1 Find the set of values of a for which $3ax^2 + 1 > ax$ for all real values of x. [3]

2 The function f is defined by $f(x) = \tan x \sec x$, where $0° \le x \le 360°$.
Find the values of x for which f is an increasing function. [4]

3 Solve the equation $\log_3(x+4) - \log_3(2x-1) + 2\log_9(x-2) = 1$. [4]

4 The curve $y^2 + 17 = 2x^2$ intersects the straight line $y + 4 = x$ at the points A and B.
Find the equation of the perpendicular bisector of AB. [6]

5 (i) Show that $\sin 2x(\tan^2 x + 1) = 2\tan x$. [3]

(ii) Hence solve the equation $\sin 4\theta(\tan^2 2\theta + 1) = 2\cot\theta$ for $0° < \theta < 360°$. [4]

6 The function f is defined, for $0 \le x \le \pi$, by

$$f(x) = 3\cos 3x - a,$$

where a is a constant.

Given that the minimum value of $f(x)$ is -4, find

(i) the value of a, [1]

(ii) the maximum value of $f(x)$, [1]

(iii) the period and the amplitude of $f(x)$. [2]

Using the value of a found in part (i),

(iv) find the exact value(s) of x for which $f(x) = \frac{1}{2}$. [3]

7 (i) Sketch the graph of $y = |x^2 - 4x| + 1$. [3]

(ii) It is given that the line $y = mx$, where $m > 0$, does not intersect the graph of $y = |x^2 - 4x| + 1$. Determine the set of possible values of m. [2]

(iii) Find the coordinates of the point(s) of intersection of the line $y = 6$ and the graph of $y = |x^2 - 4x| + 1$. [3]

8 In January 2016, Adam bought an antique vase for \$1500. It was believed that the value of the antique vase will increase continuously with time such that it doubles after every 5 years.

(i) Formulate an expression for \$$V$, the value of the vase after Adam owned it for x years. [2]

(ii) Sketch the graph of V against x. [2]

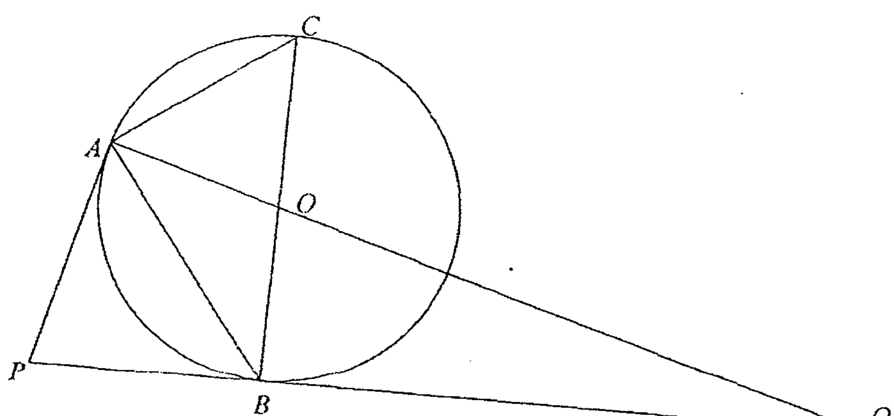
(iii) Using your answer in part (i), find the number of years that Adam has to wait before the value of the vase appreciates to one million dollars. [3]

9 The diagram shows a triangle ABC whose vertices lie on the circumference of a circle with centre O. AP and PB are tangents to the circle at A and B respectively. The tangent to the circle at B meets AO extended at Q.

(i) Show that angle $AOB = 2\times$ angle PAB. [2]

(ii) Hence determine whether it is possible to draw a circle that passes through O, A, P and B? Justify your answer with clear explanations. [3]

(iii) If triangle PAB is equilateral, prove that $OQ = 2OB$. [2]



10 The equation of a curve is $y = -\sqrt{1+3x}$.

(i) A particle P moves along the curve in such a way that the x-coordinate of P decreases at a constant rate of 0.2 units per second. Find the coordinates of P at the instant when the y-coordinate is increasing at a rate of 0.05 units per second. [4]

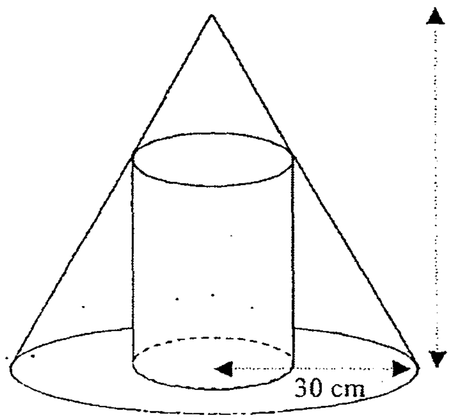
(ii) Find the area enclosed by the curve and the line $y = -3x - 1$. [5]

11 A solid cylinder is cut from a solid cone of height 60 cm and radius 30 cm as shown in the diagram. The cylinder has height h cm, radius r cm and volume V cm^3.

(i) Show that $h = 60 - 2r$. [2]

(ii) Express V in terms of r. [1]

(iii) Determine the value of r for which the volume of the cylinder is maximum. Hence find the maximum volume of the cylinder. [6]



12 A particle travels in a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 12t - 2t^2$. The particle comes to an instantaneous rest at A. Find

(i) the acceleration of the particle at A, [3]

(ii) the greatest velocity of the particle, [2]

(iii) the distance travelled by the particle between $t = 0$ and $t = 5$. [4]

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 3\mathrm{e}^{-x} + \frac{1}{x+1}$, $x > 0$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no stationary point. [2]

(ii) Given that the curve passes through the point $(0, 1)$, find an expression for $\mathrm{f}(x)$. [4]

2 (i) Differentiate $\ln(\sin x)$ with respect to x. [2]

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cot x) = \cot x - x\operatorname{cosec}^2 x$. [2]

(iii) Using the results from parts (i) and (ii), show that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cot^2 x \, \mathrm{d}x = \frac{\pi}{4} - \frac{3\pi^2}{32} - \ln\frac{\sqrt{2}}{2}.$$ [4]

3 The equation of a curve is $y = -x^3 - 2x^2 - x - 1$. The point A lies on the curve and has x-coordinate of -2. The normal to the curve at A meets the x-axis at P and the y-axis at Q.

(i) Find the area of triangle POQ, where O is the origin. [6]

The point B also lies on the curve. The tangent to the curve at B is perpendicular to the normal to the curve at A.

(ii) Find the x-coordinate of B. [3]

4 (a) (i) Write down, and simplify, the first four terms in the expansion of $(1-x)^8$ in ascending powers of x. [2]

(ii) Replacing x by $2z - z^2$, determine the coefficient of z^3 in the expansion of $(1 - 2z + z^2)^8$. [3]

(b) (i) Write down the general term in the binomial expansion of $\left(2x - \frac{1}{3x^3}\right)^6$.

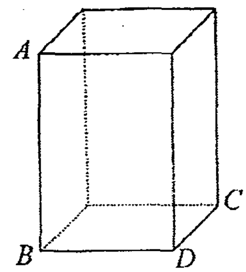
[1]

(ii) Determine whether there is a constant term in the expansion. [1]

(iii) Using the general term, or otherwise, determine the coefficient of x^2 in the binomial expansion of $\left(3x^4 + 2 - \frac{3}{x}\right)\left(2x - \frac{1}{3x^3}\right)^6$. [2]

5 **Do not use a calculator in this question.**

The diagram shows a cuboid with a square base. The area of the square base is $\left(7+4\sqrt{3}\right)\text{cm}^2$ and the volume of the cuboid is $\left(26+15\sqrt{3}\right)\text{cm}^3$.



Find

(i) the height of the cuboid in the form $a+b\sqrt{3}$, where a and b are integers, [2]

(ii) an expression for BC^2 in the form $c+d\sqrt{3}$, where c and d are integers, [2]

(iii) the value of m and of n if the length of AC is $\left(\sqrt{m}+\sqrt{n}\right)\text{cm}$, where m and n are integers. [6]

6 The equation of a curve is $y=\dfrac{\sin x}{2-\cos x}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and obtain the coordinates of the stationary point(s) of the curve for $0 \le x \le \pi$. [5]

(ii) Find an expression for $\dfrac{d^2y}{dx^2}$ and hence determine the nature of the stationary point(s) for $0 \le x \le \pi$. [4]

7 The lines $x=2$ and $y=3$ are tangents to a circle C_1.

Given that the centre of circle C_1 lies on the positive x-axis, find

(i) the equation of C_1. [4]

Circle C_2 is a reflection of circle C_1 along the line $y=x+1$, find

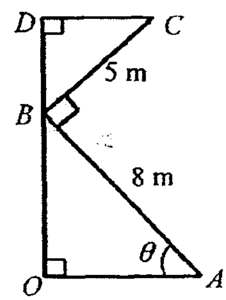
(ii) the equation of C_2. [3]

8 **(a)** **(i)** Find the remainder when $f(x)=3x^3+x^2+x-4$ is divided by $x+1$. [2]

(ii) Hence find the value of k for which $g(x)=f(x)+k$ is divisible by $x+1$ and factorise $g(x)$ completely. [3]

(b) Express $\dfrac{4x+1}{(2x+1)(x-1)^2}$ as the sum of 3 partial fractions. [5]

9 In the diagram, $AB = 8\text{ m}$, $BC = 5\text{ m}$, $\angle AOB = \angle ABC = \angle BDC = 90°$ and $\angle OAB = \theta$ where $0° < \theta < 90°$.



(i) Find OD in terms of θ. [3]

(ii) Express OD in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of θ for which OD has a maximum length. [3]

10 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [2]

(ii) Find the value of $\alpha - \beta$ given that $\alpha < \beta$. [2]

(iii) Show that $\alpha^2 - \beta^2 = -3\sqrt{7}$. [1]

(iv) Find a quadratic equation whose roots are $\alpha^2 - \beta$ and $\beta^2 - \alpha$, in the form $ax^2 + bx + c = 0$ where a, b and c are integers. [6]

11 The table below shows experimental values of two variables x and y. It is known that x and y are related by the equation $y = \dfrac{a}{x-b}$ where a and b are constants.

x	-1.0	-0.5	0.5	1.0	1.5
y	0.33	0.40	0.67	1.00	2.00

(i) Express the equation in the form suitable for drawing a straight line graph, with xy as the variable for the horizontal axis.

State clearly the variable(s) used for the vertical axis. [2]

(ii) Using variable xy for the horizontal axis and suitable variable(s) for the vertical axis, draw, on graph paper, a straight line graph and hence estimate the value of a and of b. [6]

(iii) Show that by adding another straight line on your diagram, an estimate of the solutions for the simultaneous equations $y = \dfrac{a}{x-b}$ and $y = \dfrac{2}{x}$ can be obtained.

Write down the solutions for the simultaneous equations. [3]

2016 Preliminary Examination
Secondary Four Express
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

Answer Key

1 $0 < a < 12$

2 $0 \leq x < 90°$ or $270° < x \leq 360°$

3 $x = 5$

4 $y = -x - 12$

5 **(ii)** $\theta = 35.3°, 144.7°, 215.3°, 324.7°$

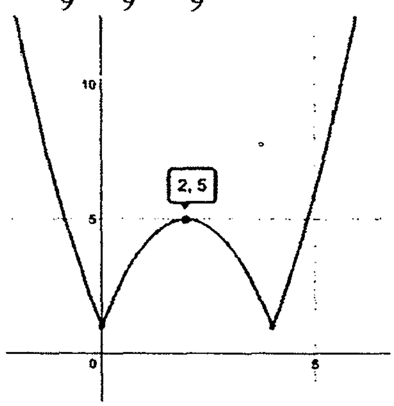
6 **(i)** $a = 1$

(ii) 2

(iii) period = $\frac{2\pi}{3}$, amplitude = 3

(iv) $x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

7 **(i)**

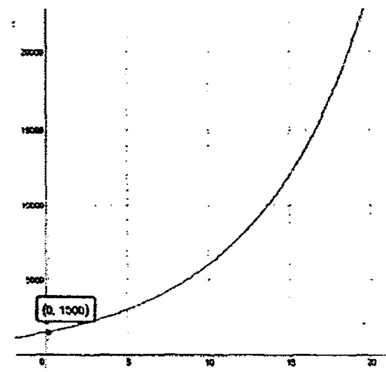


(ii) $0 < m < \frac{1}{4}$

(iii) (5, 6) and (–1, 6)

8 **(i)** $V = 1500 \times 2^{\frac{x}{5}}$

(ii)



(iii) 46.9 years

10 **(i)** $x = 11\frac{2}{3}$

(ii) $\frac{1}{18}$ units^2

11 **(ii)** $V = 60\pi r^2 - 2\pi r^3$

(iii) $r = 20$, 25100 cm^3

2016 Preliminary Examination
Secondary Four Express
ADDITIONAL MATHEMATICS PAPER 2 (4047/02)

Answer Key

1	**(ii)**	$f(x)=-3e^{-x}+\ln(x+1)+4$
2	**(i)**	$\cot x$
3	**(i)**	$4\frac{9}{10}$ units^2
	(ii)	$\frac{2}{3}$
4	**(a)(i)**	$1-8x+28x^2-56x^3+...$
	(a)(ii)	-560
	(b)(i)	$\binom{6}{r}(2^{6-r})\left(-\frac{1}{3}\right)^r x^{6-4r}$
	(b)(iii)	-48
5	**(i)**	$2+\sqrt{3}$ cm
	(ii)	$14+8\sqrt{3}$
	(iii)	$m=12$ and $n=9$, or $m=9$ and $n=12$
6	**(i)**	$\frac{dy}{dx}=\frac{2\cos x-1}{(2-\cos x)^2}$, $\left(\frac{\pi}{3},\frac{\sqrt{3}}{3}\right)$
	(ii)	$\frac{d^2y}{dx^2}=-\frac{2\sin x(1+\cos x)}{(2-\cos x)^3}$, maximum point
7	**(i)**	$(x-5)^2+y^2=9$
	(ii)	$(x+1)^2+(y-6)^2=9$
8	**(a)(i)**	-7
	(a)(ii)	$k=7$, $g(x)=(x+1)(3x^2-2x+3)$
	(b)	$-\frac{4}{9(2x+1)}+\frac{2}{9(x-1)}+\frac{5}{3(x-1)^2}$
9	**(i)**	$8\sin\theta+5\cos\theta$
	(ii)	$\sqrt{89}\sin(\theta+32.0°)$
	(iii)	$58.0°$
10	**(i)**	8
	(ii)	$-\sqrt{7}$
	(iv)	$4x^2-20x-87=0$
11	**(i)**	$y=\frac{1}{b}(xy)-\frac{a}{b}$
	(ii)	$b=2$, $a=-1$
	(iii)	$xy=2$, $y=1.5$, $x=1.33$