HOUGANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION / 2022
SECONDARY FOUR (EXPRESS)

CANDIDATE NAME: ______ CLASS: ______

CENTRE NUMBER: S ______ INDEX NUMBER: ______

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Friday 19 August 2022

2 hours 15mins

Candidates answer on the Question Paper

Instructions to students:

- Write your name, index number and class clearly in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use an HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- The use of an approved scientific calculator is expected, where appropriate.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- You are reminded of the need for clear presentation in your answers.

Information for pupils

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ______

/ 90

The Question Paper consists of 20 printed pages (including this cover page)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation :

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem :

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta ABC = \frac{1}{2} ab \sin C$$

1 The line $2y = 3x - 7$ meets the curve $y = x^2 + 3x - 8$ at two points A and B.
Find the distance between A and B. [4]

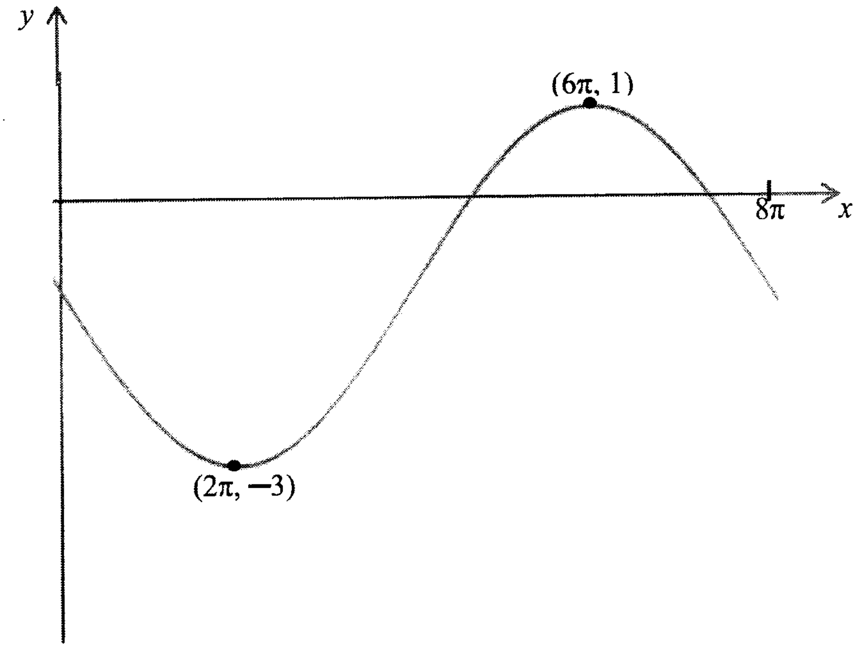
2 Express $\dfrac{7x+8}{(2x+1)(x-1)^2}$ in partial fractions. [5]

3 **Without using the calculator**, find the values of the integer a and b such that

$$\frac{4-\sqrt{3}}{a+b\sqrt{3}}=\frac{2+\sqrt{3}}{4-\sqrt{3}}.$$ [3]

4 The line of symmetry of a quadratic curve is $x=-2$ and the curve lies above the x-axis for all x. Given that the point $(-1,4)$ lies on the curve, find a possible equation of the curve in the form $y=a(x-h)^2+k$ where a, h, and k are integers. [4]

5



The diagram shows the curve $y = p\sin\dfrac{x}{q} + r$ for $0 \leq x \leq 8\pi$ radians. The curve has a minimum point at $(2\pi, -3)$ and a maximum point $(6\pi, 1)$.

(a) Show that $r = -1$. [1]

(b) Find the values of p and q. [2]

(c) Hence write down the equation of the curve. [1]

6 The function f is defined for all real values of x and is such that $f''(x) = 6x + 2$.

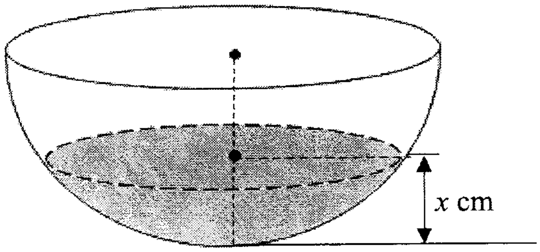
The gradient to the curve $y = f(x)$ at the point $(-1, 10)$ is 11.

(a) Find an expression for $f'(x)$. [3]

(b) Hence find the equation of the curve $y = f(x)$. [2]

(c) Determine whether the curve $y = f(x)$ have stationary point(s).
Explain with clear working. [3]

7



When the hemispherical bowl above contains water to a depth of x cm, the volume, V cm^3, of the water is given by $V = \frac{1}{3}\pi x^2(18 - x)$. The bowl is initially empty. After water has been poured into the bowl at a constant rate for 9 seconds, the depth of water is 4.5 cm.

(a) Find the constant rate of change of volume in terms of π. [3]

(b) Find the rate at which the water level is rising when the depth is 4.5cm. [4]

8 **(a)** Factorise $\sin^3 x + \cos^3 x$ completely. [1]

(b) Show that $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \dfrac{1}{2}\sin 2x$. [3]

(c) Hence solve the equation $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \dfrac{5}{4}$ for $0^\circ \le x \le 360^\circ$. [4]

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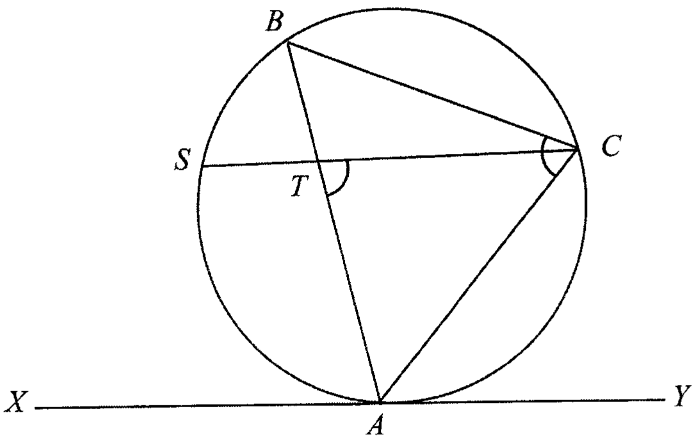


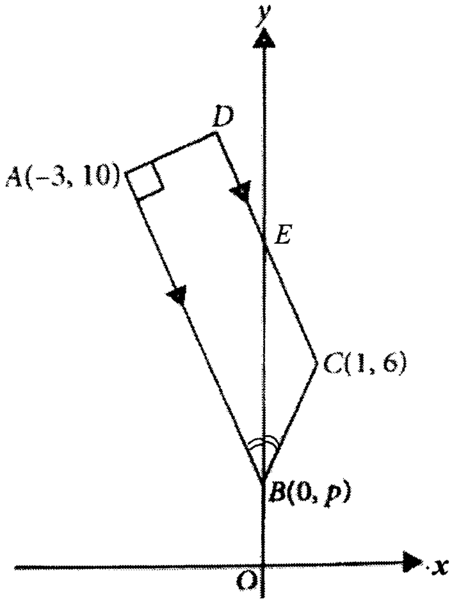
The diagram shows a point A on the circle and XAY is a tangent to the circle. Points S, B and C lie on the circle. The chords AB and SC intersect at T and angle ACB = angle ATC.

(a) Prove that triangles ABC and ACT are similar. [2]

(b) Show that $AC^2 - AT^2 = AT \times TB$. [3]

(c) Determine, with working, whether the lines *SC* and *XY* are parallel. [3]

10



The diagram shows a trapezium with vertices $A(-3,10)$, $B(0,p)$, $C(1,6)$ and D. The sides AB and DC are parallel and the angle BAD is a right angle. Angle ABE is equal to angle CBE.

(a) Express the gradients of lines AB and CB in terms of p and hence, or otherwise, show that $p = 4$. [3]

(b) Show that the equation of line AD is $y=\frac{1}{2}x+\frac{23}{2}$.

Hence find the coordinates of the point D. [5]

(c) Find the area of the trapezium $ABCD$. [2]

11 **(a)** The polynomial $P(x)=5x^3+ax^2-x+b$, where a and b are constants is exactly divisible by x^2+4x+3. Show that the value of $a=16$ and find the value of b. [4]

(b) Hence solve the equation $P(x)=0$. [3]

(c) Using a suitable substitution and your answers in **(b)**, solve the equation
$5x^6+ax^4-x^2+b=0$. [2]

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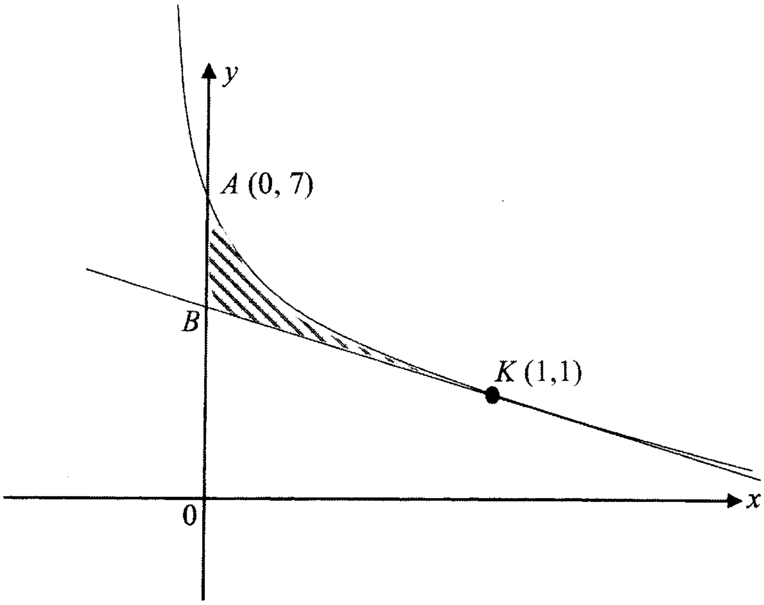


The diagram shows part of the curve $y = \dfrac{7}{6x+1}$ intersecting the y-axis at $A(0,7)$
The tangent to the curve at the point $K(1, 1)$ intersects the y-axis at B.

(a) Find the coordinates of B. [5]

(b) Find the area of the shaded region bounded by the curve, the tangent KB and the y-axis. [5]

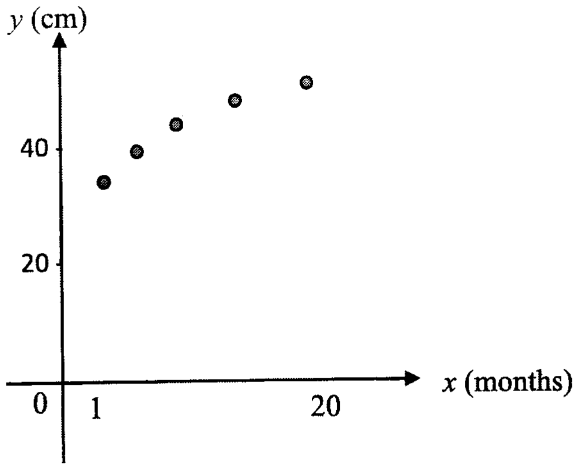
13 **(a)** Solve the equation $(\ln x)^2 + \dfrac{2}{\log_x e} = 3$. [4]

(b) It is given that $\lg p - \lg 2q = \lg(p + 2q)$.

(i) Express p in terms of q [2]

(ii) State the range of values of p and explain clearly why $0 < q < \frac{1}{2}$. [2]

(c) Mrs Tan decides to track the relationship between the age, x (in months) of her newborn baby girl and the circumference of her baby girl's head, y (in centimetres).
After plotting the data collected for 1[st], 6[th], 12[th], 18[th] and 20[th] month, the following graph was obtained.



Determine, with a reason, which of the 2 equations below is suitable to model the data plotted in the above diagram.

(A) $y = ae^{bx}$ (Exponential function)
(B) $y = a\ln x + b$ (Logarithmic function) [2]

End of paper

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SECONDARY FOUR (EXPRESS)

CANDIDATE NAME:		CLASS:	
CENTRE NUMBER:	S	INDEX NUMBER:	

ADDITIONAL MATHEMATICS 4049 / 02

Paper 2

Tuesday 23 August 2022
2 hours 15 minutes

Candidates answer on the Question Paper

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	90

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[Turn over

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where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$

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1 **(a)** Sketch the graph of $y = e^x + 2$. [2]

(b) Solve the equation $3 - e^{-x} = 2e^x$. [4]

[Turn over

2 The cubic polynomial f(x) is such that the coefficient of x^3 is -1 and the roots of f(x) = 0 are 1, k and k^2. It is given that f(x) has a remainder of -7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and explain that there are no other real values of k which satisfy this equation. [6]

3 **(i)** Given that $y = (x+2)\sqrt{x-1}$, show that $\frac{dy}{dx} = \frac{kx}{2\sqrt{x-1}}$ where k is constant. [4]

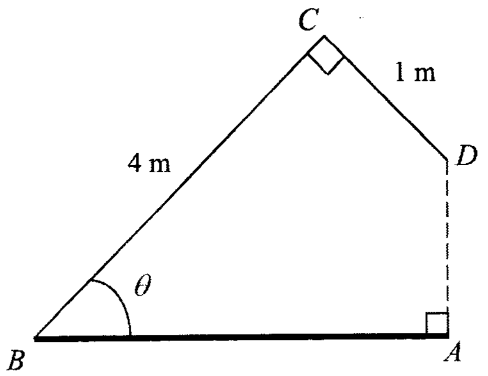
Hence

(ii) find the rate of change of x when $x = 2$, given that y is changing at a constant rate of 2 units per second, [2]

(iii) evaluate $\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$. [3]

[Turn over

4



The diagram above shows the side view of a bus stop shelter BCD such that $BC = 4$ m, $CD = 1$ m, angle $BCD = 90°$ and angle $CBA = \theta$. AB is a concrete pavement under the shelter such that DA is perpendicular to AB.

(i) Show that $AB = 4\cos\theta + \sin\theta$. [2]

(ii) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) State the maximum value of AB and find the corresponding value of θ when AB is maximum. [2]

(iv) Find the value of θ when $AB = 3$ m. [2]

[Turn over

5 **(a)** A curve has the equation $y = 2x^2 - 6x + c$, where c is a constant.
Find the value of c for which the line $y + 2x = 8$ is a tangent to the curve. [3]

(b) Represent the solution set of $3(x^2 - 5) > x - 1$ on the number line. [3]

(c) Find the greatest value of integer p for which $-2x^2 + x - p$ has real roots for all real values of x. [3]

6 **(i)** Expand and simplify $\left(\frac{1}{2}-2x\right)^5$ in ascending powers of x, up to the first 4 terms. [2]

(ii) Hence find the value of a if the coefficient of x^2 in the expansion of

$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ is $\frac{13}{2}$. [4]

(iii) Using the answer from part **(i)**, evaluate $(0.47)^5$ correct to 5 decimal places. [3]

[Turn over

7 The table shows experimental values of two variables, x and y.

x	0.5	1.0	1.5	2.0
y	15.9	19.1	23.4	30.2

It is known that x and y are related by the equation $y = 10 + Ab^x$.

(i) On **Pg 11**, draw the graph of $\lg(y-10)$ against x. [2]

(ii) Use your graph in **(i)** to estimate the value of A and of b. [4]

(ii) By drawing a suitable line on your graph, solve the equation $Ab^x = 10^{2x}$. [3]

8 A particle P moves in a straight line so that, t seconds after passing through a fixed point O, its velocity v m/s, is given by $v = 3t^2 + kt + 18$, where k is a constant. When $t = 1$, the acceleration of the particle is -9 m/s^2.

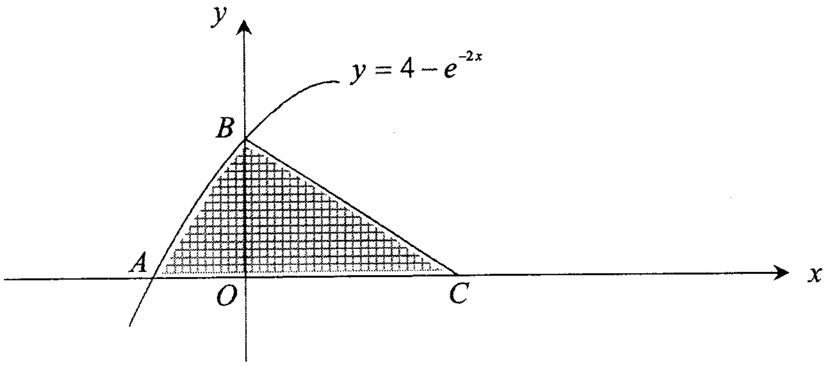
(i) Show that $k = -15$. [2]

(ii) Find the values of t for which particle P is instantaneously at rest. [2]

(iii) Find the total distance travelled by P in the first 3 seconds after passing through point O. [4]

[Turn over

9



The diagram shows part of the curve $y = 4 - e^{-2x}$ which crosses the axes at A and at B.

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region. [5]

[Turn over

10 A circle, C, has equation $x^2 + y^2 - 6x + 4y - 12 = 0$.

(i) Find the radius and the coordinates of the centre of C. [2]

The equation of the normal to the circle at the point A is $3y = m - 4x$.

(ii) Find the value of the constant m. [2]

The tangent to the circle at A cuts at the positive y-axis.

(iii) Use your answer in **(ii)** to show that A is (0, 2). [4]

(iv) B is a point on the circle. Given that the equation of tangent to the circle at B is parallel to the equation of the tangent to the circle at A, find the equation of tangent to the circle at B. [3]

End of Paper