

Centre Number	Index Number	Name	Class
S3016			

**RAFFLES INSTITUTION
2021 Preliminary Examination**

PHYSICS
Higher 2
Paper 2 Structured Questions

9749/02
September 2021
2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number, name and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.
Answer **all** questions.

The number of marks is given in brackets [] at the end of each question or part question.

For Examiner's Use	
1	/ 9
2	/ 8
3	/ 8
4	/ 10
5	/ 8
6	/ 9
7	/ 8
8	/ 20
Deduction	
Total	/ 80

This document consists of **22** printed pages.

Data

speed of light in free space
 permeability of free space
 permittivity of free space

 elementary charge
 the Planck constant
 unified atomic mass constant
 rest mass of electron
 rest mass of proton
 molar gas constant
 the Avogadro constant
 the Boltzmann constant
 gravitational constant
 acceleration of free fall

$$\begin{aligned}
 c &= 3.00 \times 10^8 \text{ m s}^{-1} \\
 \mu_0 &= 4\pi \times 10^{-7} \text{ H m}^{-1} \\
 \epsilon_0 &= 8.85 \times 10^{-12} \text{ F m}^{-1} \\
 &= (1/(36\pi)) \times 10^{-9} \text{ F m}^{-1} \\
 e &= 1.60 \times 10^{-19} \text{ C} \\
 h &= 6.63 \times 10^{-34} \text{ J s} \\
 u &= 1.66 \times 10^{-27} \text{ kg} \\
 m_e &= 9.11 \times 10^{-31} \text{ kg} \\
 m_p &= 1.67 \times 10^{-27} \text{ kg} \\
 R &= 8.31 \text{ J K}^{-1} \text{ mol}^{-1} \\
 N_A &= 6.02 \times 10^{23} \text{ mol}^{-1} \\
 k &= 1.38 \times 10^{-23} \text{ J K}^{-1} \\
 G &= 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \\
 g &= 9.81 \text{ m s}^{-2}
 \end{aligned}$$

Formulae

uniformly accelerated motion

 work done on/by a gas
 hydrostatic pressure
 gravitational potential
 temperature

 pressure of an ideal gas

 mean translational kinetic energy of an ideal gas molecule

 displacement of particle in s.h.m.
 velocity of particle in s.h.m.
 electric current
 resistors in series
 resistors in parallel

 electric potential

 alternating current/voltage

 magnetic flux density due to a long straight wire

 magnetic flux density due to a flat circular coil
 magnetic flux density due to a long solenoid
 radioactive decay

 decay constant

$$\begin{aligned}
 s &= ut + \frac{1}{2}at^2 \\
 v^2 &= u^2 + 2as \\
 W &= p\Delta V \\
 p &= \rho gh \\
 \phi &= -Gm/r \\
 T/K &= T/^\circ\text{C} + 273.15 \\
 p &= \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \\
 E &= \frac{3}{2}kT \\
 x &= x_0 \sin \omega t \\
 v &= v_0 \cos \omega t = \pm \omega \sqrt{x_0^2 - x^2} \\
 I &= Anvq \\
 R &= R_1 + R_2 + \dots \\
 1/R &= 1/R_1 + 1/R_2 + \dots \\
 V &= \frac{Q}{4\pi\epsilon_0 r} \\
 x &= x_0 \sin \omega t \\
 B &= \frac{\mu_0 I}{2\pi d} \\
 B &= \frac{\mu_0 NI}{2r} \\
 B &= \mu_0 nI \\
 x &= x_0 \exp(-\lambda t) \\
 \lambda &= \frac{\ln 2}{t_{\frac{1}{2}}}
 \end{aligned}$$

Answer **all** the questions in the spaces provided.

- 1 (a) Fig 1.1 shows two frictionless trolleys A and B of mass m_A and m_B moving horizontally towards a wall with the same speed u . The trolleys are not in contact.

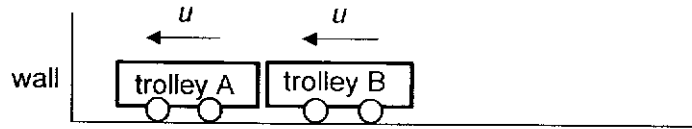


Fig. 1.1

Upon collision with the wall, trolley A rebounds with speed u and collides elastically with trolley B.

- (i) State the principle of conservation of momentum.

.....

.....

.....

..... [2]

- (ii) Taking motion to the right as positive, show that the speed of trolley B, v_B after the collision with trolley A is given by the expression

$$v_B = \frac{3m_A - m_B}{m_A + m_B} u.$$

[3]

- (b) A student performs a similar experiment with a basketball of mass 0.62 kg and a tennis ball of mass 0.059 kg. The student places the tennis ball slightly above the basketball and releases both at the same time from a height above the ground, as shown in Fig. 1.2.

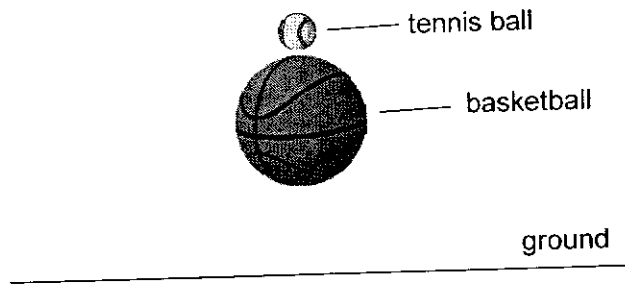


Fig. 1.2

Just before the basketball touches the ground, both the basketball and the tennis ball have the same speed of 4.4 m s^{-1} . The basketball bounces off the ground with a speed of 4.4 m s^{-1} . Its subsequent impact with the tennis ball causes the tennis ball to move up at a very large speed.

- (i) Using the expression in (a)(ii), determine the speed of the tennis ball after its collision with the basketball.

speed = m s^{-1} [1]

- (ii) Besides the assumptions that all collisions are elastic and air resistance is negligible, state one other assumption that is necessary in order to use the result in (a)(ii) to determine the speed for (b)(i).

..... [1]

- (iii) The student repeats the experiment, replacing the tennis ball with another ball of much smaller mass.

Deduce the maximum speed the ball can have after its collision with the basketball.

maximum speed = m s^{-1} [2]

- 2 Fig. 2.1 shows an airplane of mass 1.5×10^5 kg, flying horizontally at a constant velocity. The airplane has four engines, two located on each wing, which produce a combined forward thrust of 8.0×10^5 N. The other forces acting on the airplane are drag force, the combined lift of both wings and its weight. The horizontal separation of the lines of action of lift and weight is 0.75 m.

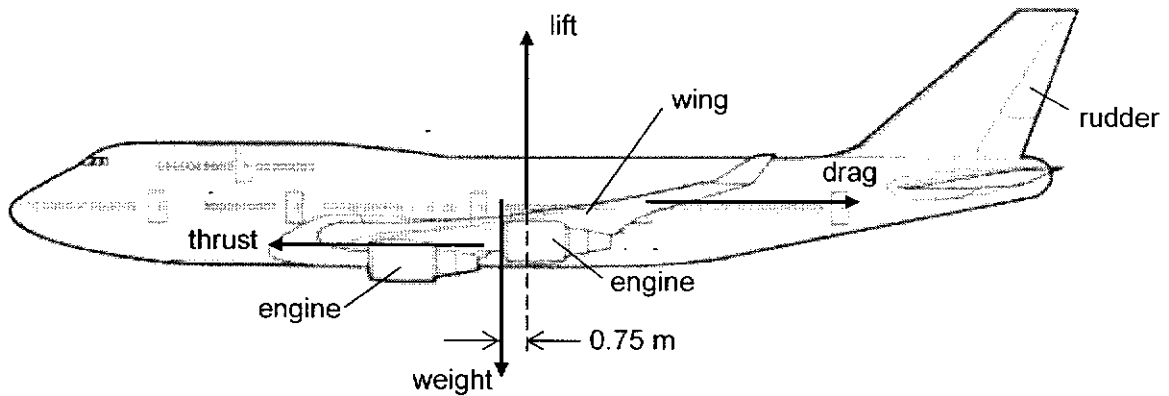


Fig. 2.1

- (a) Define the moment of a force about a point.

.....
 [1]

- (b) Determine the vertical separation of the lines of action of thrust and drag.

vertical separation = m [3]

- (c) The airplane starts to accelerate forward. Using Newton's First Law of Motion, state and explain the direction of the frictional force acting on a box that is placed on the airplane floor.

.....

 [2]

- (d) The engines of the airplane are located 10 m and 20 m perpendicularly from the midline of the airplane's body. In a training session, both engines on the right wing are shut down, leaving only the two engines on the left wing working. Each of these engines produce a forward thrust of 2.0×10^5 N. As a result, the airplane rotates in the horizontal plane. To counter this rotation, the rudder at the tail of the aircraft can be adjusted.

Fig. 2.2 shows the adjustment of the rudder to an angle such that a force P acts on the rudder at a point 30 m from centre of gravity C.G. along the midline of the airplane. P acts at an angle of 60° to the midline and is due to the airflow incident on the rudder.

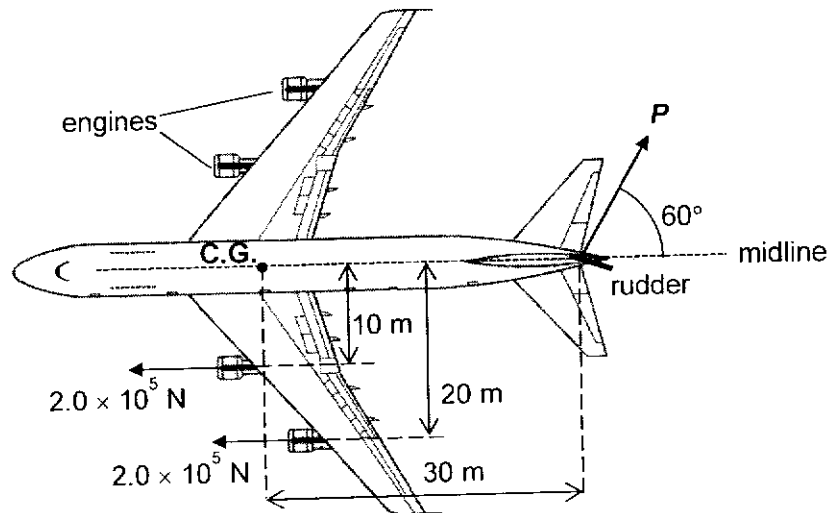


Fig 2.2

Calculate the value for P that will prevent the aircraft from rotating.

$$P = \text{.....} \text{ N} \quad [2]$$

- 3 Fig. 3.1 shows a wind turbine with a diameter of 100 m. Wind of density 1.2 kg m^{-3} is incident normally on the blades of the turbine at a speed of 20 m s^{-1} .

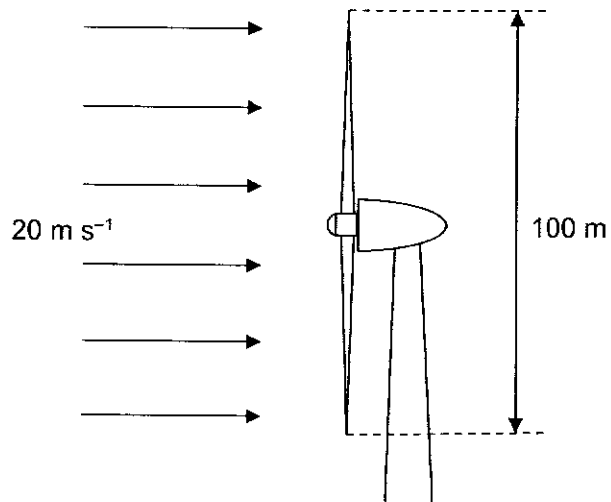


Fig. 3.1

- (a) Calculate the volume of air that passes through the area swept out by the turbine blades in one second.

volume = m^3 [2]

- (b) Hence, calculate the mass of air that passes through the area swept out by the turbine blades in one second.

mass = kg [1]

(c) After passing through the blades, the wind speed decreases to 15 m s^{-1} .

(i) Determine the rate of loss of kinetic energy of the wind.

rate of loss of kinetic energy = W [2]

(ii) Using Newton's second law, determine the force exerted by the turbine blades on the wind.

force = N [2]

(d) Explain how the answers obtained in (c)(i) and (c)(ii) are related by the equation $P = Fv$.

..... [1]

- 4 (a) A pendulum with a bob of mass 10 g is suspended from a fixed point O by an inextensible string of length 30 cm. The bob is initially held at point A, at an angle of 25° to the vertical as shown in Fig. 4.1. It is released from rest and swings towards point B, which is vertically below O.

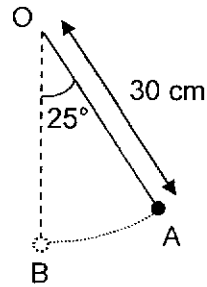


Fig. 4.1

- (i) Show that the speed of the mass at point B is 0.74 m s^{-1} .

[2]

- (ii) Hence, determine the tension in the string at point B.

tension = N [2]

- (iii) A rod is placed above point B such that part of the string remains vertical as the mass swings past B as shown in Fig. 4.2.

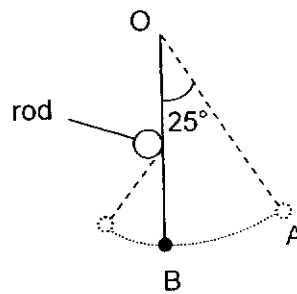


Fig. 4.2

Explain why the tension in the string just after the bob passes point B will be larger than the tension calculated in (a)(ii).

.....

.....

.....

..... [2]

- (b) The bob is now set in uniform circular motion in a horizontal plane with the string making an angle θ to the vertical as shown in Fig. 4.3. The tension in the string is 0.20 N.

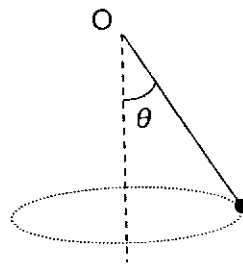


Fig. 4.3

- (i) Calculate angle θ .

$\theta = \text{.....}^\circ$ [2]

(ii) Calculate the angular speed of the bob.

angular speed = rad s⁻¹ [2]

- 5 Fig. 5.1 shows two small metal spheres X and Y each weighing 1.5×10^{-4} N carrying a charge of -3.2 nC and -1.6 nC respectively. Sphere X is fixed at its position while sphere Y is suspended from an insulating string that is attached to a fixed point P.

An external force F is applied on sphere Y in the direction shown. Sphere Y settles at equilibrium where the string makes an angle of 10° with the vertical and the centre of the two spheres are separated by a horizontal distance 0.050 m. The line joining the centres of X and Y is horizontal.

The diameter of each of the spheres is negligible compared to the separation between them.

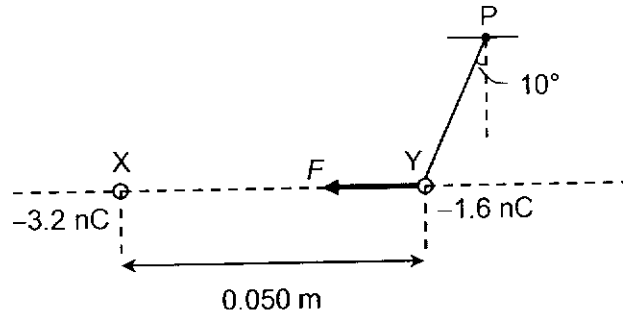


Fig. 5.1 (not to scale)

- (a) (i) Calculate the electric force acting on sphere Y.

electric force = N [2]

- (ii) Calculate the magnitude of F .

F = N [3]

- (a) (iii) State and explain whether your answer in (a)(ii) would be larger, smaller or unchanged if the diameter of each of the spheres is no longer negligible compared to the separation of the spheres.
Assume that the weight of the spheres remains unchanged.

----- [2]

- (b) The force F is now removed. Sphere Y swings downwards and moves past point Q, which is located vertically below point P as shown in Fig. 5.2.

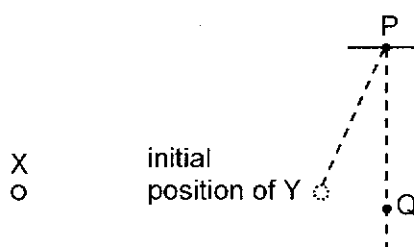


Fig. 5.2 (not to scale)

Explain why the gain in kinetic energy of sphere Y is **not** equal to the loss in gravitational potential energy as it moves from its initial position to Q.

----- [1]

- 6 A long vertical rectangular frame, of width of 2.0 m, consists of a 2.0Ω resistor and conducting wires of negligible resistance. The frame is placed in a uniform magnetic field of flux density 0.35 T. The magnetic field is directed into the plane of the frame.

A horizontal metal rod of mass 0.15 kg and negligible resistance slides along the frame downwards, as shown in Fig. 6.1. The metal rod remains in electrical contact with the frame throughout its motion.

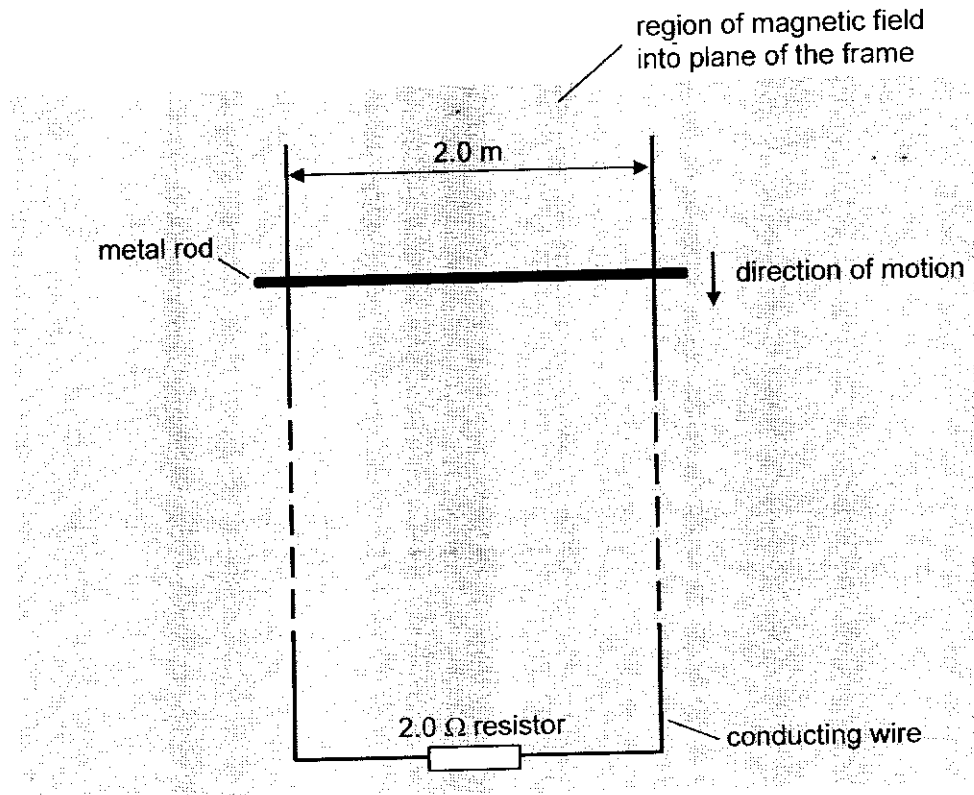


Fig. 6.1

- (a) Explain why a magnetic force acts on the metal rod.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[3]

(b) State and explain the direction of the magnetic force.

.....
.....
..... [2]

(c) Determine the highest speed the metal rod can reach. Assume that friction and air resistance are negligible.

speed = m s⁻¹ [4]

- 7 Fig. 7.1 shows an ideal transformer. The primary coil of the transformer has 4000 turns and is connected to a sinusoidal a.c. supply. The secondary coil has 200 turns and is connected to a 4.8Ω load resistor and a diode.

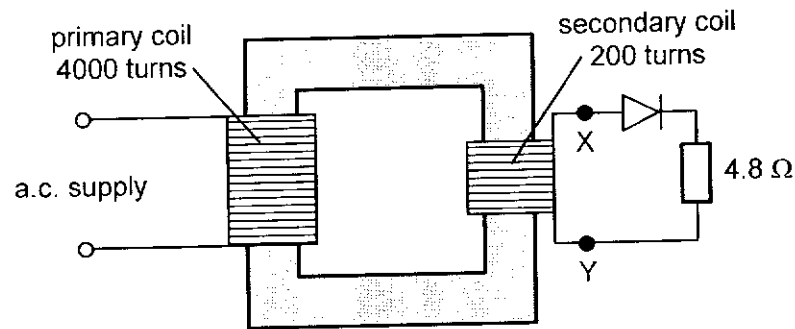


Fig. 7.1

V_{XY} is the potential of X with respect to Y. The variation with time t of V_{XY} is shown in Fig. 7.2.

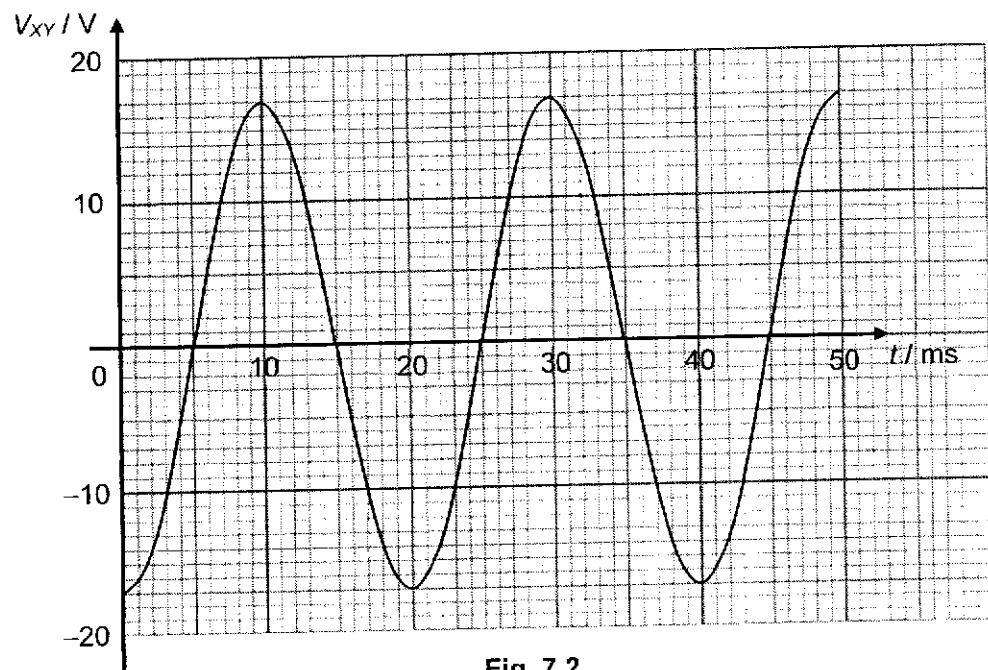


Fig. 7.2

- (a) Determine the frequency of the a.c. supply.

frequency = Hz [1]

- (b) Calculate the root-mean-square value of V_{XY} .

root-mean-square value of V_{XY} = V [1]

- (c) Determine the root-mean-square voltage of the a.c. supply.

root-mean-square voltage = V [2]

- (d) Determine the mean power dissipated in the load resistor.

mean power = W [2]

- (e) With reference to Fig. 7.2, describe and explain how the potential difference across the load resistor varies with time.

.....

.....

.....

..... [2]

- 8 Read the passage below and answer the questions that follow.

Exoplanets

An exoplanet is any planet beyond our solar system. Most orbit other stars, but free-floating exoplanets, called rogue planets, orbit the galactic centre and are not bound to any star.

The first exoplanets were discovered in the 1990s and since then we have identified thousands more using a variety of detection methods. It is pretty rare for astronomers to see an exoplanet through their telescopes the way they might see Saturn through a telescope from Earth. This method is called direct imaging, and only a handful of exoplanets have been found this way. Most exoplanets are found through indirect methods, such as the transit method.

When a planet passes directly between an observer and the star it orbits, it blocks some of that starlight, as shown in Fig. 8.1. For a brief period of time, that brightness of the star decreases. It is a tiny change, but it is enough for astronomers to detect the presence of an exoplanet around a distant star. This is known as the transit method.

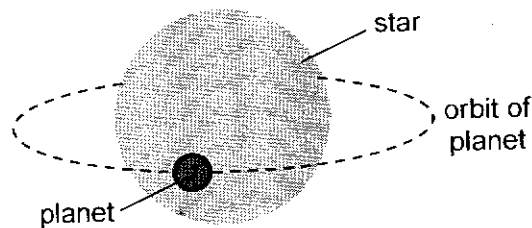


Fig. 8.1

HD 209458 b, also known as Osiris, is an exoplanet that orbits the star HD 209458. Osiris was the first exoplanet to be discovered using the transit method.

Some data of the star HD 209458 are given below.

mass = 2.28×10^{30} kg
 distance from Earth = 159 light-years
 orbital speed = 84.3 m s^{-1}
 temperature = 6070 K
 age = 3.5×10^9 years

Fig. 8.2 shows the variation with time t of the brightness of the star HD 209458.

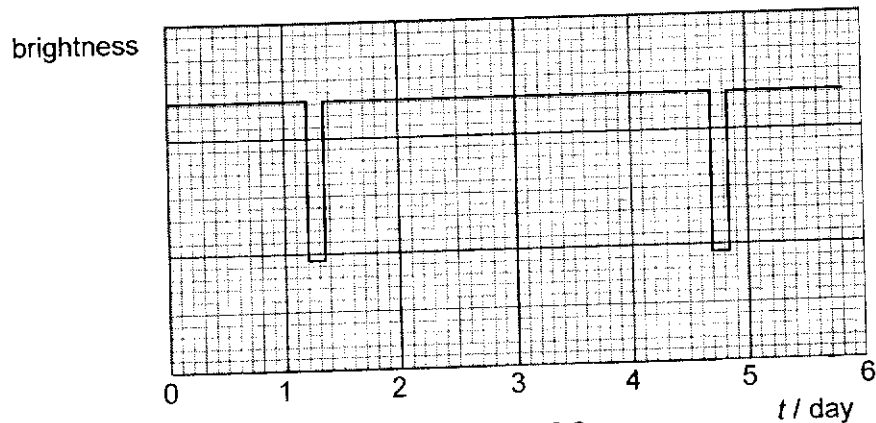


Fig. 8.2

- (a) Determine the period of orbit of Osiris.

period = days [1]

- (b) In a particular planetary system, a star of mass M_S and a planet of mass M_P move in circular orbits of radii x and y respectively, about their common centre of mass O , with a period T as shown in Fig. 8.3.

The star and planet are separated by a distance d apart.

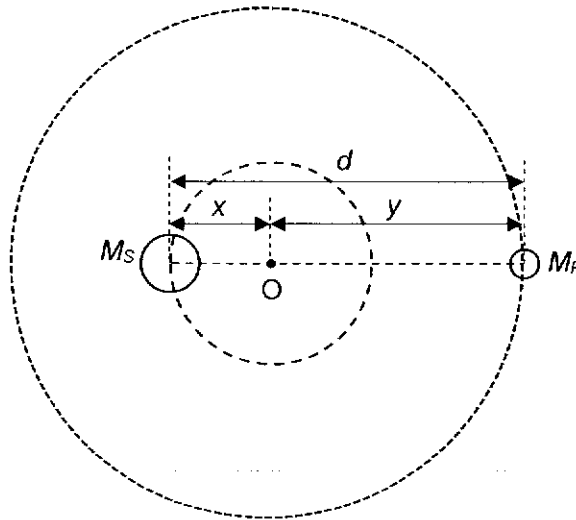


Fig. 8.3

- (i) Show that $M_S = \left(\frac{4\pi^2}{G}\right) \frac{yd^2}{T^2}$ and $M_P = \left(\frac{4\pi^2}{G}\right) \frac{xd^2}{T^2}$. Explain your working.

[3]

(ii) Hence, show that $M_S + M_P = \left(\frac{4\pi^2}{G} \right) \frac{d^3}{T^2}$.

[1]

- (c) Use the equation from (b)(ii) to calculate the distance between HD 209458 and Osiris.
Assume that the mass of Osiris is negligible compared to the mass of HD 209458.

distance = m [2]

- (d) Show that the orbital speed of Osiris about the common center of mass is $1.47 \times 10^5 \text{ m s}^{-1}$.

[1]

- (e) Hence, by considering the momentum of HD 209458 and Osiris, determine the mass of Osiris.

Assume that the centre of mass of HD 209458 and Osiris is stationary.

mass = kg [2]

- (f) Suggest two limitations of the transit method in detecting exoplanets.

1

.....

2

..... [2]

(g) A light-year is the distance that light travels in one year.

Show that one light-year is 9.5×10^{15} m.

[1]

(h) The James Webb Space Telescope (JWST) is set to launch in October 2021. It is planned to succeed the Hubble Space Telescope as the world's most powerful space telescope. The JWST will be used to observe some of the most distant events and objects in space using infrared radiation. The telescope has a large aperture size of 6.5 m.

(i) Explain what is meant by the Rayleigh criterion.

.....

.....

.....

..... [2]

- (ii) Fig. 8.4. shows a possible arrangement of the relative positions of HD 209458, Osiris and JWST.

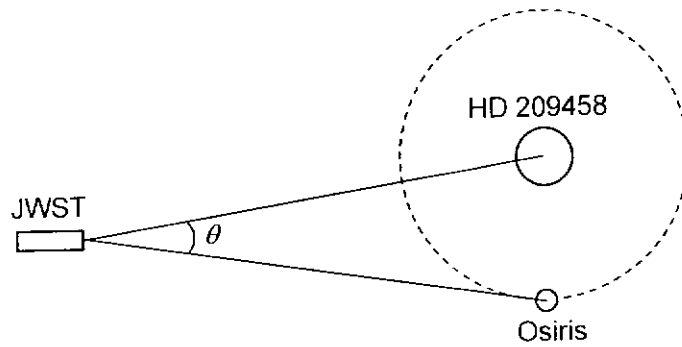


Fig. 8.4 (not to scale)

Calculate the maximum angle θ subtended by HD 209458 and Osiris at the JWST.

$$\theta = \text{.....} \text{ rad} \quad [2]$$

- (iii) Use the Rayleigh criterion to determine whether the JWST is able to distinguish Osiris from HD 209458 at a wavelength of $14 \mu\text{m}$.

[3]

[Total: 20]

2021 Raffles Institution Preliminary Examinations – H2 Physics
Paper 2 – Suggested Solutions

1 (a) (i) When bodies in a system interact, the total momentum of the system remains constant,
provided no net external force acts on it.

(ii) Relative speed of approach = relative speed of separation

$$u - (-u) = v_B - v_A$$

$$2u = v_B - v_A \quad \dots\dots\dots (1)$$

Conservation of momentum

$$m_A u - m_B u = m_A v_A + m_B v_B \quad \dots\dots\dots (2)$$

$$m_A \times (1) + (2): (2 m_A u) + (m_A u - m_B u) = (m_A v_B - m_A v_A) + (m_A v_A + m_B v_B)$$

$$3 m_A u - m_B u = m_A v_B + m_B v_B$$

$$v_B = \frac{3m_A - m_B}{m_A + m_B} u$$

(b) (i)
$$v_{\text{tennis}} = \frac{3m_A - m_B}{m_A + m_B} u = \frac{3(0.62) - (0.059)}{0.62 + 0.059} (4.4)$$

$$= 11.7 \text{ m s}^{-1}$$

(ii) The impact force is much greater than the weight of the tennis ball / basketball.
(Hence, gravitational force, which is an external force, is neglected when applying conservation of momentum.)

(iii) Basketball is much greater in mass as compared to the ball, hence m_B is negligible compared to m_A .

Using (a)(ii),

$$v_B \approx \frac{3m_A}{m_A} u$$

$$= 3u$$

$$= 3(4.4)$$

$$= 13.2 \text{ m s}^{-1}$$

2 (a) The moment of a force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the point.

(b) Since the airplane is travelling at constant speed, it is in equilibrium

Vertically: $L = W = mg = (1.5 \times 10^5)(9.81) = 1.47 \times 10^6 \text{ N}$

Horizontally: $T = D = 8.0 \times 10^6 \text{ N}$

clockwise torque = anti-clockwise torque

$$T \times y = W \times (0.75)$$

$$(8.0 \times 10^5) y = (1.47 \times 10^6)(0.75)$$

$$y = 1.38 \text{ m}$$

- (c) As the airplane accelerates forward, the box will tend to move at its original velocity. Hence, the box will tend to move backwards relative to the airplane floor. Friction on the box by the floor therefore acts forward (to oppose this relative motion).

- (d) Applying principle of moments, taking pivot at the C.G.,
 $(2.0 \times 10^5)10 + (2.0 \times 10^5)20 = P(30 \cos 30^\circ)$
 $P = 2.31 \times 10^5 \text{ N}$

- 3 (a) In one second, the volume of air that is swept by the blades is
 $V = A \times v(1)$
 $= \pi(50)^2 \times 20$
 $= 1.57 \times 10^5 \text{ m}^3$

- (b) In 1 s, mass = ρV
 $= 1.20 \times 1.57 \times 10^5$
 $= 1.88 \times 10^5 \text{ kg}$

- (c) (i) Rate of loss KE = $\frac{1}{2} \left(\frac{dm}{dt} \right) (u^2 - v^2)$
 $= \frac{1}{2} \times 1.88 \times 10^5 \times (20^2 - 15^2)$
 $= 1.65 \times 10^7 \text{ W}$

- (ii) Force = $\frac{dm}{dt} \times (u - v)$
 $= 1.88 \times 10^5 \times (20 - 15)$
 $= 9.42 \times 10^5 \text{ N}$

- (d) The rate of loss of KE is equal to the product of the force and average wind speed.

- 4 (a) (i) By the principle of conservation,
 gain in KE = loss in GPE

$$\frac{1}{2}mv^2 - 0 = mg\Delta h$$

$$\frac{1}{2}v^2 = (9.81)(0.30 - 0.30 \cos 25^\circ)$$

$$v = 0.7426$$

$$v = 0.74 \text{ m s}^{-1}$$

$$(ii) \quad T - mg = \frac{mv^2}{r}$$

$$T = \frac{(0.010)(0.74)^2}{0.30} + (0.010)(9.81)$$

$$T = 0.116 \text{ N}$$

(iii) When the bob passes point B, the radius of the circular motion is smaller and the speed of the bob is the same.

Since the centripetal force is now larger, the tension is larger.

(b) (i) Vertically,

$$T \cos \theta = mg$$

$$(0.20) \cos \theta = (0.010)(9.81)$$

$$\theta = 60.6^\circ$$

(ii) Horizontally,

$$T \sin \theta = mr\omega^2$$

$$(0.20) \sin \theta = (0.010)(0.30 \sin \theta)\omega^2$$

$$\omega = 8.16 \text{ rad s}^{-1}$$

5 (a) (i)

$$F_E = \frac{Qq}{4\pi\epsilon_0 r^2}$$

$$= \frac{(3.2 \times 10^{-9})(1.6 \times 10^{-9})}{4\pi\epsilon_0 (0.050)^2}$$

$$= 1.84152 \times 10^{-5}$$

$$= 1.84 \times 10^{-5} \text{ N}$$

(ii) $T \cos 10^\circ = W$

$$T = \frac{1.5 \times 10^{-4}}{\cos 10^\circ} = 1.523 \times 10^{-4}$$

$$F = F_E + T \sin 10^\circ$$

$$= 4.48642 \times 10^{-5}$$

$$= 4.49 \times 10^{-5} \text{ N}$$

(iii) The charges in the two spheres will be repelled/redistributed to the two far sides of the sphere.

The distance between the two charges will larger than 0.050 m and the electric force will be smaller. F will be smaller since the horizontal component of tension remains the same.

(b) The gain in kinetic energy is equal to loss in gravitational potential energy and loss in electric potential energy.

- 6 (a) As the rod falls, the area enclosed by the rod and the frame decreases and the flux linkage decreases. By Faraday's law, an e.m.f. is induced. Since the circuit is closed, there is an induced current. There will be a magnetic force acting on the current-carrying rod as it is perpendicular to the magnetic field.

- (b) The downward motion of the rod causes change in the magnetic flux linkage in the circuit. According to Lenz's law, the magnetic force acting on the induced current must oppose this downward motion.

Hence the magnetic force is upwards.

Note: one can also work out first the direction of the current flow using Lenz's law, then the direction of the magnetic force using FLHR.

(c) magnetic force = $BIL = B \times \frac{\text{e.m.f.}}{R} \times L = B \times \frac{BLv}{R} \times L = \frac{B^2L^2v}{R}$

The highest speed is reached when the upward magnetic force and weight of the rod are equal in magnitude.

$$F_B = W$$

$$BIL = mg$$

$$\frac{B^2L^2v}{R} = mg$$

$$v = \frac{mgR}{B^2L^2} = \frac{0.15 \times 9.81 \times 2.0}{0.35^2 \times 2.0^2}$$

$$v = 6.01 \text{ m s}^{-1}$$

7 (a) $f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$

(b) $V_{XY,rms} = \frac{17.0}{\sqrt{2}} = 12.0 \text{ V}$

(c) $\frac{N_P}{N_S} = \frac{V_P}{V_S}$
 $\frac{4000}{200} = \frac{V_P}{12.02}$
 $V_P = 240 \text{ V}$

- (d) There is a potential difference across the resistor for only half of each cycle.

$$\begin{aligned} \text{mean power} &= \frac{1}{2} \times \frac{V_{rms}^2}{R} \\ &= \frac{1}{2} \times \frac{12.02^2}{4.8} \\ &= 15.1 \text{ W} \end{aligned}$$

- (e) For first half of a cycle, from 5 ms to 15 ms, the diode is forward biased and the p.d. across the load resistor is the same as V_{XY} .

For the second half of a cycle, from 15 ms to 25 ms, the diode is reverse biased and the p.d. across the load resistor is zero.

8 (a) $T = 4.70 - 1.20 = 3.50$ days

- (b) (i) Gravitational force between both stars provides the centripetal force required for the circular motion. (this statement is sufficient to get 1 mark for the explanation part)

$$\frac{GM_S M_P}{d^2} = M_P y \omega^2$$

$$M_S = d^2 y \left(\frac{2\pi}{T} \right)^2 \left(\frac{1}{G} \right)$$

$$M_S = \left(\frac{4\pi^2}{G} \right) \frac{y d^2}{T^2} \quad (\text{shown})$$

$$\frac{GM_S M_P}{d^2} = M_S x \omega^2$$

$$M_P = d^2 x \left(\frac{2\pi}{T} \right)^2 \left(\frac{1}{G} \right)$$

$$M_P = \left(\frac{4\pi^2}{G} \right) \frac{x d^2}{T^2} \quad (\text{shown})$$

(ii)

$$M_S + M_P = \left(\frac{4\pi^2}{G} \right) \frac{y d^2}{T^2} + \left(\frac{4\pi^2}{G} \right) \frac{x d^2}{T^2}$$

$$= \left(\frac{4\pi^2}{G} \right) \frac{d^2}{T^2} (y + x)$$

Since $d = x + y$

$$M_S + M_P = \left(\frac{4\pi^2}{G} \right) \frac{d^3}{T^2} \quad (\text{shown})$$

(c)

$$M_S + M_P = \left(\frac{4\pi^2}{G} \right) \frac{d^3}{T^2}$$

$$2.28 \times 10^{30} = \left(\frac{4\pi^2}{6.67 \times 10^{-11}} \right) \frac{d^3}{(3.50 \times 24 \times 3600)^2}$$

$$d = 7.06 \times 10^9 \text{ m}$$

(d)

$$v_P = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 7.062 \times 10^9}{3.50 \times 24 \times 3600}$$

$$= 1.467 \times 10^5 = 1.47 \times 10^5 \text{ m s}^{-1}$$

- (e) Since the center of mass of the system is stationary, the momentum of the system must be zero.

$$2.28 \times 10^{30} \times 84.3 + M_P \times (-1.467 \times 10^5) = 0$$

$$M_P = 1.31 \times 10^{27} \text{ kg}$$

- (f)
1. The plane of orbit of the planet must be such that the transit can be observed from Earth.
 2. The size of the planet must be large enough to block enough starlight.
 3. The time of transit across the star must be long enough to be detectable.
 4. Exoplanets with very long periods may not be detected as the chances of a transit happening while the stars are under observation is very small or may not happen in our lifetime.
 5. The brightness of the star must be large enough so that a change in the brightness can be detected.
 6. Rogue planets cannot be detected as they do not orbit around any stars.

(g) One light-year = speed of light \times one year = $3.0 \times 10^8 \times 365 \times 24 \times 3600$
 $= 9.46 \times 10^{15} = 9.5 \times 10^{15} \text{ m}$

- (h) (i) Rayleigh criterion states that two images are just resolved when the central maximum of the diffraction pattern of one image falls on the first minimum of the diffraction pattern of the other image.

(ii) $S = r\theta$
 $7.06 \times 10 = (9.46 \times 10^{15} \times 159) \times \theta$
 $\theta = 4.69 \times 10^{-9} \text{ rad}$

OR

$$\tan \frac{\theta}{2} = \frac{7.06 \times 10^9 / 2}{9.46 \times 10^{15} \times 159}$$

$$\theta = 4.69 \times 10^{-9} \text{ rad}$$

- (iii) Using Rayleigh's criterion,

$$\theta_{\min} = \frac{\lambda}{b} = \left(\frac{14 \times 10^{-6}}{6.5} \right)$$

$$= 2.154 \times 10^{-6} \text{ rad}$$

Since θ is smaller than θ_{\min} , the telescope will not be able to distinguish Osiris from the star.