SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
FIRST SEMESTRAL EXAMINATION

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Name: ____________________________( )   Class: ______

ADDITIONAL MATHEMATICS

4047
Wednesday 9 May 2018
2 hours

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

SUBMIT SECTION A AND B SEPARATELY

This document consists of **5** printed pages.

[Turn over

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots.. + \binom{n}{r} a^{n-r} b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

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Answer **all** the questions.

Section A (52 marks)

1 Find the set of values of x for which $(2x-3)^2 > x$. [3]

2 Given that $s = 2+\sqrt{5}$, express $\frac{s^2+3}{s+1}$ in the form $a+b\sqrt{5}$, where a and b are integers. [5]

3 Solve, for x and y, the simultaneous equations

$$81^x \times 3^y = 1,$$

$$2^{x-5} \div 8^y = \frac{1}{4^x}.$$ [5]

4 Without using a calculator, find the values of the integers a and b for which the solution of the equation $x\sqrt{3}+\sqrt{2} = x\sqrt{18}$ is $\frac{a+\sqrt{b}}{15}$. [5]

5 Find the coordinates of the midpoint of the straight line joining the points of intersection of the curve $4x^2 - 8y^2 = 23y + 28$ and the line $2x = 3y + 4$. [6]

6 It is given that $x^3 + 6x^2 + 6x + 2 = (x-1)(x+2)Q(x) + 3Ax + B$ for all values of x, where $Q(x)$ is a polynomial.

(i) State the degree of the polynomial $Q(x)$. [1]

(ii) Find the value of A and of B. [4]

(iii) Hence, using a suitable substitution for x, find the remainder when 1662 is divided by 108. [2]

7 **(a)** The equation of a curve is $y = 2x^2 + 3x + c - 7$. Find the value of c for which the line $y - x = 8$ is a tangent to the curve. [3]

(b) Find the range of values of k for which the curve $y = 3x^2 + kx + k + 9$ lies completely above the x-axis. [4]

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8 Given that the roots of $2x^2 - 4x + 3 = 0$ are α and β, find the quadratic equation whose roots are $\alpha^3\beta$ and $\beta^3\alpha$. [7]

9 A liquid is heated up till it boils. It is then left to cool at room temperature. The temperature of the liquid, T °C, is given by $T = 25 + 75e^{-kt}$, where k is a constant and t is the time in hours since the liquid started cooling.

(i) Find the boiling temperature of the liquid. [1]

(ii) Given that the temperature of the liquid was 40 °C after 2 hours, calculate the value of k. [3]

(iii) Hence, find the temperature of the liquid after 4 hours. [1]

(iv) State the temperature that the liquid will approach after a very long time. [1]

(v) The equation $T = 25 + 75e^{-kt}$ is only valid for cooling of the liquid. The value of the constant k can change if the cooling liquid is placed in a different environment. Explain whether it will be possible for the value of k to be negative. [1]

Section B (28 marks)

Begin this section on a fresh sheet of paper.

10 Solve the equation

(i) $\lg(2y + 3) + \lg(3y - 4) = \lg 100$, [3]

(ii) $\log_x 5 - 2\log_5 x = 3$. [5]

11 The function $f(x) = x^3 - 2x^2 + ax + b$, where a and b are constants, is exactly divisible by $x - 4$ and leaves a remainder of 14 when divided by $x - 2$.

(i) Find the value of a and of b. [4]

(ii) Solve the equation $f(x) = 0$. [4]

(iii) Hence, solve the equation $8x^3 - 8x^2 + 2ax + b = 0$. [2]

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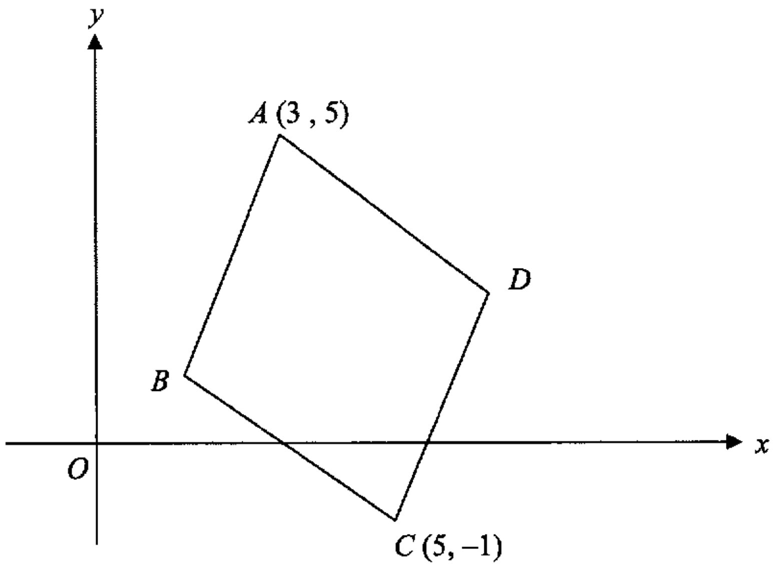
12 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ in which the point A is (3 , 5) and the point C is (5, –1). The point B lies on the perpendicular bisector of AC and the equation of the line AB is $y = 2x - 1$. Find

(i) the equation of the perpendicular bisector of AC, [4]

(ii) the coordinates of B. [2]

The point D is such that $ABCD$ is a rhombus. Find

(iii) the coordinates of D, [2]

(iv) the area of $ABCD$. [2]

END OF PAPER

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2018 3E AM SA1 Marking Scheme

S/N	Solution	Mark
1	$(2x-3)^2 > x$ $4x^2 - 12x + 9 > x$ $4x^2 - 13x + 9 > 0$ $(4x-9)(x-1) > 0$ $x < 1$ or $x > 2\frac{1}{4}$	 M1 M1 A1
2	$\frac{s^2+3}{s+1}$ $=\frac{(2+\sqrt{5})^2+3}{(2+\sqrt{5})+1}$ $=\frac{4+4\sqrt{5}+5+3}{(2+\sqrt{5})+1}$ $=\frac{12+4\sqrt{5}}{3+\sqrt{5}}$ $=\frac{12+4\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ $=\frac{36-12\sqrt{5}+12\sqrt{5}-4(5)}{9-5}$ $=4$	 M1 M1 M1: Num M1: Deno A1
3	$81^x \times 3^y = 1$ --- (1) $2^{x-5} \div 8^y = \frac{1}{4^x}$ --- (2) From (1), $3^{4x} \times 3^y = 3^0$ $4x + y = 0$ $y = -4x$ --- (3) From (2), $2^{x-5} \div 2^{3y} = 2^{-2x}$ $x - 5 - 3y = -2x$ $3x - 3y = 5$ --- (4) Subst (3) into (4), $3x - 3(-4x) = 5$ $15x = 5$ $x = \frac{1}{3}$ $y = -1\frac{1}{3}$	 M1 M1 M1 A1 A1

4	$x\sqrt{3}+\sqrt{2}=x\sqrt{18}$ $x\sqrt{3}+\sqrt{2}=3x\sqrt{2}$ $\sqrt{2}=3x\sqrt{2}-x\sqrt{3}$ $\sqrt{2}=x(3\sqrt{2}-\sqrt{3})$ $x=\frac{\sqrt{2}}{3\sqrt{2}-\sqrt{3}}$ $x=\frac{\sqrt{2}}{3\sqrt{2}-\sqrt{3}}\times\frac{3\sqrt{2}+\sqrt{3}}{3\sqrt{2}+\sqrt{3}}$ $x=\frac{6+\sqrt{6}}{9(2)-3}$ $x=\frac{6+\sqrt{6}}{15}$ $a=6, b=6$	M1 M1 M1 M1 A1
5	$4x^2-8y^2=23y+28$ --- (1) $2x=3y+4$ --- (2) Subst (2) to (1), $(3y+4)^2-8y^2=23y+28$ $9y^2+24y+16-8y^2=23y+28$ $y^2+y-12=0$ $(y+4)(y-3)=0$ $y=-4$ or $y=3$ When $y=-4$ $2x=3(-4)+4$ $x=-4$ When $y=3$ $2x=3(3)+4$ $x=6.5$ Points of intersection: $(-4,-4)$ and $(6.5,3)$ Midpoint $=\left(\frac{-4+6.5}{2},\frac{-4+3}{2}\right)$ $=(1.25,-0.5)$	M1 M1 A1: y values A1: x values M1 A1
6i	Degree = 1	B1
6ii	$x^3+6x^2+6x+2=(x-1)(x+2)Q(x)+3Ax+B$ When $x=1$, $15=3A+B$ --- (1) When $x=-2$, $-8+24-12+2=-6A+B$ $6=-6A+B$ --- (2) (1) − (2): $9A=9$ $A=1$ $15=3(1)+B$ $B=12$	M1 for either M1 A1 A1
6iii	$x^3+6x^2+6x+2=(x-1)(x+2)Q(x)+3x+12$ When $x=10$, $1000+600+60+2=(9)(12)Q(x)+30+12$ $1662=108Q(x)+42$ Remainder = 42	M1 A1

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7a	$y = 2x^2 + 3x + c - 7$ --- (1) $y - x = 8$ $y = x + 8$ --- (2) Subst (2) to (1), $x + 8 = 2x^2 + 3x + c - 7$ $2x^2 + 2x + c - 15 = 0$ For line to be tangent to the curve, $b^2 - 4ac = 0$ $2^2 - 4(2)(c - 15) = 0$ $4 - 8c + 120 = 0$ $c = 15\frac{1}{2}$	M1 M1 A1
7b	$y = 3x^2 + kx + k + 9$ For curve to lie completely above x-axis, $b^2 - 4ac < 0$ $k^2 - 4(3)(k + 9) < 0$ $k^2 - 12k - 108 < 0$ $(k - 18)(k + 6) < 0$ $-6 < k < 18$	M1: $D < 0$ M1: $b^2 - 4ac$ M1 A1
8	$2x^2 - 4x + 3 = 0$ $\alpha + \beta = -\frac{-4}{2} = 2$ $\alpha\beta = \frac{3}{2}$ New S.O.R. : $\alpha^3\beta + \beta^3\alpha$ $= \alpha\beta(\alpha^2 + \beta^2)$ $= \alpha\beta((\alpha + \beta)^2 - 2\alpha\beta)$ $= \frac{3}{2}\left((2)^2 - 2\left(\frac{3}{2}\right)\right)$ $= \frac{3}{2}$ New P.O.R. : $\alpha^3\beta \times \beta^3\alpha$ $= \alpha^4\beta^4$ $= (\alpha\beta)^4$ $= \left(\frac{3}{2}\right)^4$ $= \frac{81}{16}$ Required equation: $x^2 - \frac{3}{2}x + \frac{81}{16} = 0$	M1 M1 M1 M1 M1 M1 A1
9i	$T = 25 + 75e^0 = 100$ °C	B1
9ii	$T = 25 + 75e^{-kt}$ $40 = 25 + 75e^{-2k}$ $\frac{1}{5} = e^{-2k}$ $-2k = \ln\left(\frac{1}{5}\right)$ $k = 0.80472$ $= 0.805$ (3 s.f.)	M1 M1 A1

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9iii	$T = 25 + 75e^{-0.80472(4)}$ $= 28.0$ °C (3 s.f.)	B1
9iv	25 °C	B1
9v	No the value of k cannot be negative. If k is negative, e^{-kt} will have a positive power and $T = 25 + 75e^{-kt}$ will increase with time and the liquid will not be cooling but heating up.	B1
10i	$\lg(2y+3) + \lg(3y-4) = \lg 100$ $\lg[(2y+3)(3y-4)] = \lg 100$ $6y^2 + y - 12 = 100$ $6y^2 + y - 112 = 0$ $y = \dfrac{-1 \pm \sqrt{1^2 - 4(6)(-112)}}{2(6)}$ $y = -4.40$ or $y = 4.24$	M1 M1 A1
10ii	$\log_x 5 - 2\log_5 x = 3$ $\dfrac{\log_5 5}{\log_5 x} - 2\log_5 x = 3$ $\dfrac{1}{\log_5 x} - 2\log_5 x = 3$ $1 - 2(\log_5 x)^2 = 3\log_5 x$ $2(\log_5 x)^2 + 3\log_5 x - 1 = 0$ $\log_5 x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$ $\log_5 x = 0.28078$ or $\log_5 x = -1.78078$ $x = 1.57$ or $x = 0.0569$	M1 M1 M1 M1 A1
11i	$f(x) = x^3 - 2x^2 + ax + b$ Since $(x-4)$ is a factor, $f(4) = 0$ $(4)^3 - 2(4)^2 + a(4) + b = 0$ $4a + b = -32$ --- (1) Dividing by $(x-2)$ gives remainder of 14, $f(2) = 14$ $(2)^3 - 2(2)^2 + a(2) + b = 14$ $2a + b = 14$ --- (2) (1) – (2): $2a = -46$ $a = -23$ $2(-23) + b = 14$ $b = 60$	M1 M1 M1 A1: a & b
11ii	$x^3 - 2x^2 - 23x + 60 = (x-4)(x^2 + cx - 15)$ Comparing coefficients of x^2, $-2 = -4 + c$ $c = 2$ $f(x) = (x-4)(x^2 + 2x - 15)$ $= (x-4)(x-3)(x+5)$ $f(x) = 0$ $(x-4)(x-3)(x+5) = 0$ $x = 4$ or $x = 3$ or $x = -5$	M1 M1 M1 A1

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11iii	$8x^3-8x^2+2ax+b=0$ $8x^3-8x^2-46x+60=0$ Let $x=\frac{1}{2}y$, $8\left(\frac{1}{2}y\right)^3-8\left(\frac{1}{2}y\right)^2-46\left(\frac{1}{2}y\right)+60=0$ $y^3-2y^2-23y+60=0$ $y=4$ or $y=3$ or $y=-5$ $x=2$ or $x=1.5$ or $x=-2.5$	M1 A1
12i	Gradient of $AC=\frac{5-(-1)}{3-5}=-3$ Gradient of perpendicular bisector $=\frac{1}{3}$ Midpoint of $AC=\left(\frac{3+5}{2},\frac{5-1}{2}\right)=(4,2)$ Equation of perpendicular bisector: $y-2=\frac{1}{3}(x-4)$ $y=\frac{1}{3}x+\frac{2}{3}$	M1 M1 M1 A1
12ii	$y=\frac{1}{3}x+\frac{2}{3}$ --- (1) $y=2x-1$ --- (2) Subst (1) to (2) $2x-1=\frac{1}{3}x+\frac{2}{3}$ $x=1$ $y=2(1)-1=1$ $B=(1,1)$	M1 A1
12iii	Midpoint of BD = Midpoint of $AC=(4,2)$ Let $D=(x,y)$ $\left(\frac{x+1}{2},\frac{y+1}{2}\right)=(4,2)$ $D=(7,3)$	M1 A1
12iv	Area of ABCD $=\frac{1}{2}\begin{vmatrix}3 & 1 & 5 & 7 & 3\\5 & 1 & -1 & 3 & 5\end{vmatrix}$ $=\frac{1}{2}[(3-1+15+35)-(5+5-7+9)]$ $=20$ unit^2	M1 A1

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