Candidate Name _____	Class	Candidate Number

WESTWOOD SECONDARY SCHOOL

PRELIMINARY EXAMINATION

Secondary Four Express / Five Normal Academic

MATHEMATICS
Paper 2

4048/02
August 2021
2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

CALCULATOR
Brand : _____
Model : _____

For Examiner's Use		
Subtotal		
Presentation		
Accuracy		
Total deduction		

This question paper consists of 23 printed pages.

Setter : Mdm Sim Lay Ling

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

[Turn over

Answer all the questions.

1 (a) Simplify $\frac{1}{6x-4} \div \frac{x+1}{3x^2-5x+2}$.

Answer ..... [2]

(b) $\sqrt{2x^2+z} = y$

(i) Evaluate y when $x = -\frac{1}{2}$ and $z = \frac{5}{2}$.

Answer y = ... [2]

(ii) Express x in terms of y and z.

Answer x = ... [3]

[Turn over

(c) (i) Express $x^2 - 5x - 3$ in the form $(x+h)^2 + k$.

Answer ..... [2]

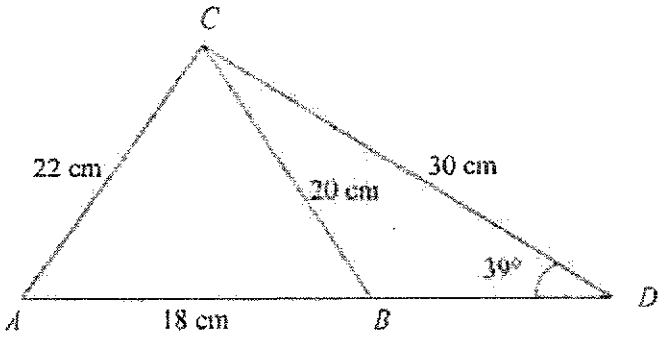
(ii) Hence, state the coordinates of the minimum point of the graph of $x^2 - 5x - 3$.

Answer (....., ...) [1]

(d) Solve $\frac{1}{5x+1} + \frac{3}{3x-1} = 2$.

Answer $x =$ ..... or ..... [3]

2 (a) The diagram shows a triangle ABC with AB produced to D.
$AB = 18$ cm, $BC = 20$ cm, $AC = 22$ cm, $DC = 30$ cm and angle $ADC = 39°$.



Calculate

(i) angle CAB,

Answer ..... ° [3]

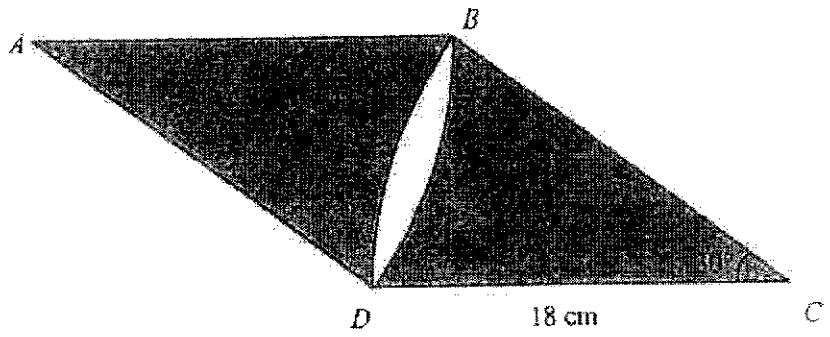
(ii) angle CBD,

Answer ..... ° [2]

(iii) the area of triangle ACD.

Answer ..... cm^2 [2]

(b) The diagram shows a rhombus $ABCD$ with sides of length of 18 cm.
CBD is a sector of the circle, centre C.
ABD is a sector of the circle, centre A.
Angle $BCD = 30°$.



(i) Change 30° to radians, leaving your answer in terms of π.

Answer ..... [1]

(ii) Calculate the shaded area, leaving your answer in the form $a + b\pi$, where a and b are integers.

Answer ..... [3]

3 (a) The table shows the currency exchange rate between Singapore dollars ($) and Thai Baht (THB) at a money changer.

Unit	Currency	Selling	Buying
100	Thai Baht	4.415	4.088

Before Adam went on a trip to Thailand, he went to the money changer.

(i) He changed $883 into Thai Baht at the money changer.
Calculate how much Thai Baht he received.

Answer ..... THB [1]

(ii) When Adam returned to Singapore, he had 2000 THB left.
He went to the money changer to change 2000 THB to Singapore dollars.
Calculate how much he received.

Answer $ ..... [1]

(b) The price of a watch at the end of 2019 was 2% lower than at the end of 2018.
The price of the watch at the end of 2020 was 5% higher than at the end of 2019.

Calculate the price of the watch at the end of 2020 as a percentage of the price at the end of 2018.

Answer ..... % [2]

(c) Water demand in Singapore is currently about 430 million gallons a day with homes (domestic) consuming 45% and the non-domestic sector taking up the rest.

Given that 250 litres is equal to 55 gallons, find the daily amount of water consumption in the domestic sector in litres.
Give your answer in standard form.

Answer ..... litres [2]

(d) 1.08×10^{-7} metres can be written as k micrometres.
Find k.

Answer $k=$ ..... [1]

4 The first four terms in a sequence of numbers are given below.

$T_1 = 1\times2$	$=2\times1$	$=2\times1$	$=2$
$T_2 = 1\times2+3\times2$	$=2\times(1+3)$	$=2\times4$	$=8$
$T_3 = 1\times2+3\times2+5\times2$	$=2\times(1+3+5)$	$=2\times9$	$=18$
$T_4 = 1\times2+3\times2+5\times2+7\times2$	$=2\times(1+3+5+7)$	$=2\times16$	$=32$
$T_5 =$			

[1]

(i) Complete the row in the above table for T_5.

(ii) Find an expression, in terms of n, for T_n.

Answer ..... [1]

(iii) Evaluate T_{500}.

Answer ..... [1]

(iv) T_p and T_{p+1} are consecutive terms in the sequence.
Find and simplify an expression, in terms, of p, for $T_{p+1} - T_p$.

Answer ..... [3]

(v) Explain why two consecutive terms of the sequence cannot have a difference of 4.

Answer .....

..... [1]

(vi) The first four terms of another sequence are 8, 18, 32 and 50.
By using (i) and (ii) or otherwise, write down an expression, in terms of n, for the n^{th} term, T_n of this sequence.

Answer ..... [1]

5 The variables x and y are connected by the equation $y = \frac{x^2}{3} + \frac{3}{x} - 4$.

Some corresponding values of x and y, correct to two decimal places, are given in the table below.

x	0.5	1	1.5	2	3	4	5	6
y	2.08	−0.67	p	−1.17	0	2.08	4.93	8.5

(a) Find the value of p.

Answer ..... [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 6$ and a vertical y-axis for $-2 \le y \le 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions to the equation $\frac{x^2}{3} + \frac{3}{x} - 5 = 0$ in the range $0 \le x \le 6$.

Answer $x =$ ..... [2]

(d) By drawing a tangent, find the gradient of the curve at $(4, 2.08)$.

Answer ..... [2]

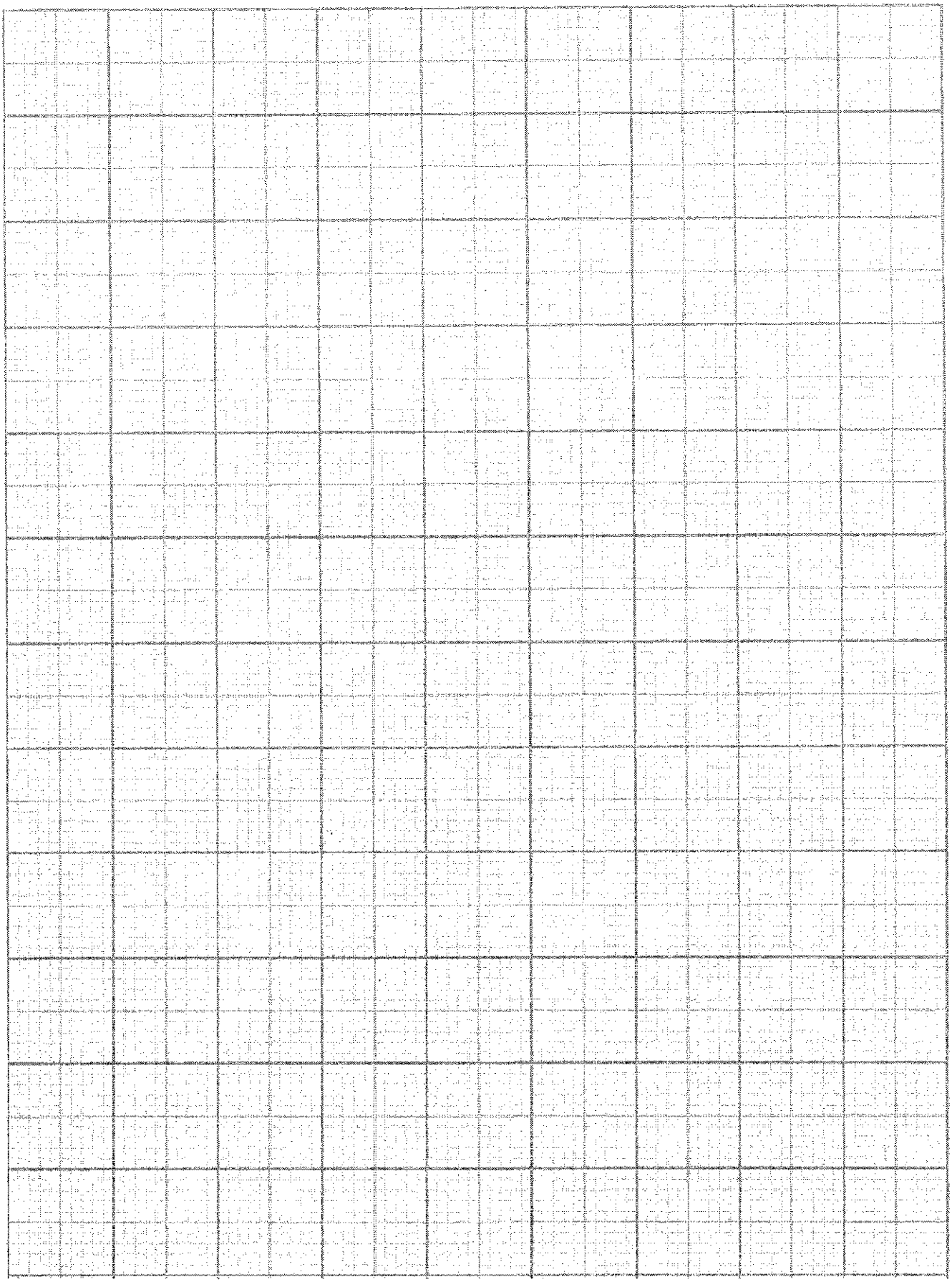
(e) (i) On the same axes, draw the line with gradient $\frac{4}{3}$ that passes through the point with coordinates $(3, 2)$. [2]

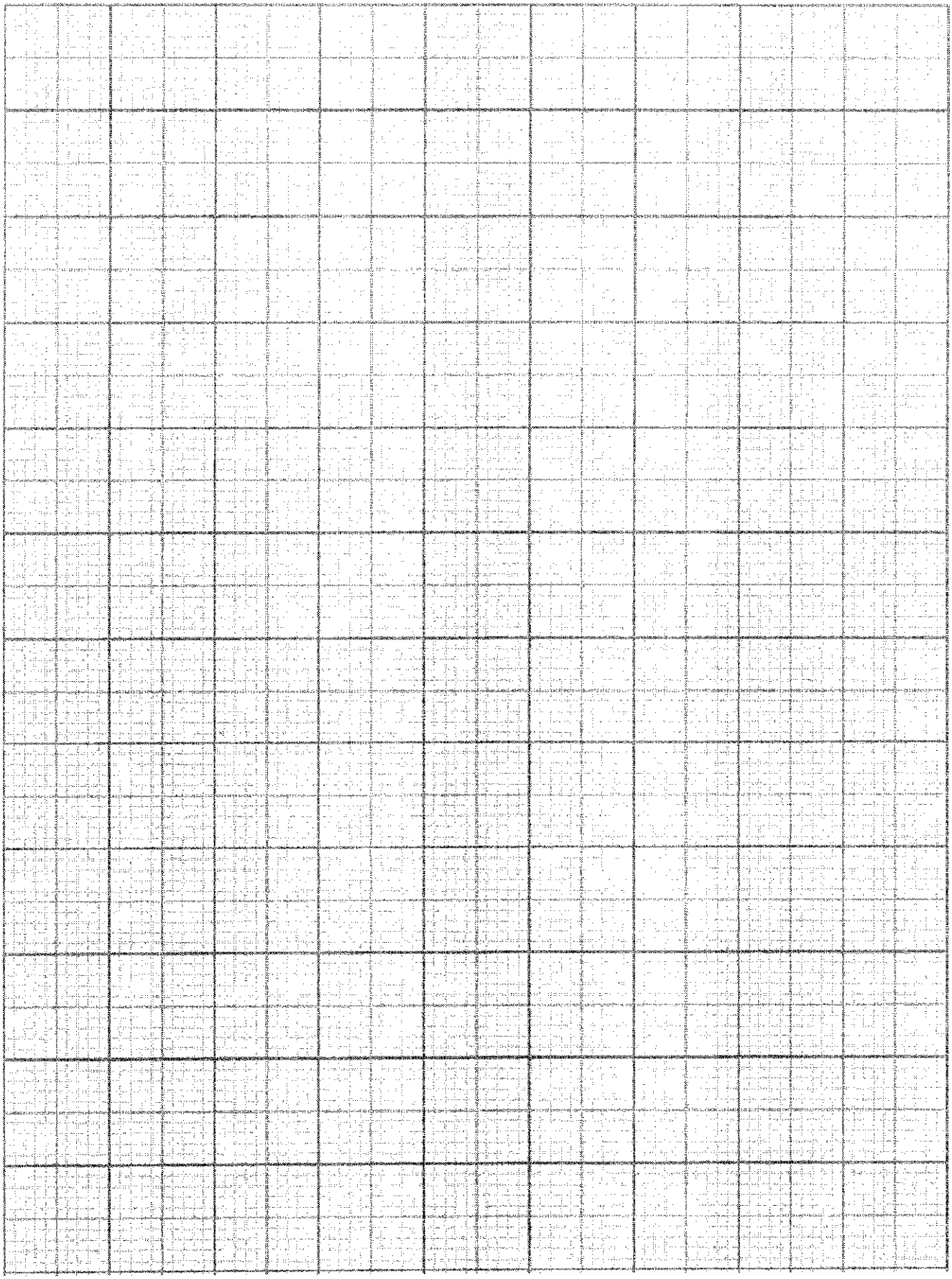
(ii) Write down the equation of this line.

Answer ..... [1]

(iii) Write down the coordinates of the points where the line intersects the curve in the range $0 \le x \le 6$.

Answer ..... [2]



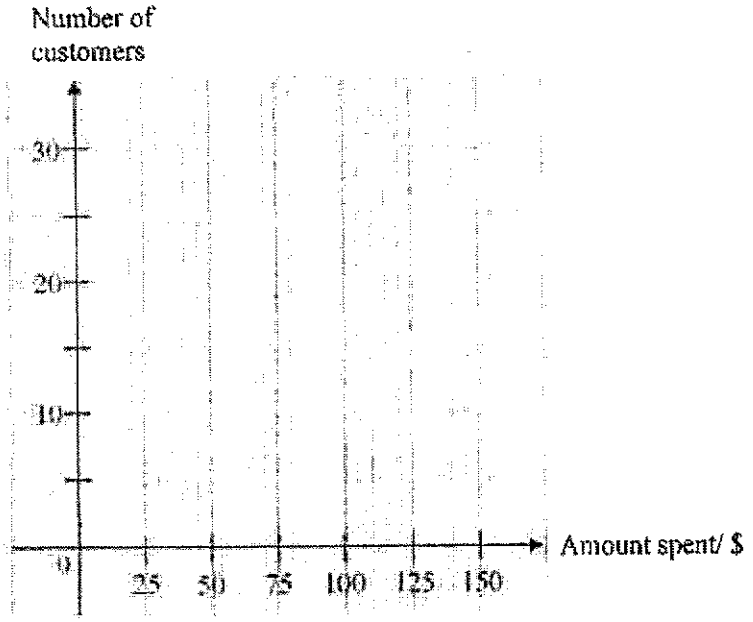


6 (a) A survey was carried out to find the amount of money spent by 100 customers in a supermarket.

This information was grouped as follows.

Amount spent (in \$$x$)	Number of customers
$0 < x \leq 25$	13
$25 < x \leq 50$	23
$50 < x \leq 75$	20
$75 < x \leq 100$	17
$100 < x \leq 125$	11
$125 < x \leq 150$	16
Total	100

(i) Complete the following histogram to represent the results of the survey.



[2]

(ii) Write down the median class.

Answer ..... $< x \leq$ ..... [1]

(iii) Write down the modal class.

Answer ..... $< x \leq$ ..... [1]

(iv) Write down an advantage of using a histogram to present the results of the survey.

Answer .....

..... [1]

(b) Jane has ten coins in a bag.
There are four 20¢ coins, three 50¢ coins and three $1 coins.
She took two coins from the bag without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

[2]

Find, as a fraction in its simplest form, the probability that,

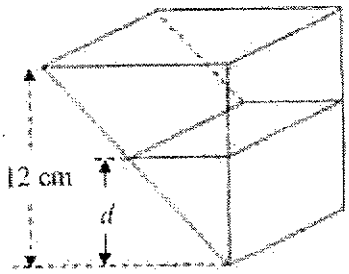
(ii) the two coins have different denominations,

Answer [1]

(iii) the total value of the two coins is more than a dollar.

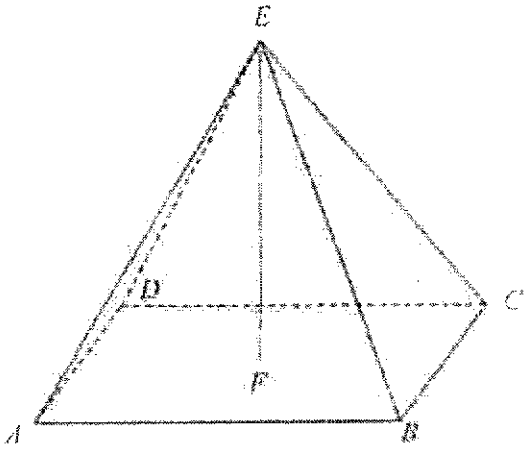
Answer [1]

7 **(a)** A container is a prism with a cross section in the shape of a right-angled triangle.
The container has a height of 12 cm.
Water is poured into the empty container at a constant rate.
It takes 8 minutes to fill the container.
After t minutes the depth of the water is d cm.
Find the value of t when $d = 6$ cm.



Answer $t =$ ..... [2]

(b) The diagram shows a pyramid with rectangular base $ABCD$ and vertex E.
$AB = 16$ cm, $BC = 12$ cm, $AE = BE = CE = DE$ and the height of the pyramid is 24 cm.



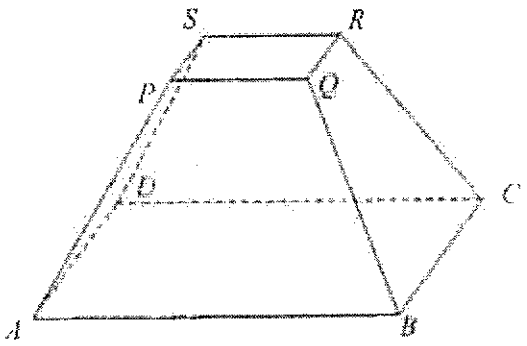
(i) Find the length of BE.

Answer ..... cm [3]

(ii) Calculate the angle DEB.

Answer ..... ° [2]

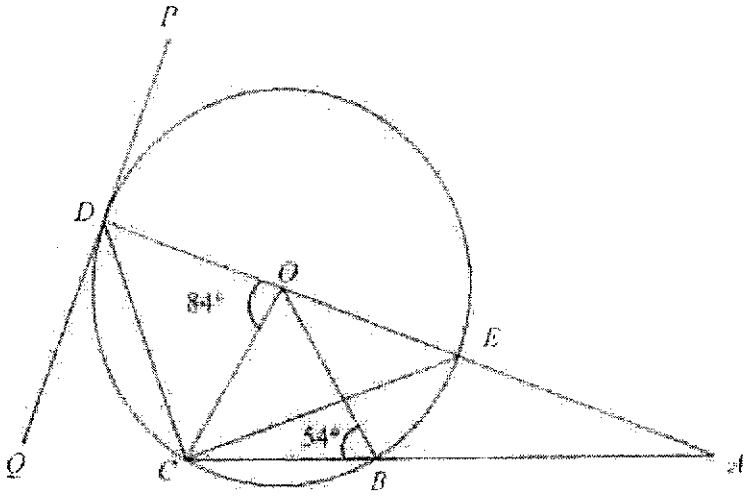
(iii) A smaller, similar pyramid of volume 24 cm^3 is removed from the top of the original pyramid.
The diagram below shows the solid remaining.



Find the total surface area of the remaining solid.

Answer ..... cm^2 [5]

8 (a)



O is the centre of the circle passing through B, E, D and C.
Angle $DOC = 84°$ and angle $CBO = 54°$.
PQ is a tangent to the circle at D.

Find, giving reasons for each answer,

(i) angle BAE,

Answer ..... [2]

(ii) angle QDC,

Answer ..... [2]

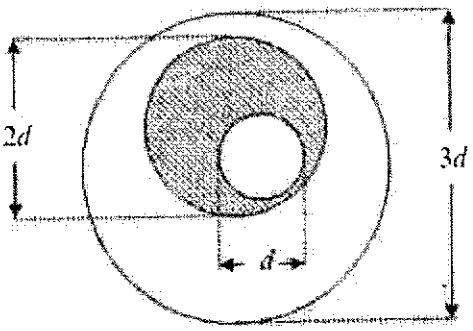
(iii) angle ECB.

Answer ..... ° [2]

(iv) angle CBE.

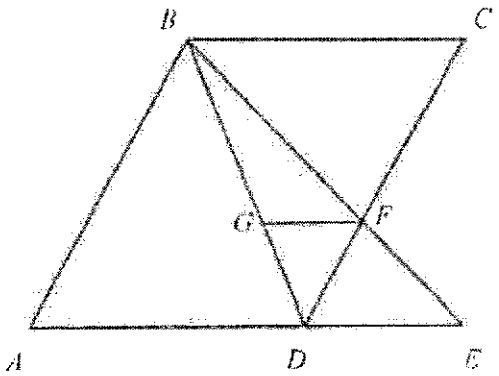
Answer ..... ° [1]

(b) The diagram shows 3 circles with diameters d cm, $2d$ cm and $3d$ cm respectively. Find, in terms of π and d, the **difference** in area between the unshaded region and the shaded region.



Answer ..... cm^2 [3]

9 $ABCD$ is a parallelogram and G is a point on BD.
The line BF produced meets DC at F and AD produced at E.
GF is parallel to AE.



(a) Show that triangles BCF and EDF are similar.

Answer .....

.....

.....

.....

..... [2]

(b) Name a triangle that is similar to triangle BDE.

Answer ..... [1]

(c) Given that $AD = 2DE$, find

(i) $\frac{DF}{AB}$.

Answer ..... [1]

(ii) $\frac{GF}{AE}$.

Answer ..... [2]

(iii) $\frac{\text{Area of } \Delta EDF}{\text{Area of } ABCD}$.

Answer ..... [3]

10 Joe is interested to buy a new car.
He has shortlisted 2 cars, which he intends to drive for 10 years.
The following tables contain specifications of the two cars, interest rates for car loans as well as the applicable road tax.

Specifications of cars

Car	Model *A*	Model *B*
Engine Capacity (cc)	1498	1691
Fuel Type	Petrol	Petrol Electric
Fuel consumption (km/L)	26.3	15.1
Fuel tank Capacity (L)	40	50
Cost	$104 999	$87 999
Annual maintenance fee	$463	$389
Simple interest rate for car loan per annum	1.88%	2.18%

Road Tax Formula

Engine Capacity (EC)	**Road Tax Formula (per annum)**
EC ≤ 600 cc	S$400 × 0.782
600 cc < EC ≤ 1000 cc	[S$400 + 0.25 × (EC − 600)] × 0.782
1000 cc < EC ≤ 1600 cc	[S$500 + 0.75 × (EC − 1000)] × 0.782
1600 cc < EC ≤ 3000 cc	[S$950 + 1.5 × (EC − 1600)] × 0.782

(a) Calculate the difference in road tax payable between Model *A* and Model *B*.

Answer $ ..... [2]

Joe estimates that he will travel approximately 20 000 km per year and petrol prices are at $2.57 per litre.

Joe owns a credit card which gives him a discount of 21% off petrol prices.
He will pay $35 000 cash for the downpayment for the car and take a car loan for 7 years.

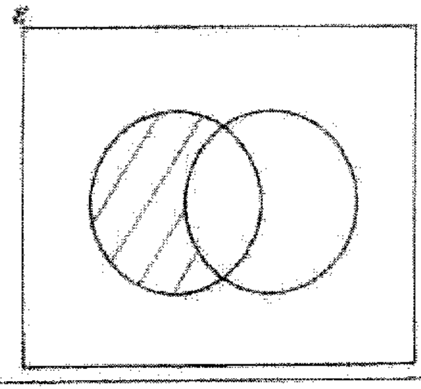
(b) Considering the above factors of owning the two cars, Joe thinks that he will be paying less if he buys Model *A* instead of Model *B*.
Is Joe correct?

[6]

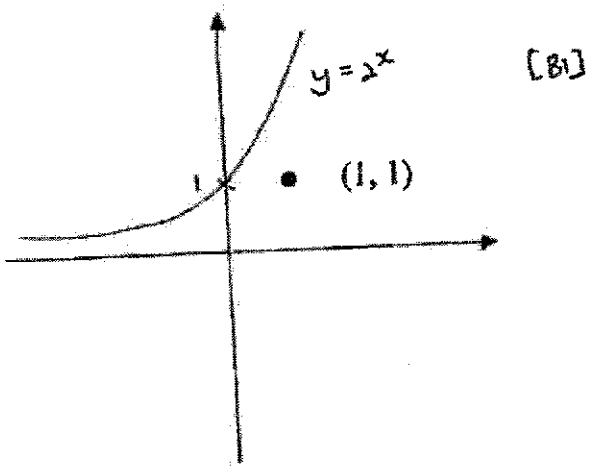
END OF PAPER

2021 4E5N EMath Prelim P1 Marking Scheme

	Solution
1.	3
2(a)	$2^2 \times 3^3 \times 7$
2(b)	784
3.	1. The title is biased ➔ It does not allow reader to make his/her own judgement OR 2. Unequal spacing of the data/ Inconsistent Scale/ Not all consecutive data are shown ➔ It exaggerates the differences between the data OR 3. Horizontal axis does not start from zero ➔ This exaggerates the differences between the data
4.	$S_{12} - S_{11}$ $= 4(12)^2 + 15(12) - [4(11)^2 + 15(11)]$ $= 756 - 649$ $= 107$
5.	$\frac{2-3(1-x)}{(x-1)^2}$ $= \frac{2-3+3x}{(x-1)^2}$ $= \frac{3x-1}{(x-1)^2}$
6.	$y = k\sqrt[3]{x}$ $\sqrt[3]{343}k - \sqrt[3]{64}k = 4$ $7k - 4k = 4$ $k = \frac{4}{3}$ $y = \frac{4}{3}\sqrt[3]{x}$
7(a)	$\frac{243}{x^{10}}$
7(b)	$2^{3n} + 2^{3n} = 2^{2+n}$ $2^{3n+1} = 2^{2+n}$ $3n + 1 = 2 + n$ $n = 0.5$
8.	$\frac{70}{100} \times 800\,000 = \$560\,000$ $560\,000\left(1 + \frac{r}{100}\right)^{30} = 1\,209\,508$ $r = 2.60$ (3 s.f)
9(a)	Diagram I

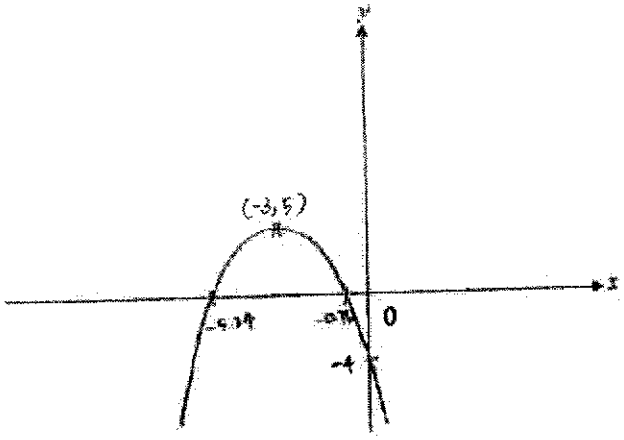
9(b)	
9(c)	10%
10(a)	( ) $\frac{1}{6}t - 3$)
10(b)	$3y(8x-5)-(8x-5)$ $=(8x-5)(3y-1)$
	$\frac{\angle DOL}{360} \times 2\pi(32) = 43$ $\angle DOL = 76.991°$ $\angle DLO = \frac{180-76.991}{2}$ $= 51.5045°$ $\angle OLN = 90°$ Bearing $= 90 - 51.5045$ $= 038.5°$ (1 d.p)
	77.7 (3 s.f)
	19.7 (3 [illegible]f)
	The answers are estimates the exact value.
13(a)	1 : 50 000
13(b)	1 cm : 500 m 1 cm^2 : 250 000 m^2 $\frac{50\,000}{250\,000} \times 1 = 0.2$ cm^2
14(a)	$\angle ECH = \angle AGD$ (given) $AG = EC$ (given) $\angle CHE = \angle ADG$ (opp $\angle s$ in a parallelogram) $\Delta ADG \equiv \Delta EHC$ (AAS)
14(b)	ABH & ADG or CBD & CHE or GHF & GAD & other acceptable answers

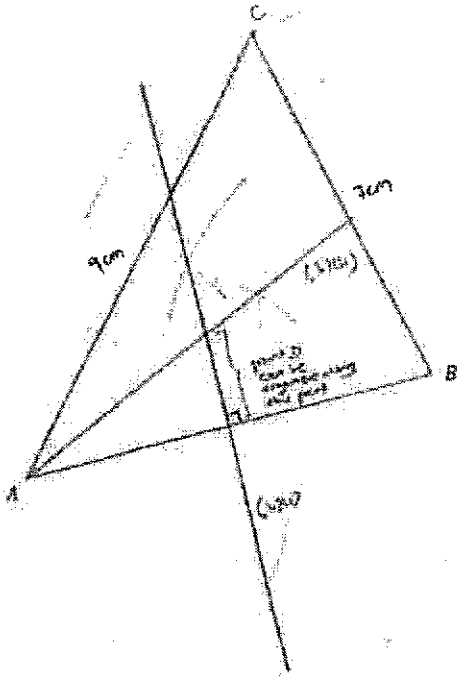
15(a)

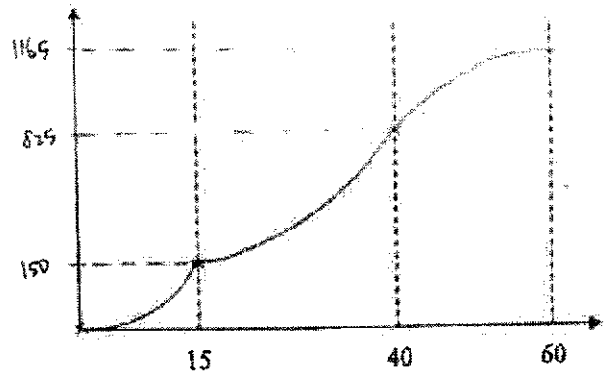


[B1]

15(b)



16.	 (a) [B1] (b)(i) [B1] (b)(ii) [B1] (c) [B1]
17(a)	$\begin{pmatrix} 0.0012 \\ 0.0025 \\ 0.0011 \\ 0.12 \end{pmatrix}$
17(b)	$\begin{pmatrix} 1.63 \\ 1.73 \\ 1.86 \end{pmatrix}$
17(c)	$\begin{pmatrix} 1.2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.95 \end{pmatrix}$
17(d)	It represents the total amount spent to make all three types of cake in dollars.
18(a)	36
18(b)	54
18(c)	$\frac{(10-2)\times 180}{10}$ $= 144$
19(a)	$\frac{1}{12} - \frac{1}{20} = \frac{1}{30}$ 30 days
19(b)	Let d be distance from house to park. $2d \div \left(\frac{d}{u} + \frac{d}{v}\right)$ $= 2d \div \frac{dv+du}{uv}$ $= 2d \times \frac{uv}{d(u+v)}$ $= \frac{2uv}{u+v}$
20(ai)	$y = 1$
20(aii)	4.47 units (3 s.f)
20(aiii)	(2,8)

20(b)	$\frac{1}{2} \times 4 \times (CH + 6) = \frac{1}{2} \times 6 \times 4 \times 2$ $CH = 6$ units $a = -4 - 6 = -10$
21(a)	$1\frac{1}{2}$ m/s^2
21(b)	$\frac{1}{2} \times 15 \times 20 + \frac{1}{2} \times 25 \times (v + 20) = 825$ $v = 34$
21(c)	Distance (m)  Time (s)
22(a)	$-9 < 2x - 1$ & $2x - 1 \leq 15$ $-8 < 2x$ & $2x \leq 16$ $-4 < x$ & $x \leq 8$ $-4 < x \leq 8$
22(b)	$3y - (y - 5) = 11$ $3y - y + 5 = 11$ $y = 3$ Subst $y = 3$: $x = 3 - 5 = -2$ $\therefore x = -2, y = 3$
22(c)	Infinite number of solutions Because $2x + 3y = 2$ and $x + 1.5y = 1$ are the same line geometrically, hence, they intersect at infinite number of points.
23(a)	$= \frac{2}{}$ $30 = 20$ cm
23(b)	$\frac{4}{9} \times 27 = \$12$
23(c)	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$ $\frac{}{22} \times 5400 = 1600$ cm^3
24(ai)	33 marks
24(aii)	$37 - 26$ $= 11$ marks
24(aiii)	32 marks
24(aiv)	30.5 marks
24(b)	Geography test was more difficult because the median marks for Geography is lower than that of History.

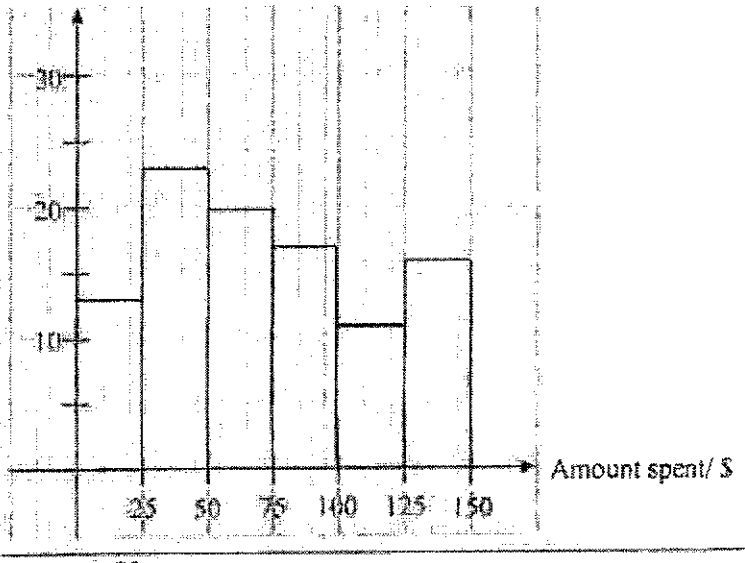
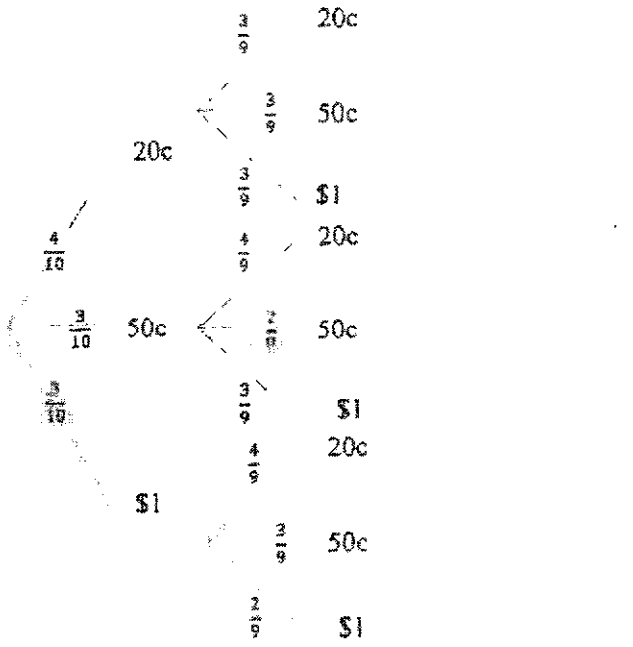
	$(x-1)(3x-2)$
	$\sqrt{(2x-\frac{1}{2})^2+\frac{3}{2}}=x$

$$\frac{22^2+18^2-30^2}{2(22)(18)}$$

Perpendicular height of rhombus

Price in 2019

3ci	$\frac{430\times10^{11}}{55}\times250=1954545455$ $45\%\times1954545455$ $=8.795\times10^{8}$ (4 s.f.) $=8.80\times10^{8}$ (3 s.f.)
3cii	$1.08\times10^{-7}\text{m}=0.108\mu\text{m}$
4i	$T_5=1\times2+3\times2+5\times2+7\times2+9\times2$ $=2\times(1+3+5+7+9)$ $=2\times25$ $=50$
4ii	$T_n=2n^2$
4iii	$T_{500}=500000$
4iv	$T_{n+1}-T_n$ $=2(n+1)^2-2n^2$ $=2(n^2+2n+1-n^2)$ $=2(2n+1)$ $=4n+2$
4v	When $4n+2=4$ $n=\frac{1}{2}$ n is not an integer
5vi	$T_n=2(n+1)^3$
5a	-1.25
5b	Drawing of graph
5c	0.55 to 0.65, 3.5 to 3.6
5d	1.98 to 2.98
5ci	Drawing of the line
5eii	$y=\frac{4}{3}x-2$
5eiii	(1, –0.67),(4.9, 4.4)

6ai	
	$50 < x \leq 75$
6aiii	$25 < x \leq 50$
6aiv	It can display a large set of grouped data clearly.
	

6bii	$1-\frac{4}{10}\times\frac{3}{9}-\frac{3}{10}\times\frac{2}{9}-\frac{3}{10}\times\frac{2}{9}$ $=\frac{11}{15}$
6biii	$\frac{4}{10}\times\frac{3}{9}+\frac{3}{10}\times\frac{3}{9}+\frac{3}{10}$ $=\frac{8}{15}$
7bi	
7bii	$\tan\angle FEB=\frac{10}{24}$ $\angle DEB=2\times\tan^{-1}(\frac{10}{24}$ $=45.2°$

7biii	Volume of original pyramid $= \frac{1}{3} \times 16 \times 12 \times 24$ $= 1536$ $(\frac{l_1}{l_2})^3 = \frac{24}{1536}$ $\frac{l_1}{l_2} = \frac{1}{4}$ Perpendicular Height of face $AEB = \sqrt{26^2 - 8^2}$ $= \sqrt{612}$ Perpendicular Height of face $BEC = \sqrt{26^2 - 6^2}$ $= \sqrt{640}$ Area of 4 triangular faces of original pyramid $= 2 \times \frac{1}{2} \times 16 \times \sqrt{612} + 2 \times \frac{1}{2} \times 12 \times \sqrt{640}$ Area of 4 triangular faces of smaller pyramid $= \frac{1}{16} \times$ Area of 4 triangular faces of original pyramid Area of 4 trapeziums $= \frac{15}{16} \times$ Area of 4 triangular faces of original pyramid Total surface area $= \frac{15}{16} \times (2 \times \frac{1}{2} \times 16 \times \sqrt{612} + 2 \times \frac{1}{2} \times 12 \times \sqrt{640}) + 4 \times 3 + 16 \times 12$ $= 860 \text{cm}^2$
8ai	$OCB = 54°$ (base $\angle$ of isos. Δ) $BAE = 84° - 54°$ (ext. $\angle$ of Δ) $= 30°$
8aii	$ODC = \frac{180° - 84°}{2}$ (base $\angle$ of isos Δ) $= 48°$ $QDC = 90° - 48°$ (tan $\perp$ rad) $= 42°$
8aiii	$DEC = \frac{84°}{2}$ ($\angle$ at centre $= 2 \times \angle$ at circumference) $= 42°$ $ECB = 54° - 42°$ (base $\angle$ of isos Δ) $= 12°$

8aiv	$CBE = 180° - 48°$ (∠in opp. segment) $= 132°$
8b	Shaded Region $= \pi d^2 - \pi(\frac{d}{2})^2$ $= \frac{3\pi d^2}{4}$ Unshaded Region $= \pi(\frac{3d}{2})^2 - \frac{3\pi d^2}{4}$ $= \frac{9\pi d^2}{4} - \frac{3\pi d^2}{4}$ $= \frac{3\pi d^2}{2}$ Difference between shaded and Unshaded Region $= \frac{3\pi d^2}{2} - \frac{3\pi d^2}{4}$ $= \frac{3\pi d^2}{4}$
9ai	$\angle BCF = \angle EDF$ (Alt. ∠s) $\angle BFC = \angle EFD$ (Vert. opp. ∠s) Triangles BCF and EDF are similar by AA similarity test.
9aii	Triangle BGF
9bi	$\frac{DF}{AB} = \frac{1}{3}$
9bii	$\frac{GF}{AE} = \frac{GF}{DE} \times \frac{DE}{AE}$ $= \frac{2}{3} \times \frac{1}{3}$ $= \frac{2}{9}$

9biii	$\frac{\text{Area of } \Delta EDF}{\text{Area of } ABCD} = \frac{\text{Area of } \Delta EDF}{\text{Area of } ABCD}$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } \Delta EAB} = \left(\frac{1}{3}\right)^2$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } \Delta EAB} = \frac{1}{9}$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } ABFD} = \frac{1}{8}$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } \Delta BCF} = \left(\frac{1}{2}\right)^2$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } \Delta BCF} = \frac{1}{4}$ $\frac{\text{Area of } \Delta EDF}{\text{Area of } ABCD} = \frac{1}{12}$
10a	$[500+0.75\times(1498-1000)]\times 0.782$ $=683.077$ $[950+1.5\times(1691-1600)]\times 0.782$ $=849.643$ $849.643-683.077$ $=166.566$ $=166.57$
10b	**Model A** $104999-35000=69999$ $\text{interest} = \frac{69999\times 1.88\times 7}{100}$ $=9211.8684$ $104999+9211.8684=114210.8684$ petrol $\frac{20000}{26.3}\times 2.57\times 79\% = 1543.95$ $1543.95\times 10 = 15439.50$ Total cost $=114210.8684+15439.50+683.077\times 10+463\times 10$ $=141111.1821$ **Model B**

	$87999 - 35000 = 52999$ $\text{Interest} = \frac{52999 \times 2.18 \times 7}{100}$ $= 8087.6474$ $87999 + 8087.6474 = 96086.6474$ petrol $\frac{20000}{15.1} \times 2.57 \times 79\% = 2689.139073$ $2689.139073 \times 10 = 26891.39073$ Total cost $= 96086.6474 + 26891.39073 + 849.643 \times 10 + 389 \times 10$ $= 135364.4681$ Joe is wrong. The total cost of owning Model A is higher than Model B.