ST. MARGARET'S SECONDARY SCHOOL
Preliminary Examinations 2021

Founded 1842

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **18 August 2021**

Secondary 4 Express **2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

This document consists of **21** *printed pages and* **1** *blank page.*

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

2 Find the range of x such that $y = 3x^2e^{5-2x}$ is a decreasing function. [3]

1 Find the range of values of m for which $x^2 - 2mx + 2m$ is greater than -3 for all real values of x. [3]

3(a) Without using a calculator, find the value of x such that $\frac{9^{2x+3}}{\sqrt{81^x}} = \left(\frac{1}{3}\right)^{1-5x}$. [4]

(b) Solve the equation $\frac{10}{3}\sqrt{3^x} = 3^x + 1$ [4]

5 (i) Given that $y = (2x-1)^3 \ln\sqrt{2x-1}$.

Show that $\frac{dy}{dx} = 6(2x-1)^2 \ln\sqrt{2x-1} + (2x-1)^2$. [3]

(ii) Hence evaluate $\int_1^5 (2x-1)^2 \ln\sqrt{2x-1}\ dx$. [4]

4 Find all values of x which satisfies the equation $(2 + 3\cos x)(4 - 5\sin x) = 0$ for $0 \leq x \leq 7$. [4]

6 (i) Show that $\tan x + \frac{\cos x}{1+\sin x} = \sec x$. [3]

(ii) Explain, without the use of calculator, why the equation $\tan x + \frac{\cos x}{1+\sin x} = -\frac{1}{3}$ has no solution. [2]

7 It is given that $f(x) = 2x^3 + ax^2 + x + b$.

(i) Find the value of a and of b for which $2x^2 + x - 1$ is a factor of $f(x)$. [4]

(ii) Solve the equation $f(x) = 0$. [2]

(iii) Hence solve $\frac{1}{4}y^3 + \frac{a}{4}y^2 + \frac{1}{2}y + b = 0$. [2]

(b) Sketch, the graph of $y = 3\sin 2x$ for $0 \le x \le \pi$. [2]

9 The equation of a circle, C_1 is $x^2 + y^2 - 4x + 6y - 3 = 0$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

8 Given that $y = 5\sin^2 t - 3\cos^2 t$,

(i) express y in the form $a\cos 2t + b$, where a and b are integers. [2]

(ii) **(a)** State the amplitude and period of $3\sin 2x$. [2]

(ii) Explain why C_1 touches the line $y = 1$. [1]

(iii) Find the equation of the circle C_2 which is a reflection of C_1 in the line $y = 1$. [1]

The line $y + 9 = x$ intersects the circle C_1 at the points $M\ (2, -7)$ and $N(6, -3)$.

(iv) Find the shortest distance from the centre of the circle to the line $y + 9 = x$. [2]

10 A bacteria sample was cultured in a laboratory. The number of bacteria in the sample, y, is related to the time elapsed, t minutes, by the equation $y = k(2)^{\frac{t}{m}}$, where k and m are constants. The table below shows measured values of y and t.

t	10	20	30	40	50
y	2 600	4 300	7 000	11 400	19 200

(i) Plot lg y against t and draw a straight line graph. [3]

(ii) Use your graph to estimate

(a) the value of k and of m. [4]

(b) the time taken for the number of bacteria to reach 15 000. [2]

11 In the expansion of $\left(x^2 - \frac{1}{2x^4}\right)^n$, the sixth term is independent of x.

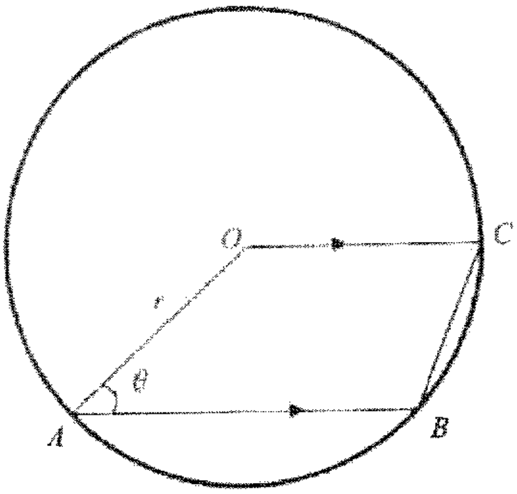
(i) Show that the value of n is 15 and find the value of the sixth term. [4]

(ii) Hence, find the term independent of x in the expansion

$$\left(x^2 - \frac{1}{2x^4}\right)^n \left(32 + 64x^6\right).$$ [5]

12 The diagram shows a trapezium $OABC$ inside a circle with centre O and radius r cm.

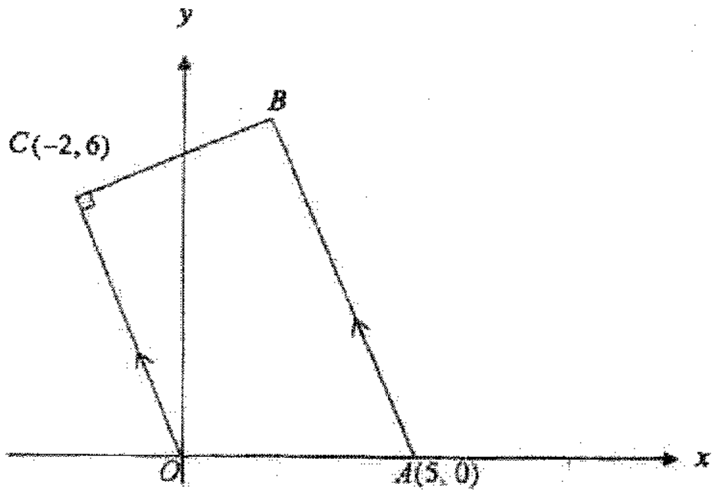
Given that OC is parallel to AB and angle $OAB = \theta$ radians.



(i) Show that the area of the trapezium $OABC$, S cm^2 is given by

$S = \frac{r^2}{2}(\sin\theta + \sin 2\theta)$. [3]

13 **Solutions to this question by accurate drawing will not be accepted.**



In the trapezium $OABC$, the point A has coordinates $(5, 0)$ and the point C has coordinates $(-2, 6)$. The sides OC and AB are parallel, and BC is perpendicular to OC.

(i) Show that the coordinates of B are $\left(2\frac{1}{2}, 7\frac{1}{2}\right)$. [5]

(ii) Given that r is a constant, calculate the value of θ for which S has a stationary value. [3]

(iii) Determine whether the stationary value of S is a maximum or minimum. [2]

(ii) OC is produced to D such that $OABD$ is a parallelogram.

Find the coordinates of D. [2]

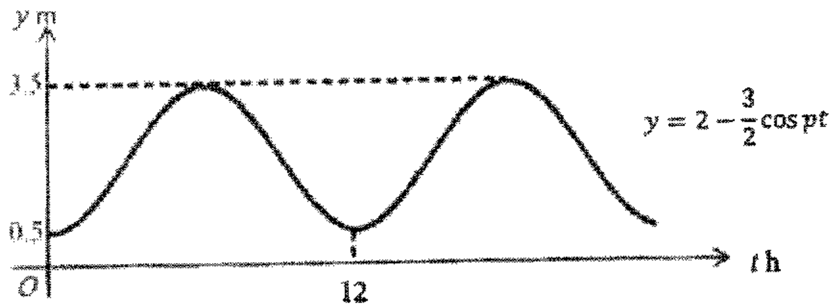
(iii) Find the equation of the perpendicular bisector of OC. [2]

(iv) E is a point which lies on the perpendicular bisector of OC such that the area of quadrilateral $OAEC$ is 15 units^2. Given that the x-coordinate of E is positive, find the coordinates of E. [4]

1 (i) Express $\frac{2x+15}{(3+x)(3-2x)}$ in partial fractions. [4]

(ii) Hence evaluate $\int_0^1 \frac{2x+15}{(3+x)(3-2x)}\,dx$. [3]

2 The graph below shows the vertical displacement of sea tidal waves, y m, represented by $y = 2 - \frac{3}{2}\cos pt$, where t is the time in hours starting from its lowest displacement and p is a constant. It takes 12 hours for the vertical displacement to be at its lowest point again.



(i) Show that $= \frac{\pi}{6}$. [1]

(ii) Find the first two values of t for which $y = 1.6$ m. [3]

(iii) It is safe to swim when the vertical displacement of sea tidal waves is below 1.6 m. Given that the time is 08 00 when $t = 0$, explain using your answers in part (ii), whether it would be safe to swim between 09 00 and 10 00. [1]

3 The line $2y + x + a = 0$ is a normal to the curve $y = 2x^2 - 3x + 1$ at the point P.

(i) Find the coordinates of P. [4]

(ii) Determine the value of a. [2]

4 (a) Differentiate with respect to x

(i) $\sqrt{2+\frac{1}{x}}$, [2]

(ii) $\frac{x^4+x^2-4}{x^2}$. [2]

(b) A curve is defined by $y = \frac{3x}{\sqrt{2-x^2}}$.

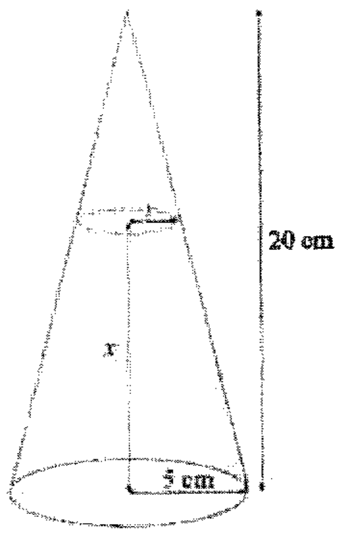
(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the x-coordinates of the points on the curve at which the gradient is 6. [2]

5 [The volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

A hollow right circular cone of base radius 5 cm and height 20 cm has a small opening at the base. Initially, the cone is fully filled with water. Due to the leak from the base, water flows out at a constant rate of 2 cm^3/s.

(i) Show that the volume of water V cm^3 in the cone when the height is x cm is given by $V = \frac{500\pi}{3} - \frac{\pi}{48}(20 - x)^3$. [3]



(ii) Find the rate of change of the height of water in the cone when $x = 8$ cm. [3]

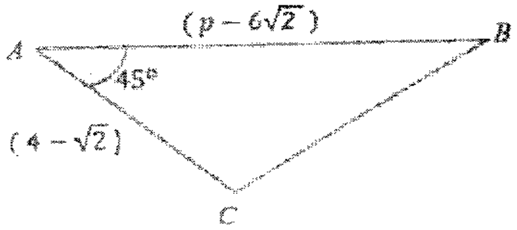
6 Solve the equation $\frac{1}{\log_9 3} + \log_3(2x+6) = \log_{\sqrt{3}}(1-x) + 1$. [5]

7 (i) Show that $\cos(A+B)\cos(A-B) = \cos^2 A + \cos^2 B - 1$. [4]

(ii) Hence, determine the value of $\cos 15^\circ \cos 75^\circ$ without the use of calculator. [3]

8 A particular curve for which $\frac{d^2y}{dx^2} = \sin 3x - \cos\frac{1}{2}x$ passes through the point $Q\left(\frac{\pi}{3}, 2\sqrt{3}\right)$. The gradient of the curve at Q is 1. Find the equation of the curve. [6]

9 In triangle ABC shown below, sides AB and AC are $(p - 6\sqrt{2})$ cm and $(4 - \sqrt{2})$ cm respectively. Angle $BAC = 45°$.

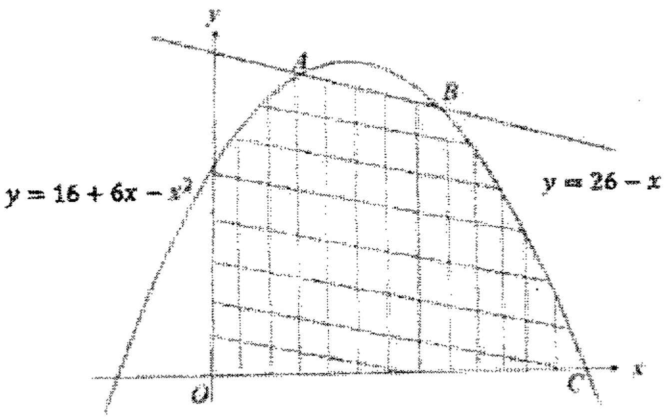


Given that the area of the triangle ABC is $(17\sqrt{2} - 19)$ cm^2,

(i) show that $p = 14$. [4]

(ii) find the perpendicular distance from B to AC, leaving your answer in the form $a\sqrt{2}+b$. [3]

10 The diagram shows part of the curve $y = 16 + 6x - x^2$ intersecting the line $y = 26 - x$ at A and B.



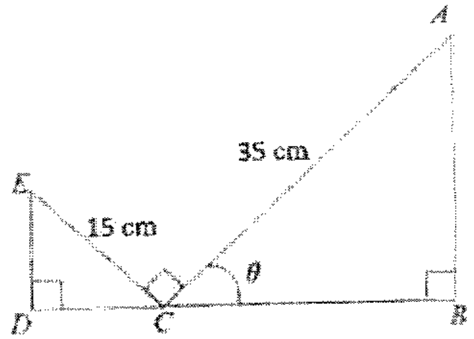
(i) Find the coordinates of A and B. [4]

[Turn Over

(ii) The curve $y = 16 + 6x - x^2$ cuts the x-axis at the point C. Show that the x-coordinate of C is 8. [1]

(iii) Find the area of the shaded region $OABC$. [3]

11 In the diagram, $AC = 35$ cm, $EC = 15$ cm and angle $ACB = \theta$ radians. ED and AB are perpendicular to DB.



(i) Explain clearly why angle $CED = \theta$ radians. [1]

(ii) Show that the sum of the perimeters, P cm, of triangles DEC and CAB is given by $P = a\cos\theta + b\sin\theta + c$ where a, b and c are constants. [2]

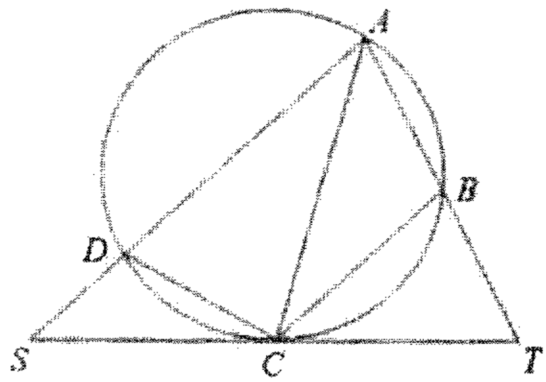
(iii) Express P in the form of $R\cos(\theta - \alpha) + c$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find the maximum value of P and the corresponding value of θ. [4]

(iv) Express A cm^2, the sum of the areas of triangles DEC and CAB in terms of θ, giving your answer in the simplest form. [2]

(v) Explain, stating your reasons, why A is maximum when P is a maximum. [2]

12 The diagram shows a circle $ABCD$ and the tangent ST to the circle at point C. The point B lies on AT such that $AB = BT$. The point C lies on the line ST such that $SC = CT$.



(i) Prove that ΔABC is similar to ΔSDC. [4]

(ii) Hence, show that $\times CD = \frac{1}{2}AS \times SC$. [2]

13 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity v ms^{-1}, is given by $v = 10\left(3 - e^{-\frac{1}{2}t}\right)$.

(i) Find the acceleration of the particle when $v = 23$. [3]

(ii) Calculate, to the nearest metre, the displacement of the particle from O when $t = 5$. [3]

(iii) State the value which v approaches as t becomes very large. [1]

Answers

1) $-1 < m < 3$ 2) $x < 0$ or $x > 1$

3a) $\frac{7}{3}$ (b) $2, -2$

4) $x = 2.30, 3.98, 0.927, 2.21$

5)(ii) 113 6(ii) Since $-1 \leq \cos x \leq 1$, the equation has no solution.

7(i) $a = 6, b = -2$ (ii) $0.5, -1, -2$ (iii) $1, -2, -4$

8(i) $1 - 4\cos 2t$ (ii) amplitude = 3, period = $\pi / 180^0$

9(i) centre$(2, -3)$, radius = 4 units

(ii) perpendicular distance from centre to line $y = 1$ is equal to the radius.

(iii) $(x-2)^2 + (y-5)^2 = 16$ (iv) $2\sqrt{2}$ or 2.83 units

10(ii)(a) $m = 13.9, k = 1510$ (b) 45 min

11(i) $-93\frac{27}{32}$ / $-\frac{3003}{32}$ / -93.84375 (ii) 2002

12(i) 0.936 rad (iii) S is maximum

13(ii) $D(-2.5, 7.5)$ (iii) $y = \frac{1}{3}x + \frac{10}{3}$ (iv) $E(0.8, 3.6)$

Answers

1(i) $\frac{2x+15}{(3+x)(3-2x)} = \frac{1}{3+x} + \frac{4}{3-2x}$

(ii) 2.48 (to 3 s.f)

2(i) $12 = \frac{2\pi}{p}$, $p = \frac{2\pi}{12} = \frac{\pi}{6}$

(ii) $t = 2.48$, 9.52

(iii) The tidal waves are at height 1.6m 2h 29 min later, that is at 1029, so it is safe to swim between 0900 and 1000 as the height would be below 1.6m

3(i) P $(1\frac{1}{4}, \frac{3}{8})$

(ii) $a = -2$

4(a)(i) $\frac{dy}{dx} =$

(ii) $\frac{dy}{dx} = 2x + \frac{8}{x^3}$

(b)(i) $\frac{dy}{dx} = \frac{6}{(2-x^2)^{\frac{3}{2}}}$

(ii) $x = 1$ or -1

5(i) $r = \frac{1}{4}(20 - x)$ by using similar triangles

$V = \frac{500\pi}{3} - \frac{\pi}{48}(20 - x)^3$

(ii) -0.0707 cms^{-1}

6 $x = -1.74$

7(ii) $\frac{1}{4}$

8 $y = -\frac{1}{9}\sin 3x + 4\cos\frac{1}{2}x + \frac{5}{3}x - \frac{5\pi}{9}$

9(i) $p = 14$

(ii) $7\sqrt{2} - 6$

10(i) A(2, 24) , B(5, 21)

(ii) show x-coordinate of C = 8

(iii) $144\frac{5}{6}$ unit^2

11(i) $\angle ECD = \pi - \frac{\pi}{2} - \theta = \frac{\pi}{2} - \theta$ (adjacent angles on a straight line)

$\angle CED = \pi - \left(\frac{\pi}{2} - \theta\right) - \frac{\pi}{2} = \theta$ (angle sum of a triangle)

(ii) $P = 50\cos\theta + 50\sin\theta + 50$

(iii) $P = 50\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) + 50$

(iv) $A = 725\ sin\theta cos\theta$

(v) since = $725\ sin\theta cos\theta = \frac{725}{2}\sin 2\theta$,

maximum A happ. .s at $\theta = \frac{\pi}{4}$ when $\sin 2\theta = \sin 2(\frac{\pi}{4}) = \sin\frac{\pi}{2} = 1$.

This is the same value for θ when P is a maximum.

13(i) 3.5 ms^{-2}

(ii) 132 m (nearest m)

(iii) v = 30