Answer Keys TANJONG KATONG

1	$1\frac{1}{9}, \left(\frac{\sqrt{2}}{2}\right)^4, -\pi, -\frac{22}{7}$		
2	$\frac{16a^{\frac{3}{2}}}{b^3}$		
3	$5(5n^2 - 2n + 1)$ is a multiple of 5 for all integers of n		
4(a)	$(6y + 5x - 5)(6y - 5x + 5)$	4(b)	$(4x - y)(3x + 2y)$
5	$x = \pm\sqrt{\frac{4y^2 + 1}{y^2 - 1}}$		
6(a)	$k = 6$	6(b)	The other solution is $x = \frac{2}{5}$
7	Angle DEB= 90°		
8	Misleading feature: The heights of the bars are not proportional to the number of covid-19 tests per million people. Effect of misleading feature: The heights of the bars suggest that Argentina tests about three-quarter the number of people per million as the *USA. However, Argentina tests 330 people per million while the USA tests 7000 people per million, which is about 21 times. *Accept correct comparison with other countries (Italy approx. 4/5 vs 43 times, Germany 2/3 vs 48 times or Norway ½ vs 68 times)		
9(a)	Elements of set B are factors of 9.	9(b)	$(A \cup B')' = A' \cap B = \{1, 3\}$
9(c)	$A' \cap B' \neq \emptyset$ $n[(A \cup B)'] = 5$ $\{3\} \subset A \cup B$ $\{9\} \notin A \cap C$		
10(a)	15.6%	10(b)	June 2021
10(c)	US$2.39		
11	Since $\frac{m}{x^3} = k$, where k=0.18 is a non-zero constant, m is directly proportional to x^3.		
12	24.9%		
13(a)	46°	13(b)	134°
13(c)	33°		
14(a)	1 : 2	14(c)	3.96×10^{-2} m^2
14(b)	1000ml bottle cost $0.0053 less per ml than 125 ml bottle. The 1000 ml bottle provides better value for money. OR 1000ml bottle provides 35.221 more ml per $1 than 125 ml bottle. The 1000 ml bottle provides better value for money.		

15(a)	$n = 8$	15(b)	$10°$
16(a)	smallest positive integer $x = 63$	16(b)	smallest positive integer $y = 44$
17(b)	36.4 cm	17(c)	13.7 cm^2
18(a)	$\binom{3}{m+3}$	18(b)	$m = -\frac{3}{4}$
18(c)	5 units		
19(a)	$x = 1.0$ or 4.0 (accept 3.9)	19(b)	$x = 0.6$
20(a)	$y = 3$	20(b)	IQR of boy's height = 178 − 168 = 10 cm
21(a)	$cos \angle LMN = -\frac{3}{5}$	21(b)	$k = 7$ or -5
21(c)	$Q(3, 6)$		
22(a)(i)	035°	22(a)(ii)	4.59km
22(b)(i)	55°	22(b)(ii)	10.2 min
23(a)	Speed = 18.75 m/s		

Sec 4 Prelim Math Paper 2 Solutions

1 (a) Express as a single fraction in its simplest form

(i) $\dfrac{24q^2}{63p^3} \div \dfrac{9q^5}{21p}$, [1]

Solutions	Skills/Concept
$= \dfrac{24q^2}{63p^3} \times \dfrac{21p}{9q^5}$ $= \dfrac{8}{9p^2q^3}$	Take reciprocal: $\dfrac{24q^2}{63p^3} \times \dfrac{21p}{9q^5}$ Laws of indices: $a^{m+n} = a^m \times a^n$ $a^{m-n} = a^m \div a^n$

(ii) $\dfrac{1}{m-4} + \dfrac{2m}{m^2-16}$. [2]

Solutions	Skills/Concept
$= \dfrac{1}{m-4} + \dfrac{2m}{(m-4)(m+4)}$ $= \dfrac{(m+4)+2m}{(m-4)(m+4)}$ $= \dfrac{3m+4}{(m-4)(m+4)}$	Quadratic Identity: $a^2-b^2 = (a+b)(a-b)$ Express as single fraction

(b) Simplify $\dfrac{3x-9}{2x-xy+3y-6}$. [3]

Solutions/Alternative Methods	Skills/Concept
$= \dfrac{3x-9}{(2x-xy)+(3y-6)} = \dfrac{3x-9}{x(2-y)+3(y-2)}$ $= \dfrac{3(x-3)}{x(2-y)-3(2-y)}$ $= \dfrac{3(x-3)}{(x-3)(2-y)}$ $= \dfrac{3}{2-y}$	Factorisation by grouping Change of sign $-\dfrac{3}{y-2}$ also acceptable

(c) Solve the equation $(x+2)(x-5) = (x-5)(4x-7)$. [3]

Solutions/Alternative Methods	Skills/Concept
$(x+2)(x-5)-(4x-7)(x-5) = 0$ $(x-5)[(x+2)-(4x-7)] = 0$ $(x-5)(9-3x) = 0$ $x = 3$ or $x = 5$	Factorisation of quadratic function Solving quadratic equation

2 In 2019, Alan and Bala decided to start a business together. Alan invested \$210 000 and Bala invested \$140 000. They agreed that all profit should be divided in the same ratio as the sums of the money they invested.

(a) In 2019, the profit was \$20 000. Calculate Alan's share of the profit. [2]

Solutions/Alternative Methods	Skills/Concept
Alan's share of profit $= \frac{3}{5} \times 20\,000$ $= \$12\,000$	Ratio: $\frac{210000}{210000+140000} = \frac{3}{5}$

(b) Due to the pandemic, the total profit in 2020 dropped to \$12 500. Calculate the percentage decrease in profit from 2019 to 2020. [1]

Solutions/Alternative Methods	Skills/Concept
percentage decrease in profit $= \frac{20\,000 - 12\,500}{20\,000} \times 100\%$ $= 37.5\%$	Percentage decrease $= \frac{\text{difference}}{\text{original}} \times 100$

(c) To expand their business, they decided to borrow \$100 000 from a bank. The bank charged an interest rate of 2.4% per annum compounded half yearly. Calculate how much interest they need to pay after 5 years. Give your answer correct to the nearest dollars. [3]

Solutions/Alternative Methods	Skills/Concept
$\text{Interest} = 100\,000\left[1+\frac{\left(\frac{2.4}{2}\right)}{100}\right]^{5\times2} - 100\,000$ $= \$12\,669.1778$ $= \$12\,669$	$n = 5\times2$ $r = 2.4 \div 2$ Round up to nearest dollars

(d) Alan and Bala can choose to import their raw materials which cost RM40 000 in Malaysia or NT\$265 000 in Taiwan. The exchange rate between Singapore and Malaysia is S\$1 = RM\$3.20 and the exchange rate between Taiwan and Singapore is NT\$100 = S\$4.60. There is a freight charge of 2% for the raw materials from Taiwan only. Determine which country they should import their raw materials from. [4]

Solutions/Alternative Methods	Skills/Concept
Amount paid in S\$ for Malaysia import $= \frac{40\,000}{3.2}$ $= \$12\,500$	Exchange rate for Malaysia Ringgit
Amount paid in S\$ for Taiwan import $= \frac{102}{100}\left(\frac{265\,000}{100} \times 4.6\right)$ $= \$12\,433.80$	Exchange rate for Taiwan dollars Include 2% freight charge
They should import their raw materials from Taiwan because the total amount paid is S\$66.20 lower compared to Malaysia.	Compare with difference in values

3 A wholesaler supplies snacks and delivers to two stalls. The matrix, **S**, shows the number of each type of snacks per delivery made to Stalls *A* and *B*. In a week, the wholesaler delivers 5 times to Stall *A* and 7 times to stall *B*.

$$\mathbf{S} = \begin{matrix} \\ \text{Stall } A \\ \text{Stall } B \end{matrix} \begin{matrix} \begin{matrix} \text{sandwich} & \text{cake} & \text{pie} \end{matrix} \\ \begin{pmatrix} 25 & 20 & 13 \\ 40 & 18 & 21 \end{pmatrix} \end{matrix}$$

(a) The wholesaler charges the stalls \$2.00, \$0.70 and \$1.50 each for sandwich, cake and pie respectively. Represent these prices in column matrix **P**. [1]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{P} = \begin{pmatrix} 2.00 \\ 0.70 \\ 1.50 \end{pmatrix}$	Column matrices 3×1

(b) Evaluate the matrix **C** = **SP**. [2]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{C} = \begin{pmatrix} 25 & 20 & 13 \\ 40 & 18 & 21 \end{pmatrix} \begin{pmatrix} 2.00 \\ 0.70 \\ 1.50 \end{pmatrix}$ $= \begin{pmatrix} 83.50 \\ 124.10 \end{pmatrix}$	Multiplying matrices, Order: $(2 \times 3) \times (3 \times 1) = (2 \times 1)$

(c) State what each of the elements of **C** represents. [1]

Solutions/Alternative Methods	Skills/Concept
The elements represent the amount collected by the wholesaler from the sales of snacks per delivery from Stall *A* and Stall *B* respectively.	

(d) The amount collected by the wholesaler in a week from Stall *A* and Stall *B* respectively is represented by a 2×1 matrix, **W**. Using **only** matrix multiplication, find **W**. [2]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{W} = \begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix} \begin{pmatrix} 83.50 \\ 124.10 \end{pmatrix}$ or $= \begin{pmatrix} 83.50 & 0 \\ 0 & 124.10 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $= \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$ $= \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$	Matrix multiplication of 2×2 with 2×1 to get 2×1

(e) Hence, find the total amount collected by the wholesaler in a week. [1]

Solutions/Alternative Methods	Skills/Concept
Total amount $= (1 \quad 1) \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$ The total amount collected in a week is \$1286.20	Unit matrix $(1 \quad 1)$

4 **(a)** These are the first four terms in a sequence.

$-2 \quad 1 \quad 4 \quad 7$

Find an expression, in terms of n, for the nth term of the sequence. [1]

Solutions/Alternative Methods	Skills/Concept
$3(n-1)-2=3n-3-2$ $=3n-5$	General term: $a+(n-1)d$ a: 1st term, d: constant difference between terms

(b) Study the following number pattern of the Pythagorean Triples.

Row	Pythagorean Triples
1	$5^2=3^2+4^2$
2	$13^2=5^2+12^2$
3	$25^2=7^2+24^2$
4	$41^2=9^2+40^2$
5	$p^2=q^2+60^2$
N	$P_N^{\,2}=Q_N^{\,2}+R_N^{\,2}$

(i) Write down the value of p and of q in Row 5. [2]

Solutions/Alternative Methods	Skills/Concept
$p=61$ $q=11$	Number patterns

(ii) Write down the Pythagorean Triples in Row 10 when $P_{10}=221$. [1]

Solutions/Alternative Methods	Skills/Concept
$221^2=21^2+220^2$	Include P, Q & R

(iii) When $Q_N=111$, find N. [2]

Solutions/Alternative Methods	Skills/Concept
$Q_N=2(N-1)+3$ $=2N+1$ $2N+1=111$ $2N=110$ $N=55$	Find number pattern for Q_N Equate $2N+1$ to 111

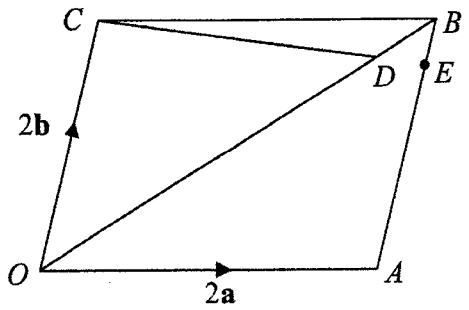
		(iv)	Given that $R_N = aN^2 + bN$, find the value of a and of b.	[4]

Solutions/Alternative Methods	**Skills/Concept**
$R_N = aN^2 + bN$ When $N = 1$, $a(1)^2 + b(1) = 4$ $a + b = 4 \quad \cdots (1)$ When $N = 2$, $a(2)^2 + b(2) = 12$ $4a + 2b = 12 \qquad (2)$ $(1) \times 2$, $2a + 2b = 8 \qquad (3)$ $(2) - (3)$, $2a = 4$ $a = 2$ Substitute $a = 2$ into (1), $(2) + b = 4$ $b = 2$	Form 1st equation Form 2nd equation Solve simultaneous equations Both a & b must be correct

		(v)	Explain with reason why it is not possible for 2021 to be a number of R_N.	[2]

Solutions/Alternative Methods	**Skills/Concept**
$2N^2 + 2N = 2(N^2 + 1)$ Since $2N^2 + 2N = 2(N^2 + 1)$ is always even for all values of N, it is not possible for 2021 which is odd to be a number of R_N.	Make R_N a multiple of 2 Multiples of 2 are even numbers

5 **(a)**



The diagram shows a parallelogram $OABC$. The point D on OB is such that $OD = 5DB$. The point E on AB is such that $AB : EB = 5:1$. Given that $OA = 2\mathbf{a}$ and $OC = 2\mathbf{b}$.

(i) Express in terms of **a** and **b**, giving each of your answers in its simplest form.

(a) OB, [1]

Solutions/Alternative Methods	Skills/Concept
$OB = OA + AB$ $= 2\mathbf{a} + 2\mathbf{b}$	Triangle Law of Vector Addition

(b) CD. [2]

Solutions/Alternative Methods	Skills/Concept
$CD = CO + OD$ $= CO + \frac{5}{6}OB$ $= -2\mathbf{b} + \frac{5}{6}(2\mathbf{a} + 2\mathbf{b})$ $= -2\mathbf{b} + \frac{5}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}$ $= \frac{5}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$	Vector addition with $OD = \frac{5}{6}OB$

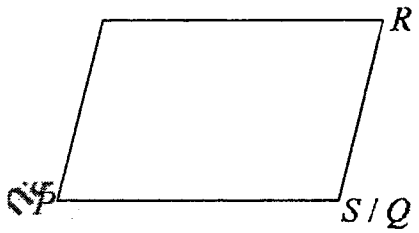
(ii) Show that C, D and E are collinear. [3]

Solutions/Alternative Methods	Skills/Concept
$CE = CB + BE$ $= 2\mathbf{a} + \frac{1}{5}BA$ $= 2\mathbf{a} + \frac{1}{5}(-2\mathbf{b})$ $= 2\mathbf{a} - \frac{2}{5}\mathbf{b}$ $CD = \frac{1}{3}(5\mathbf{a} - \mathbf{b})$ $CE = \frac{2}{5}(5\mathbf{a} - \mathbf{b})$ $\therefore CD = \frac{5}{6}CE$ Since CD is a <u>scalar multiple</u> of CE and C is a <u>common point</u>, $\therefore$ C, D and E are collinear.	Find CE using vector addition Make CD a scalar multiple of CE Or any other scalar multiple $DE = \frac{1}{3}\mathbf{a} + \frac{1}{15}\mathbf{b}$ $= \frac{1}{15}(5\mathbf{a} - \mathbf{b})$ $= \frac{1}{5\times 3}(5\mathbf{a} - \mathbf{b})$ $DE = \frac{1}{5}CD$ Conditions for collinearity

		(iii)	Find the numerical value of $\dfrac{\text{Area of } \Delta ODC}{\text{Area of parallelogram } OABC}$	[1]

Solutions/Alternative Methods	Skills/Concept
$\dfrac{\text{Area of } \Delta ODC}{\text{Area of } \Delta OBC} = \dfrac{\frac{1}{2}\times 5\times h}{\frac{1}{2}\times 6\times h}$ $= \dfrac{5}{6}$ $\dfrac{\text{Area of } \Delta ODC}{\text{Area of parallelogram } OABC} = \dfrac{5}{6}\times\dfrac{1}{2}$ $= \dfrac{5}{12}$	Use of Area of triangle $= \dfrac{1}{2}\times \text{base}\times \text{height}$ to find ratio of 2 triangles with common height Or use counting method

	(b)	It is given that $PQ = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and the coordinates of R are $(4, 0)$. Find the coordinates of the point S such that $PQRS$ is a parallelogram.	[2]

Solutions/Alternative Methods	Skills/Concept
Let the coordinates of S be (a, b) $PQ = SR$ $PQ = OR - OS$ $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix}$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\therefore S(-1, -1)$	$PQ = SR$ for equal vectors in parallelogram  Clockwise or anti-clockwise for $PQRS$ Must be coordinates

6 An aircraft flew from Town A to Town B and made a return trip to Town A from Town B. The total distance covered was 1200 km. The speed of the aircraft in still air is 200 km/h. The aircraft flew against the wind when flying from Town A to Town B, and flew wind assisted when flying back to Town A from Town B.

(a) The speed of the wind, which is constant throughout, is x km/h. The time taken by the aircraft, in hours, to fly from Town A to Town B is $\frac{600}{200-x}$. Write down an expression, in terms of x, the time taken by the aircraft, in hours, to fly from Town B to Town A. [1]

Solutions/Alternative Methods	Skills/Concept
$\frac{600}{200+x}$	

(b) The time taken to fly against the wind is 10 minutes longer than when it took to fly wind assisted. Write down an equation in terms of x and show that it reduces to $x^2 + 7200x - 40\,000 = 0$. [3]

Solutions/Alternative Methods	Skills/Concept
$\frac{600}{200-x} - \frac{600}{200+x} = \frac{10}{60}$	Forming quadratic equations
$\frac{600(200+x)-600(200-x)}{(200^2-x^2)} = \frac{1}{6}$	$(200+x)(200-x) = (200^2 - x^2)$ as denominator
$600(200+x)-600(200-x) = \frac{1}{6}(200^2-x^2)$	
$1200x = \frac{1}{6}(200^2-x^2)$	
$7200x = 200^2 - x^2$	Simplify equation to required one
$x^2 + 7200x - 40\,000 = 0$ (shown)	

(c) Showing your working clearly, solve the equation $x^2 + 7200x - 40\,000 = 0$, giving your solutions correct to 2 significant figures. [4]

Solutions/Alternative Methods	Skills/Concept
$x^2 + 7200x - 40\,000 = 0$	Solving of quadratic equation using formula or completing square **only**
$x = \frac{-7200 \pm \sqrt{7200^2 - 4(1)(-40000)}}{2(1)}$	
$= 5.55113$ or $= -7205.55113$	Leave answers in 2 sig fig
$= 5.6$ or $= -7200$ (2 s.f.)	

Solutions/Alternative Methods	
$x^2 + 7200x - 40\,000 = 0$	
$(x+3600)^2 - (3600)^2 - 40\,000 = 0$	Solve by completing the square
$(x+3600)^2 = 13000000$	
$x = -3600 \pm \sqrt{13000000}$	
$= 5.55113$ or $= -7205.55113$	
$= 5.6$ or $= -7200$ (2 s.f.)	Leave answers in 2 sig fig

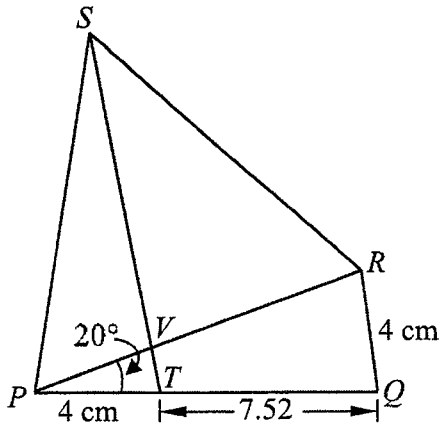
(d) Find the time taken for the whole trip. [2]

Solutions/Alternative Methods	Skills/Concept
Time taken $= 2 \times \frac{600}{200+5.55113} + \frac{10}{60}$	Use $x = 5.55113$ to find time taken to the nearest hour
$= 6.0046$	
$= 6$ hours	

Solutions/Alternative Methods	

	Time taken $= 2 \times \frac{600}{200 - 5.55113} - \frac{10}{60}$ $= 6.0046$ $= 6$ hours	
Solutions/Alternative Methods		
	Time taken $= \frac{600}{200 - 5.55113} + \frac{600}{200 + 5.55113}$ $= 6.0046$ $= 6$ hours	

7



ΔPQR is an isosceles triangle with $\angle QPR = 20°$. S is a point outside ΔPQR such that ΔPRS is an equilateral triangle and SVT is a straight line that meets PR and PQ at V and T respectively. It is given that $QR = TP = 4$ cm and $TQ = 7.52$ cm.

(a) Show that ΔPQR is congruent to ΔSTP.
Give a reason for each statement you make. [3]

Solutions/Alternative Methods	Skills/Concept
$SP = PR$ (sides of equilateral triangle) $QR = TP$ (given)	Find 2 pairs of corresponding sides
$\angle PRQ = \frac{180 - 20}{2}$ (base angle of isosceles triangle) $= 80$ $\angle SPT = \angle SPR + \angle RPQ$ $= 60 + 20$ $= 80$	Find a pair of corresponding angles
$\therefore \angle PRQ = \angle SPT$ $\therefore \Delta PQR$ is congruent to ΔSTP (SAS congruent test)	Conditions for Congruency (SAS)

(b) Show that ΔSTR is an isosceles triangle. [2]

Solutions/Alternative Methods	Skills/Concept
$SP = SR$ (sides of equilateral triangle) Since ΔPQR is congruent to ΔSTP, $SP = ST$ (sides of isosceles triangle) $\therefore ST = SR$ ΔSTR is an iososceles triangle.	Use of congruence rules Know that sides of isosceles triangle are equal

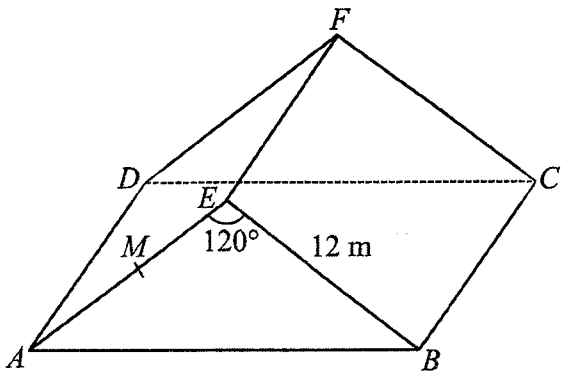
(c) Find $\angle STR$. [2]

Solutions/Alternative Methods	Skills/Concept
$\angle RST = \angle PSR - \angle PST$ $= 60 - 20$ $= 40$ Since ΔSTR is an isocesles triangle $\angle STR = \frac{180 - 40}{2}$ (base angles of isosceles triangle) $= 70$	Find $\angle RST$.

	(d)	Show that QR is parallel to ST, hence find the area of ΔPVT if area of triangle $\Delta PQR = 22.7\ \text{cm}^2$.	[4]

Solutions/Alternative Methods	**Skills/Concept**
Since ΔPQR is congruent to ΔSTP, $\angle PQR = \angle STP$ (corresponding $\angle$s of congruent triangles) $= 80^\circ$ By converse of corresponding angles, QR is parallel to ST.	Use of congruent rule Corr. angles, parallel lines
OR $\angle TSR = 40^\circ$ $\angle SRQ = 60^\circ + 80^\circ = 140^\circ$ $\angle TSR + \angle SRQ = 40^\circ + 140^\circ = 180^\circ$. By converse of interior angles, QR is parallel to ST.	
OR $\angle VRQ = 180^\circ - 40^\circ - 60^\circ = 80^\circ$ (angle sum in a triangle) $\angle SVR = \angle VRQ = 80^\circ$. By converse of alternate angles, QR is parallel to ST.	
ΔPTV is similar to ΔPQR Area of $\Delta PTV = \left(\frac{4}{11.52}\right)^2 \times 22.7$ $= 2.7368\ \text{cm}^2$ $= 2.74\ \text{cm}^2$	Areas of similar triangles

8 A roof in the shape of a triangular right prism is constructed as shown below such that $ABCD$ is a rectangle, $ADFE$ and $BCFE$ are squares. $AE = BE = 12$ m, $\angle AEB = 120°$ and M is the midpoint of AE.



Find

(a) the area of triangle ABE, [2]

Solutions/Alternative Methods	Skills/Concept
area of triangle $ABE = \frac{1}{2} \times 12 \times 12 \times \sin 120$ $= 62.354$ $= 62.4 \text{ m}^2$	Area of triangle involving sine

(b) AB^2, [2]

Solutions/Alternative Methods	Skills/Concept
$AB^2 = 12^2 + 12^2 - 2(12)(12)\cos 120$ $= 432$	Cosine rule

(c) AC, [2]

Solutions/Alternative Methods	Skills/Concept
$AC = \sqrt{432 + 12^2}$ (Pythagoras' Theorem) $= 24$ m	Pythagoras' Theorem

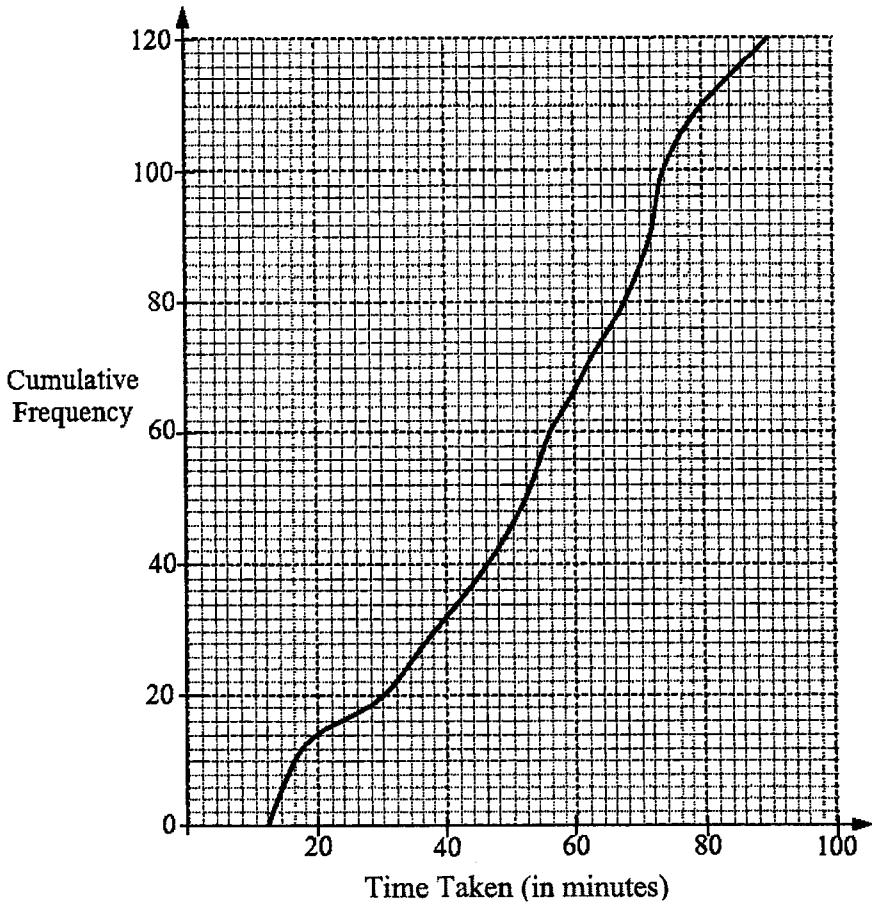
(d) $\angle AEC$, [3]

Solutions/Alternative Methods	Skills/Concept
$EC = \sqrt{12^2 + 12^2}$ (Pythagoras' Theorem) $= \sqrt{288}$ m	Pythagoras' Theorem
$\cos \angle AEC = \frac{12^2 + \left(\sqrt{288}\right)^2 - 24^2}{2(12)(\sqrt{288})}$ $= \frac{-144}{407.293506}$ $\angle AEC = 110.7048$ $= 110.7$ (1 d.p.)	Cosine rule

	(e)	the largest angle of elevation of M viewed from a point along CD.	[3]

Solutions/Alternative Methods	Skills/Concept
Let the point directly below M at AB be P and let the point be Q on CD.	
$AM = 6$ m	
$\angle PAM = 30^\circ$ (base angle of isocesles triangle)	
$PM = 6\sin 30^\circ$ $= 3$ m	Sine Trigo Ratio
$PQ = 12$ m	
$\tan \angle PQM = \frac{3}{12}$	
$\angle PQM = \tan^{-1}\left(\frac{1}{4}\right)$	Tangent Trigo Ratio
$= 14.036$	
$= 14.0$ (1 d.p.)	

9 **(a)** The cumulative frequency curve below shows the distribution of the time taken (in minutes) by 120 students in School *A* to complete an assignment.



Below is the grouped frequency table for the time taken by the students.

Time taken (in minutes)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	14	***a***	34	***b***	10

(i) Find the value of ***a*** and of ***b***. [1]

Solutions/Alternative Methods	**Skills/Concept**
$a = 18, \quad b = 44$	Convert Cumulative frequency to frequency

(ii) Estimate the mean time taken by the students. [1]

Solutions/Alternative Methods	**Skills/Concept**
$\text{mean time} = \dfrac{14\times10+18\times30+34\times50+44\times70+10\times90}{120}$ $= 53 \text{ min}$	Use calculator to find mean

		(iii)	Estimate the standard deviation of the time taken by the students.	[1]

Solutions/Alternative Methods	**Skills/Concept**
$\text{SD} = \sqrt{\frac{14\times10^2+18\times30^2+34\times50^2+44\times70^2+10\times90^2}{120} - 53^2}$ $= \sqrt{\frac{399200}{120} - 2809}$ $= 22.752289$ $= 22.8$ min (3 s.f.)	Use calculator to find S.D

		(iv)	120 students from School *B* completed the same assignment, and the analysis of their time taken is represented in the table below. Mean time taken: 60 Standard deviation: 13.6	
			Make two comments comparing the time taken by the students from the 2 schools.	[2]

Solutions/Alternative Methods	**Skills/Concept**
On average, students from school *B* took longer to complete the same assignment as their mean time taken of 60 minutes is 7 minutes longer than the mean time taken of 53 minutes by students from school *A*.	Comparing of data <u>in context</u> using mean by stating the difference
The spread of the time taken to complete the assignment for students from school *A* is wider compared to students from School *B* as their standard deviation of 22.8 minutes is 9.2 minutes higher than School *B*'s 13.6 minutes. The time taken by the students from School *B* is more homogeneous.	Comparing of data <u>in context</u> using S.D. by stating the difference

	(b)		Ali, Bryan and Chandra took part in a game of dart throwing. The probabilities that Ali, Bryan and Chandra will hit the target in a single throw are $\frac{1}{6}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively.	
		(i)	For the first game, all three of them throw the dart at the target at the same time. Find the probability that all of them hit the target.	[2]

Solutions/Alternative Methods	**Skills/Concept**
$\text{P(all of them missed)} = \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}$ $= \frac{1}{120}$	Probability of independent events

		(ii)	In the second game, they each make a single throw of the dart at the target in the order of Ali, Bryan and Chandra. For this game, once the target is hit, the game will end. Find the probability the target is hit.	[3]

Solutions/Alternative Methods	**Skills/Concept**
$\text{P(hit the target)} = \frac{1}{6} + \left(\frac{5}{6}\times\frac{1}{5}\right) + \left(\frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}\right)$ $= \frac{1}{2}$	Probability of independent events & mutually exclusive events

10 Daryl owns a concert hall with a full capacity of 120 seats. He conducted a survey to find out how much to charge for tickets. The detail of the survey is below:

Price of one ticket	Number of people who will attend the concert
\$6.00	120
\$7.50	110
\$9.00	100
\$10.50	90

(a) Write down the revenue he will get if all 120 seats are sold. [1]

Solutions/Alternative Methods	**Skills/Concept**
Revenue $= 120 \times 6$ $= \$720$	

(b) Daryl noticed that for every \$1.50 increase in the price of one ticket, the number of people who attend the concert drops by 10.

(i) Find the revenue if he makes **three** \$1.50 increases to the price from \$6. [1]

Solutions/Alternative Methods	**Skills/Concept**
Price after increase $= 6.00 + 3(1.50)$ $= \$10.50$ From the table when ticket at \$10.50, 90 people will attend Revenue $= 90 \times 10.50$ $= \$945$	

(ii) Let n be the number of \$1.50 increase in the price of the tickets, explain why the revenue R, in dollars is given by $720 + 120n - 15n^2$. [3]

Solutions/Alternative Methods	**Skills/Concept**
Amount increase $= 1.50n + 6$ Number of people who will attend $= 120 - 10n$ Revenue $= (1.50n + 6)(120 - 10n)$ $= 180n - 15n^2 + 720 - 60n$ $= 720 + 120n - 15n^2$ (shown)	Find amount increase The drop in number who attend Form expression for revenue

(iii) Explain why the number of \$1.50 increase in price should be less than 12. [1]

Solutions/Alternative Methods	**Skills/Concept**
When $n > 12$, Number of people who will attend is $120 - 10n < 0$ $\therefore$ the number of increase of \$1.50 should not >12.	

(iv) By drawing a suitable graph for $n < 12$ on the grid opposite, work out how much should Daryl charge his ticket to maximum revenue. [4]

Solutions/Alternative Methods	**Skills/Concept**
Graph below: must include table of values & scales as they are not given in the question. From the graph, since revenue is maximum at $n = 4$, He should charge $= 4(1.50) + 6 = \$12$	Sufficient points (at least 6) to draw a smooth curve Smooth curve passing through all points Know max revenue is at $n = 4$

n	0	1	2	3	4	5	6	7	8	9	10	11
R	720	825	900	945	960	945	900	825	720	585	420	225