Name: ________________________ (   )   Class: __________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 **Solution for student** **Monday 24 August 2020**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7			
Q4		Q8			

/ 80

This document consists of **15** printed pages and **1** blank page.

圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

Mathematical Formulae

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Given that $\frac{49^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{343^{x-1}}$, find the value of $\sqrt[3]{343^x}$.

2 A cuboid has a square base of length $(3+\sqrt{2})$ cm and a height of h cm. The volume of the cuboid is $(19-3\sqrt{2})$ cm^3. **Without using a calculator**, obtain an expression for h in the form $(a+b\sqrt{2})$, where a and b are integers. [4]

3 **(i)** **Without using a calculator**, show that $\cot 15° = 2 + \sqrt{3}$. [4]

(ii) Hence show that $\mathrm{cosec}^2 15° = 4\cot 15°$. [2]

4 **(i)** Express $\dfrac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}$ in partial fractions. [4]

(ii) Hence find $\displaystyle\int \frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}\,\mathrm{d}x$. [2]

5 Liquid is poured, at a constant rate of 15 cm^3/s, into a container. When the depth of liquid in the container is x cm, the volume, V cm^3, of the liquid in the container is given by

$$V = \frac{1}{4}x(x+16).$$

Find, when $V = 20$,

(i) the value of x, [3]

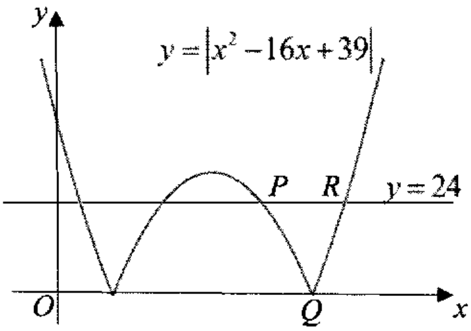
(ii) the rate of change of the depth of liquid at this time. [4]

6 Solve the equation

(i) $2\log_3(x+4)-\log_3(x+2)=2$, [5]

(ii) $\log_5 y-3\log_y 5=2$. [4]

7



The diagram shows the line $y = 24$ and part of the graph of $y = |x^2 - 16x + 39|$.

The graph crosses the line $y = 24$ at the points P and R and meets the x-axis at Q.

(i) Find the coordinates of P, Q and R. [6]

(ii) State the set of values of k for which $|x^2 - 16x + 39| = k$ has 2 solutions. [3]

8 It is given that $y_1 = \sin x - 2$ and $y_2 = -3\cos 2x$.

(i) State the amplitude and period, in degrees, of **(a)** y_1, **(b)** y_2. [2]

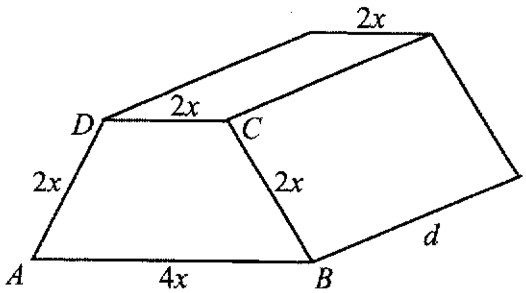
For the interval $0° \le x \le 360°$,

(ii) solve the equation $y_1 = y_2$, [4]

(iii) sketch, on the same diagram, the graphs of y_1 and y_2, [4]

(iv) find the set of values of x for which $y_2 - y_1 > 0$. [2]

9



The diagram shows a prism in which the cross-section is a trapezium *ABCD*.
$AB = 4x$ cm, $BC = DC = AD = 2x$ cm. The length of the prism is d cm.

(i) Show that the area of trapezium *ABCD* is $3\sqrt{3}x^2\ \text{cm}^2$. [2]

The volume of the prism is $3888\sqrt{3}\ \text{cm}^3$.

(ii) Express d in terms of x. [1]

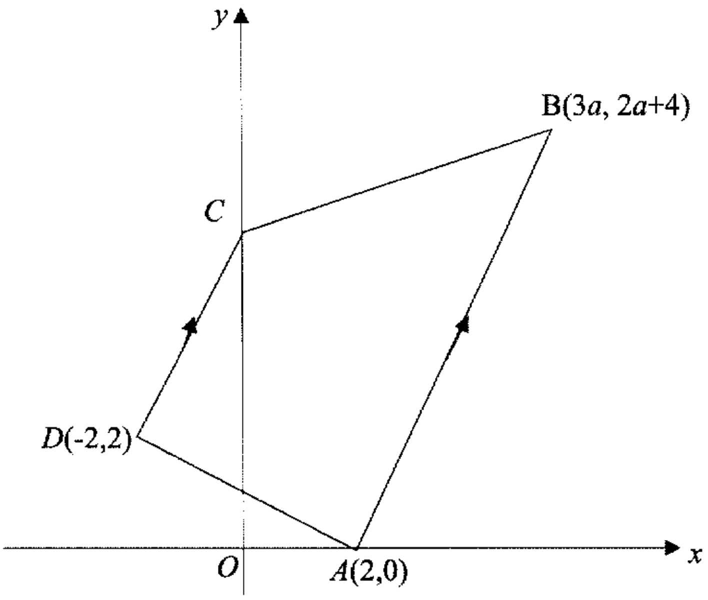
(iii) Show that the total surface area, A cm^2, of the prism is given by

$$A = 6\sqrt{3}x^2 + \frac{12960}{x}.$$ [3]

(iv) Given that x can vary, find the value of x which gives a stationary value of A. [4]

(v) [redacted] why this value of x gives the [redacted] of the prism. [1]

10 Solutions to this question by accurate drawing will not be accepted.

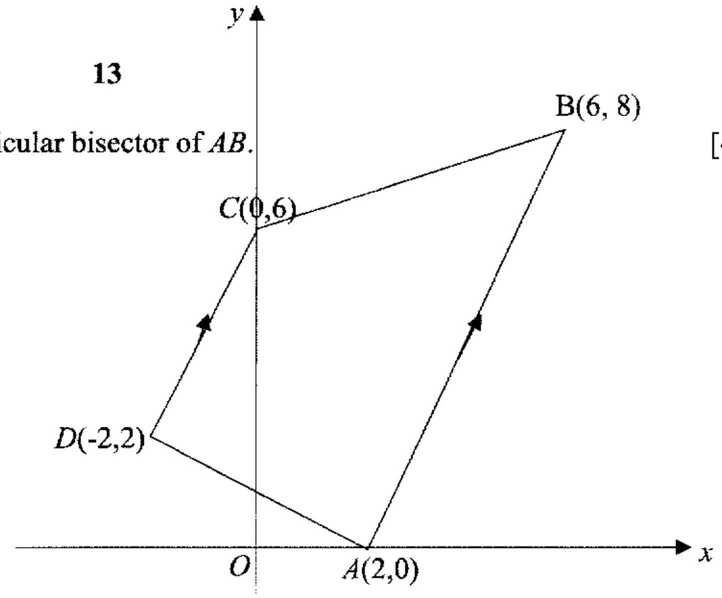


The diagram shows a trapezium *ABCD* in which *AB* is parallel to *DC*. The coordinates of the points *A*, *B* and *D* are (2, 0), ($3a$, $2a$+4) and (–2, 2) respectively, where a is a positive integer. The length of *AB* is $4\sqrt{5}$ units.

(i) Show that $a = 2$. [3]

(ii) Find the coordinates of *C*.

(iii) Find the equation of the perpendicular bisector of AB. [4]



(iv) Hence, or otherwise, determine if C lies on the perpendicular bisector of AB. [1]

(v) Find the area of the trapezium $ABCD$. [2]

Name: ________________________ (   )   Class: __________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **Tuesday 25 August 2020**

Solution for student **2 hours 30 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8			

/ 100

This document consists of **16** printed pages and **1** blank page.

圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

Mathematical Formulae

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that the roots of $3x^2 - x + 2 = 0$ are α and β, find a quadratic equation whose roots are $\dfrac{\alpha+1}{\beta}$ and $\dfrac{\beta+1}{\alpha}$. [5]

2 **(i)** Write down, and simplify, the general term in the binomial expansion of $\left(x^2 + \frac{3}{x}\right)^{15}$. [2]

(ii) Hence determine the coefficient of x^3 in the binomial expansion of $\left(x^2 + \frac{3}{x}\right)^{15}$. [3]

(iii) Explain why there is no term in $\frac{1}{x^5}$ in the expansion of $\left(x^2 + \frac{3}{x}\right)^{15}$. [2]

The highest point on a second circle, C_2, is $(3,6)$ and the equation of the tangent at the lowest point is $y = 4$.

(ii) Find the radius and the coordinates of the centre of C_2. [2]

(iii) Find the equation of C_2. [1]

Equation of C2 $(x-3)^2 + (y-5)^2 = 1$

(iv) Showing your working clearly, determine whether the circles C_1 and C_2 meet each other. [2]

3 A circle, C_1, has equation $x^2 + y^2 - 4x + 6y = 36$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

4 The equation of a polynomial is $f(x) = 2x^3 - 7x^2 + 9x - 3$.

(i) Factorise the polynomial $f(x)$. [3]

(ii) Determine the nature of the roots of the equation $f(x) = 0$. [3]

(iii) Solve the equation $2\left(3^{3y}\right) - 7\left(3^{2y}\right) + 3^{y+2} = 3$. [3]

6 In the diagram, A, B and D are fixed points such that $AB = 25\,\text{cm}$, $AD = 35\,\text{cm}$ and angle $BAD = 90°$. Angle $BAC = \theta$ and can vary. AC is perpendicular to BC, EA is perpendicular to AC, and DE is perpendicular to EA.

B

(ii) When $k < 0$, find the coordinates of the point of contact for which the line is a tangent to the curve. [3]

5 The equation of a curve is $x^2 + 4y^2 = 20$ and the equation of a line is $y = k - x$, where k is a constant.

(i) Find the range of values of k for which the line does not intersect the curve. [6]

(i) Show that $AC + BC + DE = (60\sin\theta + 25\cos\theta)$ cm. [2]

(ii) Express $AC + BC + DE$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

$60\sin\theta + 25\cos\theta = \mathbf{65\sin(\theta + 22.6°)}$

(iii) **Without evaluating** θ, explain why $AC + BC + DE$ cannot have a length of 70 cm. [1]

(iv) Find the value of θ for which $AC + BC + DE = 50$ cm. [2]

7 The equation of a curve is $y = x^2(6 - x)$.

(i) Find the coordinates of the [redacted] of the curve. [5]

(ii) Use t[redacted]est to determine t[redacted]ure of each of these points. [3]

(iii) Find the range of values of x for which [redacted]. [2]

(ii) Hence show that $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$, where c is a constant. [3]

The curve $y = f(x)$ passes through the point $\left(1, \frac{19}{4}\right)$ and is such that $f'(x) = \frac{1}{x} + x \ln x$.

(iii) Find $f(x)$. [4]

(iv) State the range of values of x for which $f(x)$ is defined. [1]

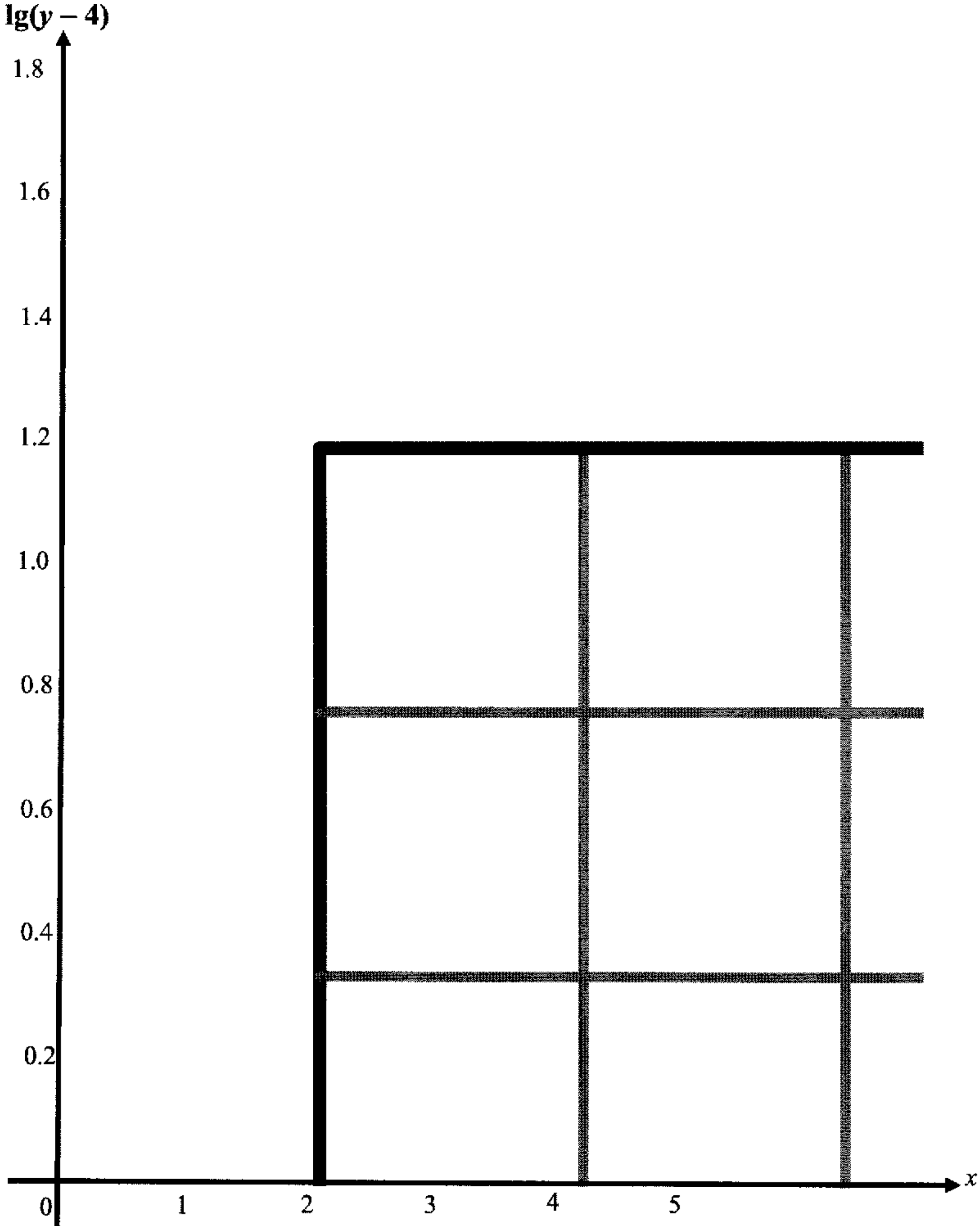
8 **(i)** Differentiate $3x^2 \ln x$. [2]

9 It is known that x and y are related by an equation, $y = ab^x + 4$ where a and b are constants.

x	1	2	3	4

(i) Draw a straight line graph of $\lg(y-4)$ against x, using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.2 units on the $\lg(y-4)$-axis. [2]

x	1	2	3	4
y	10	16	28	52
lg(y-4)	0.778	1.08	1.38	1.68



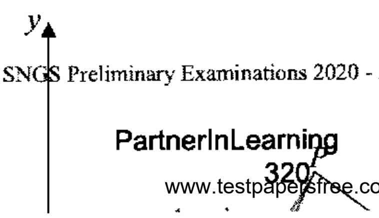
(ii) [redacted] to estimate the value of a and of b. [5]

$y = ab^x + 4$

(iii) Use your graph to estimate the value of x when $y = 33$. [2]

(iv) On the same diagram, draw the line representing the equation $y - 4 = 10^{2x}$ and hence find the value of x for which $10^{2x} = ab^x$. [2]

10



The diagram shows part of the curve $y = 1 - \sin x$ passing through the point P. The curve touches the x-axis at the point Q. The tangent and normal to the curve at P meet the x-axis at the points R and S respectively.

(i) Show that the x-coordinate of Q is $\frac{\pi}{2}$. [1]

(ii) Show that the coordinates of P are $(\pi, 1)$. [3]

(iii) Show that the x-coordinate of R is $\pi - 1$. [1]

(iv) Show that the x-coordinate of S is $\pi + 1$. [1]

[Turn over

(v) Find the exact area of the shaded region. [5]

11 A particle P moves in a straight line, so that, t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = 10e^{-2t} - 3$.

(i) Find the [redacted] velocity of P. [1]

(ii) Find the acceleration of P when $t = 1$. [2]

(iii) Find the value of t when P is at instantaneous [redacted] [3]

(iv) Find the total distance travelled by P before it comes to instantaneous rest. [4]

(v) Explain why the value of v is always greater than -3. [1]

Name: ________________________ (   )   Class: ___________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS

4047/01

Paper 1 **Solution for student** **Monday 24 August 2020**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7			
Q4		Q8			

/ 80

This document consists of **15** printed pages and **1** blank page.

圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

Mathematical Formulae

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $\frac{49^{2x-3}}{7^{3x}} = \frac{7^{2x+1}}{343^{x-1}}$, find the value of $\sqrt[3]{343^x}$. [4]

$$\frac{49^{2x-3}}{7^{3x}}=\frac{7^{2x+1}}{343^{x-1}}$$

$$\frac{\left(7^{2}\right)^{2x-3}}{7^{3x}}=\frac{7^{2x+1}}{\left(7^{3}\right)^{x-1}}$$ express as powers of 7

$$\frac{7^{2(2x-3)}}{7^{3x}}=\frac{7^{2x+1}}{7^{3(x-1)}}$$ index laws $\left(a^{m}\right)^{n}=a^{mn}$

$$7^{4x-6-3x}=7^{2x+1-(3x-3)}$$ $\frac{a^{m}}{a^{n}}=a^{m-n}$

$$x-6=-x+4$$ equate powers

$$2x=10$$

$$x=5$$

$$343^{\frac{x}{3}}=7^{x}$$

$$=16807$$

2 A cuboid has a square base of length $(3+\sqrt{2})$ cm and a height of h cm. The volume of the cuboid is $(19-3\sqrt{2})$ cm^3. **Without using a calculator**, obtain an expression for h in the form $(a+b\sqrt{2})$, where a and b are integers. [4]

$$\text{h}=\frac{19-3\sqrt{2}}{\left(3+\sqrt{2}\right)^{2}}$$

$$=\frac{19-3\sqrt{2}}{9+6\sqrt{2}+2}$$

$$=\frac{19-3\sqrt{2}}{11+6\sqrt{2}}\times\frac{11-6\sqrt{2}}{11-6\sqrt{2}}$$

$$=\frac{209-114\sqrt{2}-33\sqrt{2}+18(2)}{121-36(2)}$$

$$=\frac{245-147\sqrt{2}}{49}$$

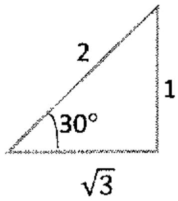
$$=5-3\sqrt{2}$$

3 **(i)** **Without using a calculator** , show that $\cot 15^{\circ}=2+\sqrt{3}$. [4]

$tan15° = \tan(45° - 30°)$ or $tan15° = \tan(60° - 45°)$

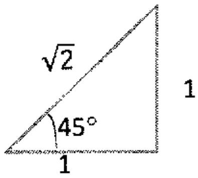
$tan15° = \frac{\tan 45° - tan30°}{1 + tan45° tan30°}$ or $tan15° = \frac{tan60° - tan45°}{1 + tan45° tan60°}$

$tan15° = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$ or $\tan 15° = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$



$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$Cot15° = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ $\cot A = \frac{1}{tanA}$



$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$

$= \frac{4 + 2\sqrt{3}}{2}$

$= \frac{2(2 + \sqrt{3})}{2} = 2 + \sqrt{3}$

(ii) Hence show that $\text{cosec}^2 15° = 4\cot 15°$. [2]

$$cosec^2 15° = 1 + \cot^2 15° = 1 + (2 + \sqrt{3})^2$$
$$= 1 + 4 + 4\sqrt{3} + 3$$
$$= 8 + 4\sqrt{3}$$
$$= 4(2 + \sqrt{3})$$
$$= 4cot15°$$

4 **(i)** Express $\frac{2x^3 - 4x^2 - 5x + 3}{(x-3)(x^2-9)}$ in partial fractions. [4]

Method 1

$$\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}=\frac{2x^3-4x^2-5x+3}{(x-3)(x-3)(x+3)}\equiv 2+\frac{A}{x+3}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$$

$$\equiv\frac{2(x+3)(x-3)^2+A(x-3)^2+B(x+3)(x-3)+C(x+3)}{(x+3)(x-3)^2}$$

$2x^3-4x^2-5x+3\equiv 2(x+3)(x-3)^2+A(x-3)^2+B(x+3)(x-3)+C(x+3)$

when $x=3$, $54-36-15+3=6C$

$C=1$

when $x=-3$, $-54-36+15+3=36A$

$A=-2$

when $x=0$, $3=2(3)(9)-2(9)+B(3)(-3)+1(3)$

$-36=-9B$

$B=4$

$\therefore\frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\equiv 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}$

Method 2

$$\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}=2+\frac{2x^2+13x-51}{(x-3)(x+3)(x-3)}$$

$$\frac{2x^2+13x-51}{(x-3)(x+3)(x-3)}=\frac{A}{x-3}+\frac{B}{x+3}+\frac{C}{(x-3)^2}$$

$2x^2+13x-51=A(x-3)(x+3)+B(x-3)^2+C(x+3)$

Let $x=3$ $2(3)^2+13(3)-51=C(3+3)$

$C=1$

Let $x=-3$ $2(-3)^2+13(-3)-51=B(-3-3)^2$

$B=-2$

Let $x=0$ $-51=A(-9)-2(9)+3$

$A=4$

$$\frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\equiv 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}$$

(ii) Hence find $\int\frac{2x^3-4x^2-5x+3}{(x-3)(x^2-9)}\,dx$. [2]

$$\int \frac{2x^3-4x^2-5x+3}{(x+3)(x-3)^2}\,\mathrm{d}x$$

$$=\int 2-\frac{2}{x+3}+\frac{4}{x-3}+\frac{1}{(x-3)^2}\,\mathrm{d}x$$

$$=\int 2-\frac{2}{x+3}+\frac{4}{x-3}+(x-3)^{-2}\,\mathrm{d}x$$

$$=2x-2\ln(x+3)+4\ln(x-3)+\frac{(x-3)^{-1}}{-1}+c$$

$$=2x-2\ln(x+3)+4\ln(x-3)-\frac{1}{x-3}+c \quad or\ 2x-\frac{1}{x-3}+\ln\frac{(x-3)^4}{(x+3)^2}$$

5 Liquid is poured, at a constant rate of 15 cm^3/s, into a container. When the depth of liquid in the container is x cm, the volume, V cm^3, of the liquid in the container is given by

$$V=\frac{1}{4}x(x+16).$$

Find, when $V=20$,

(i) the value of x, [3]

$$V=\frac{1}{4}x^2+4x$$

(i) When $V=20$, $\frac{1}{4}x^2+4x=20$

$$x^2+16x-80=0$$

$$(x+20)(x-4)=0$$

$$x=-20\text{(NA) or } x=4$$

(ii) the rate of change of the depth of liquid at this time. [4]

(ii) $\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{1}{2}x+4$

When $x=4$, $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$

$$=\frac{1}{[\frac{1}{2}(4)+4]}\times 15$$ [redacted]

$$=\frac{15}{6}$$

$$=2.5$$

The rate of change of the depth of liquid at this time is 2.5 cm/s

6 Solve the equation

(i) $2\log_3(x+4)-\log_3(x+2)=2$, [5]

$$2\log_3(x+4)-\log_3(x+2)=2$$
$$\log_3(x+4)^2-\log_3(x+2)=2$$
$$\log_3\frac{(x+4)^2}{x+2}=2$$
$$\frac{(x+4)^2}{x+2}=3^2$$
$$x^2+8x+16=9(x+2)$$
$$=9x+18$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2 \quad x=-1$$

$\log_a b^c = c\log_a b$

$\lg P - \lg Q = \lg\frac{P}{Q}$

$\log_a b = x \Longrightarrow a^x = b$

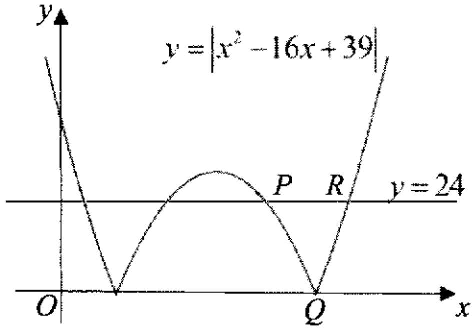
(ii) $\log_5 y - 3\log_y 5 = 2$. [4]

$$\log_5 y-3\log_y 5=2$$
$$\log_5 y-3\left(\frac{\log_5 5}{\log_5 y}\right)=2$$
$$(\log_5 y)^2-3=2\log_5 y$$
$$(\log_5 y)^2-2\log_5 y-3=0$$
$$a^2-2a-3=0$$
$$(a+1)(a-3)=0$$
$$a=-1 \qquad a=3$$
$$\log_5 y=-1 \quad \log_5 y=3$$
$$y=5^{-1} \qquad y=5^3$$
$$=\frac{1}{5} \qquad =125$$

$\log_a b = \frac{\log_c b}{\log_c a}$

$(\log_5 y)^2 \neq 2(\log_5 y)$

7



The diagram shows the line $y = 24$ and part of the graph of $y = |x^2 - 16x + 39|$.

The graph crosses the line $y = 24$ at the points P and R and meets the x-axis at Q.

(i) Find the coordinates of P, Q and R. [6]

To find P and R, y=24

$|x^2 - 16x + 39| = 24$

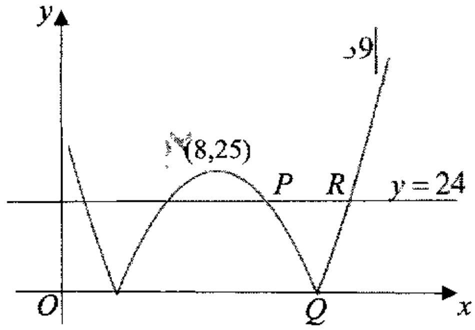
$x^2 - 16x + 39 = \pm 24$

$x^2 - 16x + 39 = -24$ $\quad$ $x^2 - 16x + 39 = 24$

$x^2 - 16x + 63 = 0$ $\quad$ $x^2 - 16x + 15 = 0$

$(x-7)(x-9) = 0$ $\quad$ $(x-1)(x-15) = 0$

$x = 7$, $x = 9$ $\quad$ $x = 1$, $x = 15$



Ans. $P(9,24)$, $R(15,24)$

To find Q , y =0

$x^2 - 16x + 39 = 0$

$(x-3)(x-13) = 0$

$x = 3$, $x = 13$

Ans. $Q(13,0)$

(ii) State the set of values of k for which $|x^2 - 16x + 39| = k$ has 2 solutions. [3]

axis of symmetry $x = \dfrac{3+13}{2}$

$= 8$

$y = |8^2 - 16(8) + 39|$

$= |-25|$

$= 25$

$y = |x^2 - 16x + 8^2 - 8^2 + 39|$

$= |(x-8)^2 - 25|$

Max pt (8,25)

Ans. $k = 0$, $k > 25$

8 It is given that $y_1 = \sin x - 2$ and $y_2 = -3\cos 2x$.

(i) State the amplitude and period, in degrees, of **(a)** y_1, **(b)** y_2. [2]

(a) $y_1 = \sin x - 2$ amplitude =1, period=360°

(b) $y_2 = -3\cos 2x$ amplitude =3, period =180°

For the interval $0° \le x \le 360°$,

(ii) solve the equation $y_1 = y_2$, [4]

$y_1 = y_2$

$sinx - 2 = -3cos2x$

$3(1 - 2sin^2 x) + sinx - 2 = 0$ $cos2x = 1 - 2\sin^2 x$

$6sin^2 x - \sin x - 1 = 0$

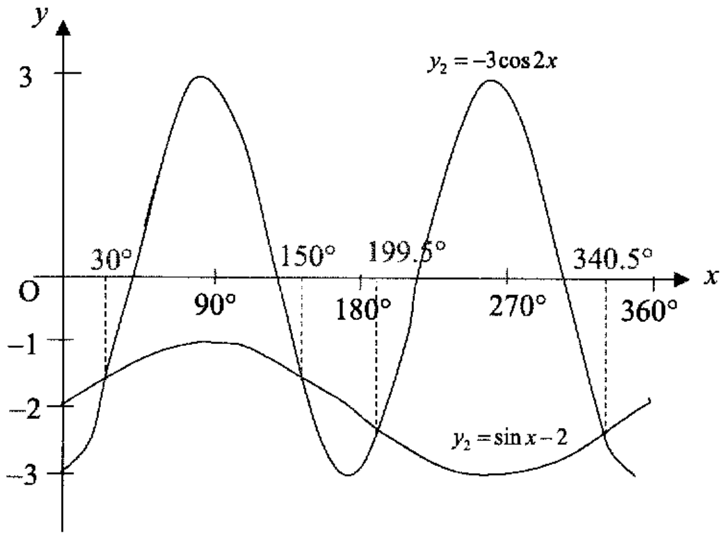
$(3\sin x + 1)(2\sin x - 1) = 0$

$sinx = -\frac{1}{3}$, $\sin x = \frac{1}{2}$

$x = 180° + 19.471°, 360° - 19.471, x = 30°, 150°$

$x = 199.5°, 340.5°$. $x = 30°, 150°$

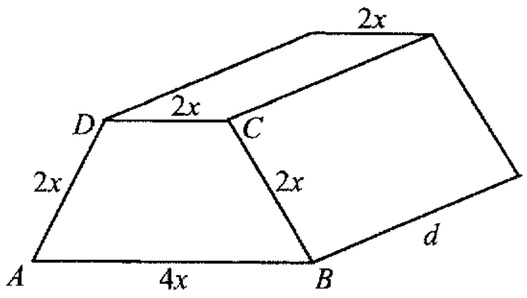
(iii) sketch, on the same diagram, the graphs of y_1 and y_2, [4]



(iv) find the set of values of x for which $y_2 - y_1 > 0$. [2]

$30° < x < 150°, 199.5° < x < 340.5°$

9



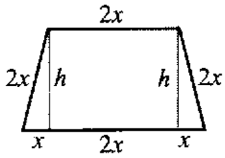
The diagram shows a prism in which the cross-section is a trapezium $ABCD$.
$AB = 4x$ cm, $BC = DC = AD = 2x$ cm. The length of the prism is d cm.

(i) Show that the area of trapezium $ABCD$ is $3\sqrt{3}x^2$ cm^2. [2]

(i) $h = \sqrt{(2x)^2 - x^2}$

$= \sqrt{3x^2}$

$= \sqrt{3}x$



Area of trapezium $ABCD$ $= \frac{1}{2}(\sqrt{3}x)(2x+4x)$

$= \frac{1}{2}(\sqrt{3}x)(6x)$

$= 3\sqrt{3}x^2$ cm^2.

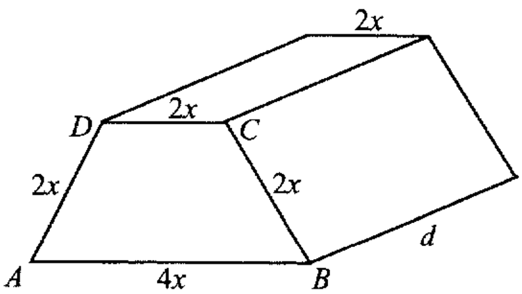
The volume of the prism is $3888\sqrt{3}$ cm^3.

(ii) Express d in terms of x. [1]

(ii) $3\sqrt{3}x^2 d = 3888\sqrt{3}$

$x^2 d = 1296$

$d = \frac{1296}{x^2}$



(iii) Show that the total surface area, A cm^2, of the prism is given by

$$A = 6\sqrt{3}x^2 + \frac{12960}{x}.$$ [3]

(iii) $A = 2\left(3\sqrt{3}x^2\right) + \left(2x+2x+2x+4x\right)\left(\frac{1296}{x^2}\right)$

$= 6\sqrt{3}x^2 + 10x\left(\frac{1296}{x^2}\right)$

$A = 6\sqrt{3}x^2 + \frac{12960}{x}$

(iv) Given that x can vary, find the value of x which gives a stationary value of A. [4]

(iv) $\frac{dA}{dx} = 12\sqrt{3}x - \frac{12960}{x^2}$

At $\frac{dA}{dx} = 0$, $\quad 12\sqrt{3}x - \frac{12960}{x^2} = 0$

$$x^3 = \frac{12960}{12\sqrt{3}}$$

$$x \approx 8.543$$

$$x = 8.54 \text{ (to 3 s.f.)}$$

(v) [redacted] why this value of x gives the [redacted] of the prism. [1]

(v) $\frac{d^2A}{dx^2} = 12\sqrt{3} + \frac{25920}{x^3}$

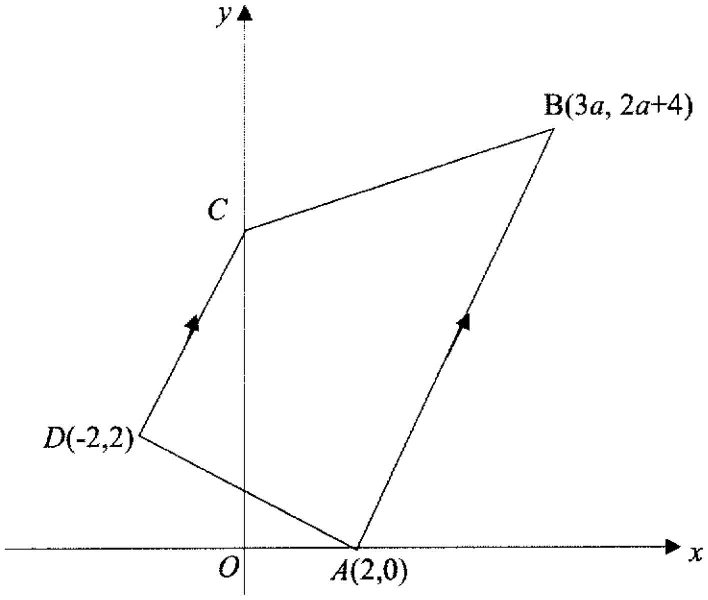
When $x \approx 8.543$, (or $x^3 = 360\sqrt{3}$)

$$\frac{d^2A}{dx^2} \approx 62.3538 \quad \text{(or } 36\sqrt{3}\text{)}$$

$$\frac{d^2A}{dx^2} > 0$$

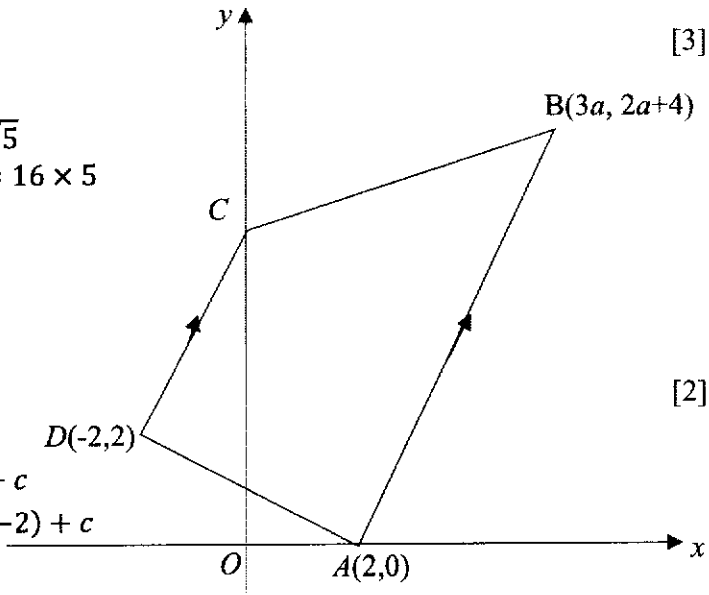
Therefore this value of x gives the smallest possible surface area of the prism.

10 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC. The coordinates of the points A, B and D are (2, 0), ($3a$, $2a$+4) and (−2, 2) respectively, where a is a positive integer. The length of AB is $4\sqrt{5}$ units.

(i) Show that $a = 2$. [3]



$$\sqrt{(3a-2)^2+(2a+4)^2} = 4\sqrt{5}$$
$$9a^2 - 12a + 4 + 4a^2 + 16a + 16 = 16 \times 5$$
$$13a^2 + 4a - 60 = 0$$
$$(13a+30)(a-2) = 0$$
$$a = -\frac{30}{13}\ (NA), a = 2$$

(ii) Find the coordinates of C. [2]

Let C be (0,y)

$\frac{y-2}{0+2} = \frac{8-0}{6-2}$ OR $y = 2x + c$

$\frac{y-2}{2} = 2$ OR $2 = 2(-2) + c$

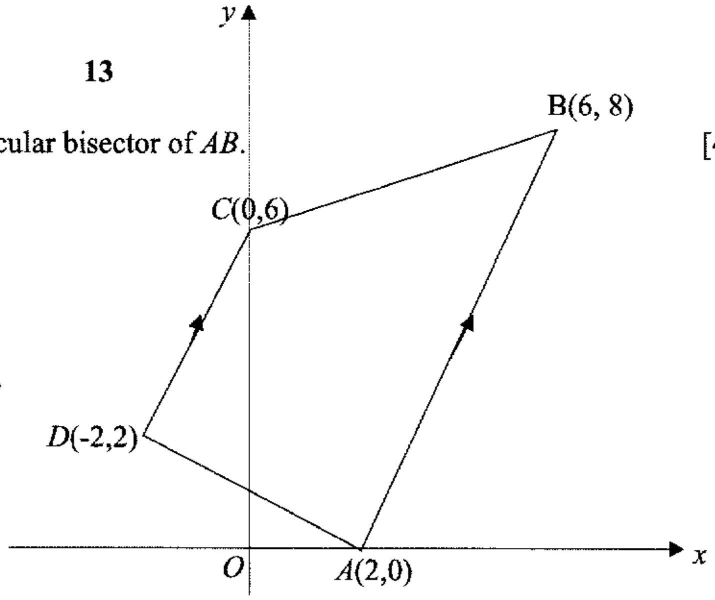
$c = 6$

$y = 2(0) + 6$

$y = 6$

C(0,6)

(iii) Find the equation of the perpendicular bisector of *AB*. [4]



A(2,0), *B* (6,8)

Mid –point of *AB*= $(\frac{6+2}{2}, \frac{0+8}{2})$

$= (4,4)$

$m_{\perp} = -\frac{1}{2}$ $m_1 \times m_2 = -1$

Equation of the perpendicular bisector of *AB*

$$4 = -\frac{1}{2}(4) + c$$

$$c = 6$$

$$y = -\frac{1}{2}x + 6$$

(iv) Hence, or otherwise, determine if *C* lies on the perpendicular bisector of *AB*. [1]

C(0,6) let *x*=0 $y = -\frac{1}{2}(0) + 6$

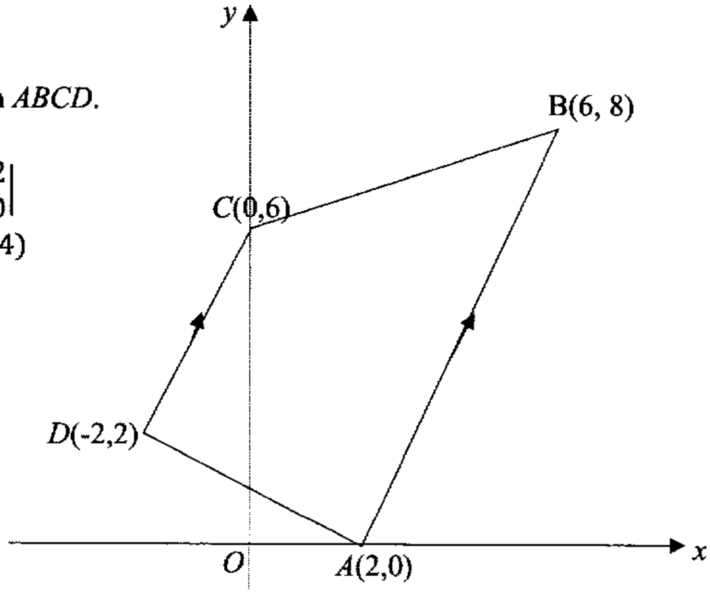
y=6

or let y=6. $6= -\frac{1}{2}x + 6$

$x = 0$

$\therefore C(0,6)$ lies on the perpendicular bisector of *AB*

(v) Find the area of the trapezium *ABCD*. [2]



Area *ABCD*= $\frac{1}{2}\begin{vmatrix} 2 & 6 & 0 & -2 & 2 \\ 0 & 8 & 6 & 2 & 0 \end{vmatrix}$

$= \frac{1}{2}(16 + 36 + 12 - 4)$

$= \frac{1}{2}(60)$

$= 30$ sq units

Name: ______________________ (   )   Class: __________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **Tuesday 25 August 2020**

Solution for student **2 hours 30 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8			

/ 100

This document consists of **16** printed pages and **1** blank page.

圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

Mathematical Formulae

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$\left(a+b\right)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$

$$\sec^2 A=1+\tan^2 A.$$

$$\operatorname{cosec}^2 A=1+\cot^2 A.$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Given that the roots of $3x^2-x+2=0$ are α and β, find a quadratic equation whose roots are

$\dfrac{\alpha+1}{\beta}$ and $\dfrac{\beta+1}{\alpha}$. [5]

$3x^2 - x + 2 = 0$
sum of roots: $\alpha + \beta = \frac{1}{3}$
product of roots: $\alpha\beta = \frac{2}{3}$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$
$$= \left(\tfrac{1}{3}\right)^2 - 2\left(\tfrac{2}{3}\right)$$
$$= -\tfrac{11}{9}$$

New roots: $\dfrac{\alpha+1}{\beta}$ and $\dfrac{\beta+1}{\alpha}$

New sum: $\dfrac{\alpha+1}{\beta} + \dfrac{\beta+1}{\alpha} = \dfrac{\alpha^2+\alpha+\beta^2+\beta}{\alpha\beta}$
$$= \frac{-11/9 + 1/3}{2/3}$$
$$= -\tfrac{4}{3}$$

New product: $\left(\dfrac{\alpha+1}{\beta}\right)\left(\dfrac{\beta+1}{\alpha}\right) = \dfrac{\alpha\beta+\alpha+\beta+1}{\alpha\beta}$
$$= \frac{2/3 + 1/3 + 1}{2/3}$$
$$= 3$$

New equation: $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$
$$x^2 - \left(-\tfrac{4}{3}\right)x + 3 = 0$$
$$x^2 + \tfrac{4}{3}x + 3 = 0$$
$$3x^2 + 4x + 9 = 0$$

2 (i) Write down, and simplify, the [redacted] in the binomial expansion of $\left(x^2 + \dfrac{3}{x}\right)^{15}$. [2]

(i) $\left(x^2+\dfrac{3}{x}\right)^{15}$

$$T_{r+1} = {}^{15}C_r\left(x^2\right)^{15-r}\left(\frac{3}{x}\right)^r$$
$$= {}^{15}C_r x^{30-2r}\left(\frac{3^r}{x^r}\right)$$
$$= {}^{15}C_r 3^r x^{30-3r}$$

(ii) Hence determine the coefficient of x^3 in the binomial expansion of $\left(x^2+\dfrac{3}{x}\right)^{15}$. [3]

(ii) $30-3r=3$

$3r=27$

$r=9$

Coefficient of $x^3 = {}^{15}C_9 3^9$

$=(5005)(19683)$

$=98\ 513\ 415$

(iii) Explain why there is no term in $\dfrac{1}{x^5}$ in the expansion of $\left(x^2+\dfrac{3}{x}\right)^{15}$. [2]

(iii) $30-3r=-5$

$3r=35$

$r=\dfrac{35}{3}$

r must be a whole number. [redacted]

3 A circle, C_1, has equation $x^2+y^2-4x+6y=36$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

Method 1 $x^2+y^2-4x+6y=36$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 36 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 49$$

Method 2 $x^2 + y^2 - 4x + 6y = 36.$

Radius $= \sqrt{(-2)^2 + 3^2 - (-36)}\ \sqrt{49} = 7$

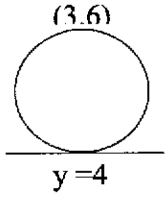
Centre $= (2,-3)$, radius $= 7$ units

The highest point on a second circle, C_2, is $(3,6)$ and the equation of the tangent at the lowest point is $y = 4$.

(ii) Find the radius and the coordinates of the centre of C_2. [2]

radius =1 unit,

Centre= (3,5)



(iii) Find the equation of C_2. [1]

Equation of C2 $(x-3)^2 + (y-5)^2 = 1$

(iv) Showing your working clearly, determine whether the circles C_1 and C_2 meet each other. [2]

Method 1

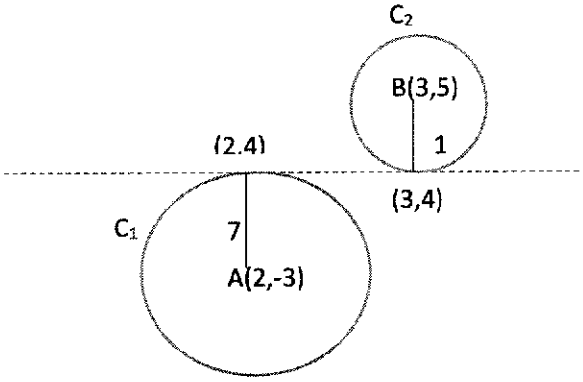
Let the centres of C_1 and C_2 be A & B respectively

$$AB = \sqrt{(2-3)^2 + (-3-5)^2}$$

$$AB = \sqrt{65} = 8.06$$

$$r_1 + r_2 = 7 + 1 = 8$$

As $AB > r_1 + r_2$ so the two circles do not meet.



Method 2

The y coordinate of the lowest point of C_2 is the same as y coordinate of the highest point of C_1.

The x coordinates of these two points are different. The two circles do not meet.

4 The equation of a polynomial is $f(x) = 2x^3 - 7x^2 + 9x - 3$.

(i) Factorise the polynomial $f(x)$. [3]

$$f(x) = 2x^3 - 7x^2 + 9x - 3$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - 7\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 3$$

$$= 0$$

$\Rightarrow (2x-1)$ is a factor

$$f(x) = (2x-1)(x^2 + bx + 3)$$

x^2: $2b - 1 = -7$

$$2b = -6$$

$$b = -3$$

$$f(x) = (2x-1)(x^2 - 3x + 3)$$

(ii) Determine the nature of the roots of the equation $f(x) = 0$. [3]

$$f(x) = (2x-1)(x^2 - 3x + 3) = 0$$

$$x = \frac{1}{2} \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{-3}}{2}$$

Ans. 1 real root and 2 imaginary/complex roots

(iii) Solve the equation $2(3^{3y}) - 7(3^{2y}) + 3^{y+2} = 3$. [3]

$$2(3^{3y}) - 7(3^{2y}) + 3^{y+2} = 3$$

$$2(3^y)^3 - 7(3^y)^2 + 3^2(3^y) - 3 = 0$$

$$3^y = \frac{1}{2}$$

$$\lg 3^y = \lg \frac{1}{2}$$

$$y = \frac{\lg \frac{1}{2}}{\lg 3}$$

$$= -0.631$$

5 The equation of a curve is $x^2 + 4y^2 = 20$ and the equation of a line is $y = k - x$, where k is a constant.

(i) Find the range of values of k for which the line [redacted] the curve. [6]

$$y = k - x \quad \_\_\_(1)$$

$$x^2 + 4y^2 = 20 \quad \_\_\_\_\_(2)$$

$$x^2 + 4(k-x)^2 = 20$$

$$x^2 + 4\left(k^2 - 2kx + x^2\right) = 20$$

$$x^2 + 4k^2 - 8kx + 4x^2 = 20$$

$$5x^2 - 8kx + 4k^2 - 20 = 0$$

$$b^2 - 4ac < 0$$

5 $$(-8k)^2 - 4(5)\left(4k^2 - 20\right) < 0$$

$$64k^2 - 80k^2 + 400 < 0$$

$$-16k^2 + 400 < 0$$

$$k^2 - 25 > 0$$

$$(k+5)(k-5) > 0$$

Ans $\quad k < -5 \text{ or } k > 5$

(ii) When $k < 0$, find the coordinates of the point of contact for which the line is a tangent to the curve. [3]

For tangent: $k = -5$

$$5x^2 - 8kx + 4k^2 - 20 = 0$$

$$5x^2 + 40x + 80 = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)^2 = 0$$

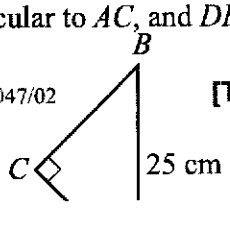
$$x = -4$$

$$y = -5 - (-4)$$

$$= -1$$

Ans. The coordinate of the point of contact is $(-4, -1)$

6 In the diagram, A, B and D are fixed points such that $AB = 25\text{ cm}$, $AD = 35\text{ cm}$ and angle $BAD = 90°$. Angle $BAC = \theta$ and can vary. AC is perpendicular to BC, EA is perpendicular to AC, and DE is perpendicular to EA.



(i) Show that $AC+BC+DE=(60\sin\theta+25\cos\theta)$ cm. [2]

[redacted]

$\angle CAD = 90° - \theta, \angle DAE = \theta,$ [redacted]

$AC + BC + DE = 25\cos\theta + 25\sin\theta + 35\sin\theta$

$= 25\cos\theta + 60\sin\theta$

(ii) Express $AC+BC+DE$ in the form $R\sin(\theta+\alpha)$, where $R>0$ and $0°<\alpha<90°$. [4]

Let $60\sin\theta + 25\cos\theta = R\sin(\theta+\alpha)$

$60\sin\theta + 25\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 60$ ----(1) $R\sin\alpha = 25$------(2)

$\frac{(2)}{(1)}$ $\frac{R\sin\alpha}{R\cos\alpha} = \frac{25}{60}$

[redacted]

$\alpha = 22.619°$

$R^2(\sin^2\alpha + \cos^2\alpha) = 60^2 + 25^2$

[redacted]

$R = 65, \quad R = -65$ (NA)

$60\sin\theta + 25\cos\theta = 65\sin(\theta + 22.6°)$

(iii) **Without evaluating** θ, explain why $AC+BC+DE$ cannot have a length of 70 cm. [1]

$-1 \le \sin(\theta + 22.6°) \le 1$

$-65 \le 65\sin(\theta + 22.6°) \le 65$

$AC+BC+DE$ cannot be 70 cm
as the maximum length is 65 cm

(iv) Find the value of θ for which $AC+BC+DE=50$ cm. [2]

$AC + BC + DE = 50$

$65\sin(\theta + 22.619°) = 50$

$\sin(\theta+22.619°) = \frac{50}{65} = 0.76923$

$\theta + 22.619 = 50.284$

$\theta = 27.7°$

7 The equation of a curve is $y=x^2(6-x)$.

(i) Find the coordinates of the [redacted] of the curve. [5]

$\frac{dy}{dx} = 12x - 3x^2$

When $\frac{dy}{dx} = 0$, $12x - 3x^2 = 0$

$3x(4 - x) = 0$

$x = 0$ or $x = 4$

When $x = 0$, $y = 0$

When $x = 4$, $y = 16(6 - 4) = 32$

Stationary points are $(0,0)$ and $(4,32)$

(ii) Use the [redacted] to determine the [redacted] of each of these points. [3]

$\frac{d^2y}{dx^2} = 12 - 6x$

At $x = 0$, $\frac{d^2y}{dx^2} = 12 > 0$

$\therefore$ $(0,0)$ is a minimum point

At $x = 4$, $\frac{d^2y}{dx^2} = 12 - 6(4)$

$= -12 < 0$

$\therefore$ $(4,32)$ is a maximum point

(iii) Find the range of values of x for which [redacted]. [2]

Method 1

$\frac{dy}{dx} > 0$

$12x - 3x^2 > 0$

$3x(4 - x) > 0$

$0 < x < 4$

Method 2

(4, 32)

(0,0)

From (ii), we see that there is a minimum point at (0,0), and a maximum point at (4,32).

Hence for $\frac{dy}{dx} > 0$, $0 < x < 4$

8 **(i)** Differentiate $3x^2 \ln x$. [2]

$$\frac{d}{dx}\left(3x^2 \ln x\right) = 3x^2\left(\frac{1}{x}\right) + (\ln x)(6x)$$
$$= 3x + 6x\ln x$$

(ii) Hence show that $\int x\ln x\, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$, where c is a constant. [3]

$$\int (3x + 6x\ln x)\, dx = 3x^2 \ln x + c$$
$$\frac{3}{2}x^2 + 6\int x\ln x\, dx = 3x^2 \ln x + c$$
$$6\int x\ln x\, dx = 3x^2 \ln x - \frac{3}{2}x^2 + c$$
$$\int x\ln x\, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

The curve $y = f(x)$ passes through the point $\left(1, \frac{19}{4}\right)$ and is such that $f'(x) = \frac{1}{x} + x\ln x$.

(iii) Find $f(x)$. [4]

$$f(x) = \int \frac{1}{x} + x\ln x\, dx \qquad \int f'(x)dx = f(x) + c$$
$$f(x) = \ln x + \int x\ln x\, dx$$
$$= \ln x + \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

Sub $\left(1, \frac{19}{4}\right)$, $\frac{19}{4} = -\frac{1}{4} + c$, $c = 5$

$$\therefore f(x) = \ln x + \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + 5$$

(iv) State the range of values of x for which $f(x)$ is defined. [1]

$x > 0$

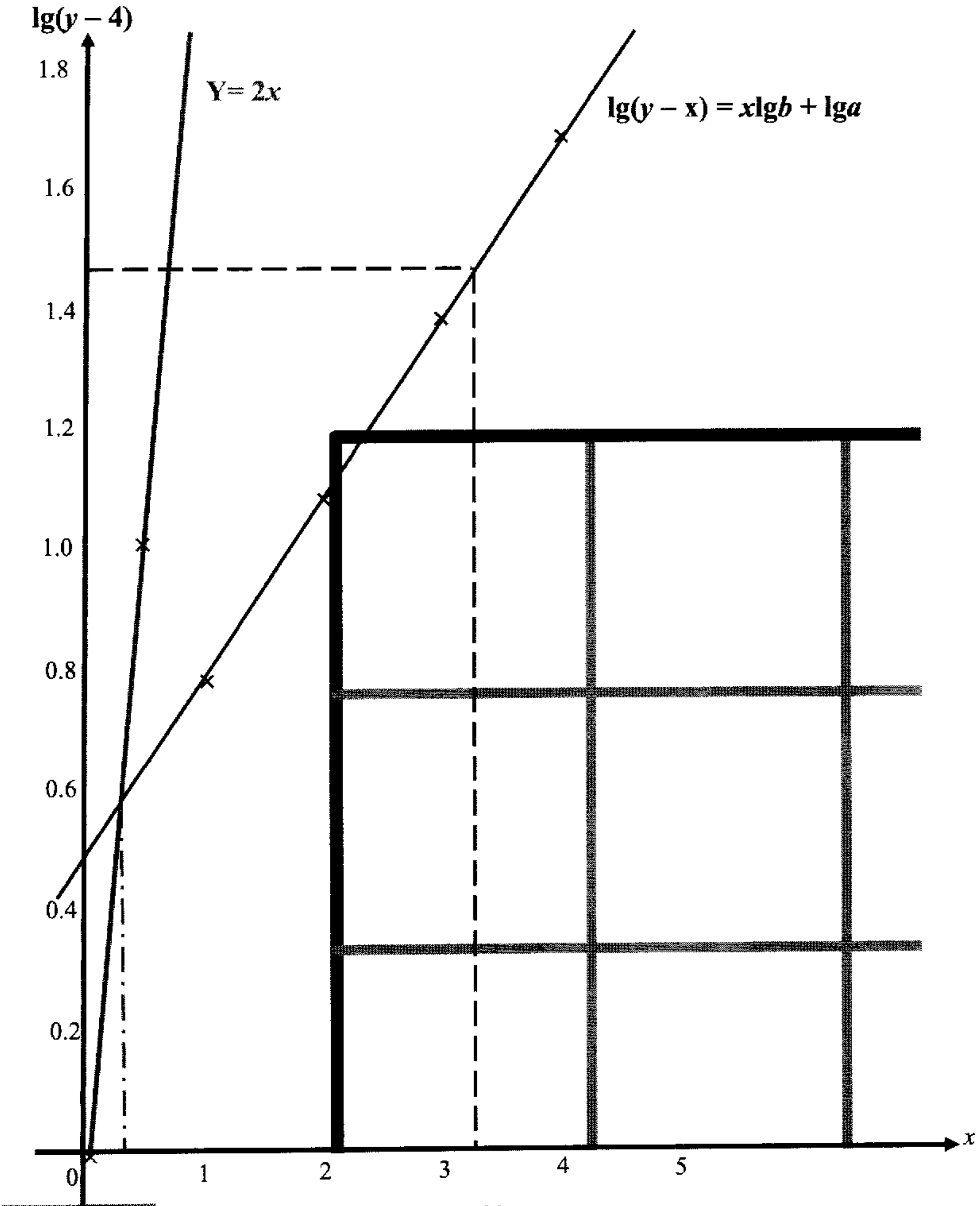
9 It is known that x and y are related by an equation, $y = ab^x + 4$ where a and b are constants.

x	1	2	3	4

y	10	16	28	52

(i) Draw a straight line graph of $\lg(y-4)$ against x, using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.2 units on the $\lg(y-4)$-axis. [2]

x	1	2	3	4
y	10	16	28	52
lg(y-4)	0.778	1.08	1.38	1.68



(ii) [redacted] to estimate the value of a and of b. [5]

$y = ab^x + 4$

$y - 4 = ab^x$
lg (y-4)=lg(ab^x)
lg (y-4)=$x$lg$b$ + lg$a$
Gradient $= \dfrac{1.68 - 1.08}{4 - 2}$
lg$b$ = 0.3
b = 2.00 (to 3 s.f.)
vertical intercept =0.5
lg$a$ = 0.5
a = 3.16

(iii) [redacted] to estimate the value of x when $y = 33$. [2]

When $y = 33$, $\lg(y-4) = \lg 29$
≈ 1.462 or 1.46
$x = 3.25$

(iv) On the same diagram, [redacted] representing the equation [redacted] and [redacted] find the value of x for which $10^{2x} = ab^x$. [2]

$y - 4 = 10^{2x}$
$\therefore \lg(y-4) = 2x$ is the equation of line representing $y - 4 = 10^{2x}$.
Draw the line $Y = 2X$
$10^{2x} = ab^x$
$\lg 10^{2x} = \lg(ab^x)$
$2x = \lg a + x \lg b$
Value of x is the x-coordinate of the point of intersection of $Y = 2X$ and $Y = \lg a + x \lg b$
$x = 0.3$ or 0.25

10 y

 P

The diagram shows part of the curve $y = 1 - \sin x$ passing through the point P. The curve touches the x-axis at the point Q. The tangent and normal to the curve at P meet the x-axis at the points R and S respectively.

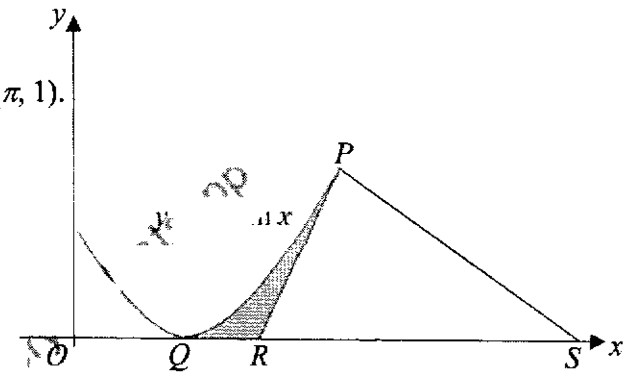
(i) Show that the x-coordinate of Q is $\frac{\pi}{2}$. [1]

At Q, $1 - \sin x = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

x-coordinate of Q is $\frac{\pi}{2}$.



The gradient of the curve at P is 1.

(ii) Show that the coordinates of P are $(\pi, 1)$. [3]

$\frac{dy}{dx} = -\cos x$

At P, $-\cos x = 1$

$\cos x = -1$

$x = \pi$

$y = 1 - \sin \pi = 1$

$\therefore$ coordinates of $P = (\pi, 1)$

(iii) Show that the x-coordinate of R is $\pi - 1$. [1]

Grad of $PR = 1$, let r be the x-coodinate of R.

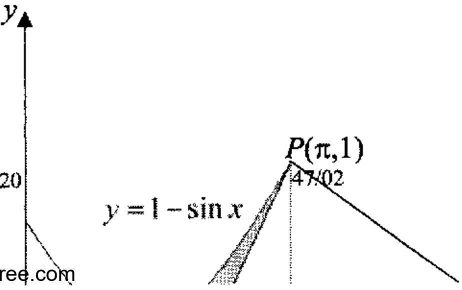
$\frac{1-0}{\pi - r} = 1$ OR $y = x + c$

$\pi - r = 1$ $(\pi, 1)$ $1 = \pi + c$

$r = \pi - 1$ $y = x + 1 - \pi$

$\therefore$ x-coordinate of R is $\pi - 1$

(iv) Show that the x-coordinate of S is $\pi + 1$. [1]



grad of $PS = -\dfrac{1}{\text{grad of } PR}$,

Let b be the x-coordinate of S.

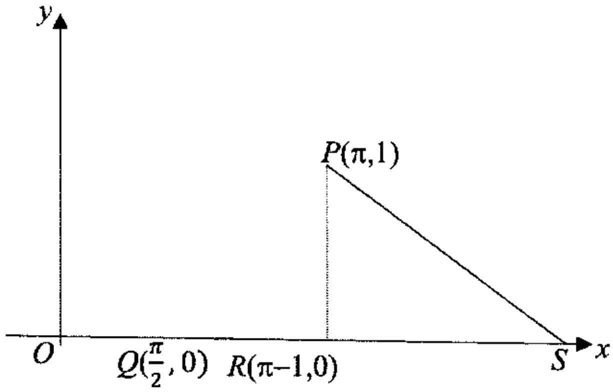
$\dfrac{1-0}{\pi - b} = -1$ OR $y = -x + c$

$\pi - b = -1$ $(\pi, 1)$ $1 = -\pi + c$

$b = \pi + 1$ $y = -x + 1 + \pi$

$\therefore$ x-coordinate of S is $\pi + 1$

(v) Find the exact area of the shaded region. [5]



Area between curve and x-axis from Q to P

$= \int_{\frac{\pi}{2}}^{\pi} (1 - \sin x)\ \mathrm{d}x$

$= \left[x + \cos x\right]_{\frac{\pi}{2}}^{\pi}$

$= (\pi + \cos \pi) - \left(\dfrac{\pi}{2} + \cos\dfrac{\pi}{2}\right)$

$= \pi + (-1) - \dfrac{\pi}{2} - 0$

$= \left(\dfrac{\pi}{2} - 1\right)$ units^2

Area of shaded region

$= \left(\dfrac{\pi}{2} - 1\right) - \dfrac{1}{2}[\pi - (\pi - 1)](1)$

$= \dfrac{\pi}{2} - 1 - \dfrac{1}{2}(1)(1)$

$= \left(\dfrac{\pi}{2} - \dfrac{3}{2}\right)$ units^2 (exact area)

Area of triangle $= \frac{1}{2}[\pi - (\pi - 1)] \times 1$

Shaded area $\neq \int_{\frac{\pi}{2}}^{\pi} (y_1 - y_2)dx$

11 A particle P moves in a straight line, so that, t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = 10e^{-2t} - 3$.

(i) Find the [redacted] velocity of P. [1]

When [redacted], $v = 10 - 3 = 7$
Initial velocity is 7 m/s.

(ii) Find the acceleration of P when $t = 1$. [2]

$a = \frac{\mathrm{d}v}{\mathrm{d}t}$ $\quad\quad$ $v = 10e^{-2t} - 3$

$a = -2(10)e^{-2t} = -20e^{-2t}$

When $t = 1$, $a = -20e^{-2} = -2.71$ m/s^2 (to 3 s.f.)

(iii) Find the value of t when P is at instantaneous [redacted] [3]

When [redacted], $10e^{-2t} - 3 = 0$

$e^{-2t} = 0.3$

$t = -\frac{1}{2}\ln 0.3$

$t \approx 0.6019864$

$t = 0.602$ (to 3 s.f.)

(iv) Find the total distance travelled by P before it comes to instantaneous rest. [4]

Method 1

Distance travelled $= \int_0^{0.6019} (10e^{-2t} - 3)\ \mathrm{d}t$

$= \left[\frac{10}{-2} e^{-2t} - 3t \right]_0^{0.6019}$

$= -5(0.3) - 3(0.6019) + 5$

≈ 1.694

$= 1.69$ m (to 3 s.f.)

Method 2

Let s be the displacement from point O.

$s = \int 10e^{-2t} - 3\ \mathrm{d}t$

$= -5e^{-2t} - 3t + c$

When $t = 0$, $s = 0$,

$0 = -5 + c$

$c = 5$

$s = -5e^{-2t} - 3t + 5$

When $v = 0$, i.e. $t = -\frac{1}{2}\ln 0.3$ or 0.60198,

$s = -5(0.3) - 3(\frac{1}{2}\ln\frac{10}{3}) + 5$

≈ 1.694

Distance travelled $= 1.694 - 0$

$= 1.69$ (to 3 s.f.)

(v) Explain why the value of v is always greater than -3. [1]

Since $10e^{-2t} > 0$