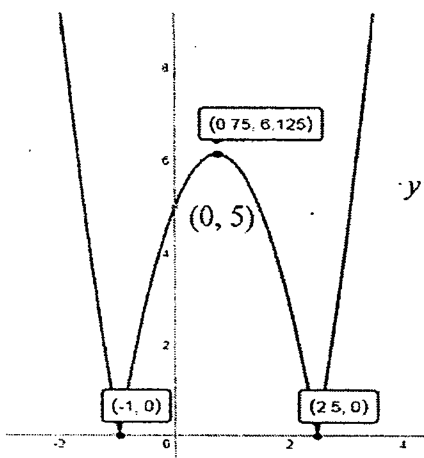
At $(-1,\ 0)$, $y=\left|2(-1)^2-a(-1)-5\right|$

$\left|a-3\right|=0$

$a=3$

At $(0.75,\ b)$, $b=\left|2(0.75)^2-3(0.75)-5\right|=6.125$

(ii)



$y=\left|2x^2-3x-5\right|$

(iii)

Line $y=mx+2$ passes through (0, 2) and cuts two points to the right of (0, 2).

The line that passes through $(-1,0)$ and (0, 2) has 3 points of intersection. Gradient

$=\dfrac{2-0}{0-(-1)}=2$

Lines with $m>2$ intersect the graph at 2 points.

5 (i)

General Term $=\binom{8}{r}(2x)^{8-r}\left(\dfrac{k}{x}\right)^r$

$=\binom{8}{r}(2)^{8-r}(k)^r x^{8-2r}$

For term in x^2:

$8-2r=2$

$r=3$

	Working
	$\text{Coefficient} = \binom{8}{3}(2)^{8-3}(k)^3$
	$= 1792k^3$
	$1792k^3 = 28$
	$k^3 = \frac{1}{64}$
	$k = \frac{1}{4}$
(ii)	$\left(6x - \frac{1}{x}\right)\left(2x + \frac{k}{x}\right)^8$
	$= \left(6x - \frac{1}{x}\right)\left(\cdots + 28x^2 + \cdots + \binom{8}{4}(2x)^4\left(\frac{1}{4x}\right)^4 + \cdots\right)$
	Term in x
	$= 6 \times 70(16)\left(\frac{1}{4^4}\right)x - 28x$
	$= -1\frac{3}{4}x$
6 **(i)**	$\frac{\pi}{2}(3r)^2 + 4rl = 90$
	$l = \frac{90 - \frac{9\pi r^2}{2}}{4r}$
	$l = \frac{45}{2r} - \frac{9}{8}\pi r$
(ii)	$P = 4r + 2l + 2r + \frac{\pi}{2}(6r)$
	$= 4r + 2\left(\frac{45}{2r} - \frac{9}{8}\pi r\right) + 2r + 3\pi r$
	$= 6r + \frac{3}{4}\pi r + \frac{45}{r}$
	$= \left(6 + \frac{3}{4}\pi\right)r + \frac{45}{r}$ (shown)

Working

(iii)

$P = \left(6 + \frac{3}{4}\pi\right)r + \frac{45}{r}$

$\frac{dP}{dr} = 6 + \frac{3}{4}\pi - \frac{45}{r^2}$

For stationary points, $\frac{dP}{dr} = 0$

$6 + \frac{3}{4}\pi = \frac{45}{r^2}$

$r^2 = \frac{45 \times 4}{24 + 3\pi}$

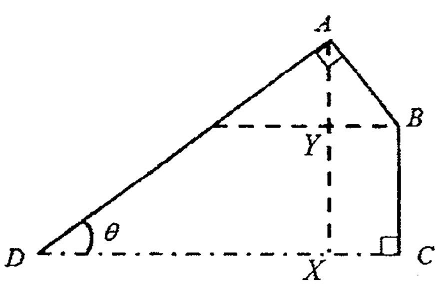
$r = \sqrt{\frac{45 \times 4}{24 + 3\pi}}$ since $r > 0$.

$r = \sqrt{\frac{60}{8 + \pi}}$ or 2.32 (3 sig. fig.)

$\frac{d^2P}{dr^2} = \frac{90}{r^3} = \frac{90}{(2.3206)^3} > 0$

Since $\frac{d^2P}{dr^2} > 0$, this gives a minimum value of P.

7 **(i)**



$\angle DAX = 90° - \theta$

$\angle XAB = \theta$

$AX = 34\sin\theta$

$BC = 34\sin\theta - 12\cos\theta$

$L = AD + AB + BC$

$= 46 + 34\sin\theta - 12\cos\theta$

(ii)

$34\sin\theta - 12\cos\theta = R\sin(\theta - \alpha)$

$= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

Comparing coefficients, $R\sin\alpha = 12$ and $R\cos\alpha = 34$

$R = \sqrt{12^2 + 34^2} = \sqrt{1300} = 10\sqrt{13}$

		Working
		$\tan\alpha = \dfrac{12}{34} \qquad \alpha = 19.440°$ $L = 46 + 10\sqrt{13}\sin(\theta - 19.4°)$ (to 1 d.p.)
	(iii)	$46 + 10\sqrt{13}\sin(\theta - 19.440°) = 62$ $10\sqrt{13}\sin(\theta - 19.440°) = 16$ $\sin(\theta - 19.440°) = \dfrac{16}{10\sqrt{13}}$ $\theta - 19.440° = 26.344°$ $\theta = 26.344° + 19.440°$ $= 45.8°$
8	(i)	$\angle BCE = \angle BAC$ (alternate segment theorem) $\angle BEC = \angle AEC$ (common angle) Triangle BEC is similar to triangle CEA (AA similarity) $\dfrac{BC}{BE} = \dfrac{AC}{CE}$ $BC \times EC = AC \times BE$ (shown)
	(ii)	Since B and C are the midpoints of AE and EF, $BC = \dfrac{1}{2}AF$ $BC // AF$ (midpoint theorem) $\dfrac{1}{2}AF \times EC = AC \times BE$ from (i) $AF \times EC = AC \times 2BE$ $AF \times EC = AC \times AE$ (shown)
	(iii)	$\angle GAD = \angle ABC$ (corr angles, $BC // AF$) $\angle ACF = \angle ABC$ (alternate segment theorem) $\angle ACF = \angle GAD$ (shown)

		Working
9	(a) (i)	LHS: $\cot 2x = \frac{1}{\tan 2x}$ $= \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$ $= \frac{1-\tan^2 x}{2\tan x}$ (RHS) (shown)
	(a) (ii)	From (i), $8\cot 2x \tan x = 4(2\cot 2x \tan x)$ $= 4(1-\tan^2 x)$ $4(1-\tan^2 x) = 1$ $4 - 4\tan^2 x = 1$ $\tan^2 x = \frac{3}{4}$ $\tan x = \pm\frac{\sqrt{3}}{2}$ Basic angle $\alpha = 40.8933°$ $x = 40.8933°, 180° + 40.8933°$ or $x = 180° - 40.8933°, 360° - 40.8933°$ $x = 40.9°, 139.1°, 220.9°, 319.1°$ (1 d.p.)
9	(b) (i)	$U = 6 - 5\cos qt$ Highest value of $-5\cos qt = 5$ when $\cos qt = -1$, highest value is 11, we are not able to measure UVI of 12.
	(b) (ii)	UVI takes 10 hours to reach its lowest again, $10q = 2\pi$ $q = \frac{\pi}{5}$
	(b) (iii)	$3 = 6 - 5\cos\frac{\pi t}{5}$ $5\cos\frac{\pi t}{5} = 3$

		Working

$\cos\frac{\pi t}{5}=\frac{3}{5}$

Basic angle, $\alpha = 0.927295$

$\frac{\pi t}{5} = 0.927295$ or 5.35589

$t = 1.47583$ or 8.52416

Duration of solar power supply

$= 8.52416 - 1.47583$

$= 7.04833$ hrs

$= 7$ hrs and 3 mins

10 (a) (i)

$y = \frac{2x^2}{4x-3}$

$\frac{dy}{dx} = \frac{(4x-3)(4x) - 2x^2(4)}{(4x-3)^2}$

$= \frac{8x^2 - 12x}{(4x-3)^2}$

$= \frac{4x(2x-3)}{(4x-3)^2}$

(a) (ii)

For decreasing function,

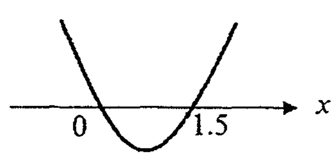
$\frac{dy}{dx} = \frac{8x^2-12x}{(4x-3)^2} < 0$

$\frac{4x(2x-3)}{(4x-3)^2} < 0$

Denominator $(4x-3)^2 > 0$ for $x > \frac{3}{4}$,

$x(2x-3) < 0$

$0 < x < \frac{3}{2}$



Since $x > \frac{3}{4}$, y is decreasing function for $\frac{3}{4} < x < \frac{3}{2}$.

	Working
	When $t=\frac{5}{2}\ln 5+\frac{1}{4}$, $a=0.2e^{0.1(\frac{5}{2}\ln 5+\frac{1}{4})}+3e^{0.1-0.3(\frac{5}{2}\ln 5+\frac{1}{4})}$ $=1.2265$ $=1.23\ \text{m/s}^2$
(iii)	$s=\int v\ dt$ $=\int 2e^{0.1t}-10e^{0.1-0.3t}\ dt$ $=20e^{0.1t}+\frac{100}{3}e^{0.1-0.3t}+c$, where c is a constant Since $s=0$ when $t=0$, $s=20+\frac{100}{3}e^{0.1}+c$ $c=-\left(20+\frac{100}{3}e^{0.1}\right)$ $OA=20e^{0.1(\frac{5}{2}\ln 5+\frac{1}{4})}+\frac{100}{3}e^{0.1-0.3(\frac{5}{2}\ln 5+\frac{1}{4})}-\left(20+\frac{100}{3}e^{0.1}\right)$ $=-15.9535$ $=-16.0$ OA is 16.0 m (3 sig. fig.)
(iv)	When $t=10$, $s_{10}=20e^{1}+\frac{100}{3}e^{(0.1-3)}-\left(20+\frac{100}{3}e^{0.1}\right)$ $=-0.63928$ m When $t=11$, $s_{11}=20e^{1.1}+\frac{100}{3}e^{(0.1-3.3)}-\left(20+\frac{100}{3}e^{0.1}\right)$ $=4.6030$ m Since the displacement of the particle changes from negative to positive, the particle passed through O during the eleventh second.

		Working
10	(b)	$\mathrm{f}(x)=\int \frac{1}{2x-5}-\frac{4}{(2x-5)^2}\,\mathrm{d}x$ $=\frac{1}{2}\ln(2x-5)+\frac{2}{2x-5}+c$, where c is a constant. Given $\mathrm{f}(3)=1.75$, $\frac{1}{2}\ln(2(3)-5)+\frac{2}{2(3)-5}+c=1.75$ $c=-0.25$ $\mathrm{f}''(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2x-5}-\frac{4}{(2x-5)^2}\right)$ $=\frac{-2}{(2x-5)^2}+\frac{16}{(2x-5)^3}$ $8\mathrm{f}(x)-(2x-5)^2\,\mathrm{f}''(x)$ $=8\left[\frac{1}{2}\ln(2x-5)+\frac{2}{2x-5}-0.25\right]-(2x-5)^2\left(\frac{-2}{(2x-5)^2}+\frac{16}{(2x-5)^3}\right)$ $=4\ln(2x-5)$ $=\ln(2x-5)^4$ (shown)
11	(i)	For instantaneous rest, $v=0$ $2e^{0.1t}-10e^{0.1-0.3t}=0$ $2e^{0.1t}=10e^{0.1-0.3t}$ $\frac{e^{0.1t}}{e^{-0.3t}}=5e^{0.1}$ $e^{0.4t}=5e^{0.1}$ Taking ln on both sides: $0.4t=\ln 5+0.1$ $t=\frac{5}{2}\ln 5+\frac{1}{4}$ (shown)
	(ii)	$a=\frac{\mathrm{d}v}{\mathrm{d}t}$ $=0.2e^{0.1t}-10(-0.3)e^{0.1-0.3t}$ $=0.2e^{0.1t}+3e^{0.1-0.3t}$