NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2021

SUBJECT : **Additional Mathematics Paper 1 (4049/01)** **DATE** : **17 August 2021**

LEVEL : **Secondary 4 Express** **DURATION** : **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index No. in the spaces at the top of this page.
Write in dark blue or back pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use	
Score	/90

This question paper consists of 23 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

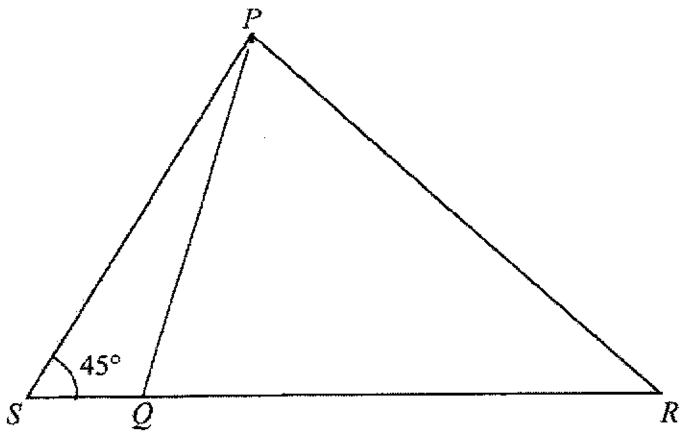
$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1. Given that $x^2 + 2x - 3$ is a factor of $f(x)$, where $f(x) = x^4 + 4x^3 + 2x^2 + ax - b$.

(a) Find the values of a and b. [4]

(b) Hence, find the other quadratic factor of $f(x)$. [2]

2.



The diagram shows a triangle PQR with an area of 18 cm^2. The base QR has a length of $(4\sqrt{2}+8)$ cm. The line RQ is extended to the point S where the angle $PSR = 45°$.

(a) Find the perpendicular distance from P to QR, leaving your answer in the form $a+b\sqrt{2}$, where a and b are rational numbers, [3]

(b) Find the exact length of *PS*, leaving your answer in the form $m+n\sqrt{2}$, where m and n are integers. [2]

3. Given the curve $y=\frac{1}{2}e^{2x}-\frac{1}{3}e^{-3x}+\frac{1}{3}$ and the normal to the curve at point $P\left(\ln 2,\ \frac{55}{24}\right)$ crosses the y – axis at Q.

(a) Show that $\frac{dy}{dx}=\frac{e^{5x}+1}{e^{3x}}$. [2]

(b) Find the equation of the normal at $P\left(\ln 2,\ \frac{55}{24}\right)$, leaving your answer in exact form. [3]

(c) Hence, find the coordinates of Q. [1]

4. (a) Find the coordinates of the stationary points on the graph of $y = 2x^3 - 9x^2 + 13$. [4]

(b) Hence, determine the nature of the stationary point(s). [3]

5. (a) Differentiate $2x\cos 2x$ with respect to x. [2]

(b) Hence, evaluate $\int_0^{\frac{\pi}{2}} (1-4x\sin 2x)\ dx$. [5]

6. It is given that $y=(x-2)(2x-5)^3$.

(a) Obtain an expression for $\frac{dy}{dx}$ in the form $(ax+b)(2x-5)^2$, where a and b are integers. [2]

(b) Determine the values of x for which y is a decreasing function. [2]

The variables x and y are such that, when $x = 3$, y is increasing at a rate of 0.35 units per second.

(c) Find the rate of change of x when $x = 3$.

It is given further that the variable z is such that $z = y^2$.

(d) Show that, when $x = 3$, z is increasing at twice the rate of y. [2]

7. (a) Given that $\sqrt{125^x} = \dfrac{5^{1-x}}{25}$, find the value of $\sqrt{125^x}$. [4]

(b) Show that $\log_2 x - \log_{16} x = \dfrac{3\lg x}{4\lg 2}$. [4]

8. (a) Write down the general term, T_{r+1}, for the binomial expansion of $\left(2x+\frac{1}{2x^3}\right)^{12}$. [1]

(b) Find the term independent of x in the expansion of $\left(\frac{1}{x^4}-1\right)\left(2x+\frac{1}{2x^3}\right)^{12}$. [5]

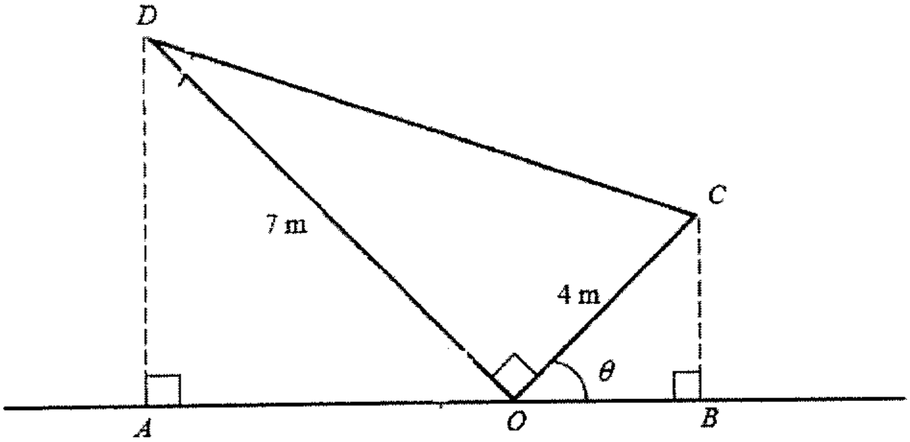
(c) Explain whether there are any terms containing x^n, where n is odd in the expansion of $\left(2x+\frac{1}{2x^3}\right)^{12}$. [2]

9. (a) Prove that $\dfrac{\sec^2 x}{1+\sec^2 x} = \dfrac{2}{\cos 2x + 3}$. [3]

(b) Hence, solve the equation $\frac{\sec^2 2x}{1+\sec^2 2x}=\frac{4}{7}$ for $0 \leq x \leq 3$. [4]

(c) State the number of solutions for $-2\pi \leq x \leq 2\pi$. [1]

10.



In the diagram, $OC = 4$ m, $OD = 7$ m and angle CBO = angle DOC = angle $DAO = 90°$.
It is also given that angle $COB = \theta$ such that $0° \leq \theta \leq 90°$.

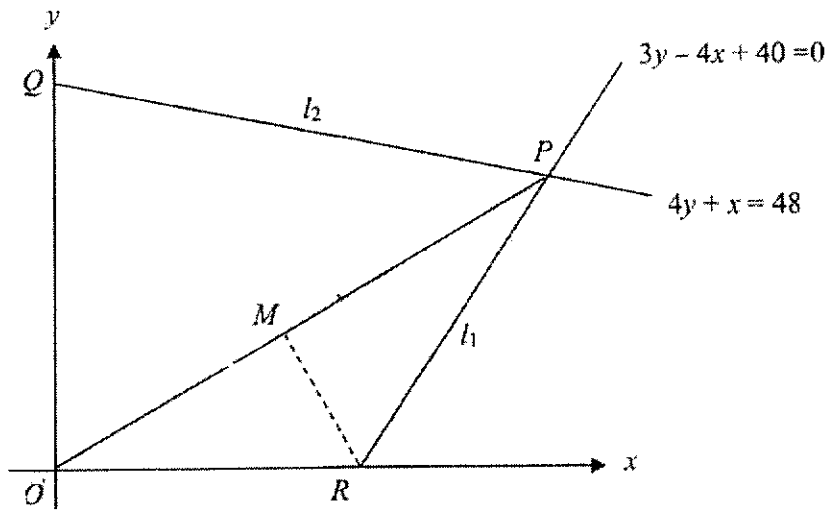
(a) Express AB in the form $p \sin \theta + q \cos \theta$ where p and q are integers. [2]

(b) Hence, express AB in the form $R \sin (\theta + \alpha)$ where R is positive and α is acute. [3]

(c) State the maximum length of AB and its corresponding value of θ. [3]

(d) Find the value of θ when $AB = \sqrt{55}$ m. [3]

11.



The diagram shows lines l_1 and l_2 intersecting at P. Line l_1 has equation $3y - 4x + 40 = 0$ and l_2 has equation $4y + x = 48$. The point M is the midpoint of OP. The line l_2 intersects the y-axis at Q and the line l_1 intersects the x-axis at R.

(a) Show that OP is perpendicular to MR. [5]

(b) Find the area of *ORPQ*. [3]

12. The table shows experimental values of two variable, x and y, which are connected by the equation $y = ae^{bx-1}$, where a and b are constants.

x	1	2	3	4	5
y	1.89	2.3	2.82	3.44	4.20

(a) On the grid given on page 23, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.2 units on the vertical axis, plot a straight line graph of ln y against x. [3]

(b) Use your graph to estimate the value of a and of b. [3]

(c) By drawing a suitable line on your graph, solve the equation $2.46 = ae^{bx-1}$. [2]

NAME	CLASS	INDEX NO.

ST. PATRICK'S SCHOOL
PRELIMINARY EXAMINATION 2021

SUBJECT	:	**Additional Mathematics Paper 2 (4049/02)**	DATE	:	**23 August 2021**
LEVEL	:	**Secondary 4 Express**	DURATION	:	**2 hours 15 minutes**

Candidates answer on the Question Paper.

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$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$Area\ of\ \Delta = \frac{1}{2}bc\sin A$$

[Turn over

1. (a) Express $y = 2x^2 - 4x + 7$ in the form $a(x - b)^2 + c$, where a, b and c are constants. [3]

(b) Hence, explain why the curve lies completely above the x-axis. [2]

[Turn over

2. A virus is spreading in a country. The number of infected cases, I, after t days, can be modelled by $I = \frac{a}{b+2^{20-t}}$, where a and b are constants. The number of infected cases was 3600 after 16 days. Four days later, the number of infected cases increased to 9000.

(a) Show that $a = 90000$ and $b = 9$. [3]

[Turn over

(b) Given that the number of infected cases first exceeds 100 after n days, where n is an integer, find the value of n. [3]

(c) Describe what happens to the number of infected cases in the long run. [1]

[Turn over

3. (a) Without the use of calculator, show that $\sin 75 = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

(b) In a city in the United Kingdom, the number of daylight hours is given by $h(t) = -4\cos(bt) + 12$, where t is an integer ranging from 1 to 12 inclusive that represents the month of January to December. It takes 12 months before the number of daylight hours reaches its lowest point again.

(i) Explain, with workings clearly shown, why $b = \frac{\pi}{6}$. [2]

(ii) Sketch the graph of $h(t)$, where $0 \le t \le 18$. [3]

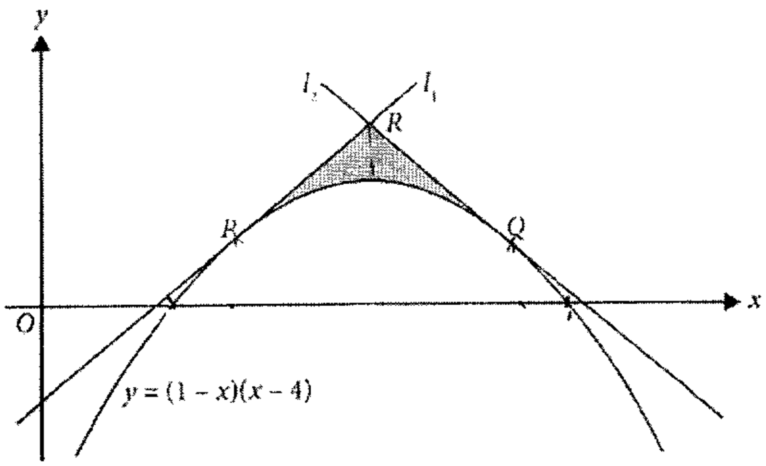
4. (a) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^4}\right) = \frac{1}{x^5} - \frac{4\ln x}{x^5}$. [3]

(c) Given that the curve $y = f(x)$ passes through the point $\left(1, \frac{3}{4}\right)$ and is such that

$f'(x) = \frac{\ln x}{x^5}$, find $f(x)$. [3]

[Turn over

5.



The diagram shows part of the curve $y=(1-x)(x-4)$ and two lines l_1 and l_2. The lines l_1 and l_2 are tangents to the curve at points P and Q and have gradients 2 and -2 respectively.

(a) Show that coordinates P and Q are $\left(\frac{3}{2},\frac{5}{4}\right)$ and $\left(\frac{7}{2},\frac{5}{4}\right)$ respectively. [3]

The lines l_1 and l_2 intersect at point R.

(b) Find the coordinate of point R. [2]

(c) Hence, find the shaded area PQR. [4]

6. (a) Find the set of values of x for which $5x^2 + 12x > 3x + 2$. [3]

[Turn over

(b) The line $y = mx + c$ does not intersect the curve $x^2 + y^2 = 8$.

(i) Prove that $8m^2 + 8 < c^2$. [4]

(ii) Hence, determine whether the line $y = 2x + 5$ intersects the curve $x^2 + y^2 = 8$. [2]

[Turn over

7. The equation of a circle is $x^2 + y^2 + 2x - 8y - 8 = 0$.

(a) Find the coordinates of the centre, C and the radius of the circle. [3]

(b) Find the equation of the tangent to the circle at the point $P(-2, 8)$. [3]

[Turn over

(c) If the circle is reflected in the line $y = 1$, find the equation of the reflected circle. [2]

(d) Find the area of the biggest square that can be drawn inside the circle. [2]

[Turn over

8. (a) Express $\dfrac{8x^3+3x+1}{x(2x+1)^2}$ as a sum of a polynomial and a proper fraction. [1]

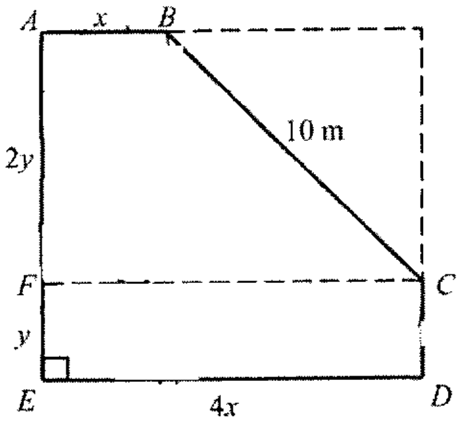
(b) Hence, express $\dfrac{8x^3+3x+1}{x(2x+1)^2}$ as a sum of a polynomial and a partial fraction. [5]

[Turn over

Space for Q8(b)

[Turn over

9.



ABCDEF is the cross-sectional plan of a temporary spectator stand, where *AB* is parallel to *FC* and *ED* and *AE* is parallel to *CD*.
Given that $AB = x$, $ED = 4x$, $FE = y$, $AF = 2y$ and $BC = 10$ m.

(a) Express y in terms of x. [2]

(b) Show that the area, S, of the cross-sectional plan of the spectator stand is given by
$S = \frac{9}{2}x\sqrt{100-9x^2}$. [3]

[Turn over

(c) Given that x varies, find the stationary value of S. [5]

(d) Hence, determine whether it is a maximum or minimum value. [1]

10. A particle moves in a straight line such that its velocity, v m/s, is given by $v = 9\sin\left(\frac{t}{2}\right)$, where t is the time in seconds after passing a fixed point O.

(a) Find the value of t for which the particle is at its first instantaneous rest after leaving O. [3]

(b) Find the value of t for which the particle is at its first maximum velocity. [2]

[Turn over

(c) Calculate the distance travelled by the particle in the first 10 seconds. [4]

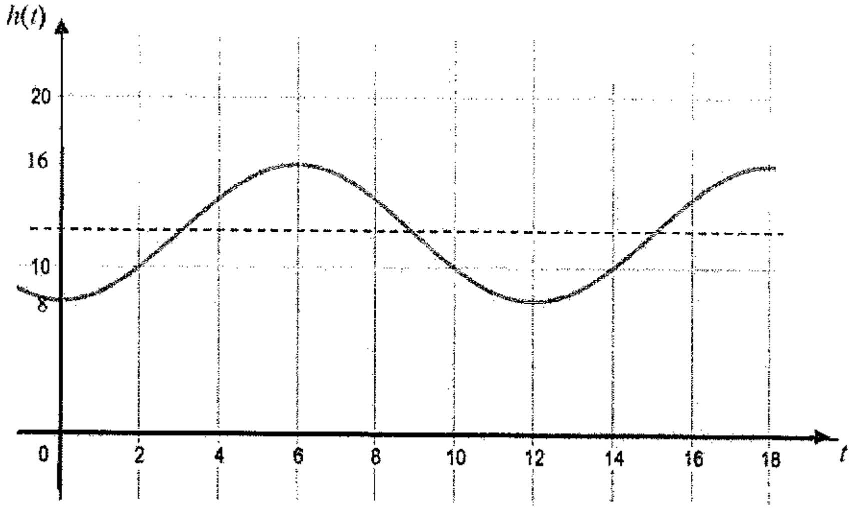
(d) Find the time that the particle passes through the point O again. [3]

End of Paper

Sec 4 Exp AM Prelim Exam Paper 2 Marking Scheme

1(a)	$y = 2x^2 - 4x + 7$ $= 2(x^2 - 2x + 7/2)$ $= 2[x^2 - 2x + (-1)^2 + 7/2 - (-1)^2]$ $= 2[(x-1)^2 + 5/2]$ $= 2(x-1)^2 + 5$	 [M1] [M1] [A1]
1(b)	$2(x-1)^2 \geq 0$ $2(x-1)^2 + 5 > 0$ => y >0 Therefore, curve lies completely above the x-axis.	 [M1] [A1]
Qn 1	(a) $y = 2(x-1)^2 + 5$	
2(a)	$I = \frac{a}{b + 2^{20-t}}$ When $t = 16, I = 3600$ $3600 = \frac{a}{b + 2^4}$ $3600b + 57600 = a$ (1) When $t = 20, I = 9000$ $9000 = \frac{a}{b+1}$ $9000b + 9000 = a$ (2) sub (1) into (2) $9000b + 9000 = 3600b + 57600$ $5400b = 48600$ $b = 9$ a = 90000	 [M1] [M2] [A1]

2(b)	$\dfrac{90000}{9+2^{20-t}}=100$ $90000=900+100(2^{20-t})$ $89100=100(2^{20-t})$ $891=2^{20-t}$ $\lg 891=(20-t)\lg 2$ $t=20-\dfrac{\lg 891}{\lg 2}$ $t=10.2007$ $\Rightarrow n=11$	[M1] [M1] [A1]
2(c)	$t\to\infty,\ 2^{20-t}\to 0$ $\Rightarrow I\to\dfrac{90000}{9}=10000$	[A1]
Qn 2	(b) n = 11 (c) $I\to 10000$	
3(a)	$\sin 75^0=\sin(30^0+45^0)$ $=\sin 30\cos 45+\cos 30\sin 45$ $=\dfrac{1}{2}\left(\dfrac{1}{\sqrt{2}}\right)+\dfrac{\sqrt{3}}{2}\left(\dfrac{1}{\sqrt{2}}\right)$ $=\dfrac{1+\sqrt{3}}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$ $=\dfrac{\sqrt{2}+\sqrt{6}}{4}$	[M1] [M1] [A1]
3(b)	$h(t)=-4\cos bt+12$ Period $=\dfrac{2\pi}{b}$ Period = 12 $\dfrac{2\pi}{b}=12$ $\Rightarrow b=\dfrac{\pi}{6}$	 [M1] [A1]

3(c)		Curve[2] axes/ label [1]
4(a)	$\frac{d}{dx}\left(\frac{\ln x}{x^4}\right) = \frac{x^4\left(\frac{1}{x}\right)-(\ln x)(4x^3)}{x^8}$	M1
	$= \frac{x^3 - 4x^3 \ln x}{x^8}$	M1
	$= \frac{1-4\ln x}{x^5}$	
	$= \frac{1}{x^5} - \frac{4\ln x}{x^5}$	A1
4(b)	$\int\left(\frac{1}{x^5}-\frac{4\ln x}{x^5}\right) dx = \frac{\ln x}{x^4}+c$	M1
	$\int \frac{1}{x^5}\, dx - \int \frac{4\ln x}{x^5}\, dx = \frac{\ln x}{x^4}+c$	
	$\int \frac{4\ln x}{x^5}\, dx = \int \frac{1}{x^5}\, dx - \frac{\ln x}{x^4}+c$	M1
	$\int \frac{\ln x}{x^5}\, dx = -\frac{1}{16x^4} - \frac{\ln x}{4x^4}+c$	A1

4(c)	$\int f'(x)\,dx = \int \frac{\ln x}{x^5}\,dx$ $f(x) = -\frac{1}{16x^4} - \frac{\ln x}{4x^4} + c$ sub $\left(1, \frac{3}{4}\right)$ into $f(x)$, $\frac{3}{4} = -\frac{1}{16} + c$ $c = \frac{13}{16}$ $\therefore f(x) = \frac{1}{16x^4} - \frac{\ln x}{4x^4} + \frac{13}{16}$	 M1 M1 A1
Qns 4	(b) $\frac{1}{16x^4} - \frac{\ln x}{4x^4} + c$ (c) $f(x) = \frac{1}{16x^4} - \frac{\ln x}{4x^4} + \frac{11}{16}$	
5(a)	$y = -x^2 + 5x - 4$ $\frac{dy}{dx} = -2x + 5$ let $\frac{dy}{dx} = 2$, then, $x = \frac{3}{2}$ and $y = \frac{5}{4}$ so, $P\left(\frac{3}{2}, \frac{5}{4}\right)$ let $\frac{dy}{dx} = -2$ then, $x = \frac{7}{2}$ and $y = \frac{5}{4}$ so, $Q\left(\frac{7}{2}, \frac{5}{4}\right)$	 M1 A1 A1

5(b)	equation of l_1 is $y-\frac{5}{4}=2\left(x-\frac{3}{2}\right)$ $y=2x-\frac{7}{4}$ equation of l_2 is $y-\frac{5}{4}=-2\left(x-\frac{7}{2}\right)$ $y=-2x+\frac{33}{4}$ $2x-\frac{7}{4}=-2x+\frac{33}{4}$ $4x=10$ $x=\frac{5}{2}$ and $y=\frac{13}{4}$ $\therefore R\left(\frac{5}{2},\frac{13}{4}\right)$	M1 A1
5(c)	Shaded area $=2\times\left[\frac{1}{2}\left(\frac{5}{4}+\frac{13}{4}\right)(1)-\int_{3/2}^{5/2}\left(-x^2+5x-4\right)\,dx\right]$ $=2\times\left[\frac{9}{4}-\left[\frac{-x^3}{3}+\frac{5x^2}{2}-4x\right]_{3/2}^{5/2}\right]$ $=2\times\left[\frac{9}{4}-\left[\frac{5}{12}-\left(-\frac{3}{2}\right)\right]\right]$ $=\frac{2}{3}$ units^2	M2 M1 A1
Qns 5	(b) $R\left(\frac{5}{2},\frac{13}{4}\right)$ (c) $\frac{2}{3}$ units^2	
6(a)	$5x^2+12x>3x+2$ $5x^2+9x-2>0$ $(5x-1)(x+2)>0$ $x<-2$ or $x>\frac{1}{5}$	[M1] [M1] [A1]

6(bi)	$y = mx + c$ (1) $x^2 + y^2 = 8$ (2) Sub (1) into (2), $x^2 + (mx+c)^2 = 8$ $x^2 + m^2x^2 + 2mcx + c^2 - 8 = 0$ $(1+m^2)x^2 + 2mcx + c^2 - 8 = 0$ Does not intersect => D=0 $(2mc)^2 - 4(1+m^2)(c^2-8) < 0$ $4m^2c^2 - 4(c^2 - 8 + m^2c^2 - 8m^2) < 0$ $4m^2c^2 - 4c^2 + 32 - 4m^2c^2 + 32m^2 < 0$ $32m^2 - 4c^2 + 32 < 0$ $8m^2 + 8 < c^2$	[M1] [M1] [M1] [A1]
6(bii)	$y = 2x + 5$ $m = 2, c = 5$ $8m^2 + 8 = 8(2)^2 + 8 = 40$ $c^2 = 25$ Since $8m^2 + 8$ is not $< c^2$, $y = 2x + 5$ intersect the curve.	} [M1] [A1]
Qn 6	(a) $x < -2$ or $x > \frac{1}{5}$ (b) The line intersect the curve.	
7(a)	$x^2 + 2x + 1 + y^2 - 8y + (-4)^2 = 8 + 1 + 16$ $(x+1)^2 + (y-4)^2 = 25$ Centre $C(-1, 4)$ Radius = 5 units	[M1] [A1] [A1]

7(b)	Grad of CP = –4 Grad of tangent at P = $\frac{1}{4}$ $y=\frac{1}{4}x+c$ (–2,8) $8=\frac{1}{4}(-2)+c$ $c=\frac{17}{2}$ $y=\frac{1}{4}x+\frac{17}{2}$	[M1] [M1] [A1]
7(c)	Centre = (–1, –2) Radius = 5 Equation: $(x+1)^2+(y+2)^2=25$	[M1] [A1]
7(d)	Diameter is the diagonal of the biggest square that can be drawn inside the circle. Let x – side of the square. $x^2+x^2=10^2$ $2x^2=100 \Rightarrow x=\sqrt{50}$ Area = $\sqrt{50}\times\sqrt{50}$ = 50 units^2	[M1] [A1]
Qn 7	(a) Centre $C(-1,4)$ (b) $y=\frac{1}{4}x+\frac{17}{2}$ Radius = 5 units (c) $(x+1)^2+(y+2)^2=25$ (d) 50 units^2	
8(a)	$\frac{8x^3+3x+1}{x(2x+1)^2}=\frac{8x^3+3x+1}{4x^3+4x^2+x}=2+\frac{1+x-8x^2}{4x^3+4x^2+x}$	B1

8(b)	so, $\dfrac{1+x-8x^2}{x(2x+1)^2}=\dfrac{A}{x}+\dfrac{B}{2x+1}+\dfrac{C}{(2x+1)^2}$ $1+x-8x^2=A(2x+1)^2+Bx(2x+1)+Cx$ let $x=0$, $1=A$ let $x=-\dfrac{1}{2}$, $-\dfrac{3}{2}=-\dfrac{C}{2}$ $C=3$ let $x=1$, $-6=9A+3B+C$ $-6=9+3B+3$ $B=-6$ $\therefore \dfrac{8x^3+3x+1}{x(2x+1)^2}=2+\dfrac{1}{x}-\dfrac{6}{2x+1}+\dfrac{3}{(2x+1)^2}$	M1 M3 A1
8(c)	$\int_1^3 \dfrac{8x^3+3x+1}{x(2x+1)^2}\,dx \quad =\int_1^3\left(2+\dfrac{1}{x}-\dfrac{6}{2x+1}+\dfrac{3}{(2x+1)^2}\right)dx$ $=\left[2x+\ln x-3\ln(2x+1)-\dfrac{3}{2(2x+1)}\right]_1^3$ $=1.046596+1.795836$ $=2.8424$ $=2.84$ (3 s.f.)	M1 M1 M1 A1
Qns 8	(a) $2+\dfrac{1+x-8x^2}{4x^3+4x^2+x}$ (b) $2+\dfrac{1}{x}-\dfrac{6}{2x+1}+\dfrac{3}{(2x+1)^2}$ (c) 2.84	
9(a)	$10^2=(2y)^2+(3x)^2$ $100=4y^2+9x^2$ $y=\sqrt{\dfrac{100-9x^2}{4}}$	M1 A1
9(b)	$S=\dfrac{1}{2}(5x)(2y)+4xy$ $=9xy$ $=9x\left(\sqrt{\dfrac{100-9x^2}{4}}\right)$ $=\dfrac{9}{2}x\sqrt{100-9x^2}$ (shown)	M1 M1 A1

9(c)	$\frac{dS}{dx} = \frac{9}{2}(100-9x^2)^{\frac{1}{2}} + \frac{9}{2}x\left(\frac{1}{2}\right)(100-9x^2)^{-\frac{1}{2}}(-18x)$	M1
	$= \frac{9}{2}(100-9x^2)^{\frac{1}{2}} - \frac{81x^2}{2}(100-9x^2)^{-\frac{1}{2}}$	
	$= \frac{9}{2}(100-9x^2)^{-\frac{1}{2}}\left[(100-9x^2)-9x^2\right]$	M1
	$= \frac{9(100-18x^2)}{2(100-9x^2)^{\frac{1}{2}}}$	
	$= \frac{9(50-9x^2)}{(100-9x^2)^{\frac{1}{2}}}$	M1
	let $\frac{dS}{dx} = 0$,	
	$\frac{9(50-9x^2)}{(100-9x^2)^{\frac{1}{2}}} = 0$	M1
	$50-9x^2 = 0$	
	$x^2 = \frac{50}{9}$	
	$x = 2.357$ or $x = -2.357$ (rej)	
	$\therefore$ stationary value of $S = 74.99999 = 75.0$ m^2 (3s.f.)	A1
9(d)	$\frac{dS}{dx} = \frac{9(50-9x^2)}{(100-9x^2)^{\frac{1}{2}}}$	
	let $x = 2$, $\frac{dS}{dx} = \frac{63}{4} > 0$, function is increasing.	
	let $x = 2.5$, $\frac{dS}{dx} = -8.504 < 0$, function is decreasing.	B1
	$\therefore$ by first derivative test, $S = 75.0$ is maximum value.	
Qns 9	(a) $y = \sqrt{\frac{100-9x^2}{4}}$ (c) 75.0 (d) maximum value	

10(a)	$9\sin\left(\frac{t}{2}\right)=0$ $\sin\left(\frac{t}{2}\right)=0$ basic angle $=0$ rad so, $\frac{t}{2}=0,\ \pi$ $t=0,\ 2\pi$ $\therefore t=2\pi$ sec	M1 M1 A1
10(b)	Maximum velocity when $\sin\left(\frac{t}{2}\right)=1$. $\frac{t}{2}=\frac{\pi}{2}$ $t=\pi$ sec	M1 A1
10(c)	$s=\int 9\sin\frac{t}{2}\ dx$ $s=9\left(\frac{-\cos\frac{t}{2}}{\frac{1}{2}}\right)+c$ $s=-18\cos\frac{t}{2}+c$ when $t=0,\ s=0,\qquad c=18$ hence, $s=-18\cos\frac{t}{2}+18$ when $t=2\pi,\quad s=-18\cos\pi+18$ $s=36$ m when $t=10,\quad s=-18\cos 5+18$ $s=12.894$ m Dist. travelled in first 10 sec $=36+(36-12.894)$ $=59.1059$ $=59.1$ m (3 s.f.)	M1 M1 M1 A1

10(d)	$s = -18\cos\frac{t}{2} + 18$ let $s = 0$, $-18\cos\frac{t}{2} + 18 = 0$ $\cos\frac{t}{2} = 1$ basic angle $= 0$ $\frac{t}{2} = 0$ (rej), 2π $\therefore t = 4\pi$	 M1 M1 A1
Qns 10	(a) $t = 2\pi$ sec (b) $t = \pi$ sec (c) 59.1 m (d) $t = 4\pi$	

1(a)	$(x+3)(x-1)$ $f(1)=0$ $1+4+2+a-b=0$ $a-b=-7$ ---------(1) $f(-3)=0$ $(-3)^4+4(-3)^3+2(-3)^2+a(-3)-b=0$ $81-108+18-3a-b=0$ $-3a-b=9$ -------(2) $(1)-(2):\quad 4a=-16$ $a=-4$ $b=3$	 [M1] [M1] [A1] [A1]
1(b)	Long division x^2+2x+1 $x^2+2x-3 \,\big)\, x^4+4x^3+2x^2-4x-3$ $-(x^4+2x^3-3x^2)$ $2x^3+5x^2-4x-3$ $-(2x^3+4x^2-6x)$ x^2+2x-3 $-(x^2+2x-3)$ 0 $f(x)=(x^2+2x-3)(x^2+2x+1)$ Other factor is x^2+2x+1.	 [M1] [A1]
Qn 1	(a) $a=-4$, $b=3$ (b) the other factor: x^2+2x+1	

2(a)	Let the perpendicular distance be x. $\frac{1}{2}\left(4\sqrt{2}+8\right)x=18$ $\left(4\sqrt{2}+8\right)x=36$ $x=\frac{36}{8+4\sqrt{2}}$ $=\frac{36}{8+4\sqrt{2}}\times\frac{8-4\sqrt{2}}{8+4\sqrt{2}}$ $=\frac{288-144\sqrt{2}}{64-32}$ $=9-\frac{9}{2}\left(\sqrt{2}\right)\ cm$	[M1] [M1] [A1]
2(b)	$\sin 45° = \frac{x}{PS}$ $\frac{1}{2}=\frac{9-\frac{9}{2}\sqrt{2}}{PS}$ $PS=\sqrt{2}\left(9-\frac{9}{2}\sqrt{2}\right)$ $=9\sqrt{2}-\frac{9}{2}(2)$ $=-9+9\sqrt{2}$	[M1] [A1]
Qn 2	(a) $9-\frac{9}{2}\left(\sqrt{2}\right)\ cm$ (b) $PS=-9+9\sqrt{2}$ cm	

3(a)	$\frac{dy}{dx}=\frac{1}{2}e^{2x}(2)-\frac{1}{3}e^{-3x}(-3)$ $=e^{2x}+\frac{1}{e^{3x}}$ $=\frac{e^{2x}(e^{3x})+1}{e^{3x}}$ $=\frac{e^{5x}+1}{e^{3x}}$ (shown)	[M1] [M1]
3(b)	at P, $\frac{dy}{dx}=\frac{e^{5\ln 2}+1}{e^{3\ln 2}}=\frac{33}{8}$ $\therefore$ gradient of normal $=-\frac{8}{33}$ equation of normal is $y-\frac{55}{24}=-\frac{8}{33}(x-\ln 2)$ $y=-\frac{8}{33}x+\frac{8}{33}\ln 2+\frac{55}{24}$	 [M1] [M1] [A1]
3(c)	$Q\left(0, \frac{8}{33}\ln 2+\frac{55}{24}\right)$	[A1]
Qn 3	(b) $y=-\frac{8}{33}x+\frac{8}{33}\ln 2+\frac{55}{24}$ (c) $Q\left(0, \frac{8}{33}\ln 2+\frac{55}{24}\right)$	

4(a)	$\frac{dy}{dx}=6x^2-18x$ let $\frac{dy}{dx}=0$, $6x^2-18x=0$ $6x(x-3)=0$ $x=0$ or $x=3$ $y=13$ $y=-14$ $\therefore (0,13)$ and $(3,-14)$	[M1] [M1] [A2]
4(b)	$\frac{d^2y}{dx^2}=12x-18$ at $x=0$, $\frac{d^2y}{dx^2}=-18<0$ $\therefore (0,13)$ is maximum point. at $x=3$, $\frac{d^2y}{dx^2}=18>0$ $\therefore (3,-14)$ is minimum point.	[M1] [A1] [A1]
Qn 4	(a) (0, 13) and (3, –14) (b) (0, 13) max point; (3, –14) min point.	

5(a)	$\frac{d}{dx}(2x\cos 2x) = 2x(-\sin 2x)(2) + 2\cos 2x$ $= 2\cos 2x - 4x\sin 2x$	[M1] [A1]
5(b)	$\int(2\cos 2x - 4x\sin 2x)\,dx = 2x\cos 2x + c$ $\int 2\cos 2x\,dx - \int 4x\sin 2x\,dx = 2x\cos 2x + c$ $\int 4x\sin 2x\,dx = \int 2\cos 2x\,dx - 2x\cos 2x + c$ $= 2\left(\frac{\sin 2x}{2}\right) - 2x\cos 2x + c$ $= \sin 2x - 2x\cos 2x + c$ so, $\int_0^{\frac{\pi}{2}}(1 - 4x\sin 2x)\,dx = [x - \sin 2x + 2x\cos 2x]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{2} - \pi$ $= -\frac{\pi}{2}$	[M1] [M1] [M1] [M1] [A1]

6(a)	$\frac{dy}{dx} = (x-2)\times 3(2x-5)^2(2)+(2x-5)^3$ $= (2x-5)^2[6x-12+2x-5]$ $= (2x-5)^2(8x-17)$	[M1] [A1]
6(b)	$\frac{dy}{dx}<0$, so $(2x-5)^2(8x-17)<0$ since $(2x-5)^2>0$ for all values of x, then $(8x-17)<0$. $8x-17<0$ $x<\frac{17}{8}$	[M1] [A1]
6(c)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.35=(2x-5)^2(8x-17)\times\frac{dx}{dt}$ when $x=3$, $0.35=7\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.35\div 7=0.05$ units/s^2	[M1] [A1]
6(d)	$z=y^2$ $\frac{dz}{dy}=2y$ $\frac{dz}{dt}=\frac{dz}{dy}\times\frac{dy}{dt}$ $=2y\times\frac{dy}{dt}$ $=2(x-2)(2x-5)^3\times\frac{dy}{dt}$ *when* $x=3$, $\frac{dz}{dt}=2(1)(1)^3\frac{dy}{dy}$ $=2\frac{dy}{dt}$	[M1] [A1]
Qn 6	(a) $(2x-5)^2(8x-23)$ (b) $x<\frac{23}{8}$ (c) 0.05 units/s^2	

7(a)	$\sqrt{125^x} = \frac{5^{1-x}}{25}$ $\sqrt{5^{3x}} = \frac{5^{1-x}}{5^2}$ $5^{\frac{3}{2}x} = 5^{1-x-2}$ $\Rightarrow \frac{3}{2}x = -x-1$ $x = -\frac{2}{5}$ $\therefore \sqrt{125^x} = 0.3807 = 0.381\ (3s.f)$	 [M1] [M1] [A1] [A1]
7(b)	$\log_2 x - \log_{16} x$ $= \log_2 x - \frac{\log_2 x}{\log_{16} x}$ $= \log_2 x - \frac{\log_2 x}{4}$ $= \log_2 x - \log_2 x^{\frac{1}{4}}$ $= \log_2 x^{\frac{3}{4}}$ $= \frac{3}{4}\log_2 x$ $= \frac{3}{4}\left(\frac{\lg x}{\lg 2}\right) = \frac{3\lg x}{4\lg 2}$	 [M1] [M1] [A1] [M1]
Qn 7	(a) 0.381	

8(a)	$T_{r-1} = {}^{12}C_r\,(2x)^{12-r}\left(\frac{1}{2x^3}\right)^r$	[B1]
8(b)	$T_{r+1} = {}^{12}C_r\,(2x)^{12-r}\left(\frac{1}{2x^3}\right)^r$ $= {}^{12}C_r\,(2)^{12-r-r}\,(x)^{12-4r}$ Term with x^4: $12-4r=4$ $r=2$ Coef. of $x^4 = {}^{12}C_2(2)^8 = 16896$ Term with x^0: $12-4r=0$ $r=3$ Coef. of $x^0 = {}^{12}C_3(2)^6 = 14080$ Term independent of $x = 1(16896) + (-1)(14080)$ $= 2816$	[M2] [M1] [M1] [A1]
8(c)	$n = 12 - 4r$ 4r is even => $n = 12 - 4r$ is even. Therefore, there are no terms containing x^n whereby n is odd.	[M1] [A1]
Qn 8	(a) $T_{r+1} = {}^{12}C_r\,(2x)^{12-r}\left(\frac{1}{2x^3}\right)^r$ (b) 2816 (c) There are no terms containing x^n whereby n is odd.	

9(a)	$LHS = \frac{\sec^2 x}{1+\sec^2 x}$ $= \frac{1}{\frac{1}{\sec^2 x}+1}$ $= \frac{1}{\cos^2 x+1}$ $= \frac{1}{\frac{\cos 2x+1}{2}+1}$ $= \frac{2}{\cos 2x+3} = RHS$	 [M1] [M1] [A1]
9(b)	$\frac{\sec^2 2x}{1+\sec^2 2x} = \frac{4}{7}$ $\frac{2}{\cos 4x+3} = \frac{4}{7}$ $14 = 4\cos 4x+12$ $\cos 4x = \frac{1}{2}$ $ref\angle = \frac{\pi}{3}$ $4x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$	 [M1] [M1] [A1] [A1]
9(c)	$-2\pi \le x \le 2\pi \Rightarrow -8\pi \le 4x \le 8\pi$ $\therefore$ No. of solutions = 16	 [A1]
Qn 9	(b) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ (c) 16 solutions	

10(a)	$\cos\theta = \dfrac{OB}{4} \Rightarrow OB = 4\cos\theta$ $\angle ADO = \theta$ $\sin\theta = \dfrac{OA}{7} \Rightarrow OA = 7\sin\theta$ $\therefore AB = OA + OB = 7\sin\theta + 4\cos\theta$	[M1] [A1]
10(b)	$AB = R\sin(\theta+\alpha)$ $R = \sqrt{7^2+4^2} = \sqrt{65}$ $\alpha = \tan^{-1}\left(\dfrac{4}{7}\right) = 29.744$ $\therefore AB = \sqrt{65}\sin(\theta + 29.7°)$	[M1] [M1] [A1]
10(c)	Max length of AB = $\sqrt{65}$ Max occurs when $\sin(\theta + 29.744°) = 1$ $\sin(\theta + 29.744°) = 1$ $ref\angle = 90°$ $\theta + 29.744° = 90°$ $\theta = 90° - 29.744$ $= 60.256° = 60.3° \quad (1dp)$	[A1] [M1] [A1]
10(d)	$\sqrt{65}\sin(\theta + 29.744°) = \sqrt{55}$ $\sin(\theta + 29.744°) = \dfrac{\sqrt{55}}{\sqrt{65}}$ $ref\angle = 66.906°$ $\theta + 29.744° = 66.906°$ $\theta = 66.906° - 29.744$ $= 37.162 = 37.2° \quad (1dp)$	[M1] [M1] [A1]
Qn 10	(b) $AB = \sqrt{65}\sin(\theta + 29.7°)$ (c) Max AB = $\sqrt{65}$; $\theta = 60.3°$ (d) $\theta = 37.2°$	

11(a)	$3y-4x+40=0$ (1) $4y+x=48$ (2) $From (2): x=48-4y$ $3y-4(48-4y)+40=0$ $19y=152$ $y=8 \Rightarrow x=16$ $P(16,8)$ $M(8,4)$ l_2: when $y=0$, $x=10$ $R(10,0)$ $\text{Grad}_{OP}=\frac{8}{16}=\frac{1}{2}$ $\text{Grad}_{MR}=\frac{4-0}{8-10}=-2$ Since $\text{Grad}_{OP}\times\text{Grad}_{MR}=-1$ $\Rightarrow$ *OP* is perpendicular to *MR*.	 [M1] [M1] [M1] [M1] [A1]
10(b)	l_2: *when* $x=0$, $y=12$ $Q(0,12)$ Area *of ORPQ* $=\frac{1}{2}\begin{vmatrix}0 & 10 & 16 & 0 & 0\\0 & 0 & 8 & 12 & 0\end{vmatrix}$ $=\frac{1}{2}[(80+192)-(0)]$ $=136$ units^2	 [M1] [M1] [A1]
Qn 11	(b) 136 units^2	

12(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>ln y</td><td>0.64</td><td>0.83</td><td>1.04</td><td>1.24</td><td>1.44</td></tr></table> Table of values [1] Plot points [1] Smooth Curve [1]	
10(b)	$y = ae^{bx-1}$ $\ln y = \ln a + \ln e^{bx-1}$ $\ln y = (bx-1) + \ln a$ $\ln y = bx + \ln a - 1$ gradient $= b$ vertical intercept $= \ln a - 1$ From the graph, $\ln a - 1 = 0.43$ $\ln a = 1.43$ $a = e^{1.43}$ $= 4.1786 = 4.18\ (3s.f)$ $Grad = \frac{1.34-0.54}{4.5-0.5} = 0.2$ $\therefore b = 0.2$	[M1] [A1] [A1]
11(c)	$2.46 = ae^{bx-1}$ $y = 2.46 \Rightarrow \ln y = 0.9$ From graph, when $\ln y = 0.9$, $x = 2.3$	[M1] [A1]
Qn 11	(b) a = 4.18; b = 0.2 (c) $x = 2.3$	