Name: ____________ Class ____ Index No ____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS	**4048/1**
Paper 1	Date: 18 Aug 2021
Candidates answer on the question paper.	Time: 1010 – 1210
	Duration: 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper has a total of 21 pages.

Setter: Chong Lin Lin **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where }\theta\text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where }\theta\text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Expand and simplify $2x-3(2x-5)$.

Answer ______________________ [1]

(b) Factorise completely $x^2-y^2-2x+2y$.

Answer [2]

2 n is a positive integer. Explain whether, for all n, $(7n+2)^2-(7n-1)^2$ is a multiple of 3.

Answer

[3]

3 Simplify $\frac{a^{-7}}{ab^0} \div \left(\frac{1}{a^3}\right)^{-2}$.

Answer [3]

4 Solve the equation $3^{3x} \times 125^x = 225$.

Answwe [3]

5 Solve the inequality $x+7<\frac{17+3x}{2}\leq 15-3x$ and show your solution on a number line.

Answer

[3]

6 Find the largest rational number $p+q^2$ if $-1<p\leq 1.5$ and $-3\leq q\leq 2$.

Answer _ [2]

7 Two integers S and T are written as products of their prime factors where p is a prime number.

$$S = 3^2 \times 5 \times p$$
$$T = 2^3 \times 3 \times p^2$$

Lcm .

(a) Express the smallest integer, which is divisible by S and T, in terms of p and as a product of its prime factors.

Answer [1]

(b) Find the value of p if

(i) S is a perfect square.

Answer ______________________ [1]

(ii) the highest common factor of S and 525 is 105.

Answer ______________________ [2]

8 The number of boys in a camp is 500, correct to the nearest hundred. State the maximum number of boys in the camp.

Answer [1]

9 The smallest of a set of five number is an even number and it is prime. The range and the mean is 7. The median is also the only mode. State the five numbers.

Answer [3]

10 y is proportional to x^n. x and y are the length and breadth of a rectangle with a fixed area.

(a) State the value of n.

(b) Find the percentage reduction in breadth when the length is doubled.

Answer (a) $n =$ [1]

(b) % [2]

11 A new tile-making company is exploring using a regular hexagon and a regular octagon to make tiles. The company will use the regular polygons to make into tiles if they can fit onto the floor with no gaps and there is no overlaps.

Explain whether the company would use

(a) a regular hexagon to make into tiles.

Answer

[2]

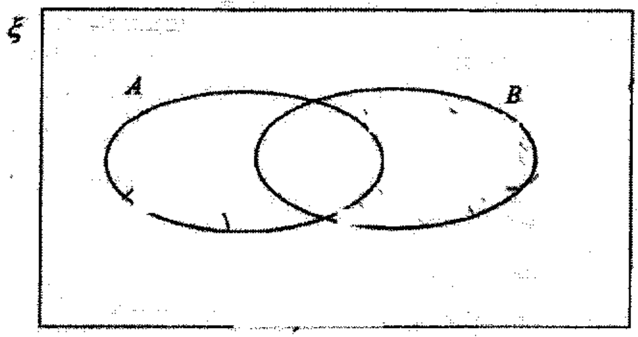
(b) a regular octagon to make into tiles.

Answer

[2]

12 (a) On the Venn diagram, shade the region which represents $A' \cap B$.

Answer



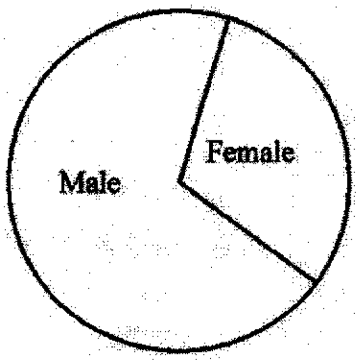
[1]

(b) Using the Venn diagram above, draw the set C such that

$$C \not\subset A, \quad A \cap C \neq \varnothing \quad \text{and} \quad n(B \cap C) = 0.$$

[2]

13 Based on the pie chart shown on the right, Mrs Goh told Mr Goh that male drivers are worse drivers than female drivers. Explain why Mrs Goh should <u>not</u> make such a statement. What could be a better mathematical measure to make a conclusion whether the male or female drivers are worse drivers?



Number of drivers involved in accidents by gender

Answer

[2]

14 The map of a park is drawn to a scale of 1: n. A garden, which has an actual area of 0.32 km^2, is represented by an area of 2 cm^2 on a map.

(a) Find the value of n.

Answer $n =$ [2]

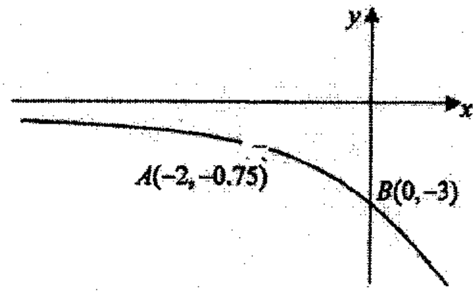
(b) Find the actual distance, in metres, if the distance between two schools on the map is 5 cm.

Answer [1]

15 The sketch shows the graph of $y = ka^x$.

The points $A(-2, -0.75)$ and $B(0, -3)$ lie on the graph.

(a) Find the values of k and a.



Answer $k =$ [1]

$a =$ [1]

(b) State the equation of the straight line that is parallel to the x-axis and passes through B.

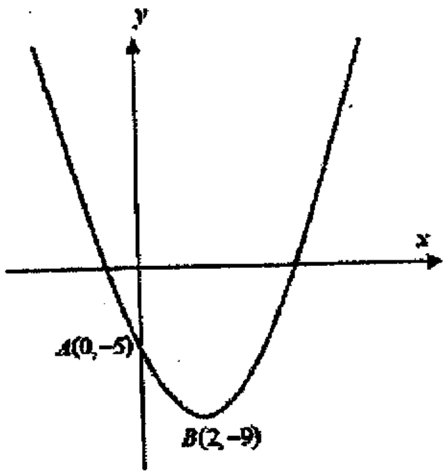
y = −3

Answer [1]

16 The diagram shows the curve

$y = x^2 + px + q$.

The points $A(0,-5)$ and $B(2,-9)$ lie on the graph. $B(2,-9)$ is the minimum point.



(a) Find the value of p and of q.

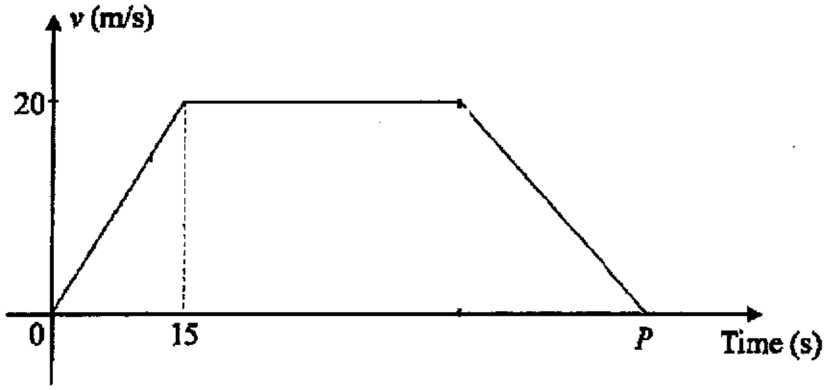
Answer $p =$

$q =$ [3]

(b) Find the equation of the straight line that is parallel to line BC and passes through A if the coordinates of C is (7,16).

Answer [2]

17 The diagram (not drawn to scale) shows the speed-time graph of a vehicle. The vehicle starts from rest and accelerates uniformly to a speed of 20 m/s in 15 seconds. It then travels at this speed before it decelerates at 1 m/s^2 until it comes to rest.



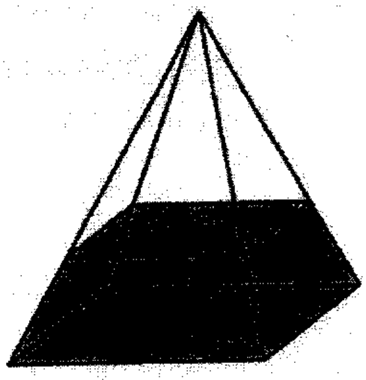
(a) Find the speed of the vehicle at the 10^{th} second.

Answer m/s [2]

(b) The total distance is 950 m and it took the vehicle P seconds. Find the value of P.

Answer $P =$ [3]

18 The diagram shows a pyramid with a rectangular base and its height is 9 cm. The volume of the liquid in the pyramid is two-third the volume of the pyramid.



Calculate the depth of the liquid.

Answer ______ cm [3]

19 If the length of a rectangle is increased by 20% and its width is decreased by 20%, explain whether there is any change in the area.

Answer

[2]

20 The table shows the colour of the pen 200 people took during a game.

Colour of pen		Black	Blue	Green	Red
Frequency	Boys	15	36	43	18
	Girls	23	32	28	5

(a) A person is selected at random. Find, as a percentage, the probability that the person took a black pen.

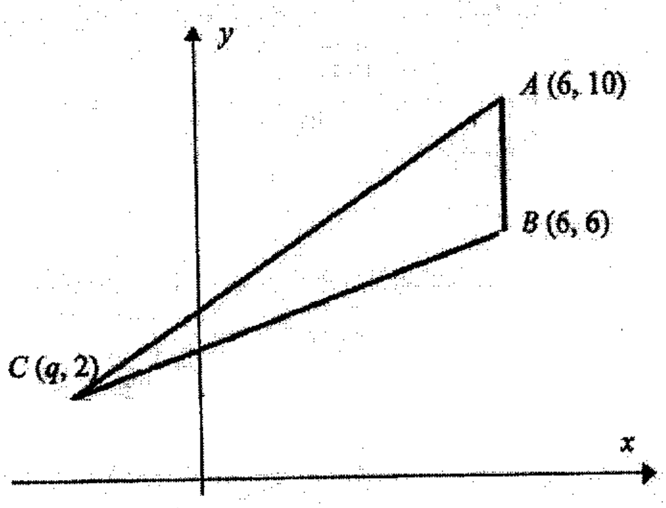
Answer % [2]

(b) Two persons are selected at random. Find the probability, as a fraction in its lowest terms, that both of them are girls who took red pens.

Answer [2]

21 A, B and C are points (6, 10), (6,6) and (q, 2) respectively.

The area of triangle ABC is 16 square units.



(a) Show that $q = -2$.

Answer

[1]

(b) Without the use of calculator and expressing in a fraction in the simplest form, find the exact value of tan $\angle CAB$.

Answer [1]

(c) **(i)** Find the length *AC*.

Answer [1]

(ii) Hence, find the perpendicular distance from *B* to *AC*.

Answer [2]

22 The table shows the height of 100 men who were at the health center.

Height (x cm)	$140 \le x < 150$	$150 \le x < 160$	$160 \le x < 170$	$170 \le x < 180$
No of men	15	31	41	13

(a) Find the estimated mean height of the men.

Answer cm [1]

(b) Find the standard deviation of the height of the men.

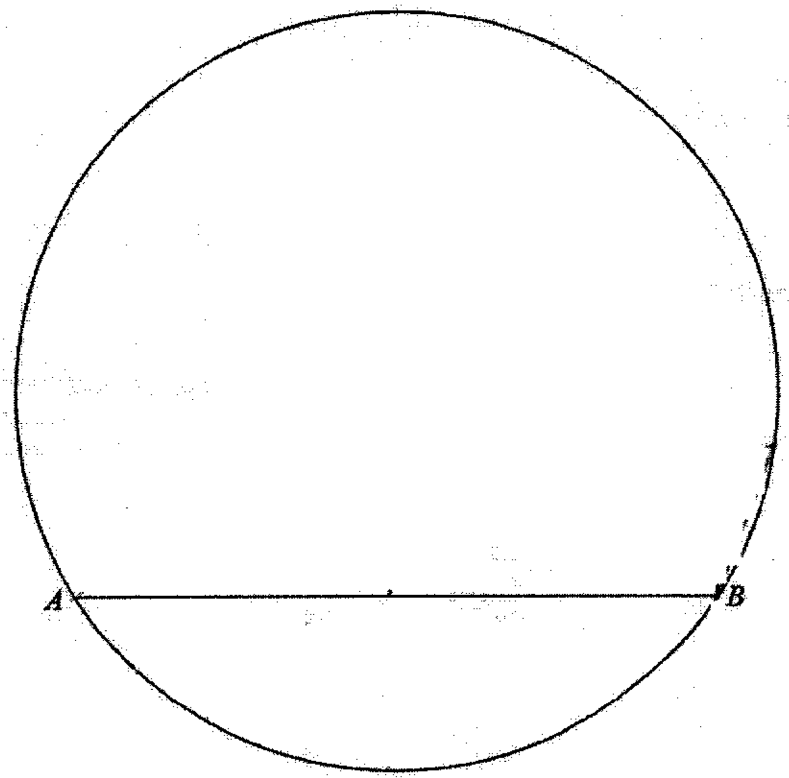
Answer cm [2]

23 The diagram shows a circle and a chord AB. B is due east of A.

(a) Locate and label a point C on the circle such that $\angle ABC = 110°$. [1]

(b) The point D is equidistant from A and B. It is also given that $\angle ADC = 70°$ and that D is above AB. Locate and label the point D. [2]

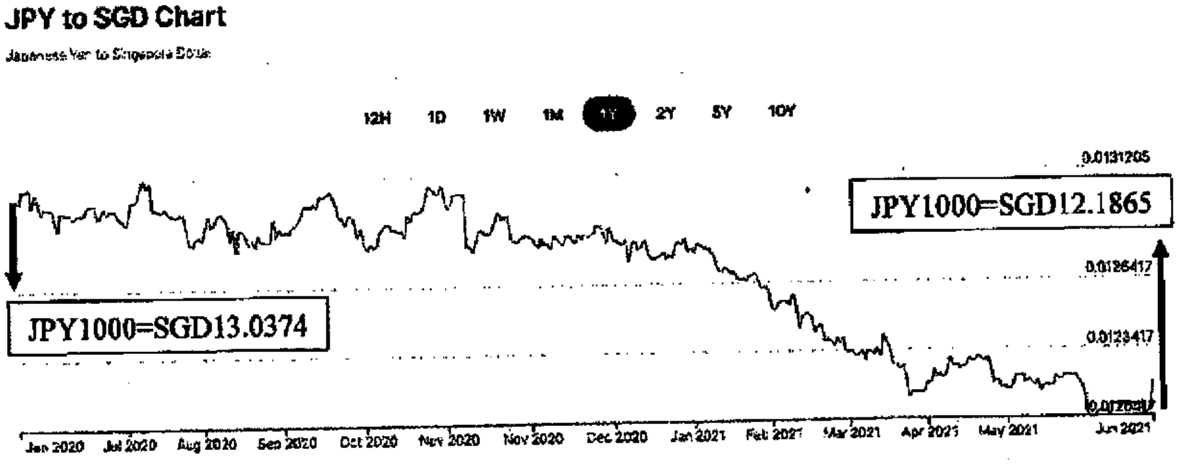
(c) By drawing suitable lines, locate and label the center of the circle as O. [1]



(d) Measure and write down the bearing of B from O.

Answer ° [1]

24 The Tan family bought a furniture, priced at 0.3 million JPY, from a furniture maker in Japan last year in June 2020 from an online store. The boxes in the conversion chart show the conversion of 1 Japanese yen (JPY) to Singapore dollar (SGD) on June 2020 and 2021. (1 million = 10^6)



(Source: https://www.xe.com/currencycharts)

(a) Find the cost of the furniture, in Singapore dollars, in June 2020.

Answer $ [2]

(b) By 2021, there is an increase of 2% in the price of the furniture. If the family had waited for a year and buy the furniture on 18 June 2021, how much Singapore dollar could they save, assuming that there are no other charges?

Answer **$** [3]

END OF PAPER

Name:______________________ Class_____ Index No___

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS | **4048/2**

Paper 2

Candidates answer on the question paper.

Date: 23 August 2021
Time: 0745 – 1015
Duration: 2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper has a total of 25 pages.

Setter: TAY KHYE PING **[Turn over]**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. a) Simplify $\dfrac{x-\dfrac{1}{x}}{x-2+\dfrac{1}{x}}$.

Answer ________________ [3]

b) Make h the subject of the formula $s=\sqrt{2h^2-r}$.

Answer . [2]

c) Given that $\frac{2a}{b}=3$, $a>0$ and $b>0$, evaluate $\frac{\sqrt{a^2-\frac{5}{4}b^2}}{b}$

Answer .... [3]

2. Chris cycled from A to ~~C in~~ a certain part of a country. He travelled for 50km from A, at a constant speed of x km/h) until he reached the point B, where his bicycle chain broke.

a) Write down the time, in hours, taken for his cycled journey from A to B.

Answer hours [1]

b) He then walked the remaining 6 km from B to C at a constant speed of $(x-16)$ km/h. Write down the time, in hours, taken for him to walk from B to C.

Answer hours [1]

c) Given that the total time for the whole journey from A to C is 4 hours, write down an equation in x and show that it reduces to $x^2-30x+200=0$. [3]

Answer

d) Solve the equation $x^2 - 30x + 200 = 0$.

Answer . . [2]

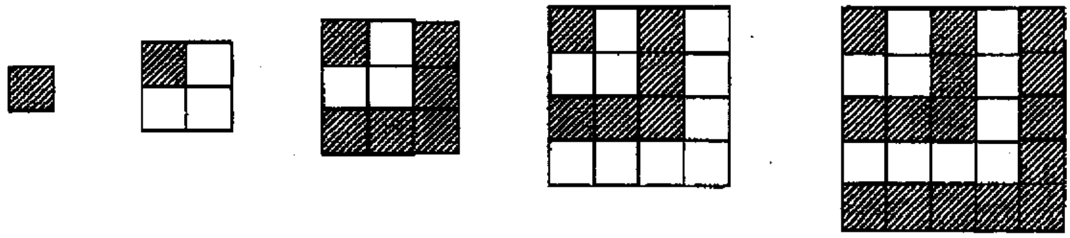
e) Explain why you would reject one of the two solutions.

Answer .... [1]

f) Find the time, in hours and minutes, Chris would have taken if he had completed the whole journey by bicycle at the original constant speed.

Answer hours ,minutes [2]

3. a) Sally is making square patterns with her square tiles using the pattern shown. She has black and white tiles.



Pattern 1 Pattern 2 Pattern 3 Pattern 4 Pattern 5

Sally starts with one black tile, then she adds 3 white tiles. The size of the square in pattern 2 is now 2×2. When she adds 5 black tiles, the size of the square is now 3×3. When she adds 7 white tiles, her Pattern 4 square is 4×4.

Find the size of her new square,

i) if she has just added 65 tiles to a pattern,

Answer . [2]

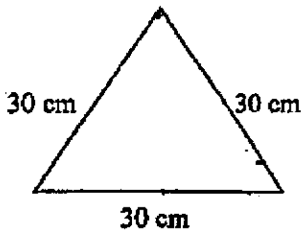
ii) if she has just added $4k+3$ tiles to a pattern. (express your answer in terms of k)

Answer [2]

b) Find the 8^{th} term of the sequence 10, 11, 13, 16, 20,........,

Answer [2]

4. John made a triangular poster for the coming mathematics competition. It was in the shape of an equilateral triangle with side 30 cm as shown in the diagram below.



His teacher liked the poster very much and asked him to make another similar one but with an area twice that of the first poster. What should be the length of each side of the new poster? Show your working clearly. [3]

Answer

5. A group of students was asked to complete a class test. The time taken to complete the test was represented as shown in the following table:

Time in minutes (x)	$25 < x \leq 35$	$35 < x \leq 45$	$45 < x \leq 55$	$55 < x \leq 65$	$65 < x \leq 75$
No. of students	12	49	81	33	25

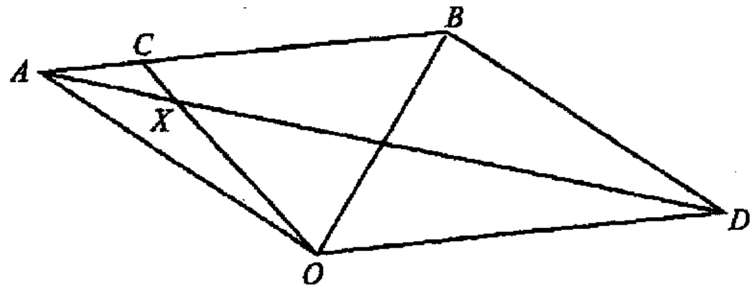
a) State the median class.

Answer [1]

b) If one more question is added to the test and each student took 5 more minutes to complete the test, comment on how this will affect the mean and standard deviation of the data.

Answer .... [2]

6. *OABD* is a parallelogram. *O* is the origin. *A* is the point (-7, 3) and $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$. If $\overrightarrow{AC} = \frac{1}{4}\overrightarrow{AB}$,



Find

a) the position vector of C,

Answer .... [2]

b) $|\overrightarrow{OD}|$,

Answer .... units [2]

c) the exact value of

i) $\dfrac{\text{area of } \Delta OAC}{\text{area of } \Delta OBC}$ and

Answer .... [1]

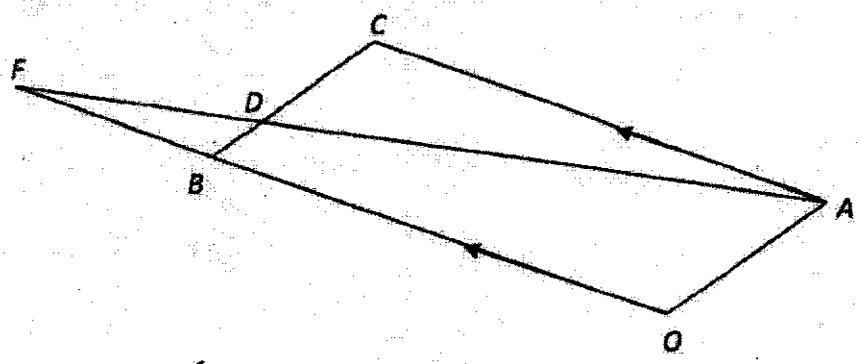
ii) $\dfrac{\text{area of } \Delta OXD}{\text{area of } \Delta AXC}$

Answer .... [1]

7. *OACB* is a quadrilateral.

OB is parallel to *AC*.

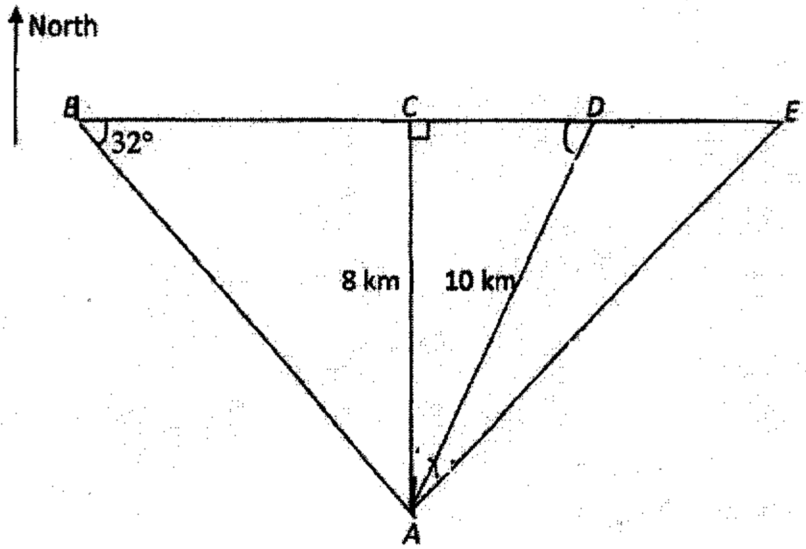
D is the point on *BC* such that $BD = \frac{1}{3}BC$. The lines *OB* and *AD* produced meet at *F*.



Explain why $FB = \frac{1}{2}AC$. [3]

Answer

8. The diagram shows five towns *A*, *B*, *C*, *D* and *E*. Towns *B*, *C*, *D* lie on a straight line. Town *E* is due east of Town *D*. It is given that $A\hat{B}C = 32°$, $AC = 8$ km and $AD = 10$ km.



a) Calculate

i) $\measuredangle ADB$,

Answer ..° [2]

ii) the distance *BD*,

Answer km [3]

iii) the value of $\cos A\hat{D}E$,

Answer [2]

iv) the bearing of A from B.

Answer ° [1]

b) Given that Town D is positioned such that it is equidistant from C and E. Calculate the distance AE.

Answer . [2]

c) Hence or otherwise, calculate the bearing of E from A.

Answer ° [3]

9. a) When Tap *A* and Tap *B* are turned on simultaneously, it takes 2.4 hours to fill tank completely. Given that Tap *A* alone takes 6 hours to fill the tank completely, calculate how long it takes in hours for Tap *B* to fill the same tank completely on its own.

Answer hours [2]

b) A distributor supplied rice in three different packages, type *A* (2 kg), type *B* (5 kg) and type *C* (10 kg) at \$6.20, \$13.50 and \$24.00 respectively.
A 3×1 matrix **P** is used to represent the above information about the weight of each type and a 1×3 matrix **Q** is used to represent the respective prices.

i) State the matrix **P**.

Answer [1]

ii) State the matrix **Q**.

Answer [1]

The orders from three shops in July 2021 were as follows:
Tasty shop ordered 100 of type A, 50 of type B and 25 of type C.
Fragrant shop ordered 120 of type A, 60 of type B and 20 of type C.
Unusual shop ordered 80 of type A, 70 of type B and 30 of type C.

c) Write down a 3×3 matrix **R** to represent this information.

Answer [1]

d) Calculate the matrix product **RP** which represents the total weight of rice ordered from each shop respectively.

Answer [2]

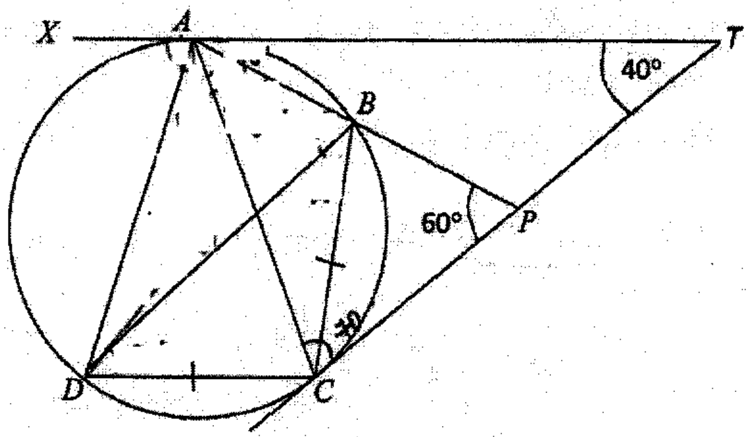
e) Given that **SRP** is a 1×1 matrix and its only element represents the total weight of rice ordered from the three shops, write down the matrix **S**.

Answer [1]

f) Using matrix multiplication, find the total revenue that the distributor earned from the sales to the three shops.

Answer $ [2]

10.



In the figure, *AT* and *CT* are tangents to the circle at *A* and *C* respectively. *AB* produced meets *TC* at *P*. Given that $\angle APC = 60°$, $\angle ATC = 40°$ and $CD = BC$, calculate the following angles and show all the reasons.

a) $\angle ACT$,

Answer ....° [1]

b) $\angle CAD$,

Answer ....° [2]

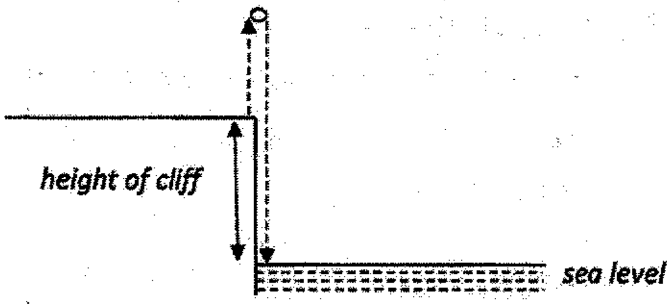
c) $\angle CBA$,

Answer .° [3]

d) $\angle ADC$.

Answer ° [1]

11. A ball was thrown up from the top of a cliff as shown below.



The height of the ball, h metre, from the sea level is measured, t seconds after leaving the cliff. The table below shows the height, h, reached by the ball after time t.

t (s)	0	0.5	1	1.5	2	2.5	3	3.5	3.8
h (m)	25	26.25	26	24.25	21	16.25	10	2.25	-3.12

a) Using a horizontal scale of 2 cm to represent 0.5 s and vertical scale of 1 cm to represent 2 m, plot the graph of h against t on a piece of graph paper.

[3]

b) From the graph, find

i) the height of the cliff,

Answer m [1]

ii) the time when the ball reached the sea level.

Answer s [1]

c) i) By drawing a tangent, find the gradient of the graph when $t = 2$ s.

Answer [2]

ii) Explain briefly what this gradient represents.

Answer .... [1]

d) A man operates a toy plane at the top of the cliff. The toy plane travels in a straight line with an equation,

$$h = 20 + 7t.$$

By adding a straight line to your graph, find the time when the toy plane collides with the ball.

Answer s [2]

e) The equation of the curve can be represented in the form

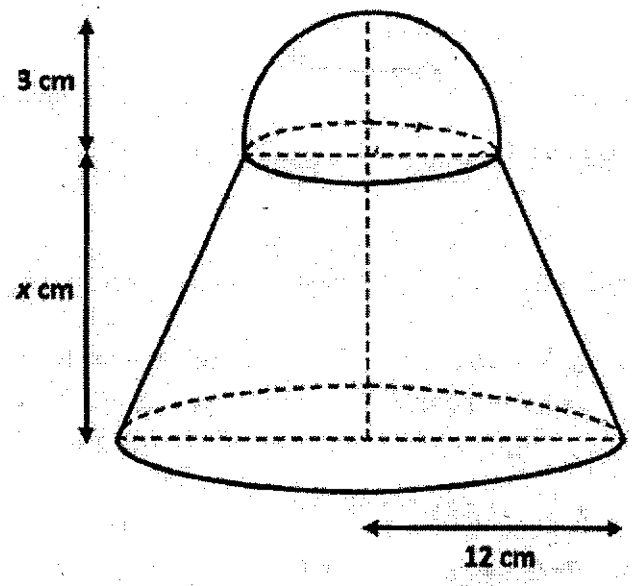
$$h = A - 3(t - B)^2,$$

where A and B are positive real numbers. Using your graph, estimate the value of A and of B.

Answer $A =$

$B =$ [2]

12. The diagram below shows a gift souvenir formed by joining the plane face of a solid hemisphere of radius 3 cm and a solid frustum, which is obtained by cutting a right circular cone of radius 12 cm into two portions by a plane parallel to its base. The height of the frustum is given by x cm.



a) Given that the height of the cone is 24 cm before it is cut to obtain the frustum, find the value of x.

Answer .... [2]

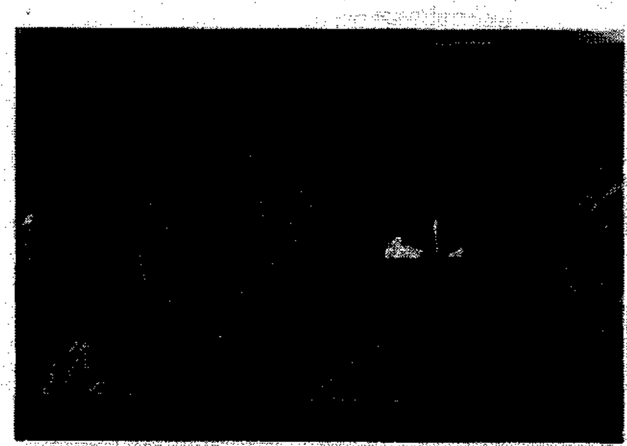
b) Find the volume of the souvenir.

Answer .... cm^3 [5]

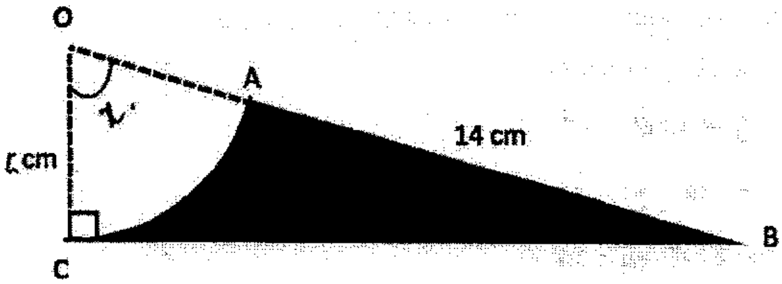
c) Find the total surface area of the entire gift souvenir.

Answer ....cm^2 [5]

13. A chock is a block placed against a wheel or rounded object, to prevent it from moving.



Jenny had to come up with a prototype for the design of a new wheel chock.
Jenny drew a model of the wheel chock and the cross section is shown below.



In the diagram, the shaded region ABC represents the cross section of the model of the wheel chock. OAC is a sector of a circle with centre O and radius r cm. AB = 14cm, $\angle AOC = x$ radians and OC is perpendicular to CB.

a) Show that $BC = \sqrt{28(7+r)}$ [2]

Answer

b) Find the area of the shaded region ABC, given that $BC = 20$ cm

Answer ....cm² [3]

c) Jenny claimed that the points O, B and C lie on the circumference of the circle. Do you agree? Explain with reasons.

Answer . [2]

End of Paper 2

Name:______________________ Class______ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4049/1**

Paper 1 Date: 24 August 2021

Candidates answer on the question paper. Time: 1015 - 1230

No Additional Materials are required. Duration: 2h 15 min

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

This document consists of **21** printed pages.

Setter: Mr Choo K L **[Turn over]**

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}bc\sin A$$

1. (a) Find the value of $\sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}}$, giving your answer in the form $a+b\sqrt{6}$. [2]

$$\sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}} = \sqrt{\frac{(5+2\sqrt{6})(5+2\sqrt{6})}{(5-2\sqrt{6})(5+2\sqrt{6})}} \quad [1]$$

$$= \sqrt{\frac{(5+2\sqrt{6})^2}{25-24}}$$

$$= 5+2\sqrt{6} \ \# \quad [1]$$

(b) Without using a calculator, evaluate $\left(2-\sqrt[3]{4}\right)\left[4+\left(2\times\sqrt[3]{4}\right)+\sqrt[3]{16}\right]$. [2]

$$\left(2-\sqrt[3]{4}\right)\left[4+\left(2\times\sqrt[3]{4}\right)+\sqrt[3]{16}\right]$$

$$= 8+4\sqrt[3]{4}+2\sqrt[3]{16}-4\sqrt[3]{4}-2\sqrt[3]{16}-\sqrt[3]{64} \quad [1]$$

$$= 8-\sqrt[3]{64}$$

$$= 8-4$$

$$= 4 \ \# \quad [1]$$

2. It is given that $2x^4 - kx^3 - 2kx^2 + 5x + 6$ leaves a remainder of 7 when divided by $(2x - k)$.

(i) Show that $k^3 - 5k + 2 = 0$. [2]

$f(x) = 2x^4 - kx^3 - 2kx^2 + 5x + 6$

$f(\frac{k}{2}) = 2(\frac{k}{2})^4 - k(\frac{k}{2})^3 - 2k(\frac{k}{2})^2 + 5(\frac{k}{2}) + 6 = 7$ [1]

$\frac{2k^4}{16} - \frac{k^4}{8} - \frac{2k^3}{4} + \frac{5k}{2} + 6 - 7 = 0$

$-\frac{1}{2}k^3 + \frac{5k}{2} - 1 = 0$

$k^3 - 5k + 2 = 0$ # (shown) [1]

(ii) Find the values of k. [3]

Let $f(k) = k^3 - 5k + 2$

$f(2) = 2^3 - 5(2) + 2 = 0$

$\therefore (k-2)$ is a factor [1]

$$\begin{array}{r} k^2 + 2k - 1 \\ k-2 \,\overline{)\, k^3 \qquad - 5k + 2} \\ k^3 - 2k^2 \qquad\quad \\ \hline 2k^2 - 5k \quad\; \\ 2k^2 - 4k \quad\; \\ \hline -k + 2 \\ -k + 2 \\ \hline 0 \\ \hline \end{array}$$

$k^3 - 5k + 2 = 0$

$(k-2)(k^2 + 2k - 1) = 0$ [1]

$k = 2$ or $\frac{-2 \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$

$k = 2$ or 0.4142 or -2.4142

$k = 2$ # or 0.414 # or -2.41 # [1]

4

(iii) Hence, solve $2y^3 - 5y^2 + 1 = 0$. [2]

$$2y^3 - 5y^2 + 1 = 0$$

Divide by y^3 throughout :

$$\frac{2y^3}{y^3} - \frac{5y^2}{y^3} + \frac{1}{y^3} = 0$$

$$2 - 5\left(\frac{1}{y}\right) + \left(\frac{1}{y}\right)^3 = 0$$

$$\left(\frac{1}{y}\right)^3 - 5\left(\frac{1}{y}\right) + 2 = 0$$ [1]

$$\therefore \frac{1}{y} = 2 \text{ or } 0.4142 \text{ or } -2.4142$$

$$y = \frac{1}{2} \text{ or } 2.41 \text{ or } -0.414$$ [1]

3. Express $\frac{-6x^3-9x^2+x-16}{(x^2-1)(2x^2+3)}$ in partial fractions. [6]

$$\frac{-6x^3-9x^2+x-16}{(x^2-1)(2x^2+3)} = \frac{-6x^3-9x^2+x-16}{(x+1)(x-1)(2x^2+3)}$$

Let $\frac{-6x^3-9x^2+x-16}{(x+1)(x-1)(2x^2+3)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{2x^2+3}$ [1]

$$-6x^3-9x^2+x-16 = A(x-1)(2x^2+3) + B(x+1)(2x^2+3) + (Cx+D)(x^2-1)$$ [1]

When $x=-1$, $6-9-1-16 = A(-2)(2+3)$
$-20 = -10A$
$A = 2$

When $x=1$, $-6-9+1-16 = B(2)(2+3)$
$-30 = 10B$
$B = -3$

When $x=0$, $-16 = A(-1)(3) + B(1)(3) + D(-1)$
$-16 = -6-9-D$
$D = 1$

A, B, C, D	
All 4 correct	[3]
2, 3 correct	[2]
1 correct	[1]
0 correct	[0]

Comparing coefficient of x:
$1 = 3A + 3B - C$
$1 = 6 - 9 - C$
$C = -4$

$$\therefore \frac{-6x^3-9x^2+x-16}{(x^2-1)(2x^3+3)} = \frac{2}{x+1} - \frac{3}{x-1} + \frac{-4x+1}{2x^2+3} \quad *$$
$$= \frac{2}{x+1} - \frac{3}{x-1} + \frac{1-4x}{2x^2+3} \quad *$$
$$= \frac{2}{x+1} - \frac{3}{x-1} - \frac{4x-1}{2x^3+3} \quad *$$
[1]

6

4. The equation of a curve is $y = x^2e^{2x}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

$$y = x^2 e^{2x}$$

$$\frac{dy}{dx} = (e^{2x})(2x) + (x^2)(2e^{2x}) \quad [1]$$

$$= 2xe^{2x} + 2x^2e^{2x} \; \#$$

$$= 2xe^{2x}(1+x) \; \# \quad [1]$$

(ii) Find the range of values of x for which y decreases as x increases. [3]

For y to decrease, $\frac{dy}{dx} < 0$ [1]

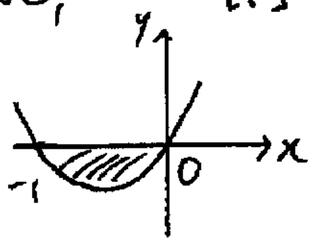
$$\frac{dy}{dx} = 2xe^{2x}(1+x)$$

$$= 2e^{2x}(x+x^2)$$

Since $2e^{2x}$ is always positive, [1]

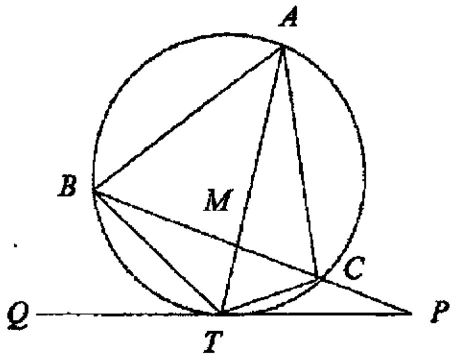
$$(x+x^2) < 0$$

$$x(x+1) < 0$$



$$-1 < x < 0 \; \# \quad [1]$$

5. In the diagram, $ABTC$ is a cyclic quadrilateral. QTP is a tangent to the circle and $\angle BAT = 2\angle CAT$. The lines AT and BC intersects at M.



Prove that

(a) $TP = TB$, [4]

(b) $TP^2 = BP \times PC$. [4]

6. The equation of a curve is given by $y = \ln\sqrt{\dfrac{3x^3}{2x-1}}$, where $x > \dfrac{1}{2}$.

(i) Find the coordinates of the stationary point. [5]

$$y = \ln\sqrt{\frac{3x^3}{2x-1}}$$

$$= \ln\left(\frac{3x^3}{2x-1}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\ln\left(\frac{3x^3}{2x-1}\right)$$

$$= \frac{1}{2}\left[\ln(3x^3) - \ln(2x-1)\right] \quad [1]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{9x^2}{3x^3} - \frac{2}{2x-1}\right] \quad [1]$$

$$= \frac{1}{2}\left(\frac{3}{x} - \frac{2}{2x-1}\right)$$

$$= \frac{1}{2}\left(\frac{3(2x-1) - 2x}{x(2x-1)}\right)$$

$$= \frac{6x-3-2x}{2x(2x-1)}$$

$$= \frac{4x-3}{2x(2x-1)}$$

$$= \frac{4x-3}{4x^2-2x} \quad [1]$$

For stationary point, $\frac{dy}{dx} = 0$

$$\frac{4x-3}{4x^2-2x} = 0$$

$$4x-3 = 0$$

$$x = \frac{3}{4} \quad [1]$$

sub value into y: $y = \ln\sqrt{\dfrac{3(\frac{3}{4})^3}{2(\frac{3}{4})-1}} = 0.464$

$\therefore$ coordinates $= (0.75, 0.464)$ # [1]

10

(ii) Determine the nature of stationary point. [3]

Method 1

$$\frac{d^2y}{dx^2} = \frac{(4x^2-2x)(4)-(4x-3)(8x-2)}{(4x^2-2x)^2}$$

$$= \frac{16x^2-8x-(32x^2-8x-24x+6)}{(4x^2-2x)^2}$$

$$= \frac{-16x^2+24x-6}{(4x^2-2x)^2} \quad [1]$$

When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2} > 0$ [1]

$\therefore$ (0.75, 0.464) is a minimum point # [1]

Method 2

x	$\frac{1}{2}$	$\frac{3}{4}$	1
$\frac{dy}{dx}$	−ve	0	+ve
gradient	\	—	/

[1]
[1]

(0.75, 0.464) is a minimum point # [1]

7. (a) Solve the equation $2\operatorname{cosec}\left(x-\frac{\pi}{3}\right)=-4$ for $0 \le x \le 2\pi$. [4]

$$2\operatorname{cosec}\left(x-\frac{\pi}{3}\right)=-4$$
$$\operatorname{cosec}\left(x-\frac{\pi}{3}\right)=-\frac{4}{2}$$
$$\frac{1}{\sin\left(x-\frac{\pi}{3}\right)}=-2$$
$$\sin\left(x-\frac{\pi}{3}\right)=-\frac{1}{2}$$ [1]

basic angle $=\frac{\pi}{6}$

$x-\frac{\pi}{3}=-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x=\frac{\pi}{6}, \frac{3}{2}\pi, \frac{13}{6}\pi$ (NA) [Reject : [1]]

$x=\frac{\pi}{6}$ or $\frac{3\pi}{2}$ #

[1] [1]

(b) Solve the equation $\cot(2x+10°)=\tan 70°$ for $0° \le x \le 360°$. [4]

$\cot(2x+10°)=\tan 70°$

$\cot(2x+10°)=\cot 20°$ [1]

$\tan(2x+10°)=\tan 20°$ [1]

$2x+10=20°, 200°, 380°, 560°$ [1]

$2x=10°, 190°, 370°, 550°$

$x=5°, 95°, 185°, 275°$ # [1]

8. (a) Given that $\cos(A+B)=-\frac{5}{13}$ and $\sin A\sin B=\frac{15}{26}$, where A and B are acute angles and angle $A >$ angle B.

(i) Find $\cos A\cos B$. [1]

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

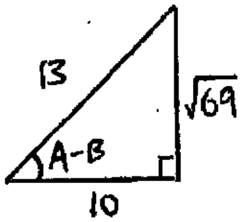
$$-\frac{5}{13} = \cos A\cos B - \frac{15}{26}$$

$$\cos A\cos B = \frac{15}{26} - \frac{5}{13} = \frac{5}{26} \quad [1]$$

(ii) Using your result in (i), find $\tan(A-B)$. Give your answer in exact form. [2]

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$= \frac{5}{26} + \frac{15}{26} = \frac{10}{13} \quad [1]$$



$$\tan(A-B) = \frac{\sqrt{69}}{10} \quad [1]$$

(b) Show that $\frac{1+\sin A+\cos A}{1+\sin A-\cos A}=\cot\frac{A}{2}$. [4]

$$\text{LHS} = \frac{1+\sin A+\cos A}{1+\sin A-\cos A}$$

$$= \frac{1+2\sin\frac{A}{2}\cos\frac{A}{2}+(2\cos^2\frac{A}{2}-1)}{1+2\sin\frac{A}{2}\cos\frac{A}{2}-(1-2\sin^2\frac{A}{2})} \quad \begin{matrix}-[1]\\-[1]\end{matrix}$$

$$= \frac{2\sin\frac{A}{2}\cos\frac{A}{2}+2\cos^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}+2\sin^2\frac{A}{2}}$$

$$= \frac{2\cos\frac{A}{2}(\sin\frac{A}{2}+\cos\frac{A}{2})}{2\sin\frac{A}{2}(\cos\frac{A}{2}+\sin\frac{A}{2})} \quad [1]$$

$$= \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$= \cot\frac{A}{2} = \text{RHS (shown)} \quad [1]$$

9. The curve $y = a + b\sin cx$ is defined for $0 \le x \le 2\pi$. The curve has a period of π, and the minimum value of $y = -6$. The curve also passes through the point $(2\pi, -2)$.

(i) Given that b is a positive integer, find the values of a, b and c. [3]

$$\text{Period} = \frac{2\pi}{c}$$

$$c = \frac{2\pi}{\pi} = 2 \quad [1]$$

$$y = a + b\sin 2x$$

When $x = 2\pi$, $y = -2$,

$$\therefore -2 = a + b\sin 2(2\pi)$$

$$-2 = a + b\sin 4\pi$$

$$-2 = a + 0$$

$$a = -2 \quad [1]$$

When $x = 0$, $y = -2$

Since minimum point is -6,

$\therefore$ Amplitude $= -2 - (-6) = 4$. $\therefore b = 4$ [1]

(ii) Find the coordinates of the maximum points. [2]

Maximum values are when $\sin 2x = 1$

$$y = -2 + 4\sin 2x$$

$$= -2 + 4$$

$$= 2$$

When $y = 2$, $-2 + 4\sin 2x = 2$

$$4\sin 2x = 4$$

$$\sin 2x = 1$$

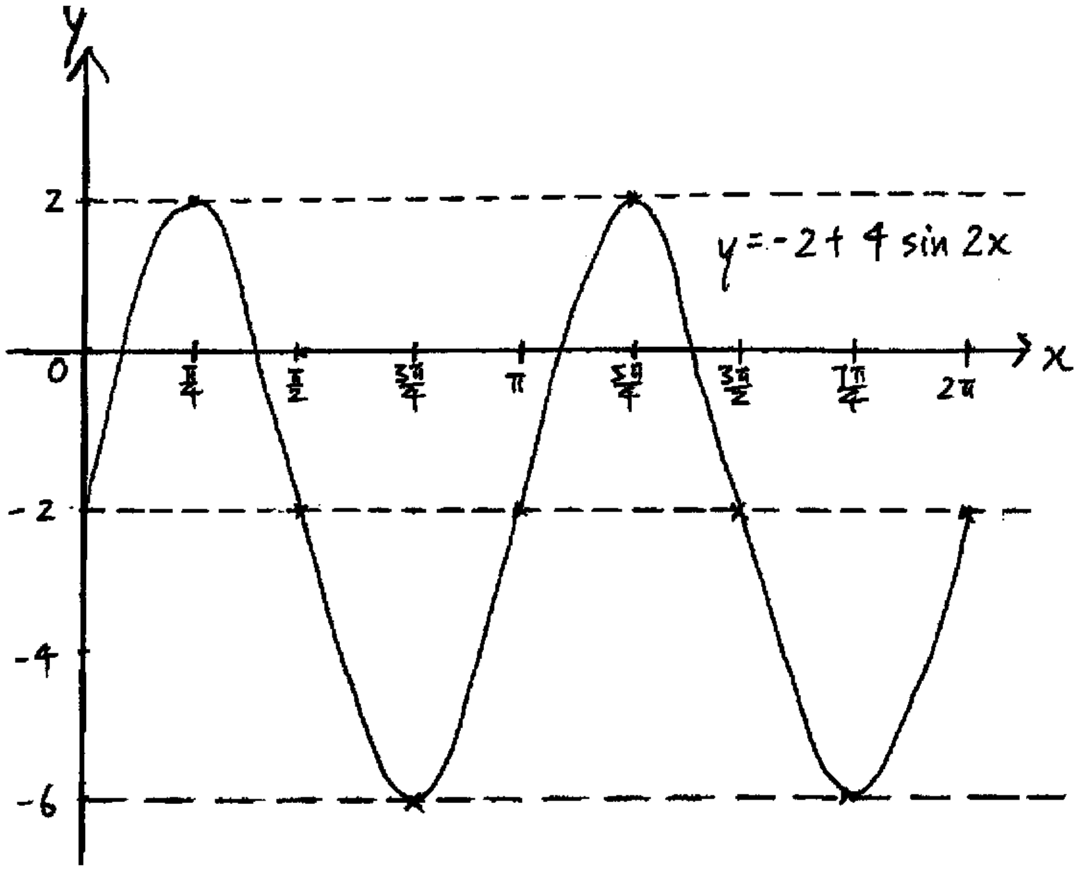
$$2x = \frac{\pi}{2}, \frac{5}{2}\pi$$

$$x = \frac{\pi}{4}, \frac{5}{4}\pi$$

Coordinates of maximum points are:

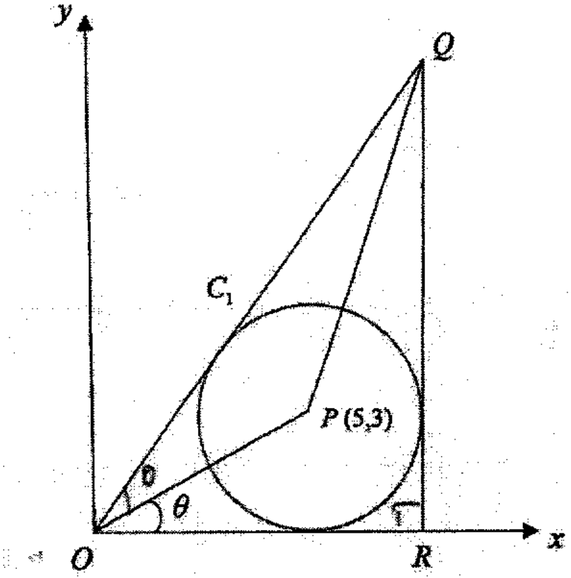
$\left(\frac{\pi}{4}, 2\right)$ [1] and $\left(\frac{5}{4}\pi, 2\right)$ [1]

(iii) Sketch $y = a + b\sin cx$ for $0 \le x \le 2\pi$. [3]



- correct shape and cut correct points at $y = -2$ [1]
- both max points correct [1]
- both min points correct [1]

10. The diagram shows a circle C_1 with centre $P(5, 3)$. The circle touches the x-axis, the tangent OQ and another line QR which is parallel to the y-axis.



(a) (i) State the equation of the circle. [2]

Centre of circle = (5, 3)
Radius of circle = 3 units [1]

Equation of circle : $(x-5)^2+(y-3)^2 = 9$

OR $x^2-10x+25+y^2-6y+9 = 9$
$x^2+y^2-10x-6y+25 = 0$ # [1]

(ii) Find the equation of the circle C_2 which is a reflection of the circle C_1 in the line QR. [2]

Centre of circle C_2 = (5+6, 3)
= (11, 3) [1]
Radius of circle C_2 = 3 units

Equation of circle C_2 : $(x-11)^2+(y-3)^2 = 9$ #

OR $x^2-22x+121+y^2-6y+9 = 9$
$x^2+y^2-22x-6y+121 = 0$ # [1]

16

(b) (i) Given that OP makes an angle θ with the x-axis, find $\tan\theta$. [1]

$\tan\theta = \frac{3}{5}$ # [1]

(ii) Find the equation of the line OQ. [3]

$\angle QOR = 2\theta$ (tangents from external point) [1]

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{5}\right)}{1-\left(\frac{3}{5}\right)^2}$ [1] $= \frac{15}{8}$

$\therefore$ Gradient of $OQ = \frac{15}{8}$

Equation of OQ : $y = \frac{15}{8}x$ # [1]

(iii) Hence, find the coordinates of Q. [2]

Equation of OQ : $y = \frac{15}{8}x$ — ①

Equation of RQ : $x = 8$ — ②

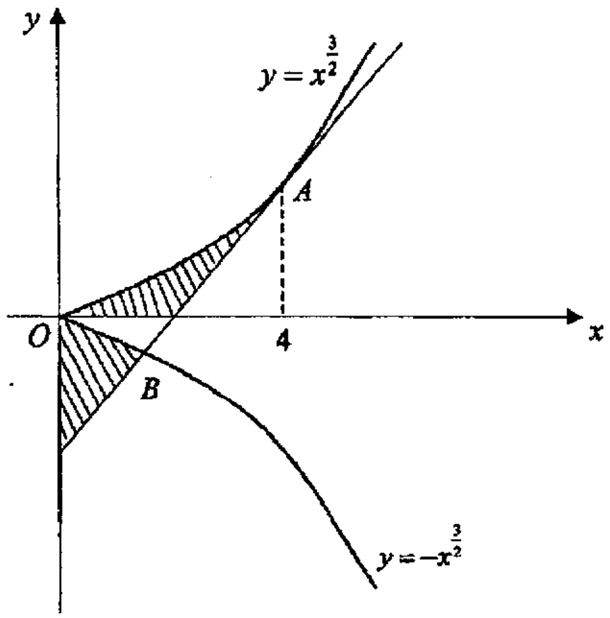
sub ② into ①: $y = \frac{15}{8}(8)$

$y = 15$

$\therefore Q(8, 15)$ #

[1] [1]

11. The diagram shows part of the curve $y = x^{\frac{3}{2}}$ and $y = -x^{\frac{3}{2}}$. The tangent meets the curve $y = x^{\frac{3}{2}}$ at the point A where $x = 4$.



(i) Find the equation of the tangent. [3]

$y = x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

When $x = 4$, $y = (4)^{\frac{3}{2}} = 8$ $\therefore A(4, 8)$ [1]

When $x = 4$, $\frac{dy}{dx} = \frac{3}{2}(4)^{\frac{1}{2}} = 3$ [1]

Gradient of tangent $= 3$

Equation of tangent : $\frac{y-8}{x-4} = 3$

$y - 8 = 3(x-4)$

$y = 3x - 12 + 8$

$y = 3x - 4$ # [1]

(ii) The tangent meets the curve $y = -x^{\frac{3}{2}}$ at B. Given that the x-coordinate of B is 1, find the y-coordinate of B. [1]

When $x = 1$, $y = 3(1) - 4 = -1$

$\therefore$ y-coordinate of B $= -1$ # [1]

(iii) Find the total area of the shaded region. [5]

For $y = 3x - 4$, when $y = 0$

$$3x - 4 = 0$$

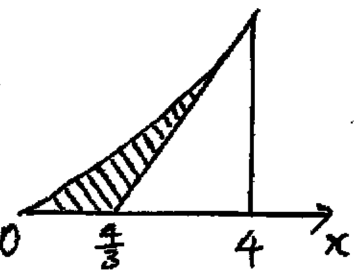
$$x = \frac{4}{3}$$

<u>Area above x-axis</u>

$$= \int_0^4 x^{\frac{3}{2}}\,dx - \frac{1}{2}\left(4 - \frac{4}{3}\right)(8) \quad [1]$$

$$= \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4 - \frac{32}{3}$$

$$= \frac{2}{5}(4)^{\frac{5}{2}} - \frac{32}{3} = \frac{32}{15} \quad [1]$$

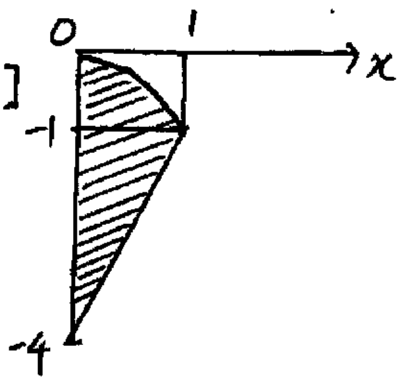


<u>Method 1:</u> <u>Area below x-axis</u>

$$= \frac{1}{2}(4+1)(1) - \int_0^1 x^{\frac{3}{2}}\,dx \quad [1]$$

$$= \frac{5}{2} - \left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1$$

$$= \frac{5}{2} - \frac{2}{5}(1)^{\frac{5}{2}} = \frac{21}{10} \quad [1]$$



<u>Method 2:</u> <u>Area below x-axis</u>

$$y = -x^{\frac{3}{2}}$$

$$y^2 = (-x^{\frac{3}{2}})^2$$

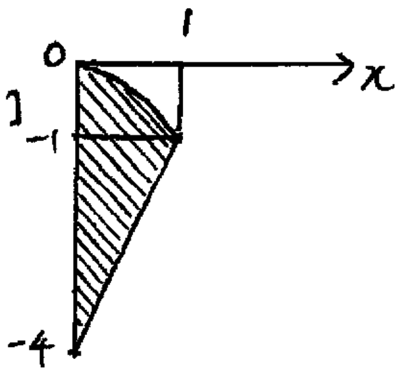
$$y^2 = x^3$$

$$y^{\frac{2}{3}} = x$$

$$= \int_{-1}^0 y^{\frac{2}{3}}\,dy + \frac{1}{2}(4-1)(1) \quad [1]$$

$$= \left[\frac{y^{\frac{5}{3}}}{\frac{5}{3}}\right]_{-1}^0 + \frac{3}{2}$$

$$= \left[(0) - \frac{3}{5}(-1)^{\frac{5}{3}}\right] + \frac{3}{2} = \frac{21}{10} \quad [1]$$



$\therefore$ Total area $= \frac{32}{15} + \frac{21}{10} = 4\frac{7}{30}$ units^2 # [1]

12. A particle moves in a straight line so that t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = t^2 + kt + 5$, where k is a constant. The particle comes to instantaneous rest first at A and then at B.

(i) Given that the acceleration is – 6 m/s² at the point O, find the value of k. [1]

$$v = t^2 + kt + 5$$

$$a = \frac{dv}{dt} = 2t + k$$

When $t = 0$, $a = -6$

$$-6 = 0 + k$$

$$k = -6 \# \quad [1]$$

(ii) Find the distance between A and B. [4]

$$v = t^2 - 6t + 5$$

$$s = \int (t^2 - 6t + 5)\, dt$$

$$s = \frac{t^3}{3} - \frac{6t^2}{2} + 5t + c$$

At $t = 0$, $s = 0$, $c = 0$

$$\therefore s = \frac{1}{3}t^3 - 3t^2 + 5t \quad [1]$$

At instantaneous rest, $v = 0$

$$t^2 - 6t + 5 = 0$$

$$(t-1)(t-5) = 0$$

$$t = 1 \text{ or } t = 5 \quad [1]$$

When $t = 1$, $s = \frac{1}{3}(1)^3 - 3(1)^2 + 5(1) = 2\frac{1}{3}$ m

When $t = 5$, $s = \frac{1}{3}(5)^3 - 3(5)^2 + 5(5) = -8\frac{1}{3}$ m [1]

$\therefore$ Distance between A and B $= 2\frac{1}{3} + 8\frac{1}{3} = 10\frac{2}{3}$ m # [1]

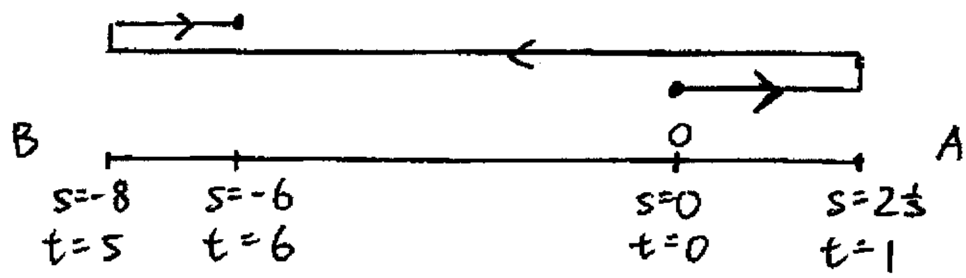
(iii) Calculate the total distance of the particle travelled in the first 6 seconds after passing O. [3]

At $t = 0$, $s = 0$

At $t = 1$, $s = 2\frac{1}{3}$ m

At $t = 5$, $s = -8\frac{1}{3}$ m

At $t = 6$, $s = \frac{1}{3}(6)^3 - 3(6)^2 + 5(6) = -6$ m [1]



Total distance travelled $= 2\frac{1}{3} + 10\frac{2}{3} + 2\frac{1}{3}$ [1]

$= 15\frac{1}{3}$ m # [1]

(iv) Given that C is the point at which the particle has minimum velocity, determine with calculation whether C is nearer to O or to B. [2]

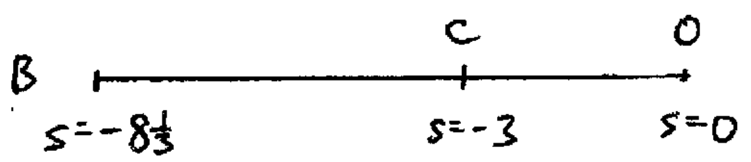
At minimum velocity, $a = 0$

$a = 2t - 6$

$0 = 2t - 6$

$t = 3$ [1]

When $t = 3$, $s = \frac{1}{3}(3)^3 - 3(3)^2 + 5(3) = -3$ m



C is nearer to O than to B # [1]

END OF PAPER

2021 Prelim Add Math Paper 2

1 (a) Solve the simultaneous equations.

$\frac{x}{2}+\frac{y}{3}=4$ ———— (1)

$2xy=45$ ———— (2) [4]

From (1) × 6 $\quad 3x+2y=24$

$y=\frac{24-3x}{2}$ ———— (3) [1]

Sub (3) into (2),

$2x\left(\frac{24-3x}{2}\right)=45$

$24x-3x^2=45$

$3x^2-24x+45=0$ [1]

$x^2-8x+15=0$

$(x-5)(x-3)=0$

$x=5$, $y=4.5$ or $4\frac{1}{2}$ [1]

$x=3$, $y=7.5$ or $7\frac{1}{2}$ [1]

(b) (i) Find the values of the constant k for which the line $y = kx - \frac{1}{3}k^2$ is a tangent to the curve $y = x^2 - \frac{1}{3}k - 1$. [3]

$x^2 - \frac{1}{3}k - 1 = kx - \frac{1}{3}k^2$

$x^2 - kx + \frac{1}{3}k^2 - \frac{1}{3}k - 1 = 0$

Use $b^2 - 4ac = 0$

$(-k)^2 - 4(1)(\frac{1}{3}k^2 - \frac{1}{3}k - 1) = 0$ [1]

$k^2 - \frac{4}{3}k^2 + \frac{4}{3}k + 4 = 0$

$-\frac{1}{3}k^2 + \frac{4}{3}k + 4 = 0$

$-k^2 + 4k + 12 = 0$

$k^2 - 4k - 12 = 0$ [1]

$(k-6)(k+2) = 0$

$k = 6$ or $k = -2$ [1]

(ii) Hence state the largest integer value of k for which the line $y = kx - \frac{1}{3}k^2$ intersects the curve $y = x^2 - \frac{1}{3}k - 1$ at two distinct points. [1]

k = 5 [1]

(iii) State, giving a reason, what can you deduce about the line $y = 9x - 27$ and the curve $y = x^2 - 4$? [2]

The line & curve does not intersect [1]
as k = 9 in the above case.
For k < -2 or k > 6, there is no real root } [1]

2 (i) Write down, and simplify the terms in the expansion of $(2+x)^5$ in ascending powers of x. [2]

$(2+x)^5$

$= 2^5 + \binom{5}{1}2^4x + \binom{5}{2}2^3x^2 + \binom{5}{3}2^2x^3 + \binom{5}{4}2x^4 + \binom{5}{5}x^5$ [1]

$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$ [1]

(ii) Hence find the coefficient of x^3 in the expansion of $(2+3x+x^2)^5$. [3]

Replace x with $3x + x^2$

$(2+3x+x^2)^5$

$= 32 + 80(3x+x^2) + 80(3x+x^2)^2 + 40(3x+x^2)^3 + \ldots$ [1]

$= 32 + 80(3x+x^2) + 80(9x^2+6x^3+x^4) + 40(27x^3+\ldots)$

Terms in $x^3 = 80(6x^3) + 40(27x^3)$ [1]

$= 1560x^3$

Coeff of $x^3 = 1560$ [1]

(iii) Using the result in part (ii), determine the coefficient of x^3 in the expansion of $\left(1+\frac{3}{2}x+\frac{1}{2}x^2\right)^5$. [2]

$= \left[\frac{1}{2}(2+3x+x^2)\right]^5$ [1]

$= \left(\frac{1}{2}\right)^5 (2+3x+x^2)^5$

Coeff of $x^3 = \left(\frac{1}{2}\right)^5 \times 1560$

$= \frac{195}{4}$ [1]

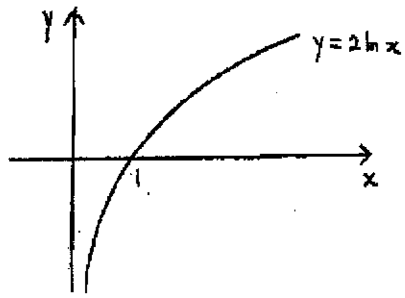
(iv) Calculate the value of the constant term in the expansion of $\left(1-\frac{1}{x^2}\right)\left(2+3x+x^2\right)^5$. [2]

$\left(1-\frac{1}{x^2}\right)(32 + \ldots + 80x^2 + 720x^2 + \ldots)$ [1]

Constant term $= 32 - 80 - 720$

$= -768$ [1]

3 (a) (i) Sketch the graph of $y = 2\ln x$. [1]



[A1]

(ii) In order to solve the equation $xe^{2x} = \frac{1}{3x}$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = 2\ln x$. Find the equation of the straight line and the number of solution(s). [3]

$x e^{2x} = \frac{1}{3x}$

$3x^2 = e^{-2x}$

$\ln 3 + 2\ln x = -2x$ [M1]

$2\ln x = -2x - \ln 3$

Eqn of str line is $y = -2x - \ln 3$ [1]

No of solution = 1 [1]

(iii) Explain why the value of x is between 0 and 1 for which $xe^{2x} = \frac{1}{3x}$. [2]

For $y = 2\ln x$ to be defined, $x > 0$ [1]

For $y = -2x - \ln 3$, $y < 0$ for $x > 0$

For $y = 2\ln x$, $y < 0$ for $0 < x < 1$ [1]

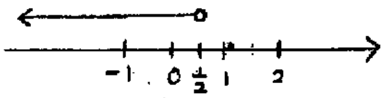
(b) Using a separate diagram for each part, represent on the number line the solution set of

(i) $3(5-x) > x+13$ [2]

$15 - 3x > x + 13$

$4x < 2$

$x < \frac{1}{2}$ [1]

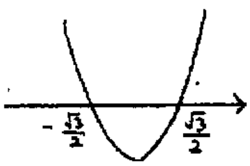


[1]

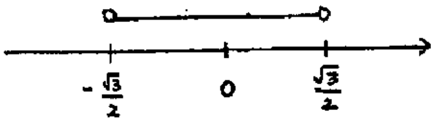
(ii) $x^2 - \frac{1}{2}x < \frac{1}{4}(3-2x)$ [3]

$4x^2 - 2x < 3 - 2x$

$4x^2 - 3 < 0$ [M1]



$-\frac{\sqrt{3}}{2} < x < \frac{\sqrt{3}}{2}$ [A1]



[1]

(-0.8660) (0.8660)

State the set of values of x which satisfy both of these inequalities. [1]

$-\frac{\sqrt{3}}{2} < x < \frac{1}{2}$ [1]

or $-0.866 < x < 0.5$

4 (a) Solve the equation $e^{x-2} - 14\left(\frac{1}{e}\right)^{x-2} = 5$. [4]

Let $p = e^{x-2}$

$p - \frac{14}{p} = 5$

$p^2 - 5p - 14 = 0$ [1]

$(p-7)(p+2) = 0$

$p = 7$ or $p = -2$ [1]

$e^{x-2} = 7$ $e^{x-2} = -2$ (rejected) [1]

$x - 2 = \ln 7$

$x = 3.95$ [1]

(b) Solve the equation $\log_2(3x+1)-\dfrac{2}{\log_{\sqrt{2}} 2}=\log_4(2-x^2)$ [4]

$$\log_2(3x+1) - \frac{2}{\frac{\log_2 2}{\log_2 \sqrt{2}}} = \frac{\log_2(2-x^2)}{\log_2 2^2} \quad [1]$$

$$\log_2(3x+1) - 1 = \frac{\log_2(2-x^2)}{2}$$

$$2\log_2(3x+1) - 2 = \log_2(2-x^2)$$

$$\log_2(3x+1)^2 - \log_2(2-x^2) = 2$$

$$\frac{(3x+1)^2}{2-x^2} = \frac{4}{1} \quad [1]$$

$$(3x+1)^2 = 4(2-x^2)$$

$$9x^2+6x+1 = 8-4x^2$$

$$13x^2+6x-7=0 \quad [1]$$

$$(13x-7)(x+1)=0$$

$$x=\frac{7}{13} \quad \text{or} \quad x=-1 \text{ (rejected)} \quad [1]$$

5 It is given that $f''(x) = e^{1-\frac{1}{2}x} + \frac{5}{(5x-9)^2}$, $x > \frac{9}{5}$.

Given that the x-axis is a tangent to the curve $y = f(x)$ at the point where $x = 2$,

(i) find an expression for $f'(x)$, [3]

$$f'(x) = \int \left[e^{1-\frac{1}{2}x} + \frac{5}{(5x-9)^2}\right] dx$$

$$= \frac{e^{1-\frac{1}{2}x}}{-\frac{1}{2}} + \frac{5(5x-9)^{-1}}{(-1)(5)} + c$$

$$= -2e^{1-\frac{1}{2}x} - \frac{1}{(5x-9)} + c \qquad [2]$$

When $x = 2$, $f'(x) = 0$

$0 = -2 - 1 + c$

$c = 3$ [1]

$$f'(x) = -2e^{1-\frac{1}{2}x} - \frac{1}{5x-9} + 3$$

deduct 1 mk if $f'(x)$ is missing

(ii) hence find an expression for f(x), [3]

$$f(x) = \int \left[-2e^{1-\frac{1}{2}x} - \frac{1}{5x-9} + 3 \right] dx$$

$$= \frac{-2e^{1-\frac{1}{2}x}}{-\frac{1}{2}} - \frac{\ln(5x-9)}{5} + 3x + c$$

$$= 4e^{1-\frac{1}{2}x} - \frac{\ln(5x-9)}{5} + 3x + c \quad [2]$$

When $x = 2$, $f(2) = 0$

$0 = 4 - 0 + 6 + c$

$c = -10$ [1]

$$f(x) = 4e^{1-\frac{1}{2}x} - \frac{\ln(5x-9)}{5} + 3x - 10$$

deduct 1 mk if f(x) is missing

(iii) explain whether the gradient of the curve f(x) is an increasing or decreasing function for $x > \frac{9}{5}$. [2]

Since $e^{1-\frac{1}{2}x} > 0$ and $\frac{5}{(5x-9)^2} > 0$ for $x > \frac{9}{5}$, therefore $f''(x) > 0$ [1]

Hence $f'(x)$ which is gradient of $f(x)$ will be an increasing function. [1]

6 It is given that $y = x\sin x + \cos^2\frac{x}{2}$.

(i) Obtain an expression for $\frac{dy}{dx}$ in the form $(x\cos x + p\sin x)$ where p is a constant. [3]

$$\frac{dy}{dx} = \underbrace{x\cos x + \sin x}_{[1]} + \underbrace{2\left(\cos\frac{x}{2}\right)\left(-\sin\frac{x}{2}\right)\left(\frac{1}{2}\right)}_{[1]}$$

$$= x\cos x + \sin x - \sin\frac{x}{2}\cos\frac{x}{2}$$

$$= x\cos x + \sin x - \frac{\sin x}{2}$$

$$= x\cos x + \frac{1}{2}\sin x \quad [1]$$

Or

used $\cos^2\frac{x}{2} = \frac{\cos x + 1}{2}$

So $y = x\sin x + \frac{\cos x + 1}{2}$

$= x\sin x + \frac{1}{2}\cos x + \frac{1}{2}$

14

(ii) Hence find the value of the constant k for which $\int_0^{\frac{\pi}{2}} 2x\cos x \, dx = \pi + k$. [4]

$$\int_0^{\frac{\pi}{2}} \left[x\cos x + \frac{1}{2}\sin x\right] dx = \left[x\sin x + \cos^2\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} x\cos x \, dx + \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin x \, dx = \left[x\sin x + \cos^2\frac{x}{2}\right]_0^{\frac{\pi}{2}} \quad [1]$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} 2x\cos x \, dx = \left[x\sin x + \cos^2\frac{x}{2}\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin x \, dx$$

$$= \left[x\sin x + \cos^2\frac{x}{2}\right]_0^{\frac{\pi}{2}} - \left[\frac{1}{2}(-\cos x)\right]_0^{\frac{\pi}{2}} \quad [1]$$

$$= \left[\left(\frac{\pi}{2} + \frac{1}{2}\right) - (0+1)\right] - \left[0 - \left(-\frac{1}{2}\right)\right]$$

$$= \left(\frac{\pi}{2} - \frac{1}{2}\right) - \frac{1}{2}$$

$$= \frac{\pi}{2} - 1 \quad [1]$$

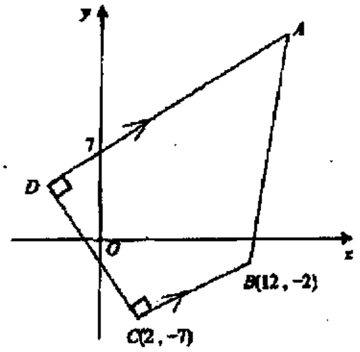
$$\int_0^{\frac{\pi}{2}} 2x\cos x \, dx = 2\left(\frac{\pi}{2} - 1\right)$$

$$= \pi - 2$$

$$k = -2 \quad [1]$$

031

7 The diagram shows a trapezium in which AD is parallel to BC and angle ADC = angle BCD = 90°. The points B and C are (12 , −2) and (2 , −7) respectively. The line AD meets the y-axis at (0 , 7).



Given that $AB = 2BC$, find

(i) the coordinates of A, [4]

Grad of AD $= \frac{-2+7}{12-2} = \frac{1}{2}$

Eqn of AD is $y = \frac{1}{2}x + 7$

$2y = x + 14$

$x = 2y - 14$ ——— (1) [1]

Given: AB = 2BC

$\sqrt{(x-12)^2 + (y+2)^2} = 2\sqrt{(12-2)^2 + (-2+7)^2}$

$(x-12)^2 + (y+2)^2 = 4(10^2 + 5^2)$ ——— (2) [1]

sub (1) into (2)

$(2y - 14 - 12)^2 + (y+2)^2 = 500$

$(2y - 26)^2 + (y+2)^2 = 500$

$4y^2 - 104y + 676 + y^2 + 4y + 4 = 500$

$5y^2 - 100y + 180 = 0$ [1]

$y^2 - 20y + 36 = 0$

$(y - 18)(y - 2) = 0$

When $y = 18$, $x = 22$

When $y = 2$, $x = -10$ (rejected)

A(22 , 18) [1]

(ii) the coordinates of D, [2]

grad of CD = −2

Eqn of CD is $\frac{y+7}{x-2} = -2$ [1]

$y = -2x - 3$

From part (i), $x = 2y - 14$

$y = -2(2y - 14) - 3$

$5y = 25$

$y = 5$

$x = -4$

$D(-4, 5)$ [1]

The point P lies on BC produced such that $PBAD$ is a parallelogram.

(iii) find the coordinates of P. [2]

Midpoint of BD = $\left(\frac{12-4}{2}, \frac{-2+5}{2}\right) = \left(4, \frac{3}{2}\right)$ [1]

Let P be (x, y)

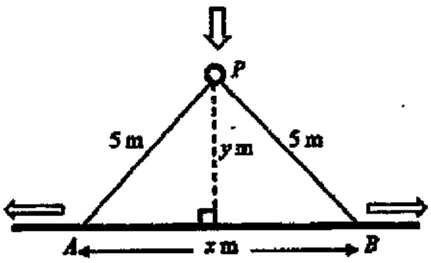
Midpoint of AP = $\left(\frac{22+x}{2}, \frac{18+y}{2}\right) = \left(4, \frac{3}{2}\right)$

$\frac{22+x}{2} = 4$ $\frac{18+y}{2} = \frac{3}{2}$

$x = -14$ $y = -15$

$P(-14, -15)$ [1]

8. The figure shows two rods *PA* and *PB* which are hinged at *P*. The rods slide such that *P*, *A* and *B* are on the same vertical plane. When *P* moves down, *A* and *B* will move in opposite directions on the horizontal ground. The horizontal distance between *A* and *B* is *x* metres and *P* is *y* metres from the ground.



(i) Express *x* in terms of *y*. [2]

$\left(\frac{x}{2}\right)^2 + y^2 = 25$ [1]

$\frac{x^2}{4} + y^2 = 25$

$x^2 = 100 - 4y^2$

$x = \sqrt{100 - 4y^2}$ [1]

(ii) Given that *P* is falling at the rate of 2 m s^{-1}, find the rate of change of *x* when *P* is 3 metres from the ground. [3]

$\frac{dy}{dt} = -2 \text{ m s}^{-1}$

$x = (100 - 4y^2)^{\frac{1}{2}}$

$\frac{dx}{dy} = \frac{1}{2}(100 - 4y^2)^{-\frac{1}{2}}(-8y)$

$= \frac{-4y}{\sqrt{100 - 4y^2}}$ [1]

Continuation of working space for question 8(ii).

$$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

$$= \frac{-4y}{\sqrt{100-4y^2}} \times (-2) \quad [1]$$

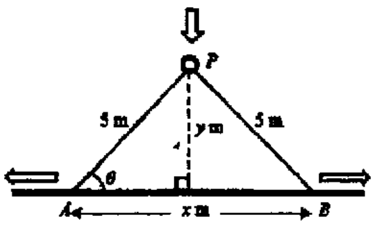
sub $y = 3$

$$\frac{dx}{dt} = \frac{-4(3)}{\sqrt{100-4(3)^2}} \times (-2)$$

$$= 3 \text{ m s}^{-1} \quad [1]$$

The rod PA makes an acute angle θ with the horizontal ground.

(iii) Using your answer to part (ii), find the rate of change of θ when P is 3 metres from the ground. [4]



$$\cos\theta = \frac{\frac{x}{2}}{5}$$

$$x = 10\cos\theta \quad [1]$$

$$\frac{dx}{d\theta} = 10(-\sin\theta)$$

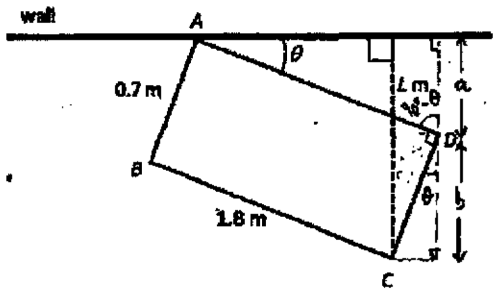
$$= -10\left(\frac{3}{5}\right) = -6 \quad [1]$$

$$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$$

$$3 = -6 \times \frac{d\theta}{dt} \quad [1]$$

$$\frac{d\theta}{dt} = -\frac{1}{2} \text{ rad s}^{-1} \quad [1]$$

9 The diagram shows a rectangular table $ABCD$ arranged such that the side AD is inclined at an angle of θ to the wall. The lengths of AB and BC are 0.7 m and 1.8 m respectively. The perpendicular distance from C to the wall is L metres.



(i) Explain clearly why $L = 1.8\sin\theta + 0.7\cos\theta$. [2]

$\sin\theta = \frac{a}{1.8}$

$a = 1.8\sin\theta$ [1]

$\cos\theta = \frac{b}{0.7}$

$b = 0.7\cos\theta$ [1]

$L = a + b$

$= 1.8\sin\theta + 0.7\cos\theta$

(ii) Express L in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$L = 0.7\cos\theta + 1.8\sin\theta$

$= \sqrt{0.7^2 + 1.8^2}\cos(\theta - \alpha)$ where $\tan\alpha = \frac{1.8}{0.7}$

$= \sqrt{3.73}\cos(\theta - 68.749°)$ [1]

$= 1.93\cos(\theta - 68.7°)$

[1] [1]

(iii) Hence, state the maximum value of L and find the corresponding value of θ. [3]

Max value of $L = 1.93$ [1]

$\cos(\theta - 68.749°) = 1$ [1]

$\theta = 68.7°$ [1]

(iv) Find the value of θ when $L = \sqrt{2.5}$. [2]

$\sqrt{3.73}\cos(\theta - 68.749°) = \sqrt{2.5}$

$\cos(\theta - 68.749°) = \sqrt{\frac{2.5}{3.73}}$

basic $\angle = \cos^{-1} 0.81868 = 35.047°$ [1]

$\theta - 68.749° = 35.047°$ $\quad$ $\theta - 68.749° = -35.047°$

$\theta = 103.8$ (rejected) $\quad$ $\theta = 33.7°$ [1]

10 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^h x^k = 10$.

x	1.2	1.6	2.0	2.4
y	2.203	1.8	1.532	1.35

(i) On graph paper, plot lg y against lg x, using a scale of 4 cm to 0.1 unit on each axis. [3]

lg x	0.079	0.204	0.301	0.380
lg y	0.343	0.255	0.185	0.130

Table [1]
Graph [2]

(ii) Use your graph to estimate the value of h and of k. [4]

$y^h x^k = 10$

$\lg y^h + \lg x^k = 1$

$\lg y = -\frac{k}{h} \lg x + \frac{1}{h}$

$\frac{1}{h} = 0.395$ [1]

$h = \frac{1}{0.395} = 2.53$ [1]

Gradient $= -\frac{k}{h} = -\frac{0.175}{0.25} = -0.7$ [1]

$k = 0.7 \times 2.5316 = 1.77$ [1]

(iii) Without plotting another graph, find the gradient of the straight line obtained when lg x is plotted against lg y. [2]

$k \lg x = -h \lg y + 1$

$\lg x = -\frac{h}{k} \lg y + \frac{1}{k}$

Grad $= -\frac{h}{k}$ [1]

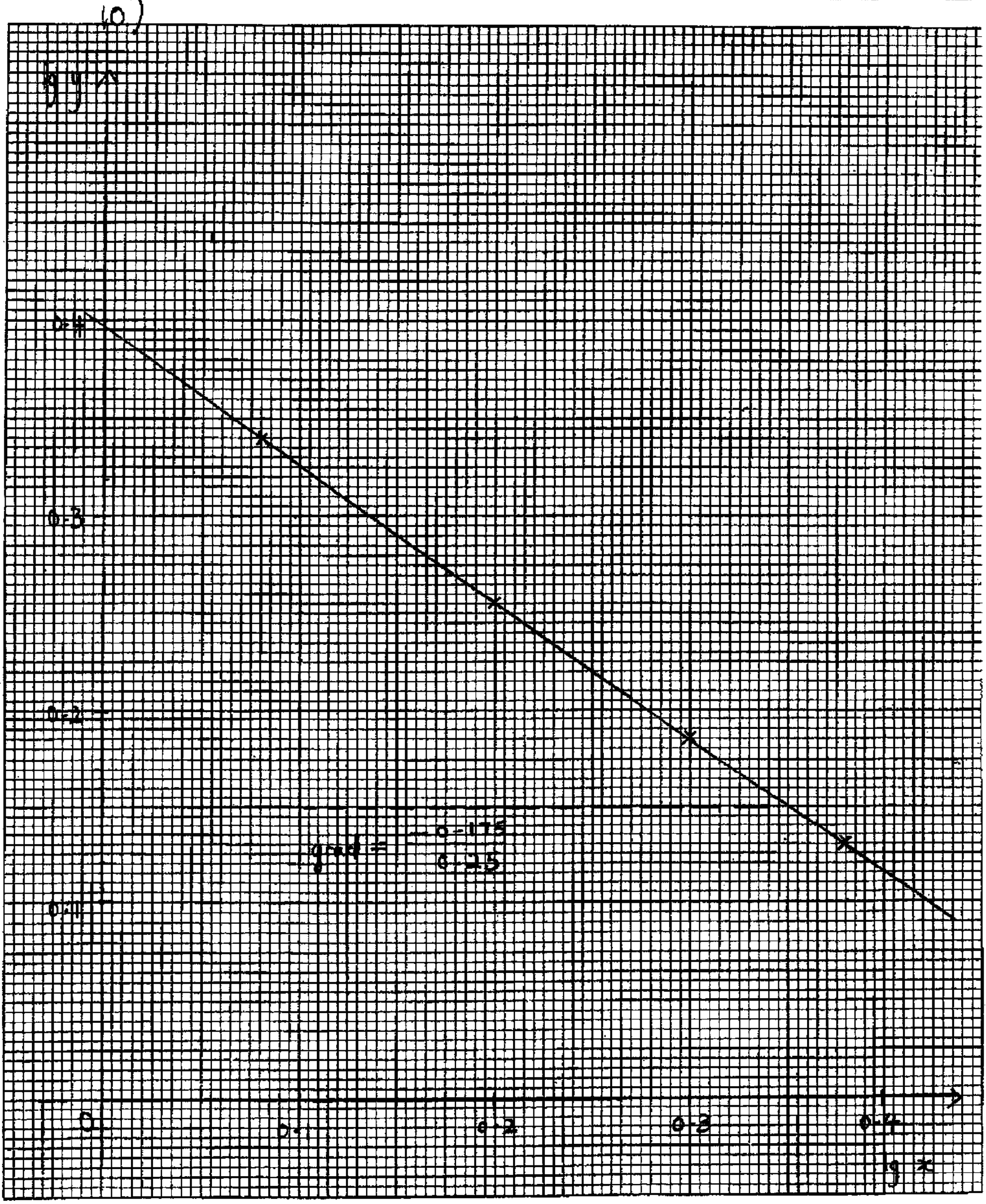
$= -\frac{1}{0.7}$

$= -1.43$ [1]

– END –

Name ______________________ Index No. __________

Subject ______________________ Class __________ Date __________



20 cm x 24 cm