NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS	**9758/01**
Paper 1	**10^{th} September 2018**
	3 Hours

Additional Materials: Answer Paper

List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

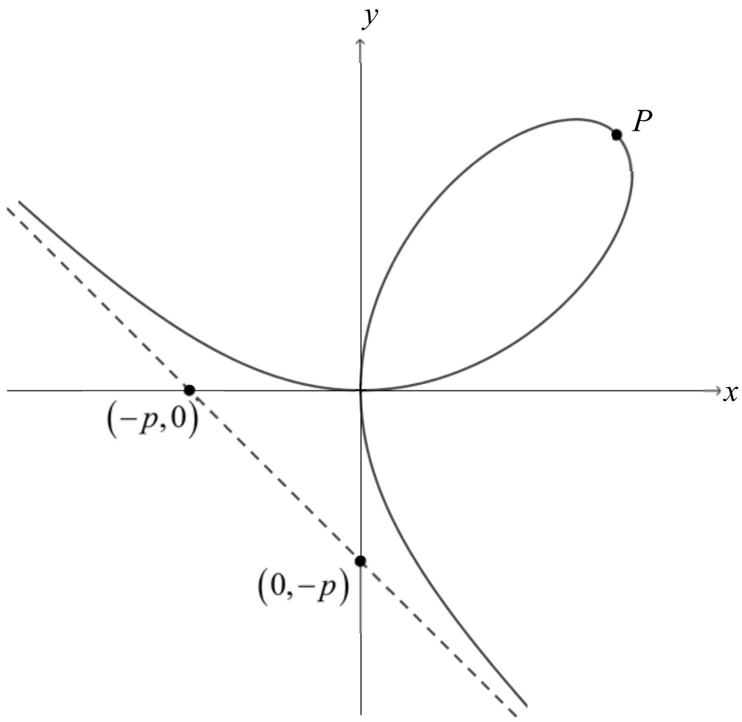
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A departmental store sells a pair of jeans at \$46.90, blouses at \$29.00 each and a pair of shoes at \$19.90. Items that are priced at more than \$25 are sold at a further discount of 15%. Betty bought twice as many blouses as jeans and she charged \$257.93 to her credit card for a total of 10 items. How many pairs of jeans, shoes and blouses did she buy? [4]

2 The diagram below shows the folium of Descartes curve with equation $x^3+y^3=3pxy$, where $p>0$, and the asymptote of the curve passes through the points $(-p,0)$ and $(0,-p)$.



Point P lies on the curve such that the tangent at P is parallel to the asymptote of the curve. Find the coordinates of point P in terms of p. [6]

3 A curve C has equation $y^2+14y+4x^2+16x+16xy+13=0$.

(i) If a real value of x is substituted into the equation, it becomes a quadratic equation in y. Given that there are two distinct values of y for this equation, show that $5x^2+8x+3>0$, and hence find the set of possible values of x. [5]

(ii) Find the coordinates of the points where C cuts the y-axis. State with a reason whether C is a graph of a function. [2]

4 A curve D has equation $y = x|x-a|, a > 0$.

(i) Describe a pair of transformations which transforms the graph of D on to the graph of $y = 2 - x|x+a|$. [2]

(ii) Sketch D, giving the coordinates of the axial intercepts and turning point in terms of a. [2]

(iii) On a separate diagram, sketch the curve $y = \dfrac{1}{x|x-a|}$, giving the coordinates of the turning point and the equations of the asymptotes in terms of a. [3]

(iv) State the range of values of k if $\dfrac{1}{x|x-a|} = k$ has exactly one solution. [1]

5 The line l passes through the points A and B with coordinates (5, 2, 4) and (4, –1, 3) respectively. The plane p has equation $4x + 7y + 5z = 24$.

(i) The point C lies on l such that the foot of perpendicular of C onto p has coordinates (3, 1, 1). Find the coordinates of C. [4]

Plane p_1 has equation $3x - 2y + \lambda z = \mu$.

(ii) What can be said about the values of λ and μ if l does not intersect p_1? [2]

(iii) Hence find the exact values of μ if the distance between p_1 and l is 2 units. [3]

6 The functions f and g are defined by

$$\mathrm{f} : x \mapsto \sin x \cos x, \quad x \in \mathbb{R}, -\frac{\pi}{2} < x \leq \frac{\pi}{2},$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \quad x \in \mathbb{R} \setminus \{0\}, -1 \leq x \leq 1.$$

(i) A function h is said to be odd if $\mathrm{h}(-x) = -\mathrm{h}(x)$ for all x in the domain of h. Show that g is odd and determine if f is odd. [2]

(ii) Explain why f does not have an inverse. If the domain of f is further restricted to $0 < x \leq a$, where $a \in \mathbb{R}$, the function f^{-1} will exist. State the largest possible exact value of a. [2]

Use the value of a in **(ii)** for the rest of the question.

(iii) Sketch the graphs of f, f^{-1} and ff^{-1} on the same diagram. [3]

(iv) State with reasons, whether the composite functions fg and gf exist. If the composite function exists, find the rule, domain and range. [3]

7 **(a)** Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors.

(i) By using scalar product, show that the vector $\mathbf{a}+\mathbf{b}$ is the bisector of angle AOB. [3]

(ii) If the area of the triangle AOB is $\dfrac{1}{\sqrt{10}}$ units^2, state the exact value of the sine of angle AOB. [1]

(b) Referred to the origin O, points C and D have position vectors $7\mathbf{i}-8\mathbf{j}+7\mathbf{k}$ and $4\mathbf{i}+7\mathbf{j}+4\mathbf{k}$ respectively.

(i) Using vector product, find the exact shortest distance of the line, passing through points C and D, from the origin. [4]

(ii) Find angle OCD. [2]

8 A curve C is represented by the parametric equations

$$x=t^2\left(t+6\right),\ y=t^2+t-6,\ \text{for}\ \ t<0.$$

(i) Find the equation(s) of the tangent to the curve C which is parallel to the y-axis. [3]

(ii) Sketch C, showing clearly the axial intercepts. [2]

(iii) Let R be the finite region bounded by C and the line $x=16$. Find the area of R. [5]

9 **(a)** The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n=\frac{6}{13}\left(1-\frac{1}{3^{3n}}\right).$$

(i) Given that the series $\sum u_r$ converges, find the smallest integer n for which S_n is within 10^{-8} of the sum to infinity. [3]

(ii) Find a formula for u_n in simplified form. [2]

(b) Using the formulae for $\sin(A\pm B)$, prove that

(i) $\sin(2r+1)\theta-\sin(2r-1)\theta=2\cos 2r\theta\sin\theta$. [1]

(ii) Hence find an expression for $\displaystyle\sum_{r=1}^{n}\sin^2 r\theta$, giving your answer in terms of $\cos(n+1)\theta$, $\sin n\theta$, $\sin\theta$ and n, where $\theta\neq k\pi, k\in$ ¢ . [5]

10 The two blades of a pair of scissors are fastened at the point A. The distance from A to the tip of the blade at point B is m cm. Let the angle formed by the line AB and the bottom edge of the blade BC be α radians and the angle between AB and AC be θ radians (Figure 1). A piece of paper resting at point C is cut. As the paper is being cut, the blades come closer to each other and the length of AC increases as shown in Figure 2.

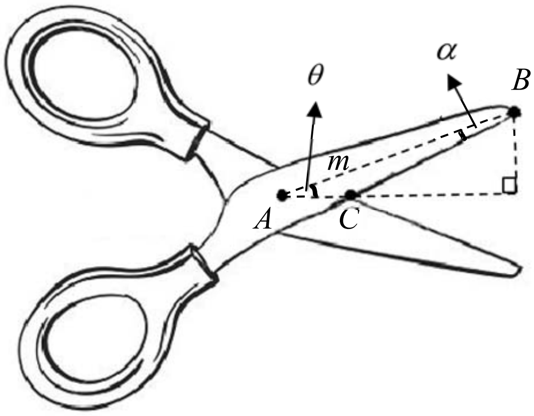


Figure 1:

Scissors when the blades are further apart

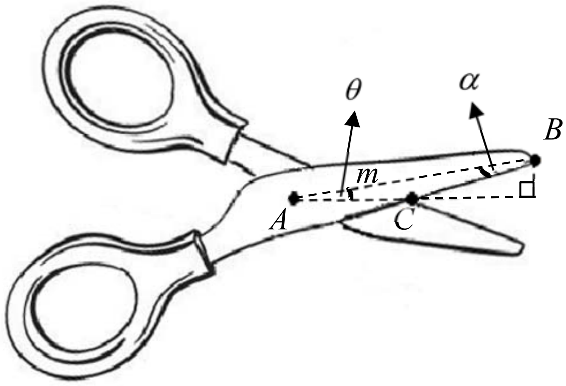


Figure 2:

Scissors when the blades are closer together

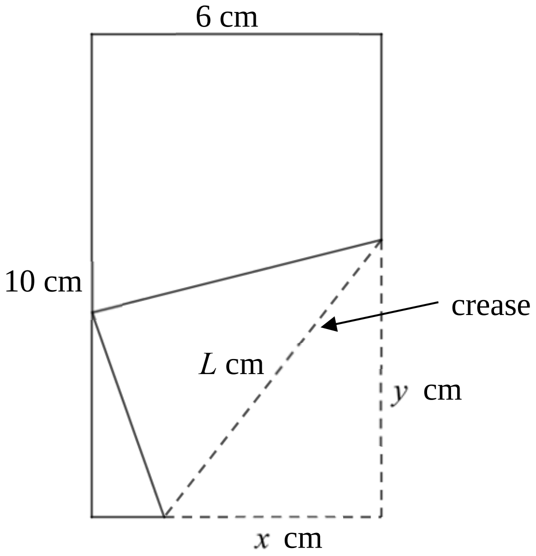
(i) By letting $AC = l$ cm, show that $l = \dfrac{m\sin\alpha}{\sin(\theta+\alpha)}$. [1]

(ii) Find $\dfrac{\mathrm{d}l}{\mathrm{d}\theta}$ in terms of m, θ and α. [1]

(iii) It is given that $m = 15$, $\alpha = \dfrac{\pi}{60}$ and θ is decreasing at a rate of $\dfrac{5\pi}{9}$ radians per second. Find the rate at which the paper is being cut at the instant when $\theta = \dfrac{\pi}{9}$. [3]

Question 10 continues on the next page

A triangle is cut out from a rectangular piece of paper measuring 10 cm by 6 cm using the scissors. To form this triangle, the right-hand corner is folded over so as to reach the left-most edge of the paper, forming a crease for the scissors to cut along, as shown in the diagram below. Let the length of the crease be L cm, the base of the triangle to be folded be x cm and the height of the triangle be y cm.



(iv) Show that $L=\sqrt{x^2+\dfrac{3x^2}{x-3}}$. [3]

(v) Using differentiation, find the minimum length of the resulting crease. [4]

11 A patient in the hospital is being administrated a certain drug through an intravenous (IV) drip at a constant rate of 30 mg per hour. The rate of loss of the drug from the patient's body is proportional to x, where x (in mg) is the amount of drug in the patient's body at time t (in hours). The amount of drug in the patient's body needs to reach 120 mg for the treatment to be effective.

(i) Explain why the rate of change of x needs to be positive. [1]

(ii) Initially, there are no traces of the drug in the patient's body, and after 4 hours, the amount of drug in the patient's body is 82.6 mg. Show that

$x=A\left(1-\mathrm{e}^{-\frac{t}{5}}\right)$, where A is a constant to be determined. [5]

(iii) Find the time needed for the amount of drug in the patient's body to reach 120 mg. [1]

A medical worker, who studied mathematical biology, proposed that the rate of change of the amount of drug in the patient's body actually satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{1}{\sqrt{2500-9t^2}}.$$

(iv) Find the general solution for the proposed differential equation, given that $x=0$ when $t=0$. [6]

NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS **9758/02**

Paper 2 **17th September 2018**

3 Hours

Additional Materials: Answer Paper

Graph paper

List of Formulae (MF26)

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Section A: Pure Mathematics [40 marks]

1 Sarah carried out a series of experiments which involved using decreasing amounts of a chemical. In the first experiment, she used 4 grams of the chemical and the amount of chemical used formed a geometric progression. In the 25th experiment, she used 1 gram of the chemical.

(i) Find the total amount of chemical she used in the first 25 experiments. [4]

(ii) Show that the theoretical maximum total amount of chemical she would use will not exceed 71.3 grams. [1]

Robert carried out the same series of experiments. He also used decreasing amounts of the same chemical but the amount of chemical used formed an arithmetic progression with common difference *d*. If the total amount of chemical that both Sarah and Robert used for the first 25 experiments were the same, and the amount of chemical Robert used for the 25th experiment was still 1 gram, find the value of *d* and the amount of chemical he used for the first experiment. [4]

2 **(a)** **(i)** Evaluate $\int \frac{2x-4}{x^2-2x+4}\,\mathrm{d}x$. [3]

(ii) Without the use of a graphic calculator, evaluate $\int_1^4 \frac{|2x-4|}{x^2-2x+4}\,\mathrm{d}x$, leaving your answer in logarithmic form. [4]

(b) Given that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\cos^2 x}\right)=\frac{2\sin x}{\cos^3 x}$, evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^3 x}\,\mathrm{d}x$, leaving your answer in exact form. [3]

3 **Do not use a calculator in answering this question.**

(i) The equation $z^3+4z^2+8z+8=0$ has a root $z=-2$. Find the other two roots z_1 and z_2 where $-\pi<\arg z_2<\arg z_1\le\pi$. [2]

(ii) Find the modulus and argument of *w*, where $w=\frac{z_1}{z_2}$. [3]

(iii) Find the set of positive integers *n* for which w^n is real, and show that, for these values of *n*, w^n is 1. [3]

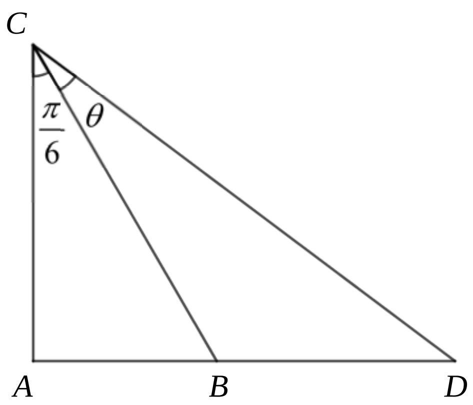
(iv) Express $w^{100}-\left(w^*\right)^{100}$ in the form *k*i, giving the exact value of *k* in non-trigonometrical form. [2]

4 **(a)** Hyperbolic functions are commonly used to study fluid dynamics and electromagnetic theory, where integrals with a $\sqrt{x^2+1}$ term occurs. The inverse function of the hyperbolic function $\sinh x$ is $\sinh^{-1} x$. It is given that $y = \sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$.

(i) Show that $e^y \dfrac{d^2y}{dx^2} + e^y \left(\dfrac{dy}{dx}\right)^2 = \left(x^2+1\right)^{-\frac{1}{2}} - x^2\left(x^2+1\right)^{-\frac{3}{2}}$. [2]

(ii) Hence, by further differentiation, find the first two non-zero terms of the Maclaurin's series for $\sinh^{-1} x$ in ascending powers of x. [4]

(b) The diagram shows a right angled triangle ABC with angle $ACB = \dfrac{\pi}{6}$ radians. D lies on AB produced such angle $BCD = \theta$ radians.



(i) Show that $\dfrac{AB}{AD} = \dfrac{1}{\sqrt{3}\tan\left(\dfrac{\pi}{6}+\theta\right)}$. [1]

(ii) Given that θ is sufficiently small for θ^3 and higher powers of θ to be neglected, show that

$$\frac{AB}{AD} \approx 1 + a\theta + b\theta^2,$$

where a and b are exact constants to be determined. [4]

Section B: Statistics [60 marks]

5 This question is about arrangements of all ten letters in the word EXCELLENCE.

(i) Find the number of arrangements in which the letters are **not** in alphabetical order. [2]

The letters are now arranged in a circle.

(ii) Find the number of arrangements that can be made with all E's together and no other adjacent letters the same. [4]

6 A game is played using a fair six-sided die, a pawn and a simple board as shown below.

S	1	2	3	4	5	**E**

Initially, the pawn is placed on square **S**. The game is played by throwing the die and moving the pawn in the following manner:

S 1 2 3 4 5 **E** 5 4 3 2 1 2 3 4 5 **E**………

Thus, for example, if the first and second throw of the die gives a "5" and "4" respectively, the final position of the pawn will be on square "3".

The game will stop when the pawn stops at square **E**.

Let X be the random variable denoting the number of throws of the die required to move the pawn such that it stops at square **E**.

(i) Show $\mathrm{P}(X=2)=\dfrac{5}{36}$. [1]

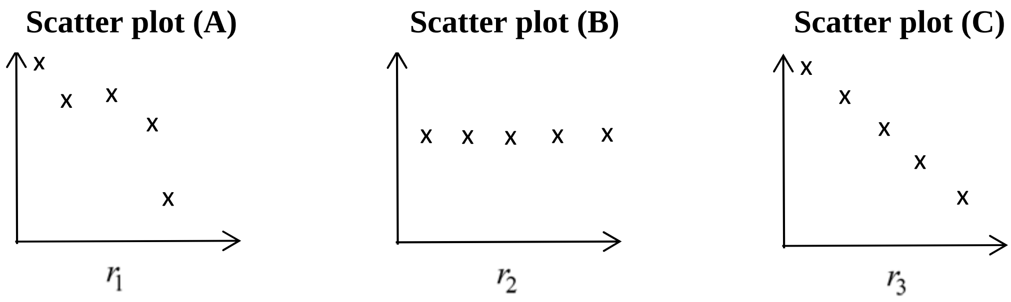
(ii) Find the probability that more than two throws of the die are needed for the pawn to stop at square **E** given that the first throw of the die gives an even number. [3]

It is now given that for each game, a player has a maximum of 3 throws of the die and a special prize is given to any player who uses not more than two throws for the pawn to stop at square **E**.

(iii) Find the probability of a player winning a special prize in at least three but not more than eight games out of ten games. [3]

(iv) Find the least number of games needed so that the probability of winning at least a special prize is at least 0.998. [3]

7 **(a)** The following three scatter plots have product moment correlation coefficients as r_1, r_2 and r_3 respectively.



State, with justifications, an inequality that relates r_1, r_2 and r_3 that best describes the correlations associated with the scatter plots (A), (B) and (C). [2]

(b) A motoring magazine published the following data on the engine capacity measured in cubic centimetres (x) and the prices in thousand dollars (y) of ten new car models.

Car Model	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*
x	1000	1270	1750	2230	1990	600	650	1500	1450	1650
y	139	142	151.6	169.8	169.3	121.9	121.9	141.6	130.5	161.5

(i) Plot a scatter diagram on graph paper for these values using appropriate scales for the x- and y-axes. On your diagram, indicate the car model for each point. [2]

(ii) Find the equation of the regression line of y on x. [1]

(iii) Calculate the product moment correlation coefficient and comment on the relationship between x and y. [2]

One particular consumer regards low price and large engine capacity as the two most important factors in choosing a car. By drawing the regression line on the scatter diagram, suggest one model which will give particularly good value for money for this consumer. Which three models would you advise the consumer not to buy? Justify your answers. [3]

8 The masses of manufactured links of a chain are normally distributed with mean 800 grams and standard deviation 20 grams.

(i) Find the probability that the mass of a randomly chosen link is more than 805 grams. [1]

To close the gate opening in a link, a locking sleeve is attached to it and it increases the mass of a link by 10%.

(ii) By writing down the distribution of the masses of links with locking sleeves, find the probability that the mass of a randomly chosen link with a locking sleeve is between 865.35 and 895.5 grams. [3]

Hooks with mean masses 750 grams are manufactured to attach to the links. The masses of hooks are normally distributed such that 15% of them have mass less than 735.6 grams.

(iii) Find the standard deviation of the masses of hooks. [2]

Five independent links with locking sleeves and a hook are packed in a wooden box with a fixed mass of 1 kilogram. The probability that the mean mass of n such wooden boxes with its contents more than 6190 grams exceeds 0.013.

(iv) Find the greatest value of n, stating the parameters of any distribution that you use. [4]

Question 9 is printed on the next page

9 During a Mathematics lesson, Ms Kim wanted her pupils to build a model. She carried two indistinguishable boxes of bricks into class. The bricks are identical and indistinguishable except for colours. The number of coloured bricks found in each box is as follows.

Colour of Bricks	Box 1	Box 2
Blue	2	4
Red	3	3
Yellow	5	3

A pupil, Donald, has to draw two bricks from these boxes randomly to build a model. He draws a brick randomly from one of the boxes. The brick is not replaced. He then draws a second brick randomly from one of the boxes.

(i) Show the probability that Donald does not draw any yellow brick is $\frac{25}{72}$. [2]

(ii) Find the probability that Donald draws two yellow bricks. [3]

(iii) Tabulate the probability distribution table for the number of yellow bricks drawn and find the expected number of yellow bricks drawn by Donald. [2]

Ms Kim decided to combine the two boxes of bricks into a single box.

(iv) Donald draws 5 bricks **with replacement** from the single box. Find the probability that he draws less than two yellow bricks. [2]

(v) Donald wanted to build the model with a yellow brick. He randomly draws a brick from the single box one at a time **with replacement** until he gets a yellow brick. Find the probability that it will not take him more than nine draws to get a yellow brick. [3]

Question 10 is printed on the next page

10 The distances thrown, x metres, by a discus thrower have been recorded over a long period. The results for a random sample of 60 throws are summarised by $\sum(x-65)=120$ and $\sum(x-65)^2=3810$.

(i) State what it means for a sample to be random in this context. [1]

(ii) Calculate the unbiased estimates of the population mean and variance of the distances thrown by the discus thrower. [2]

An analysis of all the data collected over the long period gives a mean of 68 metres and a standard deviation of 7.5 metres.

The Sports Council wishes to study if a new technique will improve the mean distance thrown by this thrower. Using this new technique, the results of a random sample of n throws give a mean of $\bar{x}$ metres. A test is carried out, at the 2% level of significance, to determine whether the mean distance thrown by the thrower has improved. You may assume that the distances thrown by the thrower follows a normal distribution.

(iii) State appropriate hypotheses for the test, defining any symbols you use. [1]

(iv) Given that n = 30, find the range of values of $\bar{x}$ for which the result of the test would be to reject the null hypothesis. [3]

(v) It is given instead that $\bar{x}=70.1$ and the result of the test is that the null hypothesis is not rejected. Obtain an inequality involving n, and hence find the set of values that n can take. [3]

A sports magazine published a claim about the mean distance thrown using this new technique. It is assumed that the distances of the discuses thrown follow a normal distribution with known population variance.

Two discus thrower coaches, Ang and Tan, decide to each conduct a hypothesis test, at the 5% level of significance, to determine whether the magazine has **overstated** the mean distance thrown using this new technique. Ang obtained a sample with mean $\bar{x}$ and concluded that there is sufficient evidence, at the 5% level of significance, that the magazine has overstated the mean distance thrown.

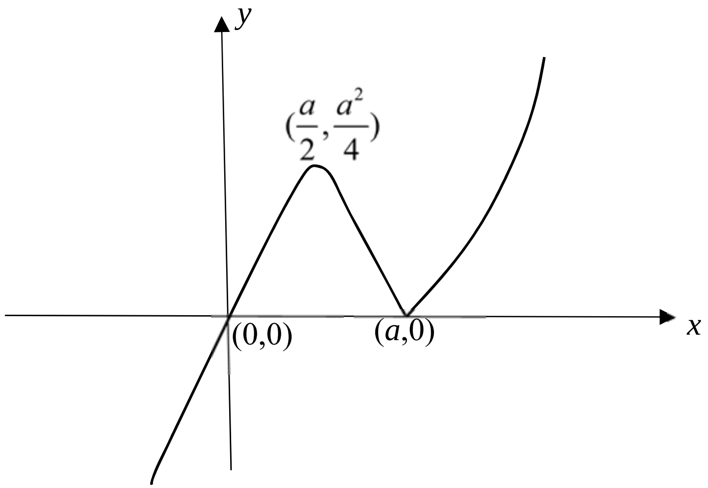
Tan took a different sample with sample size **four times** that of Ang's and obtained the same sample mean.

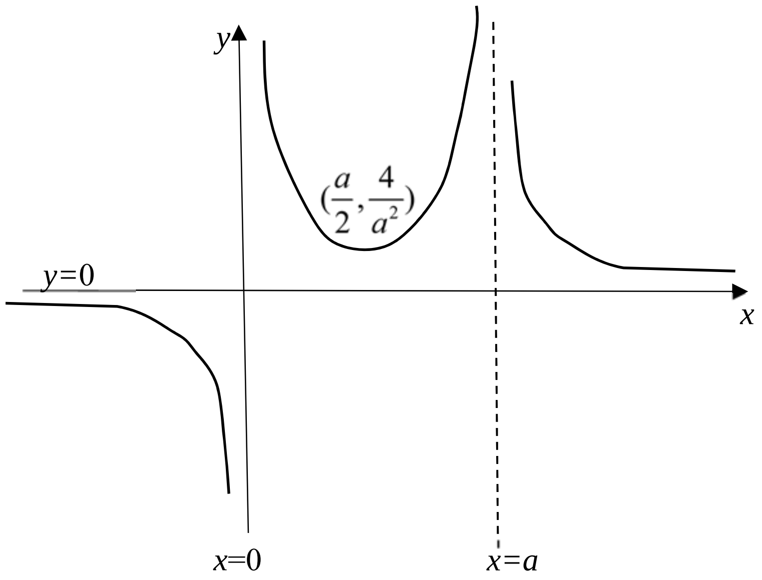
Will Tan's test yield the same conclusion? Justify your answer. [2]

Qn	Suggested Answers			Guidance
1	Let x, y and z be the no. of jeans, blouses and pairs of shoes Betty bought. $46.90\times0.85x + 29\times0.85y + 19.9z = 257.93$ $x + y + z = 10$ $2x - y = 0$ Using GC, $x = 2$, $y = 4$, $z = 4$ Hence, Betty bought 2 pairs of jeans, 4 blouses and 4 pairs of shoes.			

Qn	Suggested Answers			Guidance
2	$x^3+y^3=3pxy$ Implicit differentiation w.r.t. x: $3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3px\frac{\mathrm{d}y}{\mathrm{d}x}+3py$ Gradient of asymptote $=-1$ Let $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$. $3x^2-3y^2=-3px+3py$ $x^2-y^2=-px+py$ $(x+y)(x-y)=-p(x-y)$ $(x-y)(x+y+p)=0$ $x=y$ or $x+y=-p$ (rejected since $p>0$ and $x>0, y>0$) Since $x=y$, $x^3+x^3=3px^2$ $x^2(2x-3p)=0$ $x=0$ (rejected) or $x=\frac{3p}{2}$ Hence, P has coordinates $\left(\frac{3p}{2},\frac{3p}{2}\right)$.			

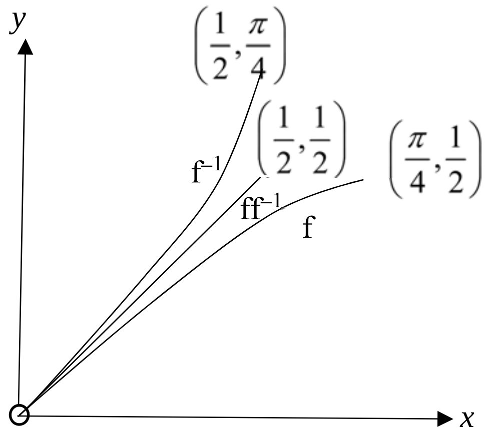
Qn	Suggested Answers			Guidance
3(i)	Let $x = k$ $y^2 + 14y + 4k^2 + 16k + 16ky + 13 = 0$ $y^2 + y(14 + 16k) + (4k^2 + 16k + 13) = 0$ For 2 distinct values of y, $(14 + 16k)^2 - 4(4k^2 + 16k + 13) > 0$ $196 + 448k + 256k^2 - 16k^2 - 64k - 52 > 0$ $240k^2 + 384k + 144 > 0$ Since $x = k$, $5x^2 + 8x + 3 > 0$ $(5x + 3)(x + 1) > 0$ $x < -1$ or $x > -\frac{3}{5}$ Therefore the set of x is $\{x : x \in \mathbb{R}, x < -1 \text{ or } x > -\frac{3}{5}\}$			
3(ii)	When $x = 0$ $y^2 + 14y + 13 = 0$ $(y + 13)(y + 1) = 0 \Rightarrow y = -1$ or -13 The coordinates are $(0, -1)$ and $(0, -13)$ Curve C is not graph of a function as the vertical line $x = 0$ cuts the curve at 2 points.			

Qn	Suggested Answers			Guidance
4(i)	Reflection in the y-axis (Replace x with $-x$) $y=-x\left\|-x-a\right\|$ $=-x\left\|x+a\right\|$ Translation of 2 units in the direction of y-axis $y=2-x\left\|x+a\right\|$.			
4(ii)	 			

4(iii)				
4(iv)	$0<k<\dfrac{4}{a^2}$ or $k<0$			

Qn	Suggested Answers			Guidance
5(i)	$l:\mathbf{r}=\begin{pmatrix}5\\2\\4\end{pmatrix}+\beta\begin{pmatrix}1\\3\\1\end{pmatrix},\beta\in\mathbb{R}$ Line perpendicular to plane passing through (3, 1, 1) is $\mathbf{r}=\begin{pmatrix}3\\1\\1\end{pmatrix}+\alpha\begin{pmatrix}4\\7\\5\end{pmatrix}$ The 2 lines will intersect at *C*. $\begin{pmatrix}5+\beta\\2+3\beta\\4+\beta\end{pmatrix}=\begin{pmatrix}3+4\alpha\\1+7\alpha\\1+5\alpha\end{pmatrix}$ $\beta-4\alpha=-2$ $3\beta-7\alpha=-1$ $\beta=2,\alpha=1$ Point *C* has coordinates (7, 8, 6)			
5(ii)	Since l does not intersect p_1 $\begin{pmatrix}1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\\lambda\end{pmatrix}=0$ $3-6+\lambda=0\Rightarrow\lambda=3$			

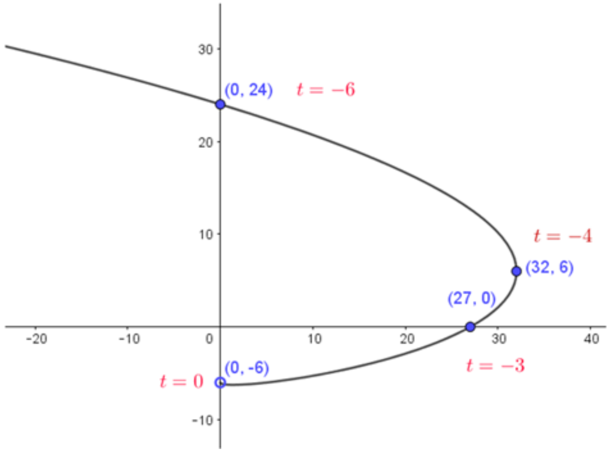
	$\begin{pmatrix}5\\2\\4\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\3\end{pmatrix}\neq\mu$ or $\begin{pmatrix}4\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\3\end{pmatrix}\neq\mu$ $\mu\neq 23$			
5(iii)	Using $\lambda=3$ $\dfrac{\left\lvert\begin{pmatrix}5\\2\\4\end{pmatrix}\cdot\begin{pmatrix}3\\-2\\3\end{pmatrix}-\mu\right\rvert}{\left\lvert\begin{pmatrix}3\\-2\\3\end{pmatrix}\right\rvert}=2$ $\lvert 23-\mu\rvert=2\sqrt{22}$ $23-\mu=2\sqrt{22}$ or $23-\mu=-2\sqrt{22}$ $\mu=23+2\sqrt{22}$ or $23-2\sqrt{22}$			

Qn	Suggested Answers			Guidance
6(i)	Since $g(-x)=\frac{1}{(-x)}=-\frac{1}{x}=-g(x)$, therefore g is odd. Although $f(-x)=\sin(-x)\cos(-x)=-\sin x\cos x=-f(x)$, but $f\left(-\frac{\pi}{2}\right)=-f\left(\frac{\pi}{2}\right)=0$ and $f\left(-\frac{\pi}{2}\right)$ is undefined. Therefore, f is not odd.			
6(ii)	Since $f(0)=f\left(\frac{\pi}{2}\right)=0$, there are two values of x that give the same value of f(x), which means f is not one-one. OR: The line y = 0 cuts the graph y = f(x) at 2 points. Therefore f has no inverse. $a=\frac{\pi}{4}$			
6(iii)				

6(iv)	Domain of f = $\left(0,\frac{\pi}{4}\right]$ and Domain of g = $\left[-1,0\right)\cup\left(0,1\right]$ Range of f = $\left(0,\frac{1}{2}\right]$ and range of g = $\left(-\infty,-1\right]\cup[\left[1,\infty\right)$ Since range of g is not a subset of domain of f, fg does not exist. Range of f is a subset of domain of g, therefore gf exists. $\mathrm{gf}(x)=\mathrm{g}(\sin x\cos x)=\dfrac{1}{\sin x\cos x}=2\mathrm{cosec}2x$ Domain of gf is $\left(0,\frac{\pi}{4}\right]$ and range of gf is $[2,\infty)$.			

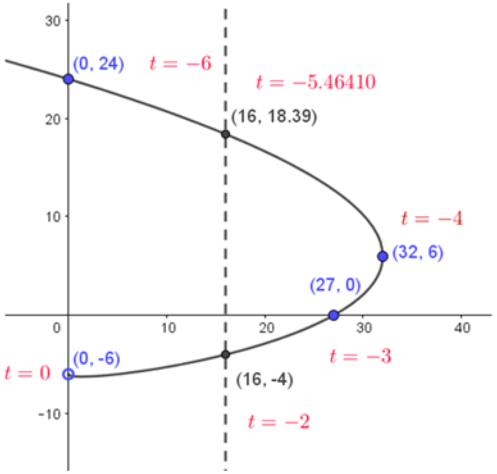
Qn	Suggested Answers			Guidance
7(a)(i)	Let the angle between **a** + **b** and **a** be α $(\mathbf{a}+\mathbf{b})\cdot\mathbf{a} = \lvert\mathbf{a}+\mathbf{b}\rvert\lvert\mathbf{a}\rvert\cos\alpha$ $\cos\alpha = \dfrac{(\lvert\mathbf{a}\rvert^2+\mathbf{b}\cdot\mathbf{a})}{\lvert\mathbf{a}+\mathbf{b}\rvert} = \dfrac{(1+\mathbf{b}\cdot\mathbf{a})}{\lvert\mathbf{a}+\mathbf{b}\rvert}$ Let the angle between $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}$ be β $(\mathbf{a}+\mathbf{b})\cdot\mathbf{b} = \lvert\mathbf{a}+\mathbf{b}\rvert\lvert\mathbf{b}\rvert\cos\beta$ $\cos\beta = \dfrac{(\mathbf{a}\cdot\mathbf{b}+\lvert\mathbf{b}\rvert^2)}{\lvert\mathbf{a}+\mathbf{b}\rvert} = \dfrac{(\mathbf{a}\cdot\mathbf{b}+1)}{\lvert\mathbf{a}+\mathbf{b}\rvert}$ Since $\cos\alpha = \cos\beta$, $\mathbf{a}+\mathbf{b}$ is the angle bisector of AOB			
7(a)(ii)	$\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin AOB = \sin AOB$ Since area of triangle $= \dfrac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert = \dfrac{1}{\sqrt{10}}$ $\therefore \sin AOB = \dfrac{2}{\sqrt{10}}$			
7(b)(i)	$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ $= \begin{pmatrix}4\\7\\4\end{pmatrix} - \begin{pmatrix}7\\-8\\7\end{pmatrix} = \begin{pmatrix}-3\\15\\-3\end{pmatrix}$			

	Perpendicular distance of O from line $= \dfrac{\left\| \begin{pmatrix} -3 \\ 15 \\ -3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} -3 \\ 15 \\ -3 \end{pmatrix} \right\|}$ $= \dfrac{\left\| \begin{pmatrix} 81 \\ 0 \\ -81 \end{pmatrix} \right\|}{\sqrt{9+225+9}}$ $= \dfrac{81\sqrt{2}}{9\sqrt{3}} = 3\sqrt{6}$			
7(b)(ii)	$\overrightarrow{CO} \cdot \overrightarrow{CD} = \left\|\overrightarrow{CO}\right\| \left\|\overrightarrow{CD}\right\| \cos\theta$ $\begin{pmatrix} -7 \\ 8 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 15 \\ -3 \end{pmatrix} = \sqrt{7^2+8^2+7^2}\sqrt{3^2+15^2+3^2}\cos\theta$ $21+120+21 = \sqrt{162}\sqrt{243}\cos\theta$ $\theta = 35.3°$ or 0.615 radians Alternatively, $\left\|\overrightarrow{OC}\right\| = \sqrt{49+64+49} = \sqrt{162}$ Let F be foot of perpendicular from O to line CD. Then $OF = 3\sqrt{6}$ $\theta = \sin^{-1}\left(\dfrac{3\sqrt{6}}{\sqrt{162}}\right) = 35.3^{\circ}$			

Qn	Suggested Answers			Guidance
8(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}/\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2t+1}{3t^2+12t}$			
	For tangent parallel to the y-axis, $3t(t+4)=0$			
	$t=-4,0$ (reject $t=0$)			
	Equation of tangent is $x=32$			
8(ii)	When $x=0$, $t=0,-6$ corresponding to (0,–6), (0,24)			
	When $y=0$, $t=2,-3$ (reject $t=2$) corresponding to (27,0)			
				
8(iii)	When $x=16$			
	$t^2(t+6)=16$			

$t = -5.46410, -2,\ 1.46410$ (reject $t = 1.46410$ since $t < 0$)

The line $x = 16$ cuts C at $(16, -4)$ and $(16.0, 18.3923)$



Required area $= \displaystyle\int_{-2}^{-5.46410} \left(t^3 + 6t^2\right)(2t+1)\,\mathrm{d}t - 16\left(18.39 - (-4)\right)$

$= 241.058$; 241 (3 s.f.)

Alternatively,

Required area

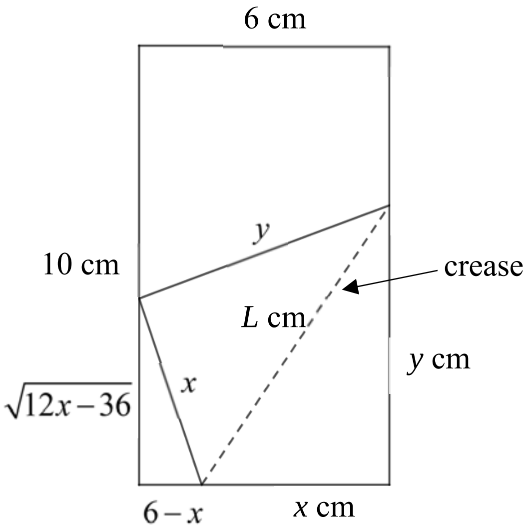
$= \displaystyle\int_{-5.46410}^{-4} \left(t^2 + t - 6\right)\left(3t^2 + 12t\right)\mathrm{d}t - \int_{-2}^{-4} \left(t^2 + t - 6\right)\left(3t^2 + 12t\right)\mathrm{d}t$

$= 241.02$; 241 (3 s.f.)

Qn	Suggested Answers			Guidance
9(a)(i)	$S_\infty = \frac{6}{13}$ $\lvert S_n - S_\infty \rvert < 10^{-8}$ $\left\lvert \frac{6}{13} \times \frac{1}{3^{3n}} \right\rvert < 10^{-8}$ Using GC, smallest $n = 6$			
9(a)(ii)	$u_n = S_n - S_{n-1}$ $= \frac{6}{13}\left(1-\frac{1}{3^{3n}}\right) - \frac{6}{13}\left(1-\frac{1}{3^{3n-3}}\right)$ $= \frac{6}{13}\left(\frac{1}{3^{3n-3}} - \frac{1}{3^{3n}}\right)$ $= \frac{6}{13} \times \frac{1}{3^{3n}}\left(\frac{1}{3^{-3}} - 1\right)$ $= \frac{6}{13} \times \frac{1}{3^{3n}}(26)$ $= \frac{12}{3^{3n}} = \frac{4}{3^{3n-1}}$			
9(b)(i)	$\sin(2r+1)\theta - \sin(2r-1)\theta$ $= \sin 2r\theta \cos\theta + \cos 2r\theta \sin\theta - \sin 2r\theta \cos\theta + \cos 2r\theta \sin\theta$ $= 2\cos 2r\theta \sin\theta$ (shown)			
9(b)(ii)	$\sum_{r=1}^{n} \sin^2 r\theta$			

	$=\frac{1}{2}\sum_{r=1}^{n}\left(1-\cos 2r\theta\right)$ $=\frac{1}{2}\sum_{r=1}^{n}1-\frac{1}{2}\sum_{r=1}^{n}\cos 2r\theta$ $=\frac{1}{2}n-\frac{1}{4\sin\theta}\sum_{r=1}^{n}\left[\sin(2r+1)\theta-\sin(2r-1)\theta\right], \theta \neq k\pi$ $=\frac{1}{2}n-\frac{1}{4\sin\theta}[\sin 3\theta-\sin\theta$ $+\sin 5\theta-\sin 3\theta$ $+\sin 7\theta-\sin 5\theta$ . . $+\sin\left(2n+1\right)\theta-\sin\left(2n-1\right)\theta]$ $=\frac{1}{2}n-\frac{1}{4\sin\theta}\left[\sin\left(2n+1\right)\theta-\sin\theta\right]$ $=\frac{1}{2}n-\frac{1}{4\sin\theta}\left[2\cos\left(n+1\right)\theta\sin n\theta\right]$ $=\frac{1}{2}n-\frac{1}{2\sin\theta}\left[\cos\left(n+1\right)\theta\sin n\theta\right]$			

Qn	Suggested Answers			Guidance
10(i)	$\dfrac{m}{\sin(\pi-(\theta+\alpha))}=\dfrac{l}{\sin\alpha}$ $l=\dfrac{m\sin\alpha}{\sin(\pi-(\theta+\alpha))}$ $=\dfrac{m\sin\alpha}{\sin(\theta+\alpha)}\text{(shown)}$			
10(ii)	$l=\dfrac{m\sin\alpha}{\sin(\theta+\alpha)}$ $\dfrac{\mathrm{d}l}{\mathrm{d}\theta}=\dfrac{-(m\sin\alpha)\cos(\theta+\alpha)}{\sin^2(\theta+\alpha)}$ Alternatively, $\dfrac{\mathrm{d}l}{\mathrm{d}\theta}=-(m\sin\alpha)\operatorname{cosec}(\theta+\alpha)\cot(\theta+\alpha)$			
10(iii)	Using chain rule, $\dfrac{\mathrm{d}l}{\mathrm{d}t}=\dfrac{\mathrm{d}l}{\mathrm{d}\theta}\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$ $=\dfrac{-15\left(\sin\dfrac{\pi}{60}\right)\cos\left(\dfrac{\pi}{9}+\dfrac{\pi}{60}\right)}{\sin^2\left(\dfrac{\pi}{9}+\dfrac{\pi}{60}\right)}\left(-\dfrac{5\pi}{9}\right)$ $=8.2611\text{ cm / sec}$ $\approx 8.26\text{ cm / sec}$			
10(iv)	$L=\sqrt{x^2+y^2}$ $\sqrt{x^2-(6-x)^2}=\sqrt{x^2-36+12x-x^2}=\sqrt{12x-36}$			



Area of rectangle = $6\times10=60\ \text{cm}^2$

$$60=\frac{1}{2}(6-x)\left(\sqrt{12x-36}\right)+2\left(\frac{1}{2}\right)(xy)$$

$$+\frac{1}{2}(6)\left(10-\sqrt{12x-36}+10-y\right)$$

$$=\frac{1}{2}(6-x)\left(\sqrt{12x-36}\right)+xy+60-3\sqrt{12x-36}-3y$$

$$0=\frac{1}{2}(6-x)\left(\sqrt{12x-36}\right)+xy-3\sqrt{12x-36}-3y$$

$$3y-xy=\left(3-\frac{x}{2}\right)\left(\sqrt{12x-36}\right)-3\sqrt{12x-36}$$

$$y=\frac{-\frac{x}{2}\left(\sqrt{12x-36}\right)}{3-x}$$

	$= \dfrac{x\left(\sqrt{12x-36}\right)}{2x-6}$ $L = \sqrt{x^2+y^2}$ $= \sqrt{x^2 + \left(\dfrac{x\left(\sqrt{12x-36}\right)}{2x-6}\right)^2}$ $= \sqrt{x^2 + \left(\dfrac{x^2(12x-36)}{(2x-6)^2}\right)}$ $= \sqrt{x^2 + \left(\dfrac{12x^2(x-3)}{4(x-3)^2}\right)} = \sqrt{x^2 + \dfrac{3x^2}{x-3}}$			
10(v)	$\dfrac{\mathrm{d}L}{\mathrm{d}x} = \dfrac{1}{2}\left(x^2+\dfrac{3x^2}{x-3}\right)^{-\frac{1}{2}}\left(2x+\dfrac{(x-3)(6x)-(3x^2)}{(x-3)^2}\right)$ $= \dfrac{2x+\dfrac{3x^2-18x}{(x-3)^2}}{2\sqrt{x^2+\dfrac{3x^2}{x-3}}} = 0$ $2x+\dfrac{3x^2-18x}{(x-3)^2} = 0$ $x\left(2+\dfrac{3x-18}{(x-3)^2}\right) = 0$			

	$2+\frac{3x-18}{(x-3)^2}=0$ or $x=0$ (rejected) $2+\frac{3x-18}{(x-3)^2}=0$ $\frac{2(x-3)^2+3x-18}{(x-3)^2}=0$ $2(x^2-6x+9)+3x-18=0$ $2x^2-9x=0$ $x=\frac{9}{2}$ or $x=0$ (rejected) By first derivative test, <table><tr><td>x</td><td>4.45</td><td>$\frac{9}{2}$</td><td>4.55</td></tr><tr><td>$\frac{\mathrm{d}L}{\mathrm{d}x}$</td><td>$-0.0604$</td><td>0</td><td>0.0553</td></tr><tr><td>Slope</td><td>\</td><td>–</td><td>/</td></tr></table> $x=\frac{9}{2}$ gives a minimum L. Hence, $\text{Min.}L=\sqrt{\left(\frac{9}{2}\right)^2+\frac{3\left(\frac{9}{2}\right)^2}{\left(\frac{9}{2}\right)-3}}=7.79\text{cm}$			

Qn	Suggested Answers			Guidance
11(i)	If the rate of change of x is not positive, it is not possible for the amount of drug to increase to reach 120 mg in the body for the treatment to be effective.			
11(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 30 - kx$, where $k > 0$ $\Rightarrow \int \frac{1}{30-kx}\,dx = \int dt$ $\Rightarrow -\frac{1}{k}\ln(30-kx) = t + C$, since $\frac{\mathrm{d}x}{\mathrm{d}t} = 30 - kx > 0$ $\Rightarrow \ln(30-kx) = -kt - kC$ $\Rightarrow (30-kx) = e^{-kt-kC} = e^{-kC}e^{-kt} = Be^{-kt}$, $B > 0$ When $x = 0, t = 0$, thus $B = 30$ When $t = 4, x = 82.6$, $30 - k(82.6) = 30e^{-4k}$, Solving using GC. $k = 0.200$ $x = 150\left(1 - \mathrm{e}^{-\frac{t}{5}}\right)$, where $A = 150$			
11(iii)	When $x = 120, k = \frac{1}{5}$, $120 = 150\left(1 - e^{-\frac{t}{5}}\right)$ $\Rightarrow t = 8.05$ hrs			
11(iv)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{1}{\sqrt{2500 - 9t^2}}$			

	$\frac{\mathrm{d}x}{\mathrm{d}t}=\int \frac{1}{\sqrt{2500-9t^2}}\,\mathrm{d}t=\frac{1}{3}\sin^{-1}\left(\frac{3t}{50}\right)+\mathrm{C}$ $x=\int \left[\frac{1}{3}\sin^{-1}\left(\frac{3t}{50}\right)+\mathrm{C}\right]\mathrm{d}t$ $=\frac{1}{3}\left[t\sin^{-1}\left(\frac{3t}{50}\right)-\int t\cdot\frac{1}{\sqrt{1-\left(\frac{3t}{50}\right)^2}}\left(\frac{3}{50}\right)\mathrm{d}t\right]+\mathrm{C}t+\mathrm{D}$ $=\frac{1}{3}\left[t\sin^{-1}\left(\frac{3t}{50}\right)-\int t\cdot\frac{3}{\sqrt{50^2-(3t)^2}}\,\mathrm{d}t\right]+\mathrm{C}t+\mathrm{D}$ $=\frac{1}{3}\left[t\sin^{-1}\left(\frac{3t}{50}\right)+\frac{1}{6}\int \frac{-18t}{\sqrt{2500-9t^2}}\,\mathrm{d}t\right]+\mathrm{C}t+\mathrm{D}$ $=\frac{1}{3}\left[t\sin^{-1}\left(\frac{3t}{50}\right)+\frac{1}{3}\sqrt{2500-9t^2}\right]+\mathrm{C}t+\mathrm{D}$ $=\frac{1}{3}t\sin^{-1}\left(\frac{3t}{50}\right)+\frac{1}{9}\sqrt{2500-9t^2}+\mathrm{C}t+\mathrm{D}$ When $t=0,\ x=0$, $\mathrm{D}=-\frac{50}{9}$ $x=\frac{1}{3}t\sin^{-1}\left(\frac{3t}{50}\right)+\frac{1}{9}\sqrt{2500-9t^2}+\mathrm{C}t-\frac{50}{9}$			

Qn	**Suggested Answers**			
1(i)	$u_1 = a = 4$, $u_{25} = 1 \Rightarrow ar^{24} = 1$ $\Rightarrow r = \sqrt[24]{0.25}$ or 0.94387 $S_{25} = \dfrac{a\left(1-r^{25}\right)}{1-r} = \dfrac{4\left[1-\left(\sqrt[24]{0.25}\right)^{25}\right]}{1-\sqrt[24]{0.25}} \approx 54.451 \approx 54.5$ or $S_{25} = \dfrac{a\left(1-r^{25}\right)}{1-r} = \dfrac{4\left[1-\left(0.94387\right)^{25}\right]}{1-0.94387} \approx 54.449 \approx 54.4$			
1(ii)	$S_\infty = \dfrac{a}{1-r} = \dfrac{4}{1-\sqrt[24]{0.25}} \approx 71.269 < 71.3$ or $S_\infty = \dfrac{a}{1-r} = \dfrac{4}{1-0.94387} \approx 71.263 < 71.3$ Hence theoretical maximum total amount of chemical she uses will not exceed 71.3 grams.			
	$u_{25} = 1 \Rightarrow a + 24d = 1$ -----(1) $S_{25} = \dfrac{25}{2}\left(2a+24d\right) = 54.451$ $\Rightarrow 25a + 300d = 54.451 - - - - - (2)$ Solving the 2 equations, $d \approx -0.0982$ and $a \approx 3.36$ or $S_{25} = \dfrac{25}{2}\left(2a+24d\right) = 54.449 - - - - - (2)$ $\Rightarrow 25a + 300d = 54.449$ Solving the 2 equations, $d \approx -0.0982$ and $a \approx 3.36$			

	or $S_{25} = \frac{25}{2}(2a + 24d) = 54.45 - - - - - (2)$ $\Rightarrow 25a + 300d = 54.45$ Solving the 2 equations, $d \approx -0.0982$ and $a \approx 3.36$			

Qn	Suggested Answers			
2(a)(i)	$\int \frac{2x-4}{x^2-2x+4}\,\mathrm{d}x = \int \frac{2x-2}{x^2-2x+4}\,\mathrm{d}x - \int \frac{2}{x^2-2x+4}\,\mathrm{d}x$ $= \int \frac{2x-2}{x^2-2x+4}\,\mathrm{d}x - \int \frac{2}{(x-1)^2+3}\,\mathrm{d}x$ $= \ln(x^2-2x+4) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x-1}{\sqrt{3}} + C$			
2(a)(ii)	$x^2-2x+4=(x-1)^2+3>0$ since $(x-1)^2 \geq 0$. Therefore, $\int_1^4 \frac{\lvert 2x-4\rvert}{x^2-2x+4}\,\mathrm{d}x = -\int_1^2 \frac{2x-4}{x^2-2x+4}\,\mathrm{d}x + \int_2^4 \frac{2x-4}{x^2-2x+4}\,\mathrm{d}x$ $= -\left[\ln(x^2-2x+4) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x-1}{\sqrt{3}}\right]_1^2 + \left[\ln(x^2-2x+4) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x-1}{\sqrt{3}}\right]_2^4$ $= -\left[\left(\ln(4) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}\right) - \left(\ln(3) - \frac{2}{\sqrt{3}}\tan^{-1}0\right)\right] +$ $\left[\left(\ln(12) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{3}{\sqrt{3}}\right) - \left(\ln(4) - \frac{2}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}\right)\right]$ $= -2\ln 4 + \ln 3 + \ln 12 + \frac{4}{\sqrt{3}}\left(\frac{\pi}{6}\right) - \frac{2}{\sqrt{3}}\left(\frac{\pi}{3}\right)$ $= \ln\left(\frac{9}{4}\right) = 2\ln\left(\frac{3}{2}\right)$			
2(b)				

	$\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^3 x}\,\mathrm{d}x = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{2\sin x}{\cos^3 x}(\sin x)\,\mathrm{d}x$ $= \frac{1}{2}\left[\left[\frac{\sin x}{\cos^2 x}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{\cos x}\,\mathrm{d}x\right]$ $= \frac{1}{2}\left[\left[\frac{\sin x}{\cos^2 x}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sec x\,\mathrm{d}x\right]$ $= \frac{1}{2}\left[\frac{\sin x}{\cos^2 x} - \ln(\sec x + \tan x)\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{\sqrt{2}} - \frac{1}{2}\ln\left(\sqrt{2}+1\right)$ $u = \sin x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2\sin x}{\cos^3 x}$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$, $v = \frac{1}{\cos^2 x}$			

Qn	Suggested Answers			
3(i)	$z^3+4z^2+8z+8=(z+2)(z^2+az+4)$ Comparing coefficient of z^2: $4 = a + 2 \Rightarrow a = 2$ $\therefore z^2+2z+4$ is a factor of z^3+4z^2+8z+8 $(z+2)(z^2+2z+4)=0$ $z+2=0$ or $z^2+2z+4=0$ $z=-2$ or $z=\dfrac{-2\pm\sqrt{4-16}}{2}=-1\pm \mathrm{i}\sqrt{3}$ $\therefore z_1=-1+\mathrm{i}\sqrt{3}$ and $z_2=-1-\mathrm{i}\sqrt{3}$			
3(ii)	$\lvert w\rvert=\dfrac{\lvert z_1\rvert}{\lvert z_2\rvert}=1$ $\arg w=\arg\left(\dfrac{z_1}{z_2}\right)=\arg z_1-\arg z_2$ $=\dfrac{2\pi}{3}-\left(-\dfrac{2\pi}{3}\right)=\dfrac{4\pi}{3}$ $\therefore \arg w=\dfrac{4\pi}{3}-2\pi=-\dfrac{2\pi}{3}$			
3(iii)	$w^n=\mathrm{e}^{-\mathrm{i}\frac{2n\pi}{3}}=\cos\left(-\dfrac{2n\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2n\pi}{3}\right)$ Since w^n is real, $\sin\left(-\dfrac{2n\pi}{3}\right)=0$ $\sin\left(\dfrac{2n\pi}{3}\right)=0$ $\dfrac{2n\pi}{3}=0,\pm\pi,\pm2\pi,\pm3\pi,\pm4\pi,\pm5\pi,.....$ Since $n\in$ ¢ $^+$, $n=3,6,9,....$			

	Set of positive integers n is $\{n : n = 3p,\ p \in ¢^{+}\}$ Or Since w^n is real, $\arg w^n = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \pm 5\pi, ...$ $-\dfrac{2n\pi}{3} = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \pm 5\pi, ...$ Since $n \in ¢^{+}$, $n = 3, 6, 9, ...$ Set of positive integers n is $\{n : n = 3p,\ p \in ¢^{+}\}$ $w^n = \mathrm{e}^{-\mathrm{i}\frac{2n\pi}{3}} = \mathrm{e}^{-\mathrm{i}2p\pi} = \cos(-2p\pi) + \mathrm{i}\sin(-2p\pi) = 1$ (shown)			
3(iv)	$w^n = \mathrm{e}^{-\mathrm{i}\frac{2n\pi}{3}} = \cos\left(-\dfrac{2n\pi}{3}\right) + \mathrm{i}\sin\left(-\dfrac{2n\pi}{3}\right)$ $w^{100} - (w^*)^{100} = w^{100} - \left(w^{100}\right)^*$ $= 2\mathrm{i}\sin\left(-\dfrac{200\pi}{3}\right)$ $= 2\mathrm{i}\left(-\dfrac{\sqrt{3}}{2}\right) = -\sqrt{3}\mathrm{i}$ Alternatively, $w^{100} - (w^*)^{100} = w^{100} - \left(w^{100}\right)^*$ $= w - w^* = \mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}} - \mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$ $= -2\mathrm{i}\sin\left(\dfrac{2\pi}{3}\right) = -2\mathrm{i}\left(\dfrac{\sqrt{3}}{2}\right) = -\sqrt{3}\mathrm{i}$			

Qn	Suggested Answers			
4(a)(i)	$y=\ln\left(x+\sqrt{x^2+1}\right)$ $e^y=x+\sqrt{x^2+1}$ Differentiating both sides w.r.t. x $e^y\frac{dy}{dx}=1+\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)$ $\Rightarrow e^y\frac{dy}{dx}=1+x\left(x^2+1\right)^{-\frac{1}{2}}$ Differentiating both sides w.r.t. x $e^y\frac{d^2y}{dx^2}+e^y\left(\frac{dy}{dx}\right)^2=\left(x^2+1\right)^{-\frac{1}{2}}+x\left(-\frac{1}{2}\right)\left(x^2+1\right)^{-\frac{3}{2}}(2x)$ $\Rightarrow e^y\frac{d^2y}{dx^2}+e^y\left(\frac{dy}{dx}\right)^2=\left(x^2+1\right)^{-\frac{1}{2}}-x^2\left(x^2+1\right)^{-\frac{3}{2}}$ (shown)			
4(a)(ii)	$e^y\frac{d^3y}{dx^3}+e^y\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)+e^y\left(\frac{dy}{dx}\right)^3+e^y2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ $=-\frac{1}{2}\left(x^2+1\right)^{-\frac{3}{2}}(2x)-2x\left(x^2+1\right)^{-\frac{3}{2}}-x^2\left(-\frac{3}{2}\right)\left(x^2+1\right)^{-\frac{5}{2}}(2x)$ $\Rightarrow e^y\frac{d^3y}{dx^3}+3e^y\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)+e^y\left(\frac{dy}{dx}\right)^3$ $=-x\left(x^2+1\right)^{-\frac{3}{2}}-2x\left(x^2+1\right)^{-\frac{3}{2}}+3x^3\left(x^2+1\right)^{-\frac{5}{2}}$ When $x=0$, $y=0,\frac{dy}{dx}=1,\frac{d^2y}{dx^2}=0,\frac{d^3y}{dx^3}=-1$ $\therefore \sinh^{-1}x=x-\frac{x^3}{6}+\ldots$			

4(b)(i)	$\frac{AB}{AD}=\frac{AC\tan\frac{\pi}{6}}{AC\tan\left(\frac{\pi}{6}+\theta\right)}=\frac{1}{\sqrt{3}\tan\left(\frac{\pi}{6}+\theta\right)}$ (shown)			
4(b)(ii)	$\frac{AB}{AD}=\frac{1}{\sqrt{3}\tan\left(\frac{\pi}{6}+\theta\right)}=\frac{1-\tan\frac{\pi}{6}\tan\theta}{\sqrt{3}\left(\tan\frac{\pi}{6}+\tan\theta\right)}$ $=\frac{1}{\sqrt{3}}\left(\frac{1-\frac{\tan\theta}{\sqrt{3}}}{\frac{1}{\sqrt{3}}+\tan\theta}\right)=\frac{1}{\sqrt{3}}\left(1-\frac{\tan\theta}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}+\tan\theta\right)^{-1}$ $=\left(1-\frac{\tan\theta}{\sqrt{3}}\right)\left(1+\sqrt{3}\tan\theta\right)^{-1}\approx\left(1-\frac{\theta}{\sqrt{3}}\right)\left(1+\sqrt{3}\theta\right)^{-1}$ $=\left(1-\frac{\theta}{\sqrt{3}}\right)\left(1+(-1)\sqrt{3}\theta+\frac{(-1)(-2)}{2}\left(\sqrt{3}\theta\right)^{2}+\ldots\right)$ $=\left(1-\frac{\theta}{\sqrt{3}}\right)\left(1-\sqrt{3}\theta+3\theta^{2}+\ldots\right)$ $=1-\sqrt{3}\theta+3\theta^{2}-\frac{\theta}{\sqrt{3}}+\theta^{2}+\ldots\approx1+\left(-\frac{4\sqrt{3}}{3}\right)\theta+4\theta^{2}$ $a=-\frac{4\sqrt{3}}{3}, b=4$			

Qn	Suggested Answers			
5(i)	No. of arrangements $= \frac{10!}{2!2!4!} - 1$ $= 37799$			
5(ii)	No. of arrangements =Total no. of ways with all E's together – No. of ways all E's together and C's and L's together – No. of ways all E's together and C's but L's separated – No. of ways all E's together and L's but C's separated $= \frac{(7-1)!}{2!2!} - (5-1)! - (4-1)!\,{}^4C_2 - (4-1)!\,{}^4C_2$ $= 180 - 24 - 36 - 36 = 84$			

Qn	Suggested Answers			
6(i)	Let Di be the random variable denoting the score of the ith throw of the die for i = 1, 2 $\mathrm{P}(X=2)$ $=\mathrm{P}(D_1=1, D_2=5)+\mathrm{P}(D_1=2, D_2=4)+\mathrm{P}(D_1=3, D_2=3)$ $+\mathrm{P}(D_1=4, D_2=2)+\mathrm{P}(D_1=5, D_2=1)$ $=\frac{1}{6}\left(\frac{1}{6}\right)5=\frac{5}{36}$			
6(ii)	$\mathrm{P}(X>2 \mid D_1 \text{ is even})$ $=\frac{\mathrm{P}(X>2 \text{ and } D_1 \text{ is even})}{\mathrm{P}(D_1 \text{ is even})}$ $=\frac{\mathrm{P}(D_1=2, D_2\neq 4)+\mathrm{P}(D_1=4, D_2\neq 2)}{\mathrm{P}(D_1 \text{ is even})}$ $=\frac{\frac{1}{6}\left(\frac{5}{6}\right)+\frac{1}{6}\left(\frac{5}{6}\right)}{\frac{3}{6}}=\frac{5}{9}$			
6(iii)	Let W be the random variable denoting number of games, out of 10, that a special prize is won. P(winning a special prize) $=\mathrm{P}(X=1)+\mathrm{P}(X=2)=\frac{1}{6}+\frac{5}{36}=\frac{11}{36}$ $W \sim \mathrm{B}\left(10, \frac{11}{36}\right)$ Probability required $=\mathrm{P}(3\leq W\leq 8)=\mathrm{P}(W\leq 8)-\mathrm{P}(W\leq 2)$ $=0.63173=0.632$			

6(iv)

Let n be the number of games needed.
Let Y be the random variable denoting number of games, out of n, that a special prize is won.

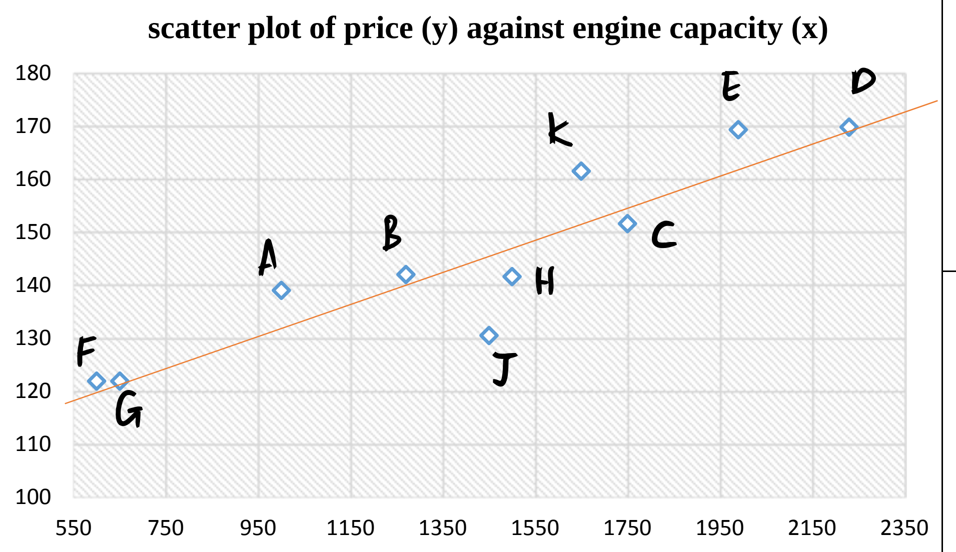
$$Y \sim \mathrm{B}\left(n, \frac{11}{36}\right)$$

$$\mathrm{P}(Y \geq 1) \geq 0.998 \Rightarrow 1 - \mathrm{P}(Y = 0) \geq 0.998$$

Using GC,

n	$1-\mathrm{P}(Y=0)$	$1-\mathrm{P}(Y=0)-0.998$
17	0.99797	-3×10^{-5}
18	0.99859	5.9×10^{-4}

Least n = 18

Qn	Suggested Answers			
7(a)	Scatter plot (C) shows perfect negative correlation, so $r_3=-1$. Scatter plot (A) shows strong, but not perfect, negative correlation, so $-1<r_1<0$. Scatter plot (B) shows no correlation, so $r_2=0$. Therefore, $r_3<r_1<r_2$.			
7(b)(i)				
7(b)(ii)	Equation of regression line of y on x is $y=0.03019188x+102.30964\approx 0.0302x+102$			
7(b)(iii)	$r=0.91336\approx 0.913$			

	The value of r implies(suggests) a strong positive linear relationship between x and y.			
	The regression line to be drawn on the scatter diagram passing through D and G.			
	Point corresponding to model J lies well below the line, which implies that its **price is lower** than it would be expected given its engine capacity. This would be good value for the consumer.			
	Points corresponding to models A, E and K are well above the line, which implies that their **prices are higher** than it would be expected given their engine capacity and would not be recommended in this case.			

Qn	Suggested Answers			
8(i)	Let L denotes the mass of a randomly chosen link. $L \sim \mathrm{N}(800, 20^2)$ $\mathrm{P}(L > 805) \approx 0.40129 \approx 0.401$			
8(ii)	Let S denotes the mass of a randomly chosen link with locking sleeve $S = 1.1L$, $S \sim \mathrm{N}(880, 22^2)$ $\mathrm{P}(865.35 < S < 895.5) \approx 0.50672 \approx 0.507$			
8(iii)	Let H denotes the mass of a randomly chosen hook $H \sim \mathrm{N}(750, \sigma^2)$ $\mathrm{P}(H < 735.6) = 0.15$ $\dfrac{735.6 - 750}{\sigma} = -1.0364$ $\sigma \approx 13.894 \approx 13.9$			
8(iv)	Let C denotes the mass of a wooden box and its contents $C = S_1 + S_2 + S_3 + S_4 + S_5 + H + 1000$ Let $\overline{C} = \dfrac{C_1 + C_2 + \ldots C_n}{n}$ $\overline{C} \sim N\left(6150, \dfrac{2613.0}{n}\right)$ $P\left(\overline{C} > 6190\right) > 0.013$ $P\left(Z > \dfrac{6190 - 6150}{\sqrt{\dfrac{2613.0}{n}}}\right) > 0.013$ $40\sqrt{\dfrac{n}{2613.0}} < 2.2262$ $n < 8.09$			

	Greatest n = 8			
Qn	**Suggested Answers**			
9(i)	Let X be the random variable denoting the number of yellow bricks drawn by Donald. P(no yellow bricks) $= \mathrm{P}(X=0)$ $= \underbrace{\frac{1}{2}\left(\frac{5}{10}\right)\frac{1}{2}\left(\frac{4}{9}\right)}_{\text{R or B from Box 1 then R or B from Box 1}} + \underbrace{\frac{1}{2}\left(\frac{5}{10}\right)\frac{1}{2}\left(\frac{7}{10}\right)}_{\text{R or B from Box 1 then R or B from Box 2}} + \underbrace{\frac{1}{2}\left(\frac{7}{10}\right)\frac{1}{2}\left(\frac{5}{10}\right)}_{\text{R or B from Box 2 then R or B from Box 1}} + \underbrace{\frac{1}{2}\left(\frac{7}{10}\right)\frac{1}{2}\left(\frac{6}{9}\right)}_{\text{R or B from Box 2 then R or B from Box 2}}$ $= \frac{25}{72}$ (shown)			
9(ii)	$\mathrm{P}(X=2)$ $= \frac{1}{2}\left(\frac{5}{10}\right)\frac{1}{2}\left(\frac{4}{9}\right)+\frac{1}{2}\left(\frac{5}{10}\right)\frac{1}{2}\left(\frac{3}{10}\right)+\frac{1}{2}\left(\frac{3}{10}\right)\frac{1}{2}\left(\frac{5}{10}\right)+\frac{1}{2}\left(\frac{3}{10}\right)\frac{1}{2}\left(\frac{2}{9}\right)$ $= \frac{53}{360}$			
9(iii)	$\begin{array}{\|c\|c\|c\|c\|} \hline x & 0 & 1 & 2 \\ \hline \mathrm{P}(X=x) & \frac{25}{72} & 1-\frac{25}{72}-\frac{53}{360}=\frac{91}{180} & \frac{53}{360} \\ \hline \end{array}$ *Direct computation P(X = 1):*			

	$\mathrm{P}(X=1)=\mathrm{P}(Y,Y')+\mathrm{P}(Y',Y)$ $=\frac{1}{2}\left(\frac{5}{10}\right)\left[\frac{1}{2}\left(\frac{5}{9}\right)+\frac{1}{2}\left(\frac{7}{10}\right)\right]+\frac{1}{2}\left(\frac{3}{10}\right)\left[\frac{1}{2}\left(\frac{5}{10}\right)+\frac{1}{2}\left(\frac{7}{9}\right)\right]$ $+\frac{1}{2}\left(\frac{5}{10}\right)\left(\frac{1}{2}\left(\frac{5}{9}\right)+\frac{1}{2}\left(\frac{3}{10}\right)\right)+\frac{1}{2}\left(\frac{7}{10}\right)\left(\frac{1}{2}\left(\frac{5}{10}\right)+\frac{1}{2}\left(\frac{3}{9}\right)\right)$ $=\frac{113}{720}+\frac{23}{240}+\frac{77}{720}+\frac{7}{48}=\frac{91}{180}$ $\mathrm{E}(X)=0+\frac{91}{180}+2\left(\frac{53}{180}\right)=\frac{4}{5}$			
9(iv)	Let W be the random variable denoting the number of yellow bricks drawn out of 5. $W \sim \mathrm{B}\left(5,\frac{8}{20}\right)$ P(draws less than 2 yellow bricks) $=\mathrm{P}(W<2)=\mathrm{P}(W\le 1)=0.33639=0.337$ (3 s.f.) Alternatively, P(draws less than 2 yellow bricks) $=\left(\frac{12}{20}\right)^5+{}^5C_4\left(\frac{12}{20}\right)^4\left(\frac{8}{20}\right)=\frac{1053}{3125}$ (or 0.33696)			
9(v)	P(not more than 9 draws to get the first yellow brick) =P(first yellow brick is drawn on the 1st , 2nd, …or 9th draw) $=\left(\frac{8}{20}\right)+\left(\frac{12}{20}\right)^1\left(\frac{8}{20}\right)+\left(\frac{12}{20}\right)^2\left(\frac{8}{20}\right)+\ldots+\left(\frac{12}{20}\right)^8\left(\frac{8}{20}\right)$ $=\frac{0.4\left(1-0.6^9\right)}{1-0.6}$			

	=0.98992 = 0.990 (to 3 sig. fig.) Alternatively: P(not more than 9 draws to get the first yellow brick) = 1 – P(10 or more draws to get the first yellow brick) $= 1-\left(\frac{12}{20}\right)^9 = 0.98992 = 0.900$ (to 3 sig.fig.)			
Qn	**Suggested Answers**			
10(i)	The sample is random would mean that the result of each throw has the same chance of being selected. The outcome of each throw is also independent of one another.			
10(ii)	Unbiased estimate of the population mean, $\bar{x} = \frac{120}{60} + 65 = 67$ Unbiased estimate of the population variance, $s^2 = \frac{1}{59}\left[\sum(x-65)^2 - \frac{\left(\sum(x-65)\right)^2}{60}\right] = 60.50847 = 60.5$ (to 3 s.f.)			
10(iii)	Let μ be the population mean distance of the throws. Null hypothesis , H_0 : $\mu = 68$ Alt hypothesis, H_1 : $\mu > 68$			
10(iv)	Let X be the random variable denoting the distances thrown by the thrower. Under H_0, $\overline{X} \sim N\left(68, \frac{7.5^2}{30}\right)$ and test statistic $Z = \frac{\overline{X} - 68}{7.5/\sqrt{30}} \sim N(0,1)$ Critical Region : Reject H_0 if $Z_{calc} \geq 2.05375$			

	Calculations : $Z_{calc} = \dfrac{\bar{x}-68}{7.5/\sqrt{30}}$ Since H_0 is rejected, $\dfrac{\bar{x}-68}{7.5/\sqrt{30}} \geq 2.05375$ $\bar{x} \geq 70.8122$ $\bar{x} \geq 70.9$			
10(v)	If H_0 is not rejected, $Z_{calc} < 2.05375$ $\dfrac{70.1-68}{7.5/\sqrt{n}} < 2.05375$ $\sqrt{n} < 7.33482$ $n < 53.79954$ Set of values of n is $\{n \in \mathbb{Z}^+ : n \leq 53\}$ or $\{n \in \mathbb{Z} : 1 \leq n \leq 53\}$			
	Null hypothesis , H_0 : $\mu = \mu_0$ Alt hypothesis, H_1 : $\mu < \mu_0$ Reject H_0 if $Z_{calc} \leq -1.64485$ For Ang's conclusion (H_0 is rejected) : $Z_{calc} = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \leq -1.64485$ For Tan's case : $Z'_{calc} = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{4n}} = 2\left(\dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right)$			

	Since $\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \le -1.64485$, $2\left(\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right) \le 2(-1.64485) \le -1.64485$, ie $Z'_{\text{calc}} \le -1.64485$, which leads to H_0 being rejected. Therefore, Tan's test would yield the same conclusion.			