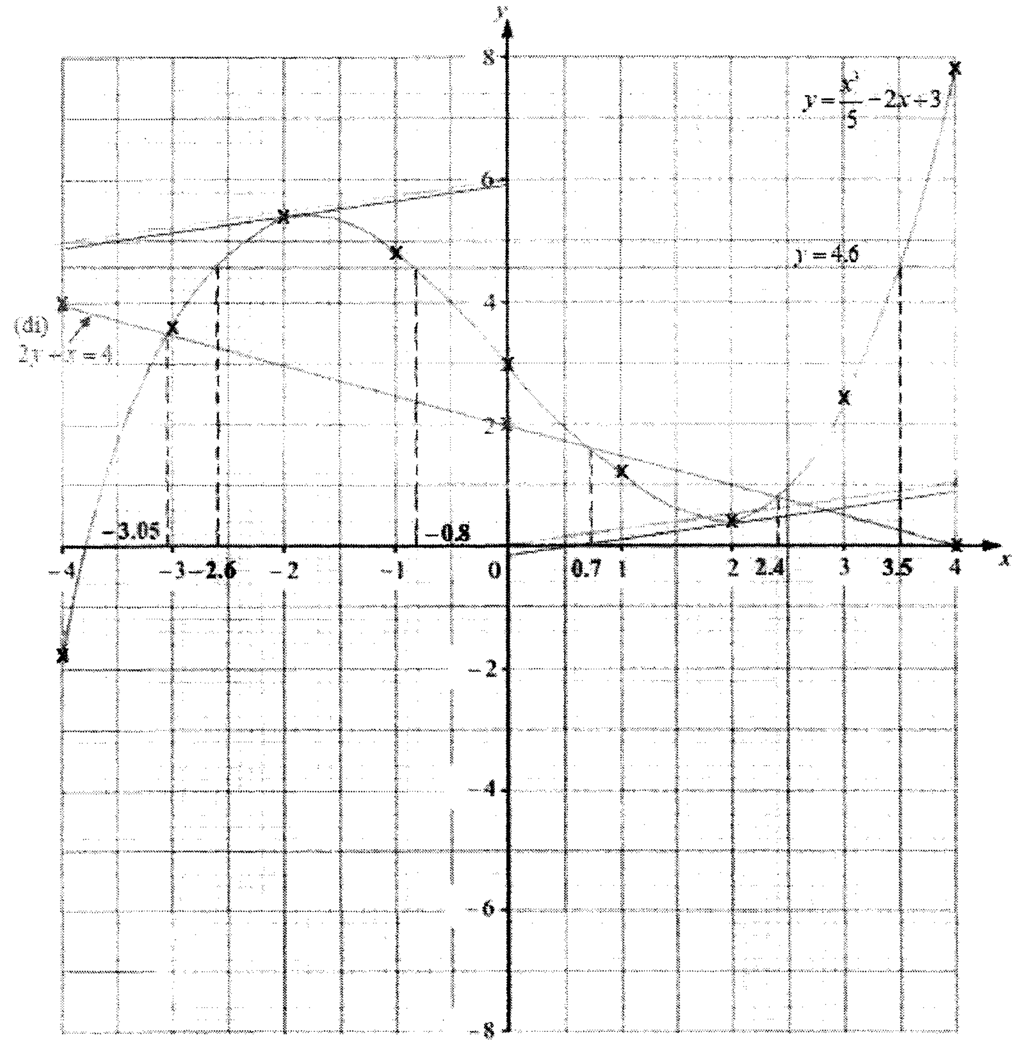
Answer Key

1		$\left(5+4\sqrt{3}\right)$ cm							
2		-2 and 6							
3	**(a)**	$-2\left(x+2\frac{1}{4}\right)^2+21\frac{1}{8}$	**(b)**	$\left(-2\frac{1}{4},\ 21\frac{1}{8}\right)$	**(c)**	Any value k where $k > 21\frac{1}{8}$			
4		$\frac{2}{5}(4+5x)^{\frac{3}{2}}-\frac{1}{x^2}+\frac{6}{7}\ln(7x-1)+c$							
5		$\frac{8x^2+4x+1}{(x+1)(x^2+4)}=\frac{1}{x+1}+\frac{7x-3}{x^2+4}$							
6	**(a)**	$h=8$	**(b)**	**(i)**	$g=-5$				
6	**(a)**	$h=8$	**(b)**	**(i)**	$g=-5$				
2	**(a)**	**(i)**	$\frac{1}{2}$	**(ii)**	$\frac{43}{138}$	**(iii)**	$\frac{95}{138}$		
	(b)	Probability $=\frac{44}{69} \neq \frac{2}{3}$ Hence, I disagree with Ben.							
3	**(a)**	$\begin{pmatrix} 350 & 420 & 280 \\ 490 & 350 & 280 \end{pmatrix}$	**(b)**	$\begin{pmatrix} 3.25 \\ 3.75 \\ 4.50 \end{pmatrix}$	**(c)**	$\begin{pmatrix} 3972.50 \\ 4165 \end{pmatrix}$	**(e)**	380.10	
	(d)	Each element represents the total cost of making cinnamon doughnuts and chocolate doughnuts respectively in a week.							
4	**(a)**	-1.8	**(c)**	**(i)**	$-2.6 < x < -0.8$				
	(c)	**(ii)**	Any value in the range: $-1.8 \le k < 0.6$ or $5.4 < k \le 7.8$						
	(c)	**(iii)**	$(2, 0.6)$ or $(-2, 5.4)$	**(d)**	**(iii)**	-3.05 or 0.7 or 2.4			
	(b)								



5	(a)	029.0°	(c)	66.3°	(d)	248.7°						
	(e)	(i) 644	(ii)	29.9°								
6	(a)	371	(b)	(i) $\left(\frac{4}{5}\right)^3 \neq \frac{4}{5}$ Thus the mass of Amy's candle is not $\frac{4}{5}$ t								
	(b)	(ii) 64	(iii)	3.59								
7	(b)	75.1	(c)	184								
8	(b)	11	(c)	5.75	(d)	10 : 3						
9	(a)	38	(b)	[illegible]	(c)	0.639	(d)	283.2°	(e)	4.16	(f)	[illegible]
	(g)	1. The mean length of service of the female staff is longer than that of the male staff. This means the female staff have worked longer at the company. 2. The standard deviation of the length of service of the female staff is smaller than that of the male staff. This means the spread of the length of service is smaller for the female staff.										
10	(a)	374										
	(b)	Company B offers a better value as the cost of the package, \$289.10, is cheaper.										

	(c)	Amount > $8956 + reason

	Class	Register Number
Name Solutions		

4049/01 **22/S4PR/AM/1**

ADDITIONAL MATHEMATICS PAPER 1

Monday **29 August 2022** **2 hours 15 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION
SECONDARY FOUR

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setters: Mdm Ernie Bte Abdullah and Ms Emmeline Lau

This paper consists of **25** printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \quad \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \quad \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A cuboid has a base area of $\left(7+6\sqrt{3}\right)$ cm^2 and a volume of $\left(107+58\sqrt{3}\right)$ cm^3.

Find, without using a calculator, the height of the cuboid, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

$$\text{Height of the cuboid} = \frac{107+58\sqrt{3}}{7+6\sqrt{3}} \times \frac{7-6\sqrt{3}}{7-6\sqrt{3}}$$

$$= \frac{749-642\sqrt{3}+406\sqrt{3}-348(3)}{49-36(3)}$$

$$= \frac{749-1044-236\sqrt{3}}{49-108}$$

$$= \frac{-295-236\sqrt{3}}{-59}$$

$$= \left(5+4\sqrt{3}\right) \text{ cm}$$

2 The line $3x-2y-12=0$ intersects the curve $xy=18$ at the points P and Q.
Find the x-coordinate of P and of Q. [3]

$3x-2y-12=0$

$y=\frac{3}{2}x-6 \qquad (1)$

$xy=18 \qquad (2)$

Sub. (1) into (2): $x\left(\frac{3}{2}x-6\right)=18$

$\frac{3}{2}x^2-6x-18=0$

$x^2-4x-12=0$

$(x+2)(x-6)=0$

$x+2=0 \quad \text{or} \quad x-6=0$

$x=-2 \qquad\qquad x=6$

The x-coordinates are -2 and 6.

Alternative working:

$3x-2y-12=0$

$x=\frac{2}{3}y+4 \qquad (1)$

$xy=18 \qquad (2)$

Sub. (1) into (2): $\left(\frac{2}{3}y+4\right)y=18$

$\frac{2}{3}y^2+4y-18=0$

$y^2+6y-27=0$

$(y+9)(y-3)=0$

$y+9=0 \quad \text{or} \quad y-3=0$

$y=-9 \qquad\qquad y=3$

Sub. $y=-9$ into (2): $-9x=18$

$x=-2$

Sub. $y=3$ into (2): $3x=18$

$x=6$

The x-coordinates are -2 and 6.

3 **(a)** Express $11-9x-2x^2$ in the form $a(x+b)^2+c$. [2]

$$
\begin{aligned}
11-9x-2x^2 &= -2\left(x^2+\frac{9}{2}x-\frac{11}{2}\right) \\
&= -2\left[\left(x+\frac{9}{4}\right)^2-\frac{11}{2}-\frac{81}{16}\right] \\
&= -2\left[\left(x+\frac{9}{4}\right)^2-\frac{169}{16}\right] \\
&= -2\left(x+\frac{9}{4}\right)^2+\frac{169}{8} \\
&= -2\left(x+2\frac{1}{4}\right)^2+21\frac{1}{8}
\end{aligned}
$$

Hence

(b) state the coordinates of the turning point of the curve $11-9x-2x^2$, [1]

The coordinates are $\left(-2\frac{1}{4},\ 21\frac{1}{8}\right)$.

(c) write down a possible value of k such that the number of real roots to the equation $11-9x-2x^2=k$ is 0. [1]

k can be any number that is greater than the maximum value of $y=21\frac{1}{8}$.

4 Integrate $3\sqrt{4+5x}+\frac{2}{x^3}+\frac{6}{7x-1}$ with respect to x. [4]

$$\int\left(3\sqrt{4+5x}+\frac{2}{x^3}+\frac{6}{7x-1}\right)\mathrm{d}x=\int\left[3(4+5x)^{\frac{1}{2}}+2x^{-3}+\frac{6}{7}\left(\frac{7}{7x-1}\right)\right]\mathrm{d}x$$

$$=\frac{3(4+5x)^{\frac{3}{2}}}{\frac{3}{2}(5)}+\frac{2x^{-2}}{-2}+\frac{6}{7}\ln(7x-1)+c$$

$$=\frac{2}{5}(4+5x)^{\frac{3}{2}}-\frac{1}{x^2}+\frac{6}{7}\ln(7x-1)+c$$

5 Express $\dfrac{8x^2+4x+1}{(x+1)(x^2+4)}$ in partial fractions. [6]

Method 1: (substitution)

$$\frac{8x^2+4x+1}{(x+1)(x^2+4)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$$

$$8x^2+4x+1=A(x^2+4)+(Bx+C)(x+1)$$

When $x=-1$, $A\left[(-1)^2+4\right]=8(-1)^2+4(-1)+1$

$$5A=8-4+1$$
$$5A=5$$
$$\therefore A=1$$

When $x=0$ and $A=1$, $4(1)+C(1)=1$

$$\therefore C=-3$$

When $x=1$, $A=1$ and $C=-3$, $1(1+4)+\left[B(1)-3\right](1+1)=8(1)+4(1)+1$

$$5+2(B-3)=13$$
$$B-3=4$$
$$\therefore B=7$$

$$\therefore \frac{8x^2+4x+1}{(x+1)(x^2+4)}=\frac{1}{x+1}+\frac{7x-3}{x^2+4}$$

Method 2: (comparing coefficients)

$$\frac{8x^2+4x+1}{(x+1)(x^2+4)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$$

$$8x^2+4x+1=A(x^2+4)+(Bx+C)(x+1)$$

$$8x^2+4x+1=Ax^2+4A+Bx^2+Bx+Cx+C$$

By comparing the coefficients of:

x^2: $\quad A+B=8 \quad (1)$

x: $\quad B+C=4 \quad (2)$

x^0: $\quad 4A+C=1 \quad (3)$

$(1)-(2)$: $\quad A-C=4 \quad (4)$

$(3)+(4)$: $\quad 5A=5$

$A=1$

Sub. $A=1$ into (1): $\quad 1+B=8$

$B=7$

Sub. $A=1$ into (3): $\quad 4(1)+C=1$

$C=-3$

$$\therefore \frac{8x^2+4x+1}{(x+1)(x^2+4)}=\frac{1}{x+1}+\frac{7x-3}{x^2+4}$$

6 A polynomial, P, is $2x^3+x^2+hx-12$, where h is an integer.

(a) Find the value of h given that P leaves a remainder of -16 when divided by $2x+1$. [2]

Let $P(x)=2x^3+x^2+hx-12$.

By the remainder theorem,

$$P\left(-\frac{1}{2}\right)=-16$$

$$2\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^2-\frac{1}{2}h-12=-16$$

$$-\frac{1}{4}+\frac{1}{4}-\frac{1}{2}h-12=-16$$

$$\frac{1}{2}h=4$$

$$h=8$$

(b) In the case where $h=-19$, the quadratic expression $2x^2+gx-4$ is a factor of P.

(i) Find the value of the constant g. [3]

Method 1: (long division)

When $h=-19$, $P(x)=2x^3+x^2-19x-12$

$$P(-3)=2(-3)^3+(-3)^2-19(-3)-12$$
$$=0$$

Since $P(-3)=0$, by factor theorem, $(x+3)$ is a factor of $P(x)$.

$$\begin{array}{r|l} & 2x^2-5x-4 \\ x+3 & 2x^3+x^2-19x-12 \\ & -(2x^3+6x^2) \\ & \quad -5x^2-19x \\ & \quad -(-5x^2-15x) \\ & \qquad -4x-12 \\ & \qquad -(-4x-12) \\ & \qquad\quad 0 \end{array}$$

$\therefore\ g=-5$

Alternative working:

$$2x^3+x^2+hx-12$$
$$=(ax+b)(2x^2+gx-4)$$

By comparing the coefficient of x^3,

$$2a=2$$
$$a=1$$

By comparing the constant terms,

$$-4b=-12$$
$$b=3$$

$\therefore$ By factor theorem, $(x+3)$ is a factor of $P(x)$.

Method 2: (comparing coefficients)

When $h=-19$, $\quad P(x)=2x^3+x^2-19x-12$

$$P(-3)=2(-3)^3+(-3)^2-19(-3)-12$$
$$=0$$

Since $P(-3)=0$, by factor theorem, $(x+3)$ is a factor of $P(x)$.

$$2x^3+x^2-19x-12=(x+3)(2x^2+gx-4)$$
$$=2x^2+gx^2-4x+6x^2+3gx-12$$

By comparing the coefficients of x^2 :

$$g+6=1$$
$$\therefore \quad g=-5$$

or By comparing the coefficients of x:

$$3g-4=-19$$
$$3g=-15$$
$$g=-5$$

(ii) Hence explain why $P=0$ has 3 real roots. [2]

$P=0$

$(x+3)(2x^2-5x-4)=0$

$x+3=0$ or $2x^2-5x-4=0$

$x=-3$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(-4)}}{2(2)}$$

$$x=\frac{5\pm\sqrt{57}}{4}$$

$x\approx 3.14$ or $x\approx -0.637$

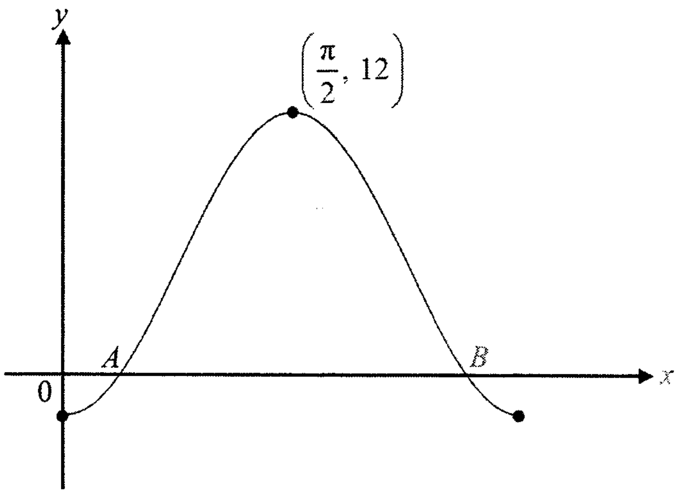
Alternative method:

$$b^2-4ac=(-5)^2-4(2)(-4)$$
$$=57>0$$

$\therefore$ The equation $2x^2-5x-4=0$ has 2 real roots.

Hence, $P=0$ has 3 real roots.

7



The diagram shows the curve $y = a\cos bx + c$ for $0 \le x \le \pi$ radians. The curve has a maximum point at $\left(\frac{\pi}{2}, 12\right)$ and two minimum points at $(0, -2)$ and $(\pi, -2)$.

(a) Explain why $c = 5$. [1]

Maximum value of $y = 12$
Minimum value of $y = -2$

$c = \frac{12-2}{2}$
$c = 5$

(b) Explain why $b = 2$. [1]

Period $= \pi$

$\frac{2\pi}{b} = \pi$

$b = \frac{2\pi}{\pi}$

$b = 2$

(c) Hence find the equation of the curve. [2]

Amplitude $= \frac{12+2}{2}$
$= 7$

$\therefore \quad y = -7\cos 2x + 5$

The curve intersects the x-axis at A and at B.

(d) Find, in radians, the values of x at A and at B for which $y = 0$. [2]

$-7\cos 2x + 5 = 0$

$\cos 2x = \frac{5}{7}$

Basic angle, $\alpha = \cos^{-1}\left(\frac{5}{7}\right)$

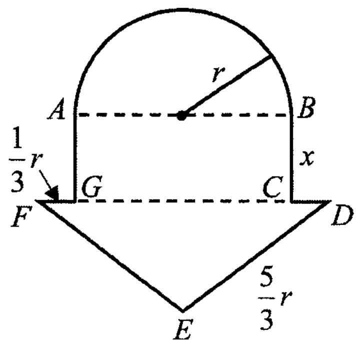
$\alpha \approx 0.77519$

$2x = \alpha,\ 2\pi - \alpha$

$x \approx 0.388,\ 2.75$

At A, $x \approx 0.388$ and at B, $x \approx 2.75$.

8



A baker uses 131 cm of wire to enclose a cake mould of the shape shown in the diagram. The shape consists of a semicircle with diameter AB, a rectangle $ABCG$ and an isosceles triangle FED such that $FE = ED$.

It is given that $AB = 2r$ cm, $BC = x$ cm, $ED = \frac{5}{3}r$ cm and $FG = CD = \frac{1}{3}r$ cm.

(a) Express x in terms of r and π. [2]

$$\frac{1}{2}(2\pi r) + 2x + 2\left(\frac{1}{3}r\right) + 2\left(\frac{5}{3}r\right) = 131$$

$$\pi r + 2x + \frac{2}{3}r + \frac{10}{3}r = 131$$

$$2x = 131 - \pi r - 4r$$

$$x = 65\frac{1}{2} - \frac{\pi}{2}r - 2r$$

(b) Show that the area of the mould, P cm^2, is given by

$$P = 131r - \frac{\pi}{2}r^2 - \frac{8}{3}r^2.$$ [3]

$$\text{Height of } \Delta DEF = \sqrt{\left(\frac{5}{3}r\right)^2 - \left(\frac{4}{3}r\right)^2}$$

$$= r \text{ cm}$$

$$P = \frac{1}{2}\pi r^2 + 2xr + \frac{1}{2}\left(2r + \frac{2}{3}r\right)(r)$$

$$= \frac{1}{2}\pi r^2 + (131 - \pi r - 4r)r + r^2 + \frac{1}{3}r^2$$

$$= \frac{1}{2}\pi r^2 + 131r - \pi r^2 - 4r^2 + r^2 + \frac{1}{3}r^2$$

$$= 131r - \frac{\pi}{2}r^2 - \frac{8}{3}r^2 \quad \text{(shown)}$$

(c) Given that r can vary, find the value of r which gives a stationary value of P. [3]

$$\frac{dP}{dr} = 0$$

$$131 - \frac{\pi}{2}(2r) - \frac{8}{3}(2r) = 0$$

$$131 - \pi r - \frac{16}{3}r = 0$$

$$\pi r + \frac{16}{3}r = 131$$

$$\left(\pi + \frac{16}{3}\right)r = 131$$

$$r \approx 15.457$$

$$r = 15.5$$

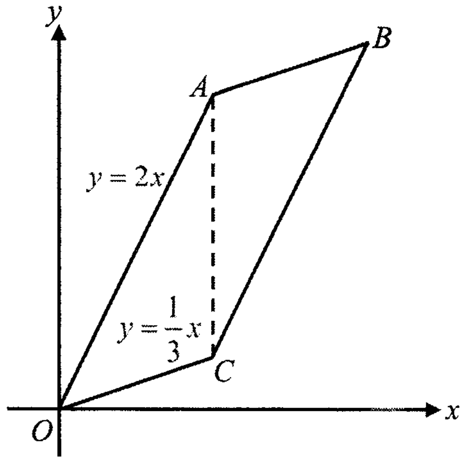
(d) The baker's son claimed that his father will be disappointed with the nature of this stationary value. Explain why you would agree or disagree with the baker's son. [2]

$$\frac{d^2P}{dr^2} = -\pi - \frac{16}{3} \quad < 0 \text{ (max)}$$

The stationary value is a maximum.

Thus, I disagree with the baker's son that his father will be disappointed.
The baker will be happy as he is able to optimize the length of the wire to obtain a maximum area to enclose the cake mould.

9



The diagram shows a parallelogram $OABC$, where O is the origin. The side OA has equation $y = 2x$ and the side OC has equation $y = \frac{1}{3}x$. The diagonal AC is parallel to the y-axis and the x-coordinate of C is k.

(a) Show that $AC = \frac{5}{3}k$ units. [1]

y-coordinate of $A = 2k$

$A(k,\ 2k)$

y-coordinate of $C = \frac{1}{3}k$

$C\left(k,\ \frac{1}{3}k\right)$

$AC = 2k - \frac{1}{3}k$

$AC = \frac{5}{3}k$ units (shown)

(b) Find the coordinates of B in terms of k. [2]

Let $B(p,\ q)$.

mid-point of OB = mid-point of AC

$$\left(\frac{p}{2}, \frac{q}{2}\right) = \left(k,\ \frac{2k + \frac{1}{3}k}{2}\right)$$

$\frac{p}{2} = k$ and $\frac{q}{2} = \frac{7}{6}k$

$p = 2k$ $\quad$ $q = \frac{7}{3}k$

$B\left(2k,\ \frac{7}{3}k\right)$

<u>Alternative working:</u>

Since $OABC$ is a parallelogram,

$OC // AB$ and $OC = AB$.

x − coordinate of $B = k + k$

$= 2k$

y − coordinate of $B = 2k + \frac{1}{3}k$

$= \frac{7}{3}k$

$B\left(2k,\ \frac{7}{3}k\right)$

It is now given that $k = 6$.

(c) Find the area of the parallelogram $OABC$. [2]

$A(6,\ 12)$, $B(12,\ 14)$ and $C(6,\ 2)$.

$$\text{Area of } OABC = 2 \times \left(\frac{1}{2} \times 6 \times 10\right)$$
$$= 60 \text{ units}^2$$

Alternative working:

$A(6,\ 12)$, $B(12,\ 14)$ and $C(6,\ 2)$.

$$\text{Area of } OABC = \frac{1}{2}\begin{vmatrix} 0 & 6 & 12 & 6 & 0 \\ 0 & 2 & 14 & 12 & 0 \end{vmatrix}$$
$$= \frac{1}{2}(84 + 144 - 24 - 84)$$
$$= \frac{1}{2}(120)$$
$$= 60 \text{ units}^2$$

D is a point such that $ABDC$ is a kite.

(d) Hence state the area of $ABDC$. [1]

BC is a diagonal of the kite.

$$\text{Area of } ABDC = 2 \times \text{area of } \Delta\, ABC$$
$$= \text{area of parallelogram } OABC$$
$$= 60 \text{ units}^2$$

10 (a) Prove the identity $\dfrac{1-\sin x}{\cos x} - \dfrac{\cos x}{\sin x - 1} = 2\sec x$. [4]

$$\begin{aligned}
\text{LHS} &= \frac{1-\sin x}{\cos x} - \frac{\cos x}{\sin x - 1} \\
&= \frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} \\
&= \frac{(1-\sin x)^2 + \cos^2 x}{(\cos x)(1-\sin x)} \\
&= \frac{1-2\sin x + \sin^2 x + \cos^2 x}{(\cos x)(1-\sin x)} \\
&= \frac{1-2\sin x + 1}{(\cos x)(1-\sin x)} \\
&= \frac{2-2\sin x}{(\cos x)(1-\sin x)} \\
&= \frac{2(1-\sin x)}{(\cos x)(1-\sin x)} \\
&= \frac{2}{\cos x} \\
&= 2\sec x \\
&= \text{RHS} \quad \text{(proven)}
\end{aligned}$$

(b) Hence solve the equation $\frac{1-\sin 2x}{\cos 2x}-\frac{\cos 2x}{\sin 2x-1}=-3$ for $-\pi \le x \le \pi$. [4]

$$\frac{1-\sin 2x}{\cos 2x}-\frac{\cos 2x}{\sin 2x-1}=-3$$

$$2\sec 2x=-3$$

$$\frac{2}{\cos 2x}=-3$$

$$\cos 2x=-\frac{2}{3}$$

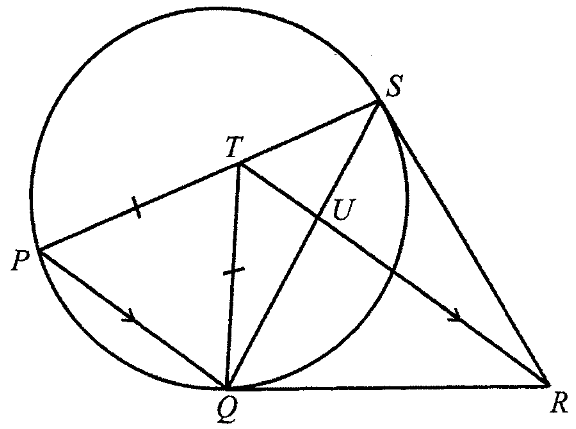
Basic angle, $\alpha=\cos^{-1}\left(\frac{2}{3}\right)$

$\alpha \approx 0.84107$

$2x=-\pi-\alpha,\ -\pi+\alpha,\ \pi-\alpha,\ \pi+\alpha$

$x \approx -1.99,\ -1.15,\ 1.15,\ 1.99$

11



In the diagram, P, Q and S lie on a circle. The tangents to the circle at Q and S meet at R and PQ is parallel to TR. SQ and TR intersect at U and $PT = QT$.

(a) Prove that ΔTQU and ΔSRU are similar. [4]

$\angle TUQ = \angle SUR$ (vert. opp. $\angle$s)

Let $\angle RQS = x$.

$\angle QPS = x$ (alternate segment theorem)

$= \angle PQT$ (base $\angle$s of isos. Δ)

$= \angle QTU$ (alt. $\angle$s, $PQ\ //\ TR$)

$\angle RSQ = \angle QPS$ (alternate segment theorem)

$= x$

or

$\angle RSQ = \angle RQS$ (tangents from ext. pt.)

$= x$

$\therefore\ \angle QTU = \angle RSU$

ΔTQU and ΔSRU are similar.

(b) **(i)** Hence show that a circle can be drawn passing through Q, R, S and T. [1]

Since $\angle QTU = \angle RSU$,
$\angle QTR = \angle RSQ$.

By the property of angles in the same segment, a circle can be drawn passing through Q, R, S and T.

(ii) Explain the conclusion that can be made for angle QTS and angle QRS. [1]

By the property of opposite angles in a cyclic quadrilateral, angle QTS and angle QRS are supplementary.

$\angle QTS + \angle QRS = 180°$ (opp. $\angle$s of a cyclicquad.)

12 **(a)** Solve the equation $6^x + 8 - 6^{2-x} = 17$. [5]

$6^x + 8 - 6^{2-x} = 17$

$6^x - \dfrac{6^2}{6^x} = 9$

$6^x - \dfrac{36}{6^x} = 9$

Let $u = 6^x$.

$u - \dfrac{36}{u} = 9$

$u^2 - 9u - 36 = 0$

$(u-12)(u+3) = 0$

$u - 12 = 0$ or $u + 3 = 0$

$u = 12$ $\quad$ $u = -3$

$6^x = 12$ $\quad$ $6^x = -3$

$x \lg 6 = \lg 12$ $\quad$ (NA)

$x = \dfrac{\lg 12}{\lg 6}$

$x \approx 1.39$

(b) Express the equation $\log_p\left(\frac{1-4x}{x}\right)=\log_{\sqrt{p}}(2-x)$, where $p>0$ and $p\neq 1$, as a cubic equation in x. [4]

$$\log_p\left(\frac{1-4x}{x}\right)=\log_{\sqrt{p}}(2-x)$$

$$\log_p\left(\frac{1-4x}{x}\right)=\frac{\log_p(2-x)}{\log_p\sqrt{p}}$$

$$\log_p\left(\frac{1-4x}{x}\right)=\frac{\log_p(2-x)}{\frac{1}{2}}$$

$$\log_p\left(\frac{1-4x}{x}\right)=2\log_p(2-x)$$

$$\log_p\left(\frac{1-4x}{x}\right)=\log_p(2-x)^2$$

$$\frac{1-4x}{x}=(2-x)^2$$

$$1-4x=x\left(4-4x+x^2\right)$$

$$1-4x=4x-4x^2+x^3$$

$$x^3-4x^2+8x-1=0$$

13 (a) *PQRS* is a rectangle with $PQ = x$ cm and $PS = (17 - x)$ cm. The sides of the rectangle vary with time such that x increases at a rate of 0.4 cm per second. Find the rate of decrease of the length of the diagonal when $x = 5$ cm. [4]

Let the diagonal be D cm.

$$D = \sqrt{x^2 + (17 - x)^2}$$

$$= \left(x^2 + 289 - 34x + x^2\right)^{\frac{1}{2}}$$

$$= \left(2x^2 - 34x + 289\right)^{\frac{1}{2}}$$

$$\frac{dD}{dx} = \frac{1}{2}\left(2x^2 - 34x + 289\right)^{-\frac{1}{2}}(4x - 34)$$

$$= \frac{2x - 17}{\left(2x^2 - 34x + 289\right)^{\frac{1}{2}}}$$

At $x = 5$, $\frac{dD}{dt} = \frac{dD}{dx} \times \frac{dx}{dt}$

$$= \frac{2(5) - 17}{\left[2(5)^2 - 34(5) + 289\right]^{\frac{1}{2}}} \times 0.4$$

$$= -\frac{7}{\sqrt{169}} \times 0.4$$

$$= -\frac{7}{13} \times 0.4$$

$$= -\frac{14}{65}$$

$$\approx -0.215$$

The rate of decrease of the length of the diagonal is $\frac{14}{65}$ cm/s.

(b) Air is pumped into a spherical balloon at a rate of 250 cm^3 per second. At a particular instant, the radius of the balloon is increasing at a rate of $\frac{5}{18\pi}$ cm per second. Find the rate of change of the surface area of the balloon at that instant. [4]

Let the radius, volume and surface area of the balloon be r cm, V cm^3 and A cm^2 respectively.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$250 = 4\pi r^2 \times \frac{5}{18\pi}$$

$$r^2 = 225$$

$$r = 15, \quad r > 0$$

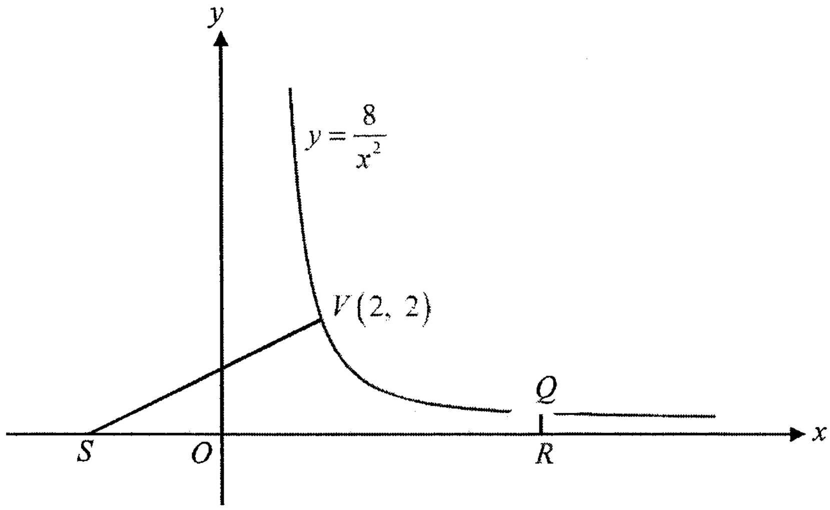
$$A = 4\pi r^2$$

$$\frac{dA}{dr} = 8\pi r$$

At $r = 15$,
$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$
$$= 8\pi(15) \times \frac{5}{18\pi}$$
$$= \frac{100}{3}$$
$$= 33\frac{1}{3}$$

The rate of change of the surface area of the balloon is $33\frac{1}{3}$ cm^2 / s.

14



The diagram shows part of the curve $y=\frac{8}{x^2}$. The point $V(2,\ 2)$ lies on the curve and the normal to the curve at V meets the x-axis at S. The x-coordinate of the points Q and R is 5.

(a) Find the coordinates of S. [5]

$$y=\frac{8}{x^2}$$

$$\frac{dy}{dx}=-\frac{16}{x^3}$$

At $x=2$, $\frac{dy}{dx}=-\frac{16}{(2)^3}$

$=-2$

At $x=2$, gradient of normal $=\frac{1}{2}$

Equation of normal at V is $y-2=\frac{1}{2}(x-2)$

$y=\frac{1}{2}x+1$

On the $x-$axis, $y=0$

$\frac{1}{2}x+1=0$

$x=-2$

$\therefore S(-2,\ 0)$

(b) Find the area of the shaded region bounded by the curve, the x-axis, the normal VS and the line QR. [5]

Area of the shaded region

$$=\frac{1}{2}(4)(2)+\int_2^5 \frac{8}{x^2}\,dx$$

$$=4+\left[-\frac{8}{x}\right]_2^5$$

$$=4+\left[-\frac{8}{5}-\left(-\frac{8}{2}\right)\right]$$

$$=4+2\frac{2}{5}$$

$$=6\frac{2}{5}\text{ units}^2$$

End of Paper

	Class	Register Number
Name Solutions		

4049/02 **22/S4PR/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Tuesday **30 August 2022** **2 hours 15 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION
SECONDARY FOUR

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setters: Ms Emmeline Lau and Mdm Ernie Bte Abdullah

This paper consists of **18** printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \quad \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \quad \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Show that $x-1$ is a factor of $2x^3 - x^2 - 3x + 2$ and hence solve the equation $2x^3 - x^2 - 3x + 2 = 0$ completely. [5]

Let $\mathrm{f}(x)$ be $2x^3 - x^2 - 3x + 2$.

$\mathrm{f}(1) = 2(1)^3 - (1)^2 - 3(1) + 2 = 0$

By factor theorem, $x-1$ is a factor of $\mathrm{f}(x)$.

$2x^3 - x^2 - 3x + 2 = (x-1)(2x^2 + bx - 2)$

Comparing coefficient of $x^2 : b - 2 = -1$

$b = 1$

$2x^3 - x^2 - 3x + 2 = 0$

$(x-1)(2x^2 + x - 2) = 0$

$x = 1$ or $2x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-2)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{17}}{4}$$

$\therefore x = 1$ or $x \approx -1.28$ or $x \approx 0.781$

2 **(a)** Show that the equation $2e^x - 1 = 3e^{-x}$ has only one solution and find its exact value. [4]

$2e^x - 1 = 3e^{-x}$

$2e^x - 1 = \frac{3}{e^x}$

Let $u = e^x$

$2u - 1 = \frac{3}{u}$

$2u^2 - u - 3 = 0$

$(2u-3)(u+1) = 0$

$u = \frac{3}{2}$ or $u = -1$

$e^x = \frac{3}{2}$ or $e^x = -1$ (rejected as $e^x > 0$ for all real x)

$\therefore 2e^x - 1 = 3e^{-x}$ has only one solution (shown)

$e^x = \frac{3}{2}$

$x = \ln\frac{3}{2}$

(b) Explain how the solution of $2e^{\ln 2x} - 1 = 3e^{\ln\frac{1}{2x}}$ can be deduced from your answer in part **(a)** and find the solution. [2]

$2e^{\ln 2x} - 1 = 3e^{\ln\frac{1}{2x}}$

$2e^{\ln 2x} - 1 = 3e^{-\ln 2x}$

The solution of $2e^{\ln 2x} - 1 = 3e^{\ln\frac{1}{2x}}$ can be found by replacing x in part **(a)** with $\ln 2x$.

$\ln 2x = \ln\frac{3}{2}$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

3 **(a)** Given that $y = x\sqrt{4x-3}$, show that $\frac{dy}{dx} = \frac{6x-3}{\sqrt{4x-3}}$. [3]

$$\frac{dy}{dx} = \sqrt{4x-3} + x \cdot \frac{1}{2}(4x-3)^{-\frac{1}{2}}(4)$$
$$= \frac{(4x-3)+2x}{\sqrt{4x-3}}$$
$$= \frac{6x-3}{\sqrt{4x-3}} \text{ (shown)}$$

(b) Hence find the value of $\int_1^7 \frac{6x}{\sqrt{4x-3}}\,dx$. [5]

From part **(a)** $\frac{d}{dx}\left(x\sqrt{4x-3}\right) = \frac{6x-3}{\sqrt{4x-3}}$

$$\int_1^7 \frac{6x-3}{\sqrt{4x-3}}\,dx = \left[x\sqrt{4x-3}\right]_1^7$$
$$\int_1^7 \frac{6x}{\sqrt{4x-3}}\,dx = \left[x\sqrt{4x-3}\right]_1^7 + \int_1^7 3(4x-3)^{-\frac{1}{2}}\,dx$$
$$= \left[x\sqrt{4x-3}\right]_1^7 + \left[\frac{3(4x-3)^{\frac{1}{2}}}{\frac{1}{2}(4)}\right]_1^7$$
$$= \left[x\sqrt{4x-3}\right]_1^7 + \left[\frac{3(4x-3)^{\frac{1}{2}}}{2}\right]_1^7$$
$$= \left[7\sqrt{4(7)-3} - (1)\sqrt{4(1)-3}\right] + \frac{3}{2}\left[\sqrt{4(7)-3} - \sqrt{4(1)-3}\right]$$
$$= 40$$

4 An ant moves in a straight line such that, t seconds after leaving a fixed point O, its velocity is modelled by $v = 8 + 2t - t^2$.

(a) Find the velocity of the ant when its acceleration is 1 cm/s^2. [3]

Let the acceleration of the ant be a.

$v = 8 + 2t - t^2$

$a = 2 - 2t$

$2 - 2t = 1$

$t = \frac{1}{2}$ s

$v = 8 + 2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2 = 8.75$ cm/s

(b) Find the distance travelled by the ant in the first 5 seconds. [5]

$v = 8 + 2t - t^2 = 0$

$t^2 - 2t - 8 = 0$

$(t+2)(t-4) = 0$

$t = -2$ (reject) or $t = 4$

Distance travelled in the first 5 s

$= \int_0^4 8 + 2t - t^2 \, dt - \int_4^5 8 + 2t - t^2 \, dt$

$= \left[8t + t^2 - \frac{t^3}{3}\right]_0^4 - \left[8t + t^2 - \frac{t^3}{3}\right]_4^5$

$= \left[\left(8(4) + 4^2 - \frac{4^3}{3}\right) - 0\right] - \left[\left(8(5) + 5^2 - \frac{5^3}{3}\right) - \left(8(4) + 4^2 - \frac{4^3}{3}\right)\right]$

$= 30$ cm

Alternative for distance travelled

$s = \int 8 + 2t - t^2 \, dt$

$= 8t + t^2 - \frac{t^3}{3} + c$

When $t = 0$, $s = 0 \Rightarrow c = 0$

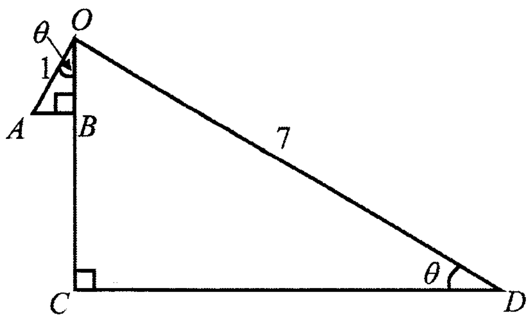
$s = 8t + t^2 - \frac{t^3}{3}$

When $t = 4$, $s = 8(4) + (4)^2 - \frac{(4)^3}{3} = 26\frac{2}{3}$

When $t = 5$, $s = 8(5) + (5)^2 - \frac{(5)^3}{3} = 23\frac{1}{3}$

Total distance travelled $= 26\frac{2}{3} + \left(26\frac{2}{3} - 23\frac{1}{3}\right) = 30$ cm

5



The diagram above shows the plan of a yard.
It is given that angle ODC = angle $AOB = \theta$, $OD = 7$ m and $OA = 1$ m.
AB and CD are each perpendicular to OC. A fence is to be built along AB, BC and CD.

(i) Show that $AB + BC + CD = (8\sin\theta + 6\cos\theta)$ m. [3]

$$\sin\theta = \frac{AB}{1} = AB$$
$$\cos\theta = \frac{OB}{1} = OB$$
$$\sin\theta = \frac{OC}{7}$$
$$OC = 7\sin\theta$$
$$\cos\theta = \frac{CD}{7}$$
$$CD = 7\cos\theta$$

$$AB + BC + CD = AB + (OC - OB) + CD$$
$$= \sin\theta + (7\sin\theta - \cos\theta) + 7\cos\theta$$
$$= (8\sin\theta + 6\cos\theta) \text{ m (shown)}$$

(ii) Express $AB + BC + CD$ in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [2]

$$AB + BC + CD = 8\sin\theta + 6\cos\theta$$
$$= R\sin(\theta + \alpha)$$
$$R = \sqrt{8^2 + 6^2} = 10$$
$$\tan\alpha = \frac{6}{8}$$
$$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$$
$$\approx 36.870°$$
$$\approx 36.9°$$
$$\therefore AB + BC + CD \approx 10\sin(\theta + 36.9°)$$

(iii) Explain why the length of the fence needed can never be 11 m. [1]

Since the maximum value of $\sin(\theta+36.9°)$ is 1,
the maximum value of $AB+BC+CD \approx 10\sin(\theta+36.9°) = 10\,(1) = 10$.
Therefore $AB+BC+CD$ can never be 11 m.

(iv) Find the values of θ for which the length of the fence is 8.5 m. [3]

$10\sin(\theta+36.870°) = 8.5$
$\sin(\theta+36.870°) = 0.85$
$\left.\begin{aligned}\text{Basic angle } &= \sin^{-1}(0.85)\\ &\approx 58.212°\end{aligned}\right\}$
Since $\sin(\theta+36.870°)$ is positive, θ is acute and $36.870° \le \theta+36.870° \le 126.870°$,
$\theta+36.870° \approx 58.212°, 180°-58.212°$
$\theta \approx 21.3°, 84.9°$

6 (a) Show that the second term in the expansion, in ascending powers of x, of $\left(2+\frac{x}{8}\right)^n$, is $n2^{n-4}x$, where n is a positive integer greater than 2 and find the third term in a similar form. [4]

$$\left(2+\frac{x}{8}\right)^n = 2^n + \binom{n}{1}2^{n-1}\left(\frac{x}{8}\right) + \binom{n}{2}2^{n-2}\left(\frac{x}{8}\right)^2 + \ldots$$

$$= 2^n + \binom{n}{1}2^{n-1}\left(\frac{x}{2^3}\right) + \binom{n}{2}2^{n-2}\left(\frac{x^2}{2^6}\right) + \ldots$$

$$= 2^n + n2^{n-1-3}x + \frac{n(n-1)}{2}2^{n-2-6}x^2 + \ldots$$

$$= 2^n + n2^{n-4}x + n(n-1)2^{n-9}x^2 + \ldots$$

Second term $= n2^{n-4}x$ (shown)

Third term $= n(n-1)2^{n-9}x^2$

(b) The first two terms in the expansion, in ascending powers of x, of $(1-x)\left(2+\frac{x}{8}\right)^n$ are $p+qx^2$, where p and q are constants.

(i) Show that the value of n is 16. [3]

$(1-x)\left(2+\frac{x}{8}\right)^n$

$=(1-x)\left[2^n+n2^{n-4}x+...\right]$

$=2^n+n2^{n-4}x-x2^n+...$

$=2^n+\left(n2^{n-4}-2^n\right)x+...$

Comparing coefficient of x:

$n2^{n-4}-2^n=0$

$n2^{n-4}=2^n$

$n=\frac{2^n}{2^{n-4}}$

$=16$ (shown)

(ii) Hence find the value of p and of q. [2]

$p=2^{16}=65\,536$

$(1-x)\left(2+\frac{x}{8}\right)^{16}$

$=(1-x)\left[2^{16}+(16)2^{12}x+16(15)2^7x^2+...\right]$

Comparing coefficient of x^2:

$q=16(15)2^7-(16)2^{12}$

$=-34816$

7 **(a)** The population of cheetahs, P, in n years, can be modelled by $P = ab^n$, where a and b are constants. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [3]

$$P = ab^n$$
$$\ln P = \ln(ab^n)$$
$$= \ln a + \ln b^n$$
$$\ln P = (\ln b)n + \ln a$$

When we plot **ln *P* against *n***,
a straight line graph can be drawn to represent the formula.

$\ln b =$ gradient of the line
$b = \mathrm{e}^{\text{gradient of the line}}$
$\ln a = \ln\text{-}P$ intercept
$a = \mathrm{e}^{\ln\text{-}P \text{ intercept}}$

Alternative Plot lg *P* against *n*

(b) Drone A moves along a horizontal straight line. Its displacement, s m, from a fixed point O, t seconds after it passes through O is recorded in the table below.

s	10	32	66	112
t	2	4	6	8

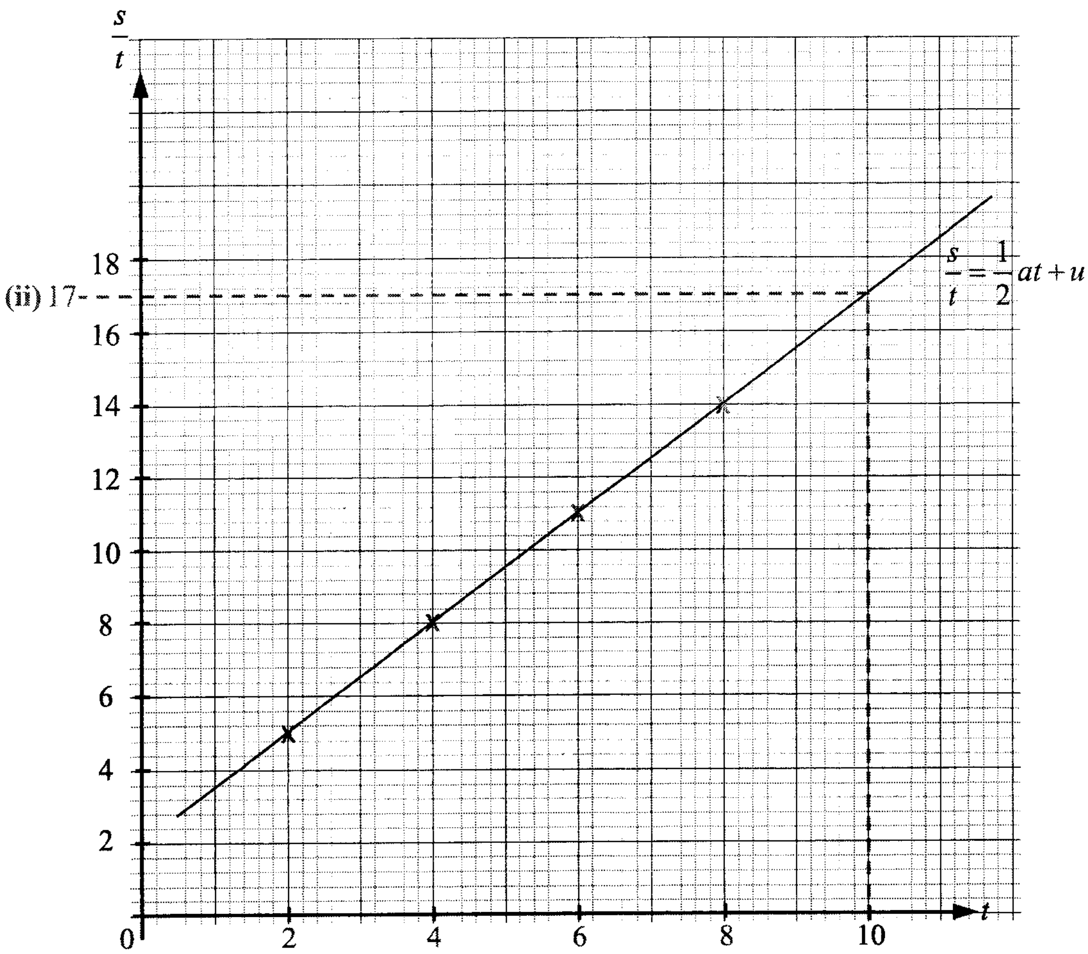
A physicist believed that these figures can be modelled by $s = ut + \frac{1}{2}at^2$, where u is the initial velocity of Drone A and a is its constant acceleration.

$$s = ut + \frac{1}{2}at^2$$
$$\frac{s}{t} = \frac{1}{2}at + u$$

Plot $\frac{s}{t}$ against t.

$\frac{s}{t}$	5	8	11	14
t	2	4	6	8

(i) Draw a straight line graph to show that the model is reasonable. [4]



(ii) Use your graph to estimate the displacement of Drone A when $t = 10$. [1]

When $t = 10$, $\frac{s}{t} = 17$

$s = 170$ m

(iii) Drone B moves along the same horizontal straight line as Drone A from O four seconds after Drone A. Its displacement, s m, from O, t seconds after Drone A passes through O can be modelled by $s = 3t^2 - 12t$. By using your graph in part **(i)**, explain how you can estimate when the drones will meet. [2]

$s = 3t^2 - 12t$

$\frac{s}{t} = 3t - 12$

Add the line $\frac{s}{t} = 3t - 12$ onto the graph in part **(i)**

The ***t*-coordinate** of the point of intersection of the two lines will be when the drones will meet.

8 **(a)** Show that the equation

$$(p+1)x^2+(p+3)x-(p+2)=0$$

has two real roots for all real values of p. [4]

$(p+1)x^2+(p+3)x-(p+2)=0$

$(p+1)x^2+(p+3)x+[-(p+2)]=0$

Discriminant:

$(p+3)^2-4(p+1)[-(p+2)]$

$=(p+3)^2+4(p+1)(p+2)$

$=p^2+6p+9+4p^2+12p+8$

$=5p^2+18p+17$

$=5\left(p^2+\frac{18}{5}p\right)+17$

$=5\left(p+\frac{9}{5}\right)^2-5\left(\frac{9}{5}\right)^2+17$

$=5\left(p+\frac{9}{5}\right)^2+\frac{4}{5}$

Since $\left(p+\frac{9}{5}\right)^2\geq 0$ for all real values of p,

Discriminant $=5\left(p+\frac{9}{5}\right)^2+\frac{4}{5}>0$ for all real values of p.

Thus the equation has two real roots for **all real values of p.**

(b) The equation of a curve is $y = 3x^2 - 5ax + 2a^2$, where a is a positive constant.

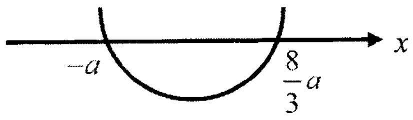
(i) Find, in terms of a, the set of values of x for which the curve lies below the line $y = 10a^2$ and represent this set on a number line. [4]

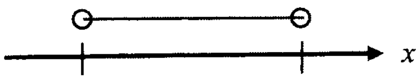
$3x^2 - 5ax + 2a^2 < 10a^2$

$3x^2 - 5ax - 8a^2 < 0$

$(3x - 8a)(x + a) < 0$

$-a < x < \frac{8}{3}a$





(ii) Find the value of a for which the curve touches the line $y = 1 - 3ax$. [3]

$3x^2 - 5ax + 2a^2 = 1 - 3ax$

$3x^2 - 2ax + 2a^2 - 1 = 0$

Discriminant = 0

$(-2a)^2 - 4(3)(2a^2 - 1) = 0$

$4a^2 - 24a^2 + 12 = 0$

$a^2 = \frac{3}{5}$

$a = \sqrt{\frac{3}{5}}$

$= 0.775$ (to 3sf)

9 The equation of a circle C, with centre O, is $x^2+y^2-4x-6y-5=0$.

(i) Find the coordinates of O and the exact radius of C. [3]

$x^2+y^2-4x-6y-5=0$

$2g=-4 \qquad 2f=-6 \qquad c=-5$

$g=-2 \qquad f=-3$

Coordinates of $O=(2,3)$

Radius of $C=\sqrt{(-2)^2+(-3)^2-(-5)}=\sqrt{18}=3\sqrt{2}$ units

OR

$x^2+y^2-4x-6y-5=0$

$\left(x^2-4x\right)+\left(y^2-6y\right)-5=0$

$\left[(x-2)^2-2^2\right]+\left[(y-3)^2-3^2\right]-5=0$

$(x-2)^2+(y-3)^2=18$

Coordinates of $O=(2,3)$

Radius of $C=\sqrt{18}=3\sqrt{2}$ units

The line l is a tangent to the circle at the point $P(5,6)$.

(ii) Find the equation of l. [3]

Gradient of $OP=\dfrac{6-3}{5-2}=\dfrac{3}{3}$

Gradient of $l=-1$

Equation of l:

$y-6=-(x-5)$

$y=-x+11$

(iii) Points A and B are on C such that AB is a diameter of C and is also parallel to l. Find the equation of AB. [2]

Equation of AB :

$y-3=-(x-2)$

$y=-x+5$

(iv) Hence find the coordinates of A and of B. [4]

$x^2+y^2-4x-6y-5=0$ ---(1)

$y=-x+5$ --- (2)

Substitute (2) into (1):

$x^2+(5-x)^2-4x-6(5-x)-5=0$

$x^2+25-10x+x^2-4x-30+6x-5=0$

$2x^2-8x-10=0$

$x^2-4x-5=0$

$(x+1)(x-5)=0$

$x=-1$ or $x=5$

From (2):

$y=6 \qquad y=0$

The coordinates of A and B are $(-1,6)$ and $(5,0)$.

Alternative

$y=-x+5$ ---(1)

$\sqrt{(x-2)^2+(y-3)^2}=\sqrt{18}$ ---(2)

Substitute (1) into (2),

$\sqrt{(x-2)^2+(-x+5-3)^2}=\sqrt{18}$

$(x-2)^2+(2-x)^2=18$

$2(x-2)^2=18$

$(x-2)^2=9$

$x-2=\pm 3$

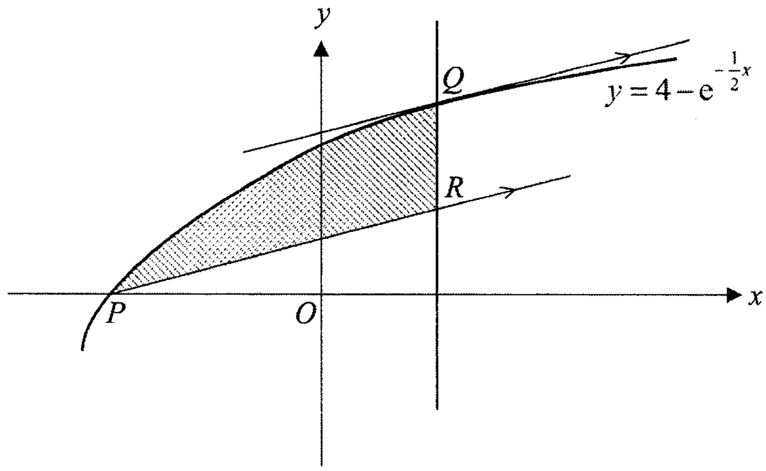
$x=-1$ or $x=5$

$y=-(-1)+5 \qquad y=-(5)+5$

$=6 \qquad =0$

The coordinates of A and B are $(-1,6)$ and $(5,0)$.

10



The diagram shows part of a curve with equation $y = 4 - e^{-\frac{1}{2}x}$ meeting the x-axis at the point P. A line $x = 2\ln 2$ intersects the curve at the point Q. R is a point on the line $x = 2\ln 2$ such that PR is parallel to the tangent to the curve at Q. Show that the area of the shaded region is $a(\ln 2)^2 + b\ln 2 - c$, where a, b and c are constants to be determined. [12]

$y = 4 - e^{-\frac{1}{2}x}$

At P, $4 - e^{-\frac{1}{2}x} = 0$

$$e^{-\frac{1}{2}x} = 4$$

$$-\frac{1}{2}x = \ln 4$$

$$x = -2\ln 4 = -4\ln 2$$

$\frac{dy}{dx} = \frac{1}{2}e^{-\frac{1}{2}x}$

When $x = 2\ln 2$, $\frac{dy}{dx} = \frac{1}{2}e^{-\frac{1}{2}(2\ln 2)} = \frac{1}{4}$

Equation of PR:

$y - 0 = \frac{1}{4}(x + 2\ln 4)$

$y = \frac{1}{4}x + \frac{1}{2}\ln 4$ or $y = \frac{1}{4}x + \ln 2$

When $x = 2\ln 2$, $y = \frac{1}{4}(2\ln 2) + \ln 2 = \frac{3}{2}\ln 2$

Alternative to find the y-coordinate of R Let $R(2\ln 2, y_R)$.

$\frac{y_R - 0}{2\ln 2 - (-4\ln 2)} = \frac{1}{4}$

$$y_R = \frac{6\ln 2}{4}$$

$$= \frac{3}{2}\ln 2$$

Continuation of working space for Question 10.

Area of shaded region

$$= \int_{-4\ln 2}^{2\ln 2} 4 - e^{-\frac{1}{2}x}\, dx - \frac{1}{2}(2\ln 2 + 4\ln 2)\left(\frac{3}{2}\ln 2\right)$$

$$= \left[4x - \frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}} \right]_{-4\ln 2}^{2\ln 2} - \frac{9}{2}(\ln 2)^2$$

$$= \left[4x + 2e^{-\frac{1}{2}x} \right]_{-4\ln 2}^{2\ln 2} - \frac{9}{2}(\ln 2)^2$$

$$= \left(4(2\ln 2) + 2e^{-\frac{1}{2}(2\ln 2)} \right) - \left(4(-4\ln 2) + 2e^{-\frac{1}{2}(-4\ln 2)} \right) - \frac{9}{2}(\ln 2)^2$$

$$= \left(-\frac{9}{2}(\ln 2)^2 + 24\ln 2 - 7 \right) \text{ units}^2$$

Alternative

Area of shaded region

$$= \int_{-4\ln 2}^{2\ln 2} \left(4 - e^{-\frac{1}{2}x}\right) - \left(\frac{1}{4}x + \ln 2\right) dx$$

$$= \left[4x - \frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}} \right]_{-4\ln 2}^{2\ln 2} - \left[\frac{1}{8}x^2 + (\ln 2)x \right]_{-4\ln 2}^{2\ln 2}$$

$$= \left[4x + 2e^{-\frac{1}{2}x} \right]_{-4\ln 2}^{2\ln 2} - \left[\left(\frac{1}{8}(2\ln 2)^2 + (\ln 2)(2\ln 2) \right) - \left(\frac{1}{8}(-4\ln 2)^2 + (\ln 2)(-4\ln 2) \right) \right]$$

$$= \left(4(2\ln 2) + 2e^{-\frac{1}{2}(2\ln 2)} \right) - \left(4(-4\ln 2) + 2e^{-\frac{1}{2}(-4\ln 2)} \right) - \frac{9}{2}(\ln 2)^2$$

$$= \left(-\frac{9}{2}(\ln 2)^2 + 24\ln 2 - 7 \right) \text{ units}^2$$

End of Paper