1 A parabola, P with equation $(y-a)^2 = ax$, where a is a constant, undergoes in succession, the following transformations:

A: A translation of 2 units in the positive x-direction

B: A scaling parallel to the y-axis by a factor of $\frac{1}{3}$

The resulting curve, Q passes through the point with coordinates $\left(2, \frac{4}{3}\right)$.

(i) Show that $a = 4$. [3]

(ii) Find the range of values of k for which the line $y = kx$ does not meet P. [3]

2 The region bounded by the curve $y = \frac{1}{\sqrt{x}-2}$, the x-axis and the lines $x = 9$ and $x = 16$ is rotated through 2π radians about the x-axis. Use the substitution $t = \sqrt{x}$ to find the exact volume of the solid obtained. [6]

3 **(i)** Express $\frac{r+1}{(r+2)!}$ in the form $\frac{A}{(r+1)!} + \frac{B}{(r+2)!}$, where A and B are integers to be found. [2]

(ii) Find $\sum_{r=1}^{n} \frac{r+1}{3(r+2)!}$. [3]

(iii) Hence, evaluate $\sum_{r=0}^{\infty} \frac{r+1}{3(r+2)!}$. [2]

4 Kumar wishes to purchase a gift priced at $280 for his mother.

Starting from January 2017,

- Kumar saves $100 in his piggy bank on the 1st day of each month;
- Kumar donates 30% of his money in his piggy bank to charity on the 15th day of each month and
- Kumar's father puts an additional $20 in Kumar's piggy bank on the 25th day of each month.

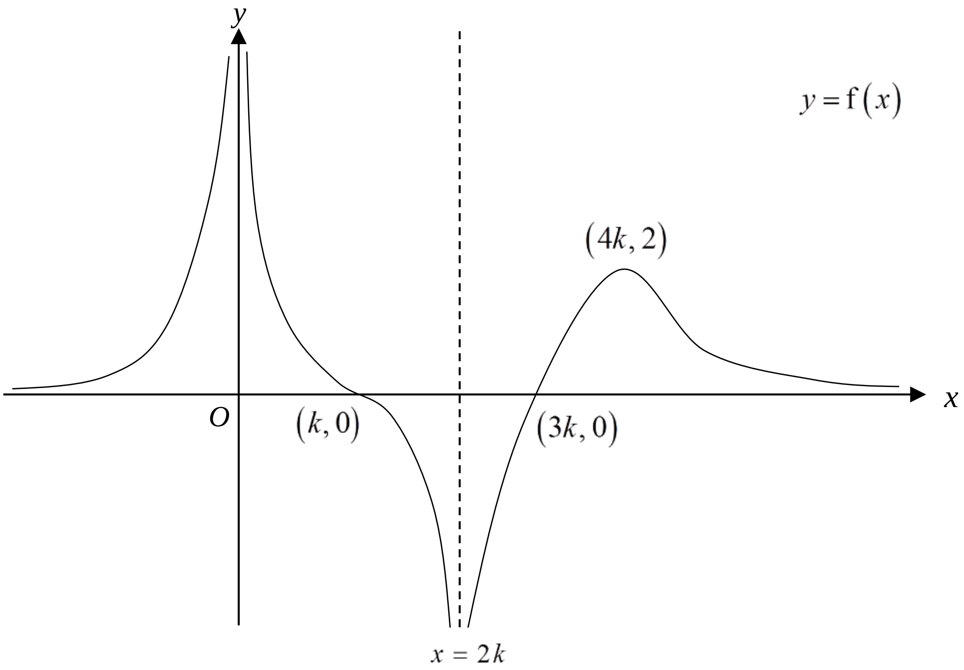
(i) Find the amount of money in Kumar's piggy bank at the end of March 2017. [2]

(ii) Show that the amount of money in Kumar's piggy bank at the end of n months is

$$300\left(1-0.7^n\right).$$ [3]

(iii) At the end of which month will Kumar first be able to purchase the gift for his mother? [2]

5 The diagram below shows the sketch of the graph of $y = \mathrm{f}(x)$ for $k > 0$. The curve passes through the points with coordinates $(k, 0)$ and $(3k, 0)$, and has a maximum point with coordinates $(4k, 2)$. The asymptotes are $x = 0$, $x = 2k$ and $y = 0$.



Sketch on separate diagrams, the graphs of

(i) $y = \mathrm{f}(-x-k)$, [2]

(ii) $y = \mathrm{f}'(x)$, [2]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

showing clearly, in terms of k, the equations of any asymptote(s), the coordinates of any turning point(s) and any points where the curve crosses the x- and y-axes.

6 A straight line passes through the point with coordinates $(4,\ 3)$, cuts the positive x-axis at point P and the positive y-axis at point Q. It is given that $\angle PQO = \theta$, where $0 < \theta < \frac{\pi}{2}$ and O is the origin.

(i) Show that the equation of line PQ is given by $y = (4-x)\cot\theta + 3$. [2]

(ii) By finding an expression for $OP + OQ$, show that as θ varies, the stationary value of $OP + OQ$ is $a + b\sqrt{3}$, where a and b are constants to be determined. [5]

7 A curve C has parametric equations

$$x = \frac{4}{t+1} \quad \text{and} \quad y = t^2 - 3, \quad t \neq -1.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [2]

(ii) Find the equation of the normal to C at P where $x=-2$. [3]

(iii) Find the other values of t where the normal at P meets the curve C again. [3]

8 The curve C has equation

$$y=\frac{2x^2-3x+5}{x-5}.$$

(i) Express y in the form $px+q+\frac{r}{x-5}$ where p, q and r are constants to be found. [3]

(ii) Sketch C, stating the equations of any asymptotes, the coordinates of any stationary points and any points where the curve crosses the x- and y-axes. [4]

(iii) By sketching another suitable curve on the same diagram in part **(ii)**, state the number of roots of the equation

$$\left(2x^2-3x+5\right)^2=5x\left(x-5\right)^2.$$ [3]

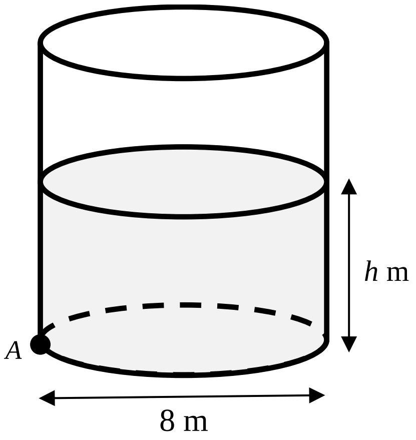
9 **(a)** Given that the first two terms in the series expansion of $\sqrt{4-x}$ are equal to the first two terms in the series expansion of $p+\ln\left(q-x\right)$, find the constants p and q. [5]

(b)(i) Given that $y=\tan^{-1}\left(ax+1\right)$ where a is a constant, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=a\cos^2 y$. Use this result to find the Maclaurin series for y in terms of a, up to and including the term in x^3. [5]

(ii) Hence, or otherwise, find the series expansion of $\frac{1}{1+\left(4x+1\right)^2}$ up to and including the term in x^2.

[3]

10



The figure above shows a cylindrical water tank with base diameter 8 metres. Water is flowing into the tank at a constant rate of $0.36\pi\,\mathrm{m}^3$/min. At time t minutes, the depth of water in the tank is h metres. However, the tank has a small hole at point A located at the bottom of the tank. Water is leaking from point A at a rate of $0.8\pi h$ m^3/min.

(i) Show that the depth, h metres, of the water in the tank at time, t minutes satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{400}(9-20h).$$ [3]

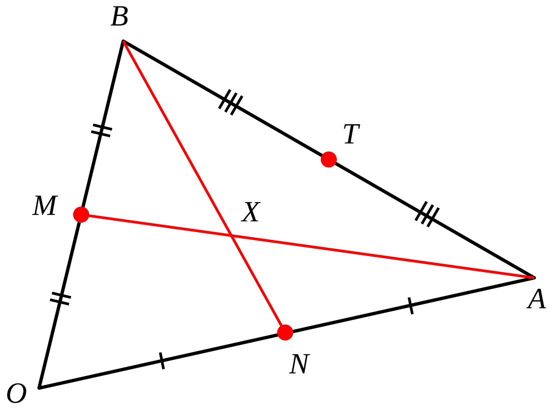
(ii) Given that $h=0.4$ when $t=0$, find the particular solution of the above differential equation in the form $h=\mathrm{f}(t)$. [6]

(iii) Explain whether the tank will be emptied. [1]

(iv) Sketch the part of the curve with the equation found in part **(ii)**, which is relevant in this context. [2]

11 A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side.

For the triangle shown below, O, A and B are vertices, where O is the origin, $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$. The midpoints of OB, OA and AB are M, N and T respectively.



It is given that X is the point of intersection between the medians of triangle OAB from vertices A and B.

(i) Show that $\overrightarrow{OX}=\frac{1}{3}(\mathbf{a}+\mathbf{b})$. [4]

(ii) Prove that X also lies on OT, the median of triangle OAB from vertex O. [2]

The **centroid** of triangle OAB is the common point of intersection X between all three medians of the triangle.

Ray tracing is a technique in computer graphics rendering used to realistically capture the lighting effect in a scene being modelled. Starting from a chosen viewpoint, different rays are being traced backwards towards different parts of an object in the scene and reflected off the object. For each ray, if it reflects off the object and intersects a light source, then the part of the object at which the ray is reflected off would be made to appear brighter.

In a particular scene depicting a dolphin jumping out of the ocean, a ray is being traced back from a chosen viewpoint at V to the **centroid** X of a particular triangular facet defined by the vertices comprising the origin O, $A(5,\ 4,\ 6)$ and $B(-2,\ 2,\ 3)$, and then reflected off the facet at X, as shown in Figure 1.

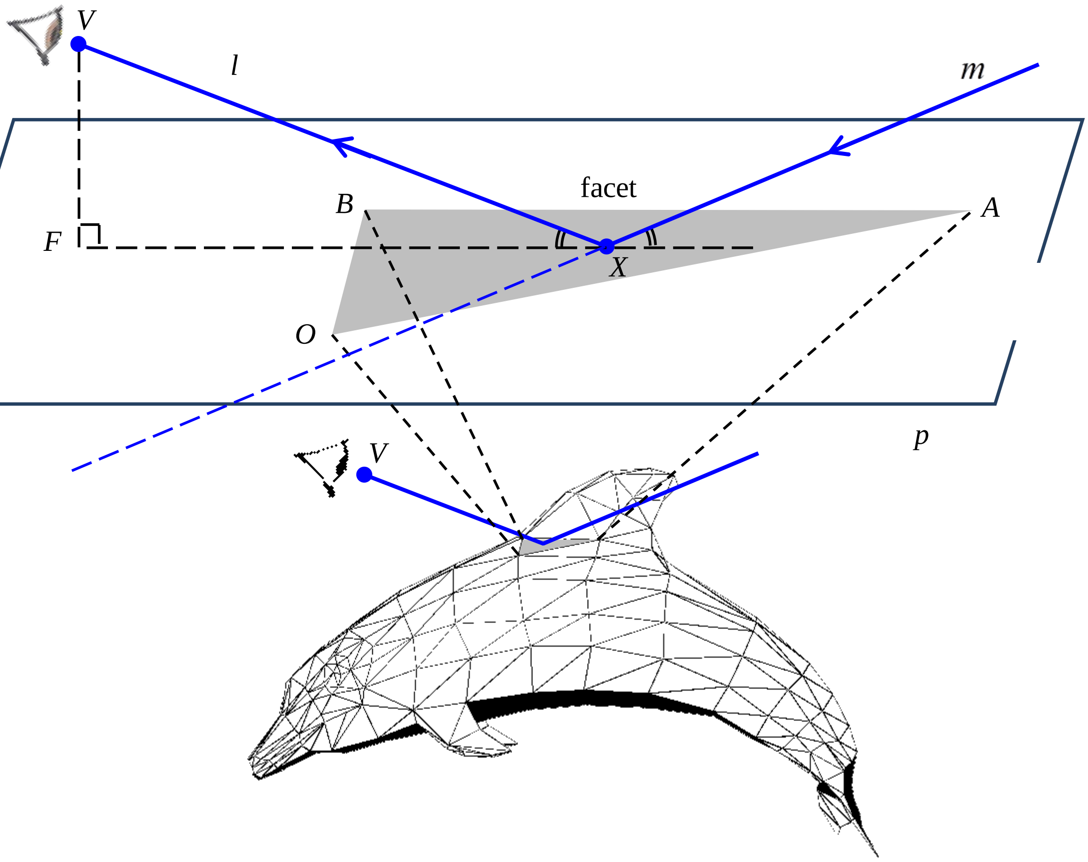


Figure 1

(iii) Show that the plane p which contains the triangular facet OAB can be represented by the cartesian equation $-3y+2z=0$. [2]

(iv) Given $V(1,-68,-37)$, determine the coordinates of the foot of perpendicular F from V to plane p. [4]

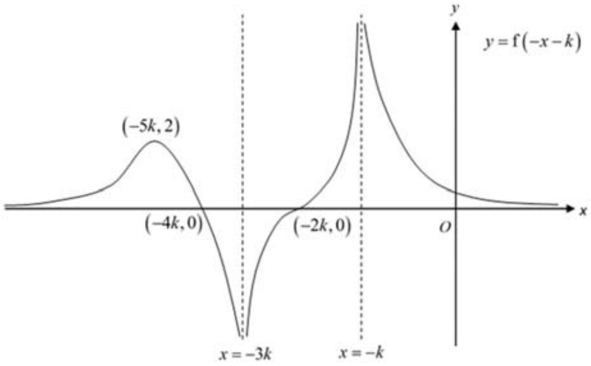
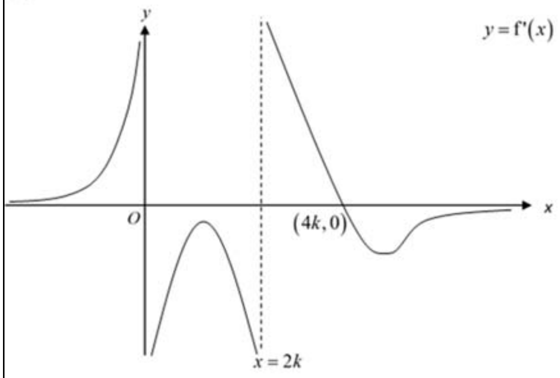
The reflected ray travels along a line m such that:

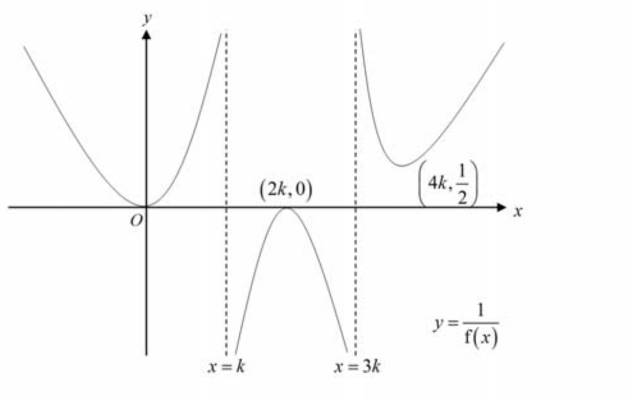
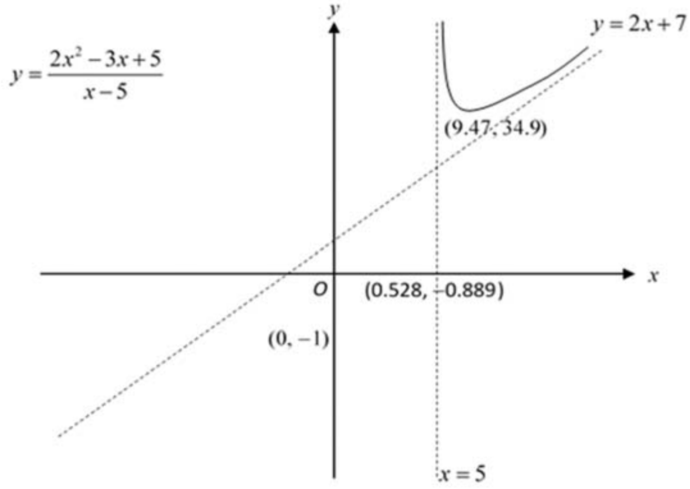
- both line VX (denoted by l) and line m lie in a plane that is perpendicular to plane p, and
- the angle between line l and plane p equals the angle between line m and plane p.

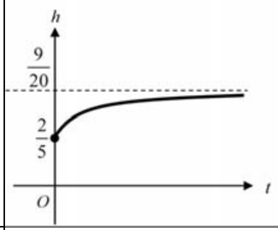
(v) By first finding two suitable points lying on line m, or otherwise, find a cartesian equation for line m. [5]

ANNEX B

CJC H2 Math JC2 Preliminary Examination Paper 1

QN	Topic Set	Answers
1	Graphs and Transformation	(ii) $k < -\frac{1}{4}$
2	Application of Integration	$\pi\left(2\ln 2+2\right)$
3	Sigma Notation and Method of Difference	(i) $\frac{1}{(r+1)!}-\frac{1}{(r+2)!}$ (ii) $\frac{1}{3}\left[\frac{1}{2}-\frac{1}{(n+2)!}\right]$ (iii) $\frac{1}{3}$
4	AP and GP	(i) \$197.10 (iii)August 2017
5	Graphs and Transformation	(i)  (ii)  (iii)

		
6	Differentiation & Applications	(ii) $7 + 4\sqrt{3}$
7	Differentiation & Applications	(i) $-\frac{t(t+1)^2}{2}$ (ii) $y = -\frac{1}{6}x + \frac{17}{3}$ or $6y + x = 34$ (iii) $t = -3$ (given) or -0.915 or 2.91
8	Graphs and Transformation	(i) $p = 2,\ q = 7,\ r = 40$ (ii)  (iii)2
9	Maclaurin series	(a) $p = 2 - \ln 4$ (b)(i) $\frac{\pi}{4} + \frac{1}{2}ax - \frac{1}{4}a^2x^2 + \frac{1}{12}a^3x^3 + \ldots$ (ii) $\frac{1}{2} - 2x + 4x^2$

10	Differential Equations	(ii) $h=\frac{1}{20}\left(9-\mathrm{e}^{-\frac{1}{20}t}\right)$ (iv) 
11	Vectors	(iv) $(1,-38,-57)$ (v) $l_m : x=1,\ y-2=\frac{z-3}{8}$

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS
2017 JC2 PRELIMINARY EXAMINATION PAPER I SOLUTION

Q1. Transformations, Conics and Inequalities

Assessment Objectives	Solution	Examiner's Feedback
Determine the transformations on the graph of $y=\mathrm{f}(x)$ as represented by $y=\mathrm{f}(x)+a$ and $ay=\mathrm{f}(x)$.	**(i)** $(y-a)^2=ax$ $\downarrow A$ $(y-a)^2=a(x-2)$ $\downarrow B$ $(3y-a)^2=a(x-2)$ Since resulting curve passes through point $\left(2,\frac{4}{3}\right)$, $(4-a)^2=a(2-2)$ $(4-a)^2=0$ $a=4$ (shown)	Most candidates were able to answer this part correctly. Some forgot that scaling to a variable is achieved by dividing the variable by the scaling factor.
Applying the concept no real roots $\Rightarrow b^2-4ac<0$	**(ii)** **Method ①:** Parabola: $(y-4)^2=4x$ — ① Line: $y=kx$ — ② Substitute ② into ①: $(kx-4)^2=4x$ $k^2x^2-8kx+16=4x$ $k^2x^2+(-8k-4)x+16=0$ For the line not to meet the parabola, $b^2-4ac<0$	Many presented satisfactory answers. Some students failed to link the intersection of linear/quadratic curves to solving simultaneous and subsequently quadratic equations, and that the number of common points can be inferred from the sign of the determinant. Some also had algebraic slips when handling inequalities. They need to practise more.

$$\left(-8k-4\right)^2-4k^2\left(16\right)<0$$

$$64k^2+64k+16-64k^2<0$$

$$64k+16<0$$

$$k<-\frac{1}{4}$$

Method ②:

Parabola: $\left(y-4\right)^2=4x$ — ①

Line: $y=kx \Rightarrow x=\frac{k}{y}$ — ②

Substitute ② into ①:

$$\left(y-4\right)^2=\frac{4y}{k}$$

$$ky^2-8ky+16k=4y$$

$$ky^2+\left(-8k-4\right)y+16k=0$$

For the line not to meet the parabola, $b^2-4ac<0$

$$\left(-8k-4\right)^2-4k\left(16k\right)<0$$

$$64k^2+64k+16-64k^2<0$$

$$64k+16<0$$

$$k<-\frac{1}{4}$$

Q2. Definite Integral

Assessment Objectives	Solution	Examiner's Feedback
Find the volume of solid formed by revolution. Perform integration by a given substitution.	**Method ①:** Volume $=\pi\int_{9}^{16}\left(\frac{1}{\sqrt{x}-2}\right)^{2}\ \mathrm{d}x$ $=\pi\int_{3}^{4}\left(\frac{1}{t-2}\right)^{2}(2t)\,\mathrm{d}t$ $=\pi\int_{3}^{4}\frac{2t}{t^{2}-4t+4}\ \mathrm{d}t$ $=\pi\int_{3}^{4}\frac{2t-4}{t^{2}-4t+4}+\frac{4}{(t-2)^{2}}\ \mathrm{d}t$ $=\pi\Big[\ln\lvert t^{2}-4t+4\rvert\Big]_{3}^{4}+\pi\int_{3}^{4}4(t-2)^{-2}\ \mathrm{d}t$ $=\pi\left[\ln\lvert t^{2}-4t+4\rvert+4\frac{(t-2)^{-1}}{-1}\right]_{3}^{4}$ $=\pi\left[\ln\lvert t^{2}-4t+4\rvert-\frac{4}{(t-2)}\right]_{3}^{4}$ $=\pi\big[(\ln 4-2)-(\ln 1-4)\big]$ $=\pi(\ln 4+2)$ units^{3} <u>Substitution:</u> $t=\sqrt{x}$ $t^{2}=x$ $2t=\frac{\mathrm{d}x}{\mathrm{d}t}$ When $x=9, t=3$ When $x=16, t=4$	Most candidates were able to setup the correct integral for the volume of revolution. However, many failed make the correct substitution of dx by $\frac{dx}{dt}dt$ and thus $2tdt$. Another group of students forgot to change the upper and lower limits to the respective values of t when the variable was changed. Many students were also stuck at the integration of $\frac{4}{(t-2)^{2}}$ as it is not very easy for those who don't practise much to identify the fraction as a power function of power -2.

Method ②:

Volume

$$= \pi \int_{9}^{16} \left(\frac{1}{\sqrt{x}-2} \right)^2 \, \mathrm{d}x$$

$$= \pi \int_{3}^{4} \left(\frac{1}{t-2} \right)^2 (2t) \, \mathrm{d}t$$

$$= \pi \int_{3}^{4} \frac{2t}{(t-2)^2} \, \mathrm{d}t$$

$$= \pi \int_{3}^{4} \frac{2}{(t-2)} + \frac{4}{(t-2)^2} \, \mathrm{d}t$$

$$= \pi \left[2\ln|t-2| \right]_3^4 + \pi \int_3^4 4(t-2)^{-2} \, \mathrm{d}t$$

$$= \pi \left[2\ln|t-2| + 4\frac{(t-2)^{-1}}{-1} \right]_3^4$$

$$= \pi \left[2\ln|t-2| - \frac{4}{(t-2)} \right]_3^4$$

$$= \pi \left[(2\ln 2 - 2) - (2\ln 1 - 4) \right]$$

$$= \pi (2\ln 2 + 2) \text{ units}^3$$

Substitution:

$t = \sqrt{x}$

$t^2 = x$

$2t = \frac{\mathrm{d}x}{\mathrm{d}t}$

When $x = 9, t = 3$

When $x = 16, t = 4$

Q3. Sigma Notation		
Assessment Objectives	**Solution**	**Examiner's Feedback**
Apply concept of factorial	**(i)** **Method ①:** $\frac{r+1}{(r+2)!}=\frac{A}{(r+1)!}+\frac{B}{(r+2)!}$ $r+1=\frac{A(r+2)!}{(r+1)!}+\frac{B(r+2)!}{(r+2)!}$ $r+1=A(r+2)+B$ When $r=-1$, $A+B=0$ — ① When $r=0$, $2A+B=1$ — ② Solving, $A=1$ and $B=-1$ $\therefore \frac{1}{(r+2)r!}=\frac{1}{(r+1)!}-\frac{1}{(r+2)!}$ **Method ②:** $\frac{r+1}{(r+2)!}=\frac{A}{(r+1)!}+\frac{B}{(r+2)!}$ When $r=1$, $\frac{A}{2}+\frac{B}{6}=\frac{2}{6}$ $3A+B=2$ — ① When $r=0$, $A+\frac{B}{2}=\frac{1}{2}$ $2A+B=1$ — ② Solving, $A=1$ and $B=-1$ $\therefore \frac{1}{(r+2)r!}=\frac{1}{(r+1)!}-\frac{1}{(r+2)!}$	Most students were able to get the values of A and B correctly. There were a variety of methods used to get the correct answers.

Apply summation of series by the method of differences.	**(ii)** $\sum_{r=1}^{n}\frac{r+1}{3(r+2)!}=\frac{1}{3}\sum_{r=1}^{n}\frac{r+1}{(r+2)!}$ $=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{(r+1)!}-\frac{1}{(r+2)!}\right]$ $=\frac{1}{3}\left[\frac{1}{2!}-\cancel{\frac{1}{3!}}\right.$ $+\cancel{\frac{1}{3!}}-\cancel{\frac{1}{4!}}$ $+\cancel{\ldots}\ \cancel{\ldots}$ $+\cancel{\frac{1}{n!}}-\cancel{\frac{1}{(n+1)!}}$ $\left.+\cancel{\frac{1}{(n+1)!}}-\frac{1}{(n+2)!}\right]$ $=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{(n+2)!}\right]$	Most students were able to get this part correct.
Understand convergence of a series and the sum to infinity.	**(iii)** From (ii), $\sum_{r=1}^{n}\frac{r+1}{3(r+2)!}=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{(n+2)!}\right]$ As $n\to\infty$, $\frac{1}{(n+2)!}\to 0$, thus $\sum_{r=1}^{n}\frac{r+1}{3(r+2)!}\to\frac{1}{6}$ $\sum_{r=0}^{\infty}\frac{r+1}{3(r+2)!}=\frac{1}{3(2)(1)}+\sum_{r=1}^{\infty}\frac{r+1}{3(r+2)!}$ $=\frac{1}{6}+\frac{1}{6}$ $=\frac{1}{3}$	Most students were able to get the sum to infinity correct but failed to realized that the starting value of r had change.

<table>
<tr><th colspan="3">Q4. Geometric Progression</th></tr>
<tr><th>Assessment Objectives</th><th>Solution</th><th>Examiner’s Feedback</th></tr>
<tr>
<td rowspan="2">Determine sum of a finite geometric series</td>
<td>

(i)

<table>
<tr><th colspan="2"></th><th colspan="3">Amount of $ Kumar has @ the …</th></tr>
<tr><th colspan="2"></th><th>Beginning</th><th>Middle</th><th>End</th></tr>
<tr><td>Jan 2017</td><td>1</td><td>100</td><td>$0.7(100)$</td><td>$0.7(100)+20$</td></tr>
<tr><td>Feb 2017</td><td>2</td><td>$100+0.7(100)+20$</td><td>$0.7[100+0.7(100)+20]$</td><td>$0.7(100)+0.7^2(100)$
$+0.7(20)+20$</td></tr>
<tr><td>Mar 2017</td><td>3</td><td>$100+0.7(100)$
$+0.7^2(100)$
$+0.7(20)+20$</td><td>$0.7[100+0.7(100)$
$+0.7^2(100)+0.7(20)+20]$</td><td>$0.7(100)+0.7^2(100)$
$+0.7^3(100)+0.7^2(20)$
$+0.7(20)+20$</td></tr>
<tr><td></td><td></td><td>…</td><td>…</td><td>…</td></tr>
<tr><td></td><td>n</td><td></td><td></td><td>$0.7(100)+0.7^2(100)$
$+0.7^3(100)+...+0.7^n(100)$
$+0.7^{n-1}(20)+...+0.7^2(20)$
$+0.7(20)+20$</td></tr>
</table>

Amount of money Kumar has at the end of March 2015

$$=\$\left[0.7(100)+0.7^2(100)+0.7^3(100)+0.7^2(20)+0.7(20)+20\right]$$

$$=\$197.10$$

</td>
<td>Most students were able to get the value correct.</td>
</tr>
<tr>
<td>

(ii) Amount of money Kumar has at the end of *n* months

$$=0.7(100)+0.7^2(100)+0.7^3(100)+...+0.7^n(100)$$

$$+0.7^{n-1}(20)+...+0.7^2(20)+0.7(20)+20$$

$$=100\left(0.7+0.7^2+...+0.7^n\right)+20\left(1+0.7+0.7^2+...+0.7^{n-1}\right)$$

$$=100\left[\frac{0.7\left(1-0.7^n\right)}{1-0.7}\right]+20\left[\frac{1-0.7^n}{1-0.7}\right]$$

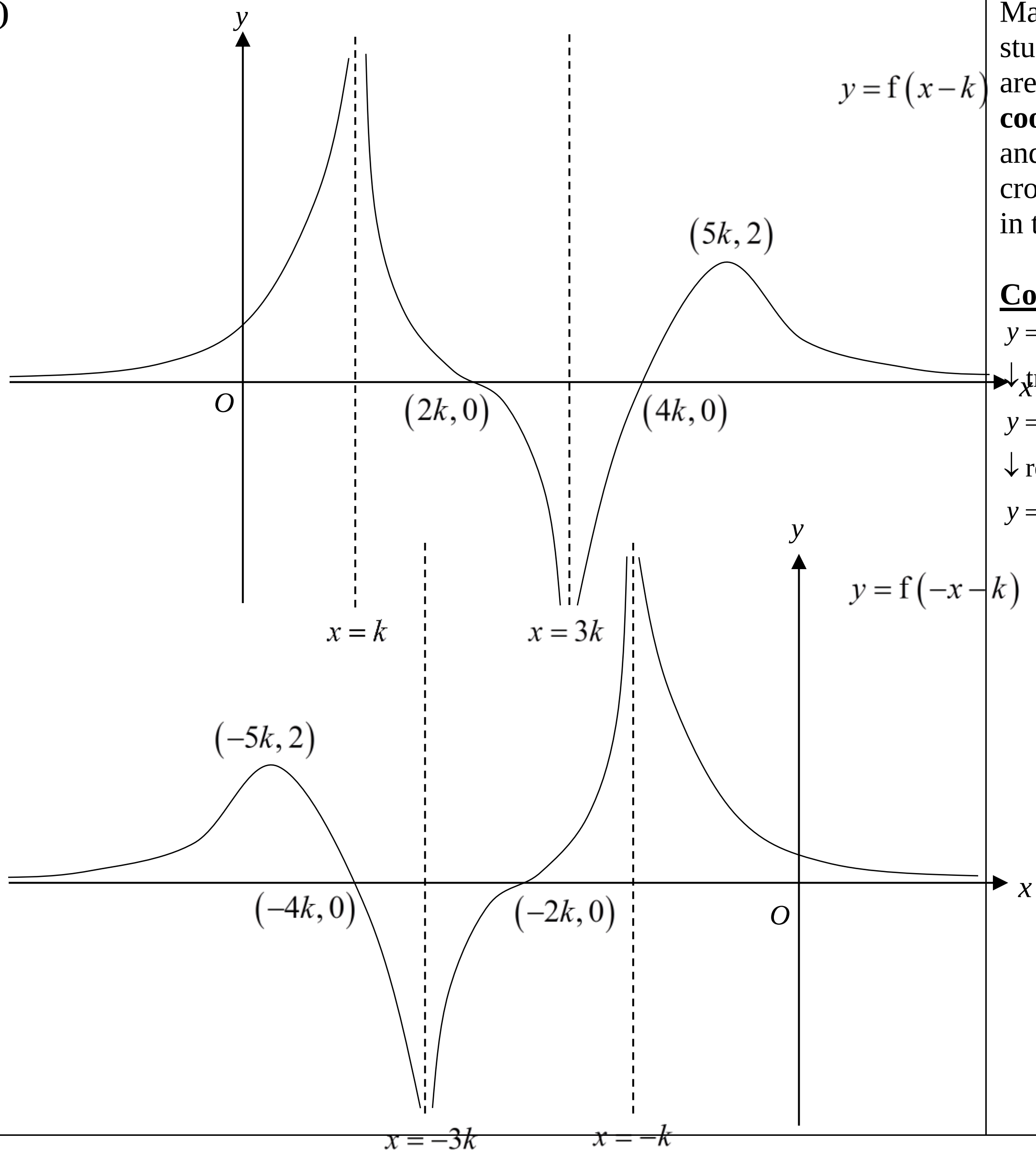
$$=\frac{700}{3}\left(1-0.7^n\right)+\frac{200}{3}\left(1-0.7^n\right)$$

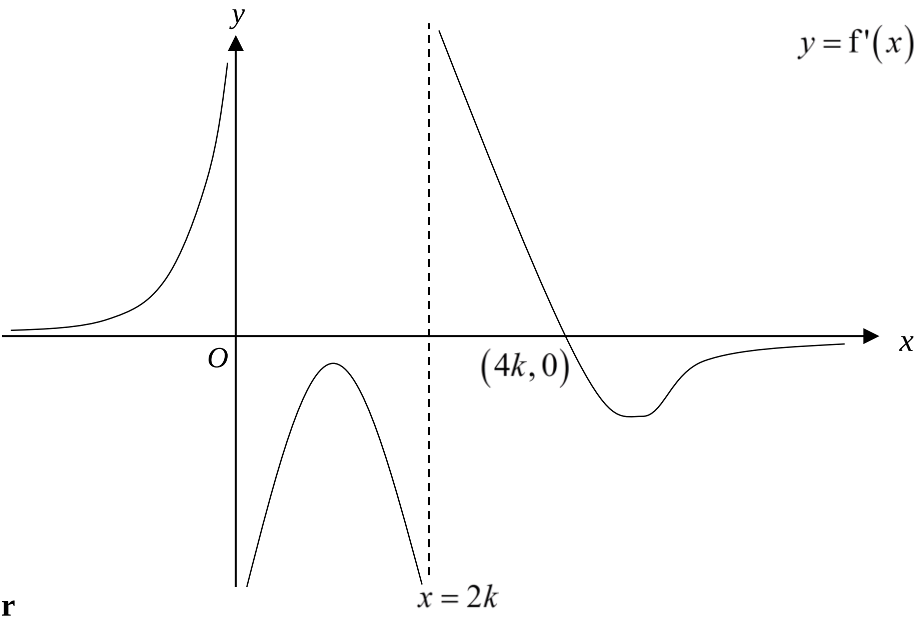
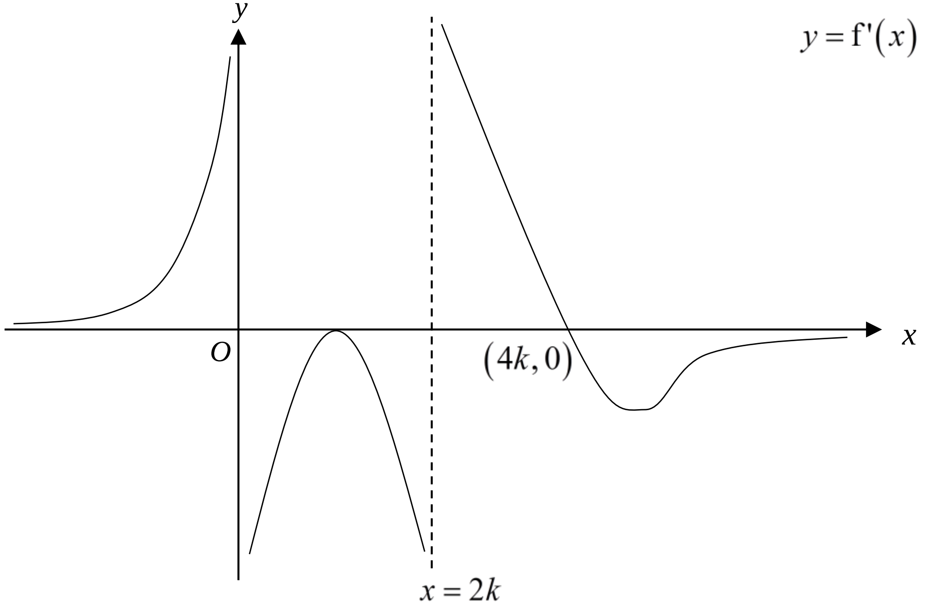
$$=300\left(1-0.7^n\right)\text{ (shown)}$$

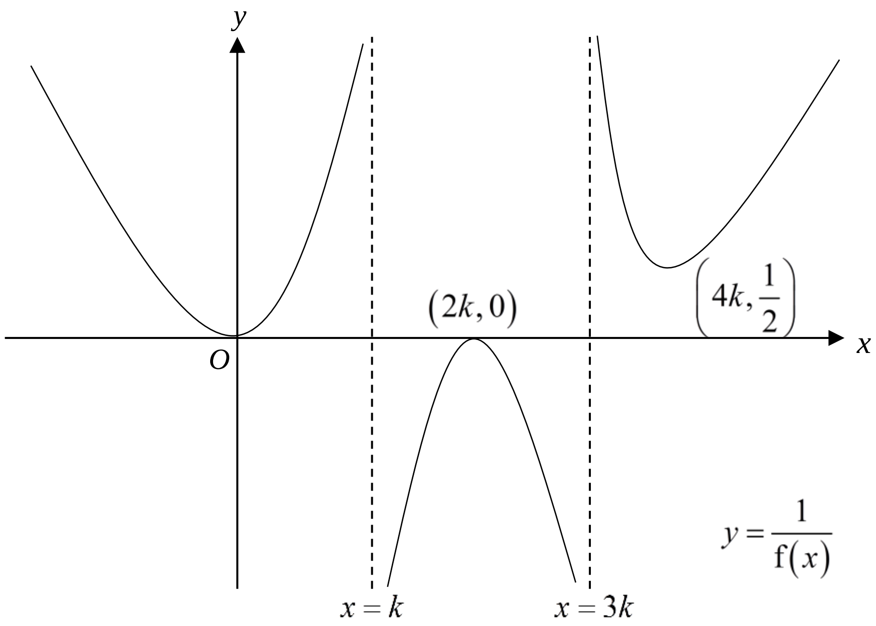
</td>
<td>There were a many methods presented by students. However, because it is a show question, their working must be clear. Credit were not given to students who just state that $a=90, r=0.7$ unless the explanation on why $a=90$ is clear.</td>
</tr>
</table>

Solve inequality	**(iii)** $300\left(1-0.7^{n}\right)\geq 280$ $1-0.7^{n}\geq\frac{14}{15}$ $0.7^{n}\leq\frac{1}{15}$ $n\geq 7.59$ Kumar will first be able to purchase the gift for his mother at the 8th month. (or August 2017)	Quite badly done by students wo did not use the GC table method with many students not realized that $\ln 0.7 < 0$ and hence there is a need to change the inequality sign when dividing by $\ln 0.7$ on both sides of the unequalities.

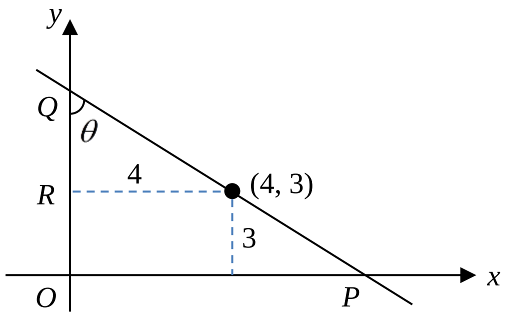
Q5. Transformations

Assessment Objectives	Solution	Examiner's Feedback
Apply effect of transformation on the graph of $y=\mathrm{f}\left(x\right)$ as represented by $y=\mathrm{f}\left(-x-k\right)$.	**(i)**  	Many good answers. However, students should take note that they are supposed to state the **coordinates** of any turning point(s) and any points where the curve crosses the *x*- and *y*-axes as given in the question. **Correct sequence:** $y=\mathrm{f}\left(x\right)$ $\downarrow$ translate k units in the +ve *x*-direction $y=\mathrm{f}\left(x-k\right)$ $\downarrow$ reflection in the $y-$axis $y=\mathrm{f}\left(-x-k\right)$

Relate the graph of $y = \mathrm{f}\,'(x)$ to the graph of $y = \mathrm{f}\,(x)$.	**(ii)**  **or**  	Many good answers. However, students should take note that they are supposed to state the **coordinates** of any turning point(s) and any points where the curve crosses the *x*- and *y*-axes as given in the question. There were a number of students who mixed up the sketching of $y = \mathrm{f}\,'(x)$ and $y = \dfrac{1}{\mathrm{f}\,(x)}$.

Relate the graph of $y=\frac{1}{\mathrm{f}(x)}$ to the graph of $y=\mathrm{f}(x)$.	**(iii)**  	Many good answers. However, students should take note that they are supposed to state the **coordinates** of any turning point(s) and any points where the curve crosses the *x*- and *y*-axes as given in the question. There were a number of students who mixed up the sketching of $y=\mathrm{f}\,'(x)$ and $y=\frac{1}{\mathrm{f}(x)}$.

Q6. Application of Differentiation

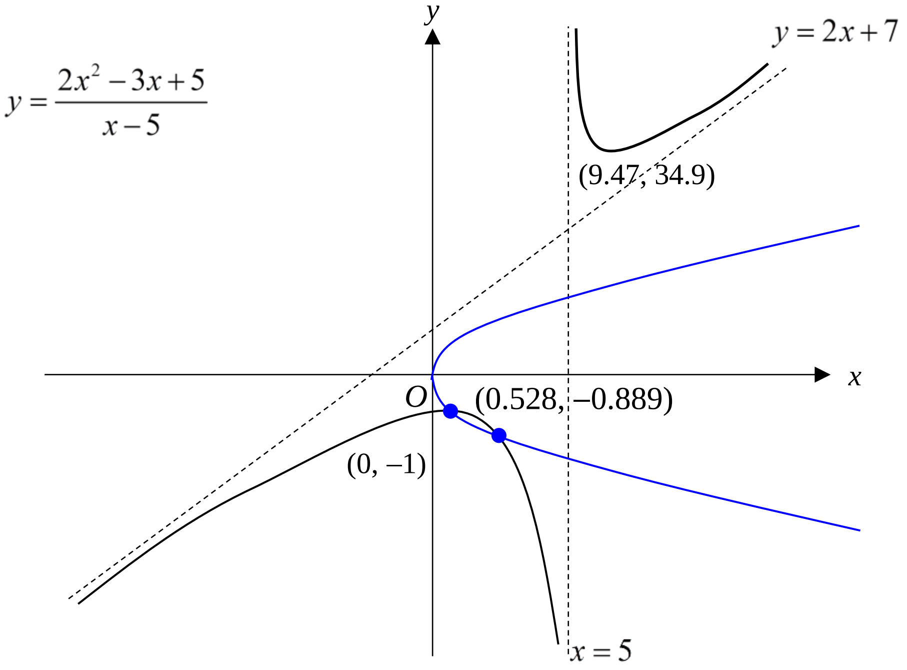
Assessment Objectives	Solution	Examiner’s Feedback
Use of trigonometric ratio to express gradient and *y*-intercept in terms of θ.	**(i)**  $\text{Gradient} = -\frac{1}{\frac{OP}{OQ}} = -\frac{1}{\tan\theta} = -\cot\theta$ $\tan\theta = \frac{4}{QR} \Rightarrow QR = 4\cot\theta$ $y-\text{intercept} = 3+4\cot\theta$ Equation of line *PQ* is $y = -(\cot\theta)x+3+4\cot\theta$ $y = (4-x)\cot\theta+3$ (shown)	Poorly attempted. Many students could identify that they needed to find gradient but did not realized that gradient in this question is in fact negative. Students who attempted to ‘work backwards’ but did not show sufficient and accurate working were penalized.

Find axial intercepts using equation of line. Find stationary value using first derivative.	**(ii)** When $x=0,\ y=4\cot\theta+3$ When $y=0,\ 0=\left(4-x\right)\cot\theta+3$ $x=4-\dfrac{-3}{\cot\theta}=4+3\tan\theta$ $OP+OQ=4+3\tan\theta+4\cot\theta+3$ $=7+3\tan\theta+4\cot\theta$ Let $L=OP+OQ$ $\dfrac{\mathrm{d}L}{\mathrm{d}\theta}=3\sec^2\theta-4\operatorname{cosec}^2\theta$ $\dfrac{\mathrm{d}L}{\mathrm{d}\theta}=0\Rightarrow 3\sec^2\theta=4\operatorname{cosec}^2\theta$ $\dfrac{3}{\cos^2\theta}=\dfrac{4}{\sin^2\theta}$ $\tan^2\theta=\dfrac{4}{3}$ $\tan\theta=\dfrac{2}{\sqrt{3}}\left(\text{rej.}-\dfrac{2}{\sqrt{3}}\because 0<\theta<\dfrac{\pi}{2}\right)$ Stationary value of $OP+OQ=7+3\left(\dfrac{2}{\sqrt{3}}\right)+4\left(\dfrac{\sqrt{3}}{2}\right)$ $=7+4\sqrt{3}$	Many students were unable to find the *x*-coordinate of point *P*. For students who found the expression for *OP* + *OQ*, they were unable to differentiate the expression. Students should know the following: (1) $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left(\tan\theta\right)=\sec^2\theta$ (2) $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left(\cot\theta\right)=-\operatorname{cosec}^2\theta$

Q7. Parametric Equations

Assessment Objectives	**Solution**	**Examiner's Feedback**
Find first derivative of a function defined parametrically.	**(i)** $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{2t}{\frac{-4}{(t+1)^2}}=-\frac{t(t+1)^2}{2}$	Most students able to get the correct answers. Those who were unable to do made careless mistake in dx/dt.
Find equation of normal.	**(ii)** When $x=-2,\ \frac{4}{t+1}=-2$ $t=-3$ $y=6$ Gradient of normal $=\frac{2}{-3(-3+1)^2}=-\frac{1}{6}$ Equation of normal at $P(-2,6)$ is $y-6=-\frac{1}{6}(x+2)$ $y=-\frac{1}{6}x+\frac{17}{3}$ or $6y+x=34$	Most students got the correct concept to solve for the eqn of normal but lost the accuracy mark due of the wrong expression in part (i).
Find *t*-values at points of intersection of a Cartesian line and a parametric curve.	**(iii)** $t^2-3=-\frac{1}{6}\left(\frac{4}{t+1}\right)+\frac{17}{3}$ $6(t+1)(t^2-3)=-4+34(t+1)$ $3(t+1)(t^2-3)=17t+15$ $3t^3+3t^2-9t-9=17t+15$ $3t^3+3t^2-26t-24=0$ Using GC, $t=-3$ (given) or $t=-0.915$ (3 s.f.) or $t=2.91$ (3 s.f.)	Many students attempt to convert the parametric eqn of the curve to cartesian form first then solve for the *x* values, then solve for the *t* values which lead to a longer method. Please note that this method may not work for all questions as it may be hard/impossible to convert to cartesian form. Also, many students didn't make use of their **GC** to solve and hence wasted their time to solve algebraically.

Q8. Graphing Techniques

Assessment Objectives	Solution	Examiner's Feedback
Perform long division	**(i)** $y=\frac{2x^2-3x+5}{x-5}=2x+7+\frac{40}{x-5}$ $\therefore p=2$ $q=7$ $r=40$ $\begin{array}{r} 2x+7 \\ x-5\overline{)2x^2-3x+5} \\ -(2x^2-10x) \\ \hline 7x+5 \\ -(7x-35) \\ \hline 40 \end{array}$	This part was generally well done.
Identify characteristic of asymptotes, turning points and axial intercepts. Use of a G.C. to graph a given function.	**(ii)** Asymptotes: $y=2x+7$ and $x=5$  	Candidates need to use a ruler for axis and asymptotes to provide an accurate sketch. Many students did not write down intercept in coordinate form.

Understand that the number of intersections is equivalent to the number of roots in an equation.	**(ii)** $\left(2x^2-3x+5\right)^2=5x\left(x-5\right)^2$ $\left(\frac{2x^2-3x+5}{x-5}\right)^2=5x$ $y^2=5x$ $y=\pm\sqrt{5x}$ Sketch $y=\pm\sqrt{5x}$ in part **(ii).** From the diagram, there are 2 points of intersections. Hence, there are 2 roots.	Many students did not include $y=-\sqrt{5x}$. Some students drew a sketch of $y=\pm\sqrt{5x}$ which did not touch the origin.

Q9. Maclaurin's Series

Assessment Objectives	Solution	Examiner's Feedback
Use of series expansion formula in MF26.	**(a)** **Method ①:** $\sqrt{4-x}=4^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ $=2\left(1-\frac{x}{8}+\dots\right)$ $=2-\frac{x}{4}+\dots$ $p+\ln\left(q-x\right)=p+\ln\left[\left(q\right)\left(1-\frac{x}{q}\right)\right]$ $=p+\ln q+\ln\left(1-\frac{x}{q}\right)$ $=\left(p+\ln q\right)-\frac{x}{q}+\dots$ Comparing, $2=p+\ln q$ and $-\frac{x}{4}=-\frac{x}{q}$ $2=p+\ln 4$ $q=4$ $p=2-\ln 4$ **Method ②:** Let $\mathrm{f}(x)=\sqrt{4-x}\Rightarrow \mathrm{f}'(x)=\frac{-1}{2\sqrt{4-x}}$, $\therefore \mathrm{f}(0)=2\ \&\ \mathrm{f}'(0)=-\frac{1}{4}$ Let $\mathrm{g}(x)=p+\ln(q-x)\Rightarrow \mathrm{g}'(x)=\frac{-1}{q-x}$, $\therefore \mathrm{g}(0)=p+\ln q\ \&\ \mathrm{f}'(0)=-\frac{1}{q}$ Comparing, $q=4$ and $p=2-\ln 4$	Quite a majority of the students attempted this question successfully with a variety of methods. The most successful method being the use of repeated derivatives to form equations in *p* and *q*. Common errors include erroneous use of the standard series expansions and also not knowing how to convert the expressions into the standard form required in their use. A significant number of students also made arithmetic errors on the rules of logarithms, resulting in many marks lost.

Implicit differentiation involving trigonometric expressions. Use of formula given in MF26 to find Maclaurin series.	**(i)** $y=\tan^{-1}\left(ax+1\right)$ $\tan y=ax+1$ $\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=a$ $\frac{\mathrm{d}y}{\mathrm{d}x}=a\cos^2 y$ (shown) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2a\cos y(-\sin y)\frac{\mathrm{d}y}{\mathrm{d}x}=-a\sin 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^2}=-2a\cos 2y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-a\sin 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ When $x=0$, $y=\tan^{-1}(1)=\frac{\pi}{4}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=a\left(\cos\frac{\pi}{4}\right)^2=\frac{1}{2}a$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-a\left(\sin\frac{\pi}{2}\right)\left(\frac{1}{2}a\right)=-\frac{1}{2}a^2$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-2a\left(\cos\frac{\pi}{2}\right)\left(\frac{1}{2}a\right)^2-a\left(\sin\frac{\pi}{2}\right)\left(-\frac{1}{2}a^2\right)=\frac{1}{2}a^3$ $\tan^{-1}\left(ax+1\right)=\frac{\pi}{4}+\frac{1}{2}ax+\frac{-\frac{1}{2}a^2}{2!}x^2+\frac{\frac{1}{2}a^3}{3!}x^3+\dots$ $=\frac{\pi}{4}+\frac{1}{2}ax-\frac{1}{4}a^2x^2+\frac{1}{12}a^3x^3+\dots$	Most students performed badly for this question as they are unclear about the process of implicit differentiation, often omitting the multiplication of the first derivative. Students who attempted direct differentiation are rarely successful due to the complexity of the equations. Most students who are successful with the repeated differentiation ended up with the correct expression, except a few who made arithmetic errors on the coefficients.

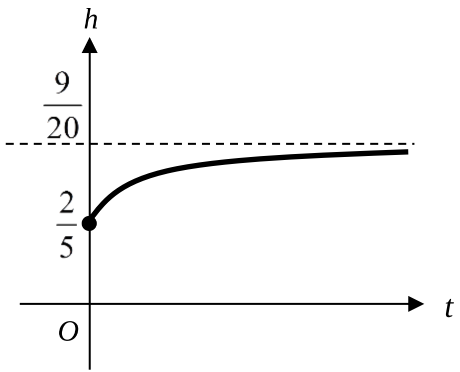
Use of chain rule and formula given in MF26 to differentiate $\tan^{-1}(4x+1)$, and make use of expression found in **(i)**.	**(ii)**	**Method ① (HENCE: direct differentiation using MF26)** $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}(4x+1)\right]=\frac{4}{1+(4x+1)^2}$ $\frac{1}{1+(4x+1)^2}=\frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}(4x+1)\right]$ $=\frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\pi}{4}+2x-4x^2+\frac{16}{3}x^3+\dots\right]$ $=\frac{1}{4}\left[2-8x+16x^2+\dots\right]$ $=\frac{1}{2}-2x+4x^2+\dots$ **Method ② (OTHERWISE: binomial expansion)** $\frac{1}{1+(4x+1)^2}=\left[1+\left(16x^2+8x+1\right)\right]^{-1}$ $=2^{-1}\left[1+\left(4x+8x^2\right)\right]^{-1}$ $=\frac{1}{2}\left[1-\left(4x+8x^2\right)+\frac{(-1)(-2)}{2!}(4x)^2+\dots\right]$ $=\frac{1}{2}\left[1-4x-8x^2+16x^2+\dots\right]$ $=\frac{1}{2}-2x+4x^2+\dots$ **Method ③ (OTHERWISE: repeated differentiation)** $\mathrm{f}(x)=\frac{1}{1+(4x+1)^2}\Rightarrow \mathrm{f}'(x)=\frac{-8(4x+1)}{\left[1+(4x+1)^2\right]^2}\Rightarrow \mathrm{f}(0)=\frac{1}{2}\ \&\ \mathrm{f}'(0)=-2$ $\Rightarrow \mathrm{f}''(x)=\frac{-32\left[1+(4x+1)^2\right]^2+128(4x+1)^2\left[1+(4x+1)^2\right]}{\left[1+(4x+1)^2\right]^4}\Rightarrow \mathrm{f}''(0)=8$ $\therefore \mathrm{f}(x)=\frac{1}{2}-2x+4x^2+\dots$	Most students were unable to see the link necessary for the “hence” method and adopted the otherwise methods. The most successful methods were those which involved repeated differentiation as it does not depend on the previous answers. Many students attempted to use the series expansion for $(1+x)^n$ using $(4x+1)^2$ in place of *x*, but failing to realize that all powers of $(4x+1)$ will result in terms which have to be include (i.e. constant, *x* and x^2).

<table>
<tr><th colspan="3">Q10. Differential Equations</th></tr>
<tr><th>Assessment Objectives</th><th>Solution</th><th>Examiner’s Feedback</th></tr>
<tr><td>Use of chain rule

Formulate differential equation from a problem situation</td><td>(i) $V=\pi\left(4^2\right)h$
$\frac{\mathrm{d}V}{\mathrm{d}h}=16\pi$
$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V_{\text{in}}}{\mathrm{d}t}-\frac{\mathrm{d}V_{\text{out}}}{\mathrm{d}t}$
$=0.36\pi-0.8\pi h$
$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\cdot\frac{\mathrm{d}h}{\mathrm{d}t}$
$0.36\pi-0.8\pi h=16\pi\cdot\frac{\mathrm{d}h}{\mathrm{d}t}$
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{0.36\pi-0.8\pi h}{16\pi}$
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{400}\left(9-20h\right)$ (shown)</td><td>This part is usually well done.

Some candidates introduced <i>t</i>, representing time, in an attempt to establish an equation of <i>h</i> in terms of <i>t</i>. Followed by wrong differentiation of <i>h</i> with respect to <i>t</i> .This approach earn no mark.</td></tr>
</table>

Solve differential equations to find particular solution.	**(ii)** $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{400}(9-20h)$ $\int \frac{1}{9-20h}\,\mathrm{d}h=\frac{1}{400}\int 1\,\mathrm{d}t$ $-\frac{1}{20}\int \frac{-20}{9-20h}\,\mathrm{d}h=\frac{1}{400}\int 1\,\mathrm{d}t$ $-\frac{1}{20}\ln\lvert 9-20h\rvert=\frac{1}{400}(t+A)$ $\ln\lvert 9-20h\rvert=-\frac{1}{20}(t+A)$ $\lvert 9-20h\rvert=\mathrm{e}^{-\frac{1}{20}(t+A)}$ $9-20h=\pm\mathrm{e}^{-\frac{1}{20}(t+A)}$ $9-20h=\pm\mathrm{e}^{-\frac{1}{20}t}\cdot\mathrm{e}^{-\frac{1}{20}A}$ $9-20h=B\mathrm{e}^{-\frac{1}{20}t}$ where $B=\pm\mathrm{e}^{-\frac{1}{20}A}$ When $t=0$, $h=0.4$, $9-20(0.4)=B\mathrm{e}^{-\frac{1}{20}(0)}$ $B=1$ $\therefore\ 9-20h=\mathrm{e}^{-\frac{1}{20}t}$ $20h=9-\mathrm{e}^{-\frac{1}{20}t}$ $h=\frac{1}{20}\left(9-\mathrm{e}^{-\frac{1}{20}t}\right)$	Careless mistakes in writing the numbers are unusually frequent in this part and resulted in marks loss. Generally well done.

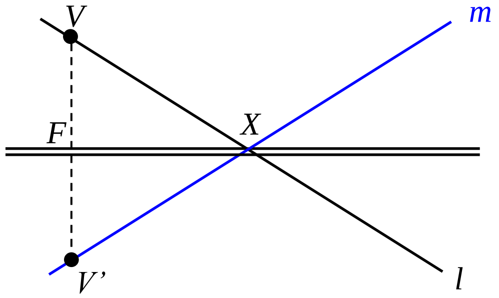
Interpret a differential equation and its solution in terms of a problem situation.	**(iii)** If the tap is on indefinitely, the tank will not be empty. In the long run, there will be $\frac{9}{20}$ m of water in the tank.	No marks awarded to candidates who attempted to explain in words without clear reference to the mathematical equation obtained earlier.
Interpret a differential equation and sketch a graph.	**(iv)** $h=\frac{1}{20}\left(9-\mathrm{e}^{-\frac{1}{20}t}\right)$ 	Many candidates often overlooked the presence of a horizontal asymptote. Lacks proper labelling of axes or asymptote.

Q11. Vectors		
Assessment Objectives	**Solution**	**Examiner's Feedback**
Solve a two-dimensional vector geometry problem involving abstract vectors, by : • Using the collinearity theorem to formulate expressions for the position vector of an unknown point, in terms of two non-zero non-parallel base vectors, and • Compare and equate corresponding coefficients of respective base vectors in a vector equation, to solve the problem.	**(i)** $\overrightarrow{AM} = -\mathbf{a}+\tfrac{1}{2}\mathbf{b}$ $\overrightarrow{BN} = -\mathbf{b}+\tfrac{1}{2}\mathbf{a}$ $\overrightarrow{OX} = \mathbf{a}+\overrightarrow{AX} \qquad = \mathbf{b}+\overrightarrow{BX}$ $= \mathbf{a}+\lambda(-\mathbf{a}+\tfrac{1}{2}\mathbf{b}) \quad = \mathbf{b}+\mu(-\mathbf{b}+\tfrac{1}{2}\mathbf{a})$ for some scalars $\lambda,\ \mu$ $(1-\lambda)\mathbf{a}+\tfrac{\lambda}{2}\mathbf{b} \ = \tfrac{\mu}{2}\mathbf{a}+(1-\mu)\mathbf{b}$ $\therefore \begin{cases} 1-\lambda=\frac{\mu}{2} \\ \frac{\lambda}{2}=1-\mu \end{cases}$ Solving, $\lambda=\mu=\tfrac{2}{3}$ $\therefore\ \overrightarrow{OX} = \mathbf{a}+\tfrac{2}{3}(-\mathbf{a}+\tfrac{1}{2}\mathbf{b}) \ = \tfrac{1}{3}\mathbf{a}+\tfrac{1}{3}\mathbf{b}$ (shown) **Method ①: (Using Equation of Lines)** $l_{AM}: \mathbf{r} = \mathbf{a}+\lambda\left(-\mathbf{a}+\frac{1}{2}\mathbf{b}\right),\ \lambda\in\mathbb{R}$ $l_{BN}: \mathbf{r} = \mathbf{b}+\mu\left(-\mathbf{b}+\frac{1}{2}\mathbf{a}\right),\ \mu\in\mathbb{R}$ Since X lies on both lines, $\mathbf{a}+\lambda\left(-\mathbf{a}+\frac{1}{2}\mathbf{b}\right) = \mathbf{b}+\mu\left(-\mathbf{b}+\frac{1}{2}\mathbf{a}\right)$ $\begin{cases} 1-\lambda=\dfrac{\mu}{2} \\ \dfrac{\lambda}{2}=1-\mu \end{cases}$ Solving, $\lambda=\mu=\tfrac{2}{3}$ $\therefore\ \overrightarrow{OX} = \mathbf{a}+\tfrac{2}{3}(-\mathbf{a}+\tfrac{1}{2}\mathbf{b}) \ = \tfrac{1}{3}\mathbf{a}+\tfrac{1}{3}\mathbf{b}$ (shown)	Most students were able to obtain at least 2 out of 4 marks by using ratio theorem to find $\overrightarrow{AM}$ and $\overrightarrow{BN}$, but some were unsure how to continue. Since **a** and **b** are non-parallel, we can compare the coefficients of the 2 vectors to obtain the 2 equations.

	<u>Method ②:</u> (Using Ratio Theorem) Using triangle OAM, $\overrightarrow{OX}=\dfrac{\lambda(\mathbf{a})+(1-\lambda)\left(\frac{1}{2}\mathbf{b}\right)}{\lambda+(1-\lambda)}=\lambda\mathbf{a}+\dfrac{(1-\lambda)}{2}\mathbf{b}$ Using triangle ONB, $\overrightarrow{OX}=\dfrac{\mu\left(\frac{1}{2}\mathbf{a}\right)+(1-\mu)\mathbf{b}}{\mu+(1-\mu)}=\dfrac{1}{2}\mu\mathbf{a}+(1-\mu)\mathbf{b}$ $\lambda\mathbf{a}+\dfrac{(1-\lambda)}{2}\mathbf{b}=\dfrac{1}{2}\mu\mathbf{a}+(1-\mu)\mathbf{b}$ $\begin{cases}\lambda=\dfrac{\mu}{2}\\ \dfrac{1-\lambda}{2}=1-\mu\end{cases}$ Solving, $\lambda=\dfrac{1}{3},\ \mu=\dfrac{2}{3}$ $\overrightarrow{OX}=\dfrac{1}{3}\mathbf{a}+\dfrac{1}{3}\mathbf{b}$ (shown)	
Apply the midpoint theorem. Apply the collinearity theorem to determine whether three distinct points are collinear.	**(ii)** $\overrightarrow{OT}=\frac{1}{2}(\mathbf{a}+\mathbf{b})$ using the midpoint theorem $\overrightarrow{OX}=\frac{1}{3}(\mathbf{a}+\mathbf{b})$ $=\frac{2}{3}\left[\frac{1}{2}(\mathbf{a}+\mathbf{b})\right]=\frac{2}{3}\overrightarrow{OT}$ Since $\overrightarrow{OX}=k\overrightarrow{OT}$ for some scalar k where $0<k<1$, $\overrightarrow{OX}$ is parallel to $\overrightarrow{OT}$ with a common point *O*, hence *X* lies on *OT*.	Most students gave an incomplete proof for *X* lying on *OT*. It is essential to show that $\overrightarrow{OX}$ is a scalar multiple of $\overrightarrow{OT}$ and hence the 2 vectors are parallel.

Find a normal vector for a plane given three non-collinear points on the plane. Formulate a vector equation of a plane in scalar product form, using a point on the plane and a normal vector to the plane.	**(iii)** $\overrightarrow{OA}=\begin{pmatrix}5\\4\\6\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}-2\\2\\3\end{pmatrix}$ $\overrightarrow{OA}\times\overrightarrow{OB}=\begin{pmatrix}5\\4\\6\end{pmatrix}\times\begin{pmatrix}-2\\2\\3\end{pmatrix}=\begin{pmatrix}(4)(3)-(6)(2)\\(6)(-2)-(5)(3)\\(5)(2)-(4)(-2)\end{pmatrix}$ $=\begin{pmatrix}0\\-27\\18\end{pmatrix}=9\begin{pmatrix}0\\-3\\2\end{pmatrix}$ Since $\begin{pmatrix}0\\-3\\2\end{pmatrix}$ is perpendicular to the plane, and origin *O* is on the plane, it is represented by $\mathbf{r}\cdot\begin{pmatrix}0\\-3\\2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\cdot\begin{pmatrix}0\\-3\\2\end{pmatrix}=0.$ $\therefore\ -3y+2z=0$ (shown)	Most students were able to obtain the normal of the plane. Since *O* is on the plane, the most direct method is to cross $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Find the foot of the perpendicular from a given point to a given plane, by: • Formulate an equation for the perpendicular line passing through the point, and • Find the point of intersection between this perpendicular line and the plane.	**(iv)** Line *VF*, $l_{VF}: \mathbf{r} = \begin{pmatrix} 1 \\ -68 \\ -37 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}, \ \lambda \in \mathbb{R}$. Since *F* is on l_{VF}, $\overrightarrow{OF} = \begin{pmatrix} 1 \\ -68 \\ -37 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$, for some $\lambda \in \mathbb{R}$. Since *F* is on *p*, $\overrightarrow{OF} \cdot \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} = 0$. $\Rightarrow \left[\begin{pmatrix} 1 \\ -68 \\ -37 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} \right] \cdot \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} = 0$ $130 + 13\lambda = 0$ $\lambda = -10$ $\overrightarrow{OF} = \begin{pmatrix} 1 \\ -68 \\ -37 \end{pmatrix} + (-10) \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -38 \\ -57 \end{pmatrix}$ The coordinates of *F* is $(1, -38, -57)$.	Some students had the **misconception** that $\overrightarrow{VF} = \begin{pmatrix} 1 \\ -68 \\ -37 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$ and that since $\overrightarrow{VF}$ is parallel to the normal of plane, $\overrightarrow{VF} \cdot \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} = 0$. The 2 vectors are parallel, **not** perpendicular. A number of students were careless in solving for the value of λ.

Given a line and a plane that intersects at a point, construct a vector equation for the reflection of a line in a plane, by : - Locating the point of intersection between the line and the plane, - Finding the point of reflection of another point on the line in the plane, and - Constructing a vector equation of the reflected line containing these two points. Convert a vector equation for the line into a Cartesian equation.	**(v)** $\overrightarrow{OX}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB})=\begin{pmatrix}1\\2\\3\end{pmatrix}$  Let V' be the reflection of V in plane p. $\overrightarrow{OF}=\dfrac{\overrightarrow{OV}+\overrightarrow{OV'}}{2}$ $\left[\text{or use } \overrightarrow{VF}=\overrightarrow{FV'}\right]$ $\overrightarrow{OV'}=2\overrightarrow{OF}-\overrightarrow{OV}$ $\overrightarrow{OV'}=2\begin{pmatrix}1\\-38\\-57\end{pmatrix}-\begin{pmatrix}1\\-68\\-37\end{pmatrix}=\begin{pmatrix}1\\-8\\-77\end{pmatrix}$ $\overrightarrow{V'X}=\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}1\\-8\\-77\end{pmatrix}=\begin{pmatrix}0\\10\\80\end{pmatrix}=10\begin{pmatrix}0\\1\\8\end{pmatrix}$ Line $m:\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+k\begin{pmatrix}0\\1\\8\end{pmatrix},\ k\in\mathbb{R}$ Line $m:\ x=1,\ y-2=\dfrac{z-3}{8}$	Some students failed to notice that X is the centroid of triangle OAB although it was mentioned in the question. Common mistakes include using $\overrightarrow{OX}=\dfrac{\overrightarrow{OV}+\overrightarrow{OV'}}{2}$ instead of $\overrightarrow{OF}$. The above mistake could have been avoided if the student had **drawn a diagram**. The question asked for a cartesian equation of line m, hence students were penalized for giving the vector equation form as the final answer.

1 The curve with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a cubic polynomial, has a maximum point with coordinates $\left(-2, \frac{34}{3}\right)$ and a minimum point with coordinates $\left(3, -\frac{19}{2}\right)$. Find the equation of the curve. [4]

2 Referred to the origin O, the points A, B, P and Q have position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{p}$ and $\mathbf{q}$ respectively, such that $|\mathbf{a}| = 2$, $\mathbf{b}$ is a unit vector, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$.

(i) Give a geometrical interpretation of $|\mathbf{b} \cdot \mathbf{a}|$. [1]

(ii) Find $|\mathbf{a} \times \mathbf{b}|$, leaving your answer in exact form. [2]

It is also given that $\mathbf{p} = 3\mathbf{a} + (\mu + 2)\mathbf{b}$ and $\mathbf{q} = (\mu + 3)\mathbf{a} + \mu\mathbf{b}$, where $\mu \in \mathbb{R}$.

(iii) Show that $\mathbf{p} \times \mathbf{q} = \left(\mu^2 + 2\mu + 6\right)(\mathbf{b} \times \mathbf{a})$. [3]

(iv) Hence find the smallest area of the triangle OPQ as μ varies. [3]

3 The function f is defined by

$$\mathrm{f} : x \mapsto \frac{1}{3}\tan\left(\frac{x}{3}\right) \text{ for } x \in \mathbb{R},\ 0 \le x < \frac{3\pi}{2}.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$, indicating clearly the vertical asymptote. [2]

(ii) State the equation of the line of reflection between the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, and hence sketch the graph of $y = \mathrm{f}^{-1}(x)$ on the same diagram, indicating clearly the horizontal asymptote. [2]

The solutions to the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ are $x = 0$ and $x = \alpha$, where $0 < \alpha < \frac{3\pi}{2}$.

(iii) Using the diagram drawn, find, in terms of α, the area of the region bounded by the curves $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [5]

Another function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{e}^x \text{ for } x \in \mathbb{R},\ x \ge -2.$$

(iv) Show that the composite function gf exists and define gf in a similar form. [3]

4 **(a)** The complex numbers z and w satisfy the simultaneous equations

$$z + w^* + 5\mathrm{i} = 10 \quad \text{and} \quad |w|^2 = z + 18 + \mathrm{i}.$$

Find z and w. [4]

(b) **(i)** It is given that $2+\mathrm{i}$ is a root of the equation $z^2-5z+7+\mathrm{i}=0$. Find the second root of the equation in cartesian form, showing your working clearly. [2]

(ii) Hence find the roots of the equation $-\mathrm{i}w^2+5w+7\mathrm{i}-1=0.$ [2]

(c) The complex number z is given by $z=-a+a\mathrm{i}$, where a is a positive real number.

(i) It is given that $w=-\dfrac{\sqrt{2}z^*}{z^4}$. Express w in the form $re^{\mathrm{i}\theta}$, in terms of a, where $r>0$ and $-\pi<\theta\leq\pi$. [4]

(ii) Find the two smallest positive whole number values of n such that $\mathrm{Re}\,(w^n)=0.$ [3]

5 A planning committee of 12 students consisting of one male and one female student from each of the 6 Arts stream classes (Class A to Class F) in a junior college is to be formed for the Humanities Seminar. There are 10 male and 10 female students in Class A.

(i) How many ways can the representatives from Class A be chosen? [1]

The committee meets for their first planning meeting and is seated at a round table.

(ii) How many ways can the committee be seated if all the members need to be seated together with the member from the same class? [2]

At the seminar, the committee members are to be seated in a row of 14 seats in the theatre together with the Principal and the Guest of Honour. The chairperson and the secretary are selected from the committee and they are both from Class F.

(iii) How many ways can this be done if the Principal and the Guest of Honour occupy the middle seats and the committee members are seated together with the member from the same class except for the chairperson and the secretary? [4]

6 The table below shows the petrol mileage, y km/L and the weight, x kg in thousands for various car models in the year 1995.

x	3.5	3	2.75	2.5	2.25	2	1.75	1.5	1.25
y	7.5	8.0	8.5	8.7	10.0	k	13.5	16.8	18.0

(i) The equation of the regression line of y on x is $y=22.51355-4.908387x.$ Show that $k=11.0$. [2]

(ii) Draw a scatter diagram to illustrate the data. [1]

(iii) With reference to the scatter diagram and context of the question, explain why model (C) below is the most appropriate for modelling the data as compared to the other 2 models.

(A) $y = a + bx$, where a is positive and b is negative,

(B) $y = a + b\ln x$, where a is positive and b is negative,

(C) $y = a + \frac{b}{x}$, where a and b are positive. [1]

(iv) Calculate the least squares estimates of a and b for model (C). [1]

(v) Predict the weight of the car if the petrol mileage is 12 km/L. Comment on the reliability of your prediction. [2]

(vi) Suppose there was an error in recording the y values and all the y values must be increased by a constant M km/L, state any change you would expect in the values of

(a) $\bar{y}$, [1]

(b) standard deviation of y and [1]

(c) the correlation coefficient. [1]

7 **(a)** The random variable X follows a binomial distribution $\mathrm{B}(10, p)$.

(i) Given that X has two modes, $X = 4$ and $X = 5$, find the exact value of p. [2]

(ii) Given instead that $\mathrm{P}(X \le 9) = \frac{1023}{1024}$, find the exact value of p. [2]

(b) The random variable Y follows a binomial distribution $\mathrm{B}(500, 0.5)$.

A sample of 30 independent values of Y is recorded.

(i) Find the probability that all the values recorded are less than or equal to 256. [2]

(ii) The mean of the 30 values is calculated. Estimate the probability that this sample mean is less than or equal to 256, stating clearly the approximation used. [3]

(iii) Explain why the probability found in part **(ii)** is larger than that found in part **(i)**. [1]

8 A trading card game has rectangular cards of nominal size 64 mm wide and 89 mm long. However, due to the limited precision of the machine used to cut the cards to size, the widths of the trading cards follow a normal distribution with mean 64 mm and standard deviation 0.3 mm. The lengths of the trading cards follow an independent normal distribution with mean 89 mm and standard deviation 0.45 mm. The perimeter of the trading cards is twice the sum of its length and width.

(i) Trading cards with length 90 mm and above are called "tall" cards. Find the percentage of trading cards that are "tall". [1]

(ii) Write down the distribution of the perimeter of the trading cards, in mm, and find the perimeter that is exceeded by 8% of the trading cards. [4]

A brand of rectangular card sleeves are manufactured for the trading cards and the widths of the card sleeves follow a normal distribution with mean 66 mm and standard deviation 0.45 mm, whereas the lengths of the card sleeves follow an independent normal distribution with mean 91 mm and standard deviation 0.675 mm.

For a card sleeve to fit the trading card nicely, the dimensions of the sleeves must be larger than the dimensions of the trading card, but there should only be a maximum allowance of 1.2 mm on each side.

(iii) Find the probability that a randomly chosen card sleeve fits a randomly chosen trading card nicely, stating clearly the parameters of any distribution used. [5]

9 A computer hard drive manufacturer claims that the mean usage hours before failure of their R series hard drives is 50 thousand hours. A technology columnist wishes to investigate this claim and collected the usage hours, t thousand hours for each of the 50 randomly chosen hard drives which were submitted to the local service centre for drive failures. The data is summarized as follows.

$$n = 50 \qquad \Sigma t = 2384.5 \qquad \Sigma t^2 = 115885.23$$

The technology columnist wants to use hypothesis testing to test whether the mean usage hours before failure of a hard drive is different from what the manufacturer has stated.

(i) Explain whether it is necessary for the columnist to know about the distribution of the usage hours before failure of the drives in order to carry out a hypothesis test. [1]

(ii) Find the unbiased estimates of the population mean and variance and carry out the test at 1% level of significance for the columnist. [6]

The columnist published the data and the results of the hypothesis testing in an online article.

(iii) Suggest a reason why the test result might not be useful to a reader of the article who is deciding whether to buy an R series hard drive from the manufacturer. [1]

(iv) State an alternative hypothesis that is more relevant to the decision making process and explain whether the result will differ from the earlier test carried out by the columnist at 1% level of significance. [2]

(v) State a necessary assumption that was made for all the tests carried out. [1]

10 A box contains 2 red balls, 3 green balls and x blue balls, where $x \in \mathbb{Z}, x \geq 5$. A game is played where the contestant picks 5 balls from the box without replacement. The total score, S, for the contestant is the sum of the number of green balls chosen and thrice the number of red balls chosen. The blue balls will not contribute any points, unless all 5 balls are blue. If all the 5 balls are blue, the score will be 25 points.

(i) Show that $\mathrm{P}(S=6)=\dfrac{20x(x^2-3x+14)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$. [2]

(ii) Given that $\mathrm{P}(S=6)=\dfrac{5}{63}$, calculate x. [2]

(iii) Complete the probability distribution table for S. [4]

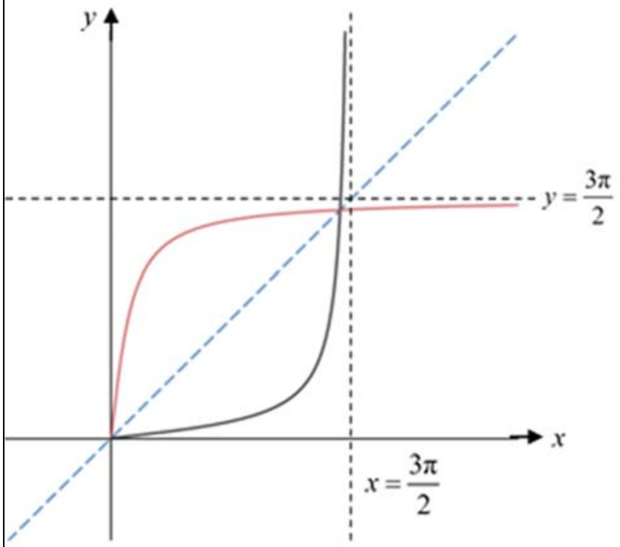
s	1	2	3	4	5	6	7	8	9	25
$\mathrm{P}(S=s)$		$\frac{5}{42}$	$\frac{5}{63}$		$\frac{5}{21}$	$\frac{5}{63}$		$\frac{5}{84}$	$\frac{1}{252}$	

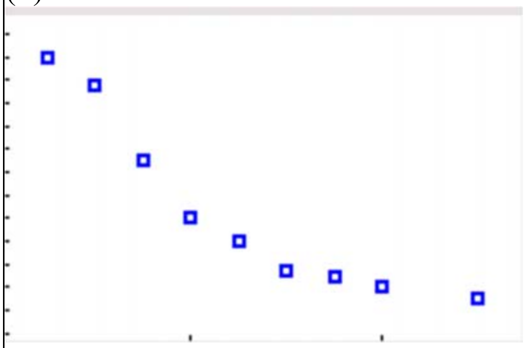
(iv) Evaluate $\mathrm{E}(S)$ and find the probability that S is more than $\mathrm{E}(S)$. [2]

(v) Find the probability that there are no green balls drawn given that S is more than $\mathrm{E}(S)$.[2]

ANNEX B

CJC H2 Math JC2 Preliminary Examination Paper 2

QN	Topic Set	Answers
1	Equations and Inequalities	$y=\frac{1}{3}x^3-\frac{1}{2}x^2-6x+4$
2	Vectors	(ii) $\sqrt{2}$ (iv) $\frac{5\sqrt{2}}{2}\text{unit}^2$
3	Functions	(i)  (ii) $y=x$ (iii) $\alpha^2+2\ln\left[\cos\left(\frac{\alpha}{3}\right)\right]$ (iv) $\mathrm{g\,f}: x\mapsto \mathrm{e}^{\frac{1}{3}\tan\left(\frac{x}{3}\right)}$ for $x\in\mathbb{R}$, $0\le x<\frac{3\pi}{2}$
4	Complex numbers	(a) $w=3+4\mathrm{i},\ z=7-\mathrm{i}$ $w=-4+4\mathrm{i},\ z=14-\mathrm{i}$ (b)(i) $3-\mathrm{i}$ (ii) $w=1-2\mathrm{i},\ w=-1-3\mathrm{i}$ (c)(i) $\frac{1}{2a^3}\mathrm{e}^{\mathrm{i}\left(-\frac{3\pi}{4}\right)}$ (ii) 2, 6
5	P&C, Probability	(i) 100 (ii)7680 (iii)92160

6	Correlation & Linear Regression	(ii)  (iv) $a = 0.257$, $b = 22.8$ (v) 1860kg (vi)(a) $\overline{y}$ will be increased by a. (b) remain unchanged. (c) remain unchanged.
7	Binomial Distribution	(a)(i) $p = \frac{5}{11}$ (ii) $p = \frac{1}{2}$ (b)(i) $= 0.0000514$ (ii) 0.998
8	Normal Distribution	(i) 1.31%. (ii) $t = 307.51$mm (iii)0.525
9	Hypothesis Testing	(ii) $\overline{t} = 47.69$ thousand hours $s^2 = 44.3$ (iv)Yes
10	DRV	(ii) $x = 5$ (iv) $\frac{127}{252}$ or 0.504 (v) $\frac{11}{127}$

CATHOLIC JUNIOR COLLEGE
H2 MATHEMATICS
2017 JC2 PRELIMINARY EXAMINATION PAPER II SOLUTION

Q1. System of Linear Equations

Assessment Objectives	**Solution**	**Examiner's Feedback**
Formulate a system of linear equations. Solve a system of linear equations using a G.C.	**(i)** $y = ax^3 + bx^2 + cx + d$ Curve passes through $\left(-2, \frac{34}{3}\right)$: $a(-2)^3 + b(-2)^2 + c(-2) + d = \frac{34}{3}$ $-8a + 4b - 2c + d = \frac{34}{3}$ — ① Curve passes through $\left(3, -\frac{19}{2}\right)$: $a(3)^3 + b(3)^2 + c(3) + d = -\frac{19}{2}$ $27a + 9b + 3c + d = -\frac{19}{2}$ — ② $\frac{\mathrm{d}y}{\mathrm{d}x} = 3ax^2 + 2bx + c$ Curve has maximum point $\left(-2, \frac{34}{3}\right)$: $3a(-2)^2 + 2b(-2) + c = 0$ $12a - 4b + c = 0$ — ③ Curve has minimum point $\left(3, -\frac{19}{2}\right)$: $3a(3)^2 + 2b(3) + c = 0$ $27a + 6b + c = 0$ — ④	Most common mistake: - Some students assumed the coeff of x^3 is 1, eg, $y = x^3 + bx^2 + cx + d$ Some attempt to form ONLY 2 or 3 equations to solve for 4 unknowns; note that at least 4 eqns are needed to solve for 4 unknowns. A few students left their eqn as $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$ instead of $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$

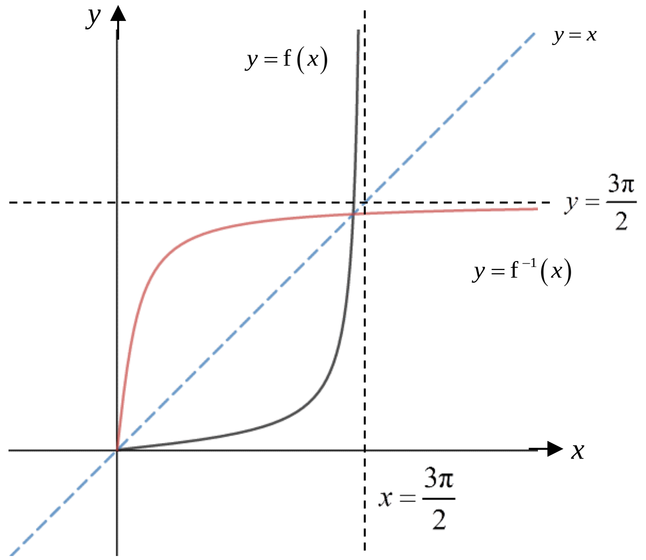
	Solving, $a=\frac{1}{3}, b=-\frac{1}{2}, c=-6, d=4$ $\therefore\ y=\frac{1}{3}x^3-\frac{1}{2}x^2-6x+4$	

Q2. Vectors

Assessment Objectives	Solution	Examiner’s Feedback
Concept of geometrical interpretation.	**(i)** Length of projection of $\mathbf{a}$ on to $\mathbf{b}$	Generally OK, but many gave the answer as length of projection of $\mathbf{b}$ onto $\mathbf{a}$.
Concept of unit vector and cross product formula.	**(ii)** $\lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$ $=(2)(1)\sin\frac{\pi}{4}$ $=\sqrt{2}$	Many students mixed up the definition of dot and cross product, although $\sin\frac{\pi}{4}$ is the same as $\cos\frac{\pi}{4}$ which some students ended up with the correct final answer, but they still get penalized as they are using the wrong definition.
Expansion of cross product.	**(iii)** $\mathbf{p}\times\mathbf{q}$ $=\left[3\mathbf{a}+(\mu+2)\mathbf{b}\right]\times\left[(\mu+3)\mathbf{a}+\mu\mathbf{b}\right]$ $=3(\mu+3)(\mathbf{a}\times\mathbf{a})+3\mu(\mathbf{a}\times\mathbf{b})+(\mu^2+5\mu+6)(\mathbf{b}\times\mathbf{a})+\mu(\mu+2)(\mathbf{b}\times\mathbf{b})$ $=(-3\mu+\mu^2+5\mu+6)(\mathbf{b}\times\mathbf{a})$ $[\because \mathbf{a}\times\mathbf{a}=\mathbf{0}$ and $\mathbf{b}\times\mathbf{b}=\mathbf{0}]$ $=(\mu^2+2\mu+6)(\mathbf{b}\times\mathbf{a})$	Common mistakes: - $\mathbf{a}\times\mathbf{a}=\lvert\mathbf{a}\rvert^2$ - The third term in the expansion was $(\mu^2+5\mu+6)(\mathbf{a}\times\mathbf{b})$ instead of $(\mu^2+5\mu+6)(\mathbf{b}\times\mathbf{a})$; note that the direction of cross product is important. - $\underset{\sim}{a}\times\underset{\sim}{a}=\underset{\sim}{0}\rightarrow$ null vector not $\underset{\sim}{a}\times\underset{\sim}{a}=0$
Finding stationary value.	**(iv)** Area $OPQ=\frac{1}{2}\lvert(\mu^2+2\mu+6)\rvert\lvert(\mathbf{b}\times\mathbf{a})\rvert$ $=\frac{1}{2}\lvert(\mu^2+2\mu+6)\rvert\sqrt{2}$ $=\frac{\sqrt{2}}{2}\lvert(\mu+1)^2+5\rvert$	Common mistake: $\frac{1}{2}(\mu^2+2\mu+6)\mathbf{b}\times\mathbf{a}$ Note that the above expression is a vector, not magnitude.

	Smallest Area $OPQ = \ = \frac{5\sqrt{2}}{2}\text{unit}^2$	

Q3. Functions & Definite Integrals

Assessment Objectives	Solution	Examiner's Feedback
Understand the relationship between a function and its inverse.	**(i)** 	Many students did not fully extend the curve past $x=\frac{3\pi}{2}$ and/or $y=\frac{3\pi}{2}$
	(ii) $y=x$	
Find area bounded by 2 curves.	**(iii)** **Method ①:** Area $=2\int_0^{\alpha} x-\frac{1}{3}\tan\left(\frac{x}{3}\right)\mathrm{d}x$ $=2\left[\frac{x^2}{2}-\ln\left\|\sec\left(\frac{x}{3}\right)\right\|\right]_0^{\alpha}$ $=2\left[\frac{\alpha^2}{2}-\ln\left\|\sec\left(\frac{\alpha}{3}\right)\right\|-0+0\right]$ $=\alpha^2+2\ln\left[\cos\left(\frac{\alpha}{3}\right)\right]$	Most students did not use this method, opting for the more tedious alternative. Students can use area of triangle formula $\frac{1}{2}\alpha(\alpha)$ instead of $\int_0^{a} x\,\mathrm{d}x$ (Some used $\frac{1}{2}\alpha\left(\frac{1}{3}\tan\left(\frac{\alpha}{3}\right)\right)$ which is not simplified

	Method ②: Area $=\int_0^\alpha 3\tan^{-1}3x-\frac{1}{3}\tan\left(\frac{x}{3}\right)\mathrm{d}x$ $=\left[3x\tan^{-1}3x\right]_0^\alpha-\int_0^\alpha 3x\frac{3}{1+(3x)^2}\mathrm{d}x-\left[\ln\left(\sec\frac{x}{3}\right)\right]_0^\alpha$ $=3\alpha\tan^{-1}3\alpha-\frac{1}{2}\int_0^\alpha\frac{18x}{1+9x^2}\mathrm{d}x-\ln\left(\sec\frac{\alpha}{3}\right)+\ln 1$ $=3\alpha\tan^{-1}3\alpha-\frac{1}{2}\left[\ln\left(1+9x^2\right)\right]_0^\alpha-\ln\left(\sec\frac{\alpha}{3}\right)$ $=3\alpha\tan^{-1}3\alpha-\frac{1}{2}\ln\left(1+9x^2\right)-\ln\left(\sec\frac{\alpha}{3}\right)$	Note the two answers are equal. Most students did this method, not utilizing the symmetry of the curves.
Determine if the composite function exists. Find the rule and domain of a composite function.	**(iv)** $\mathrm{R_f}=[0,\infty)$ $\mathrm{D_g}=[-2,\infty)$ Since $\mathrm{R_f}\subseteq\mathrm{D_g}$, gf exists. $\mathrm{gf}(x)=\mathrm{g}\left[\frac{1}{3}\tan\left(\frac{x}{3}\right)\right]$ $=\mathrm{e}^{\frac{1}{3}\tan\left(\frac{x}{3}\right)}$ $\mathrm{D_{gf}}=\mathrm{D_f}=\left[0,\frac{3\pi}{2}\right)$ $\mathrm{gf}:x\mapsto\mathrm{e}^{\frac{1}{3}\tan\left(\frac{x}{3}\right)}$ for $x\in\mathbb{R}$, $0\le x<\frac{3\pi}{2}$.	Common mistakes: $\mathrm{D_{gf}}=\mathrm{D_g}=[-2,\infty)$ Many students did not put in similar form

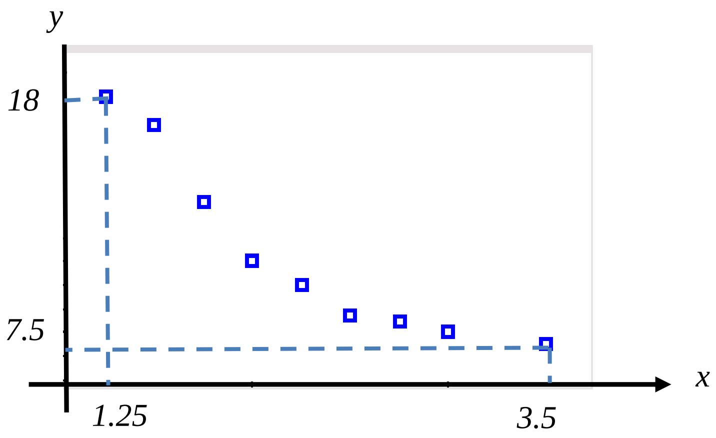
Q4. Complex Numbers		
Assessment Objectives	**Solution**	**Examiner's Feedback**
Solving simultaneous equations involving complex numbers.	**(a)** $z = 10 - w^* - 5\text{i}$ $\lvert w\rvert^2 = 10 - w^* - 5\text{i} + 18 + \text{i}$ $\lvert w\rvert^2 + w^* = 28 - 4\text{i}$ Let $w = a + b\text{i}$, $a^2 + b^2 + a - b\text{i} = 28 - 4\text{i}$ By comparing, $b = 4$, $a^2 + (4)^2 + a = 28$ $a^2 + a - 12 = 0$ $(a+4)(a-3) = 0$ $a = -4$ or $a = 3$ $\therefore w = 3 + 4\text{i}$ or $w = -4 + 4\text{i}$ When $w = 3 + 4\text{i}$, $z = 10 - (3 - 4\text{i}) - 5\text{i}$ $= 7 - \text{i}$ When $w = -4 + 4\text{i}$, $z = 10 - (-4 + 4\text{i}) - 5\text{i}$ $= 14 - 9\text{i}$	Most students were able to do this question except for the occasional slips in algebraic manipulation. A number of students mistook $\lvert w\rvert^2$ for w^2. Presentation for simultaneous equation is unclear.
	(b)(i) $z^2 - 5z + 7 + \text{i} = [z - (2+\text{i})][z - k]$ By comparing coefficient of z: $z^2 - 5z + 7 + \text{i} = [z - (2+\text{i})][z - k]$ $-5 = -k - (2 + \text{i})$ $k = 3 - \text{i}$ The second root is $3 - \text{i}$	Generally well done.

	(ii) $-\mathrm{i}w^2+5w+7\mathrm{i}-1=0$ $-w^2-5\mathrm{i}w+7+\mathrm{i}=0$ $(\mathrm{i}w)^2-5(\mathrm{i}w)+7+\mathrm{i}=0$ $\mathrm{i}w=2+\mathrm{i}$ or $\mathrm{i}w=3-\mathrm{i}$ $w=1-2\mathrm{i}$ or $w=-1-3\mathrm{i}$	Badly done. Most students fail to identify the term to replace.
Property of modulus and argument.	**(c)(i)** **Method ①:** $z=-a+a\mathrm{i}$ $=a\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{3\pi}{4}\right)}$ $w=\left(\mathrm{e}^{\mathrm{i}(\pi)}\right)\dfrac{\sqrt{2}a\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{3\pi}{4}\right)}}{4a^4\mathrm{e}^{\mathrm{i}(3\pi)}}$ $=\dfrac{1}{2a^3}\mathrm{e}^{\mathrm{i}\left(-\frac{11\pi}{4}+2\pi\right)}$ $=\dfrac{1}{2a^3}\mathrm{e}^{\mathrm{i}\left(-\frac{3\pi}{4}\right)}$	Badly done. Most students prefer to simplify the denominator but had problems with the algebraic manipulation. Most students got the argument wrong as they left their answer as $-\dfrac{1}{2a^3}\mathrm{e}^{\mathrm{i}\alpha}$, or they mistook $\arg(-\sqrt{2}z^*)=-\sqrt{2}\arg(z^*)$. Another common mistake was that many students left the argument of z as $\frac{\pi}{4}$.
Concepts of argument to find purely imaginary roots.	**(ii)** If $\operatorname{Re}(w^n)=0$, $n\left(-\dfrac{3\pi}{4}\right)=\dfrac{\pi}{2}+k\pi$, where $k\in\mathbb{Z}$ $n=-\dfrac{2}{3}(1+2k)$, where $k\in\mathbb{Z}$ Three smallest positive whole number values of n are 2, 6.	Most students got the method marks but lost the last mark as their argument was wrong.

Q5. Permutations and Combinations

Assessment Objectives	Solution	Examiner's Feedback
Solving simple counting problems involving multiplication principle	**(i)** No. of ways $= {}^{10}C_1 \times {}^{10}C_1 = 100$	Generally well done except for some who did ${}^{10}C_1 + {}^{10}C_1$ instead.
Solving counting problems involving circular arrangements	**(ii)** No. of ways $= (6-1)! \times (2!)^6 = 7680$	Some students used $(6-1)! \times 2!$ or $(6-1)! \times 6(2)!$ instead. Since there are 6 couples, and for each couple there are 2! ways to arrange them, we do $2! \times 2! \times 2! \times 2! \times 2! \times 2!$, which is different from $6(2)!$.
Solving complex arrangement problems involving restrictions	**(iii)** **<u>Method ①:</u>** (1) Arrange P and GoH $= 2!$ (2) Choose side where CP and Sec are on (left or right) $= {}^2C_1$ (3) Choose 2 classes to be seated with CP and Sec $= {}^5C_2$ (4) Arrange the 2 classes and the people within each class $= 2! \times (2!)^2$ (5) Slot in CP and Sec $= {}^3C_2 \times 2!$ (6) Arrange the 3 other classes and the people within each class $= 3! \times (2!)^3$ No. of ways = $2! \times 2 \times {}^5C_2 \times 2! \times (2!)^2 \times {}^3C_2 \times 2! \times 3! \times (2!)^3 = 92160$ **<u>Method ②:</u> (Arrange the 5 classes at one go)** No. of ways = $2! \times 2 \times 5! \times (2!)^5 \times {}^3C_2 \times 2! = 92160$ **<u>Method ③:</u> (Complement)** No. of ways = $\text{n}(\text{CP/S on 1 side but may be tog}) - \text{n}(\text{CP/S tog})$ $= \left\{2 \times \left[{}^5C_2 \times 3! \times 2^3\right] \times 2 \times (2)^2 \times 4!\right\} - \left[6! \times (2)^6 \times 2\right] = 92160$	Most students were able to get 1 or 2 marks for this part. Key is for the GoH and P to be seated in the middle, and for the CP and S to be separated, they must be on the same side. Otherwise with 5 students on one side and 7 students on the other side, GoH and P will not be in the middle. So the remaining 5 classes will be split to 3-2, with CP and S joining the side with 2 classes. Hence, using the 2 classes, we will choose 2 out of 3 slots for CP and S. Bear in mind that the 2 classes can either be on the left, or on the right.

Q6. Correlation and Linear Regression

Assessment Objectives	Solution	Examiner’s Feedback
Concept of $\left(\bar{x},\bar{y}\right)$ lies on the regression line.	**(i)** $\bar{y}=22.51355-4.908387\bar{x}$ $\left(\frac{91+k}{9}\right)=22.51355-4.908387\left(\frac{20.5}{9}\right)$ $k=11.0$	Poorly attempted. Many students simply substituted *x* = 2 into the equation of the regression line and hoped that the resulting *y*, i.e. *k* will be 2. They failed to understand that the point *x* = 2 may not pass through the regression line. Students must understand the concept that $\left(\bar{x},\bar{y}\right)$ lies on the regression line.
Scatter diagram	**(ii)**  	Most students handled this part accurately. There were students who carelessly wrote the *x*-intercepts as *y*-intercepts and vice versa. Others thought that 1.25 > 3.5 and 7.5 > 18. All these could have been avoided if students made an effort to check their scatter diagram before proceeding.

Concept of linearization of non-linear model.	**(iii)**	As x increases, y decreases at a decreasing rate and tends towards a limit.	Poorly attempted. Students merely says that the graph of model (C) is similar to the graph in the scatter diagram. This warrants no marks. Students are reminded that they need to describe the shape of the graph. Students are advised against describing the gradient of the scatter diagram as it is prone to careless mistakes. In this question, as x increases, the gradient actually increases because it becomes less negative.
Use of GC to find the regression line.	**(iv)**	$a = 0.257$ $b = 22.8$	Many students failed to leave their final answer in 3 s.f.
Estimation and its reliability.	**(v)**	$y = 0.25681 + \frac{22.837}{x}$ $12 = 0.25681 + \frac{22.837}{x}$ $x = 1.94$ (3 s.f.) The weight of the car is 1940kg. The prediction is reliable as $y = 12$ is within the data range of y and the $\|r\|$-value is close to 1.	Many students left their answer as x = 1.94. They did not conclude that the weight of the car is 1940kg or 1.94 kg in thousands. Many students failed to mention that the $\|r\|$-value is close to 1 when stating that the prediction is reliable.
Concept of mean and standard deviation	**(vi)**	(a) $\bar{y}$ will be increased by a. (b) Standard deviation of y remain unchanged. (c) Correlation coefficient remain unchanged.	Well-attempted by students.

Q7. Binomial Distribution and Sampling Distribution

Assessment Objectives	Solution	Examiner's Feedback
Setting up and solving equations using the formula for a Binomial random variable	**(a)(i)** $\mathrm{P}(X=4)=\mathrm{P}(X=5)$ $\frac{10!}{4!6!}p^4(1-p)^6=\frac{10!}{5!5!}p^5(1-p)^5$ $5(1-p)=6p$ $p=\frac{5}{11}$	Most students could identify that the probabilities for the outcomes of 4 and 5 should be equal and wrote the expressions according to the formula. Some failed to solve the equation due to inadequate skills in algebra.
Setting up and solving equations using the formula for a Binomial random variable	**(ii)** $\mathrm{P}(X\le 9)=\frac{1023}{1024}$ $\mathrm{P}(X=10)=\frac{1}{1024}$ $p^{10}=\left(\frac{1}{2}\right)^{10}$ $p=\frac{1}{2}$	Many students did not realise that the complementary case is simply 10. Once this hurdle was overcome most were able to find the final answer.
Solving simple problems based on random samples from a binomial random variable	**(b)(i)** $\mathrm{P}(Y\le 256)=0.719485301$ P(all 100 values are less than or equal to 256) $=0.719485301^{30}$ $=0.0000514$	Many students immediately dived into the irrelevant routine of using CLT to find the sampling distribution once they saw the conditions given, without analyzing the question carefully. Majority of the students left the first probability as the answer. Their understanding of the term "sample" may be in question.

Applying Central Limit Theorem for the sampling distribution of a random sample from a binomial random variable	**(ii)** $\mathrm{E}(Y) = 500(0.5) = 250$, and $\mathrm{Var}(Y) = 500(0.5)(0.5) = 125$ Since the sample size is sufficiently large, $\bar{Y} \sim \mathrm{N}\left(250, \frac{125}{30}\right)$ approximately by CLT $\mathrm{P}\left(\bar{Y} \le 256\right) = 0.998$	Most were able to follow the routine to write down the expectation and variance of Y. However, half of them did not show clear understanding of sampling distribution and central limit theorem in their subsequent presentation of the solution. The most common mistake is that quoting CLT to write down $Y \sim N(250,125)$, which is WRONG! It is the mean of samples of large size may be considered as normally distributed approximately, not the individual observation. Other common mistakes include forgetting to divide the variance by sample size, or using a wrong notation for the random variable of sample mean.
Making comparison between probabilities that are calculated based on the context	**(iii)** The probability in part **(ii)** included cases where some of the values can be larger than 256, but the final average is still at most 256.	Many students were able to give the correct reason though the phrasing can still be improved. For example many casually wrote “probability in (i) is a subset of probability in (ii)”, which showed understanding but failed to make sense mathematically when it is the collection of “events/outcomes” in one being subset of the other.

Q8. Normal Distribution		
Assessment Objectives	**Solution**	**Examiner's Feedback**
Practical application of Normal distribution to obtain percentages of items with certain properties	**(i)** Let L be the random variable denoting length of a trading card in mm. $L \sim \text{N}(89,\ 0.2025)$ $\text{P}(L > 90) = 0.0131$, hence the percentage is 1.31%.	Badly done by students. Mistakes: 1) Confusing CRV and DRV by writing $\text{P}(L \le 90) = \text{P}(L \le 89)$ 2) Not answering in %
Practical application of Normal distribution to obtain the critical value of a certain property satisfied by a given percentage of items	**(ii)** Let T be the random variable denoting the perimeter of a trading card, in mm. $T \sim \text{N}\left(2(64)+2(89),\ 2^2(0.3^2)+2^2(0.45^2)\right)$ $\sim \text{N}\left(306,\ 1.17\right)$ $\text{P}(T > t) = 0.08$ $\text{P}(T < t) = 0.92$ Hence $t = 307.51\text{mm}$	Badly done by students. Most common mistake: Taking invNorm with RHS area 0.2.
Practical application of Normal distribution to obtain probability of randomly selected items fulfilling certain independent physical properties	**(iii)** Let X and Y be random variable denoting the width and length of a card sleeve subtracting away the width and length of a trading card respectively in mm. Hence $X \sim \text{N}\left(66-64,\ (0.45^2)+(0.3^2)\right)$ $\sim \text{N}\left(2,\ 0.2925\right)$ and $Y \sim \text{N}\left(91-89,\ (0.45^2)+(0.675^2)\right)$ $\sim \text{N}\left(2,\ 0.658125\right)$ $\text{P(width fits nicely)} = \text{P}(0 < X \le 2.4) = 0.7701200999$ $\text{P(length fits nicely)} = \text{P}(0 < Y \le 2.4) = 0.6821730404$ $\text{P(sleeve fits nicely)} = \text{P(width fits nicely)}\,\text{P(length fits nicely)}$ $= 0.7701200999 \times 0.6821730404$ $= 0.525$	Badly done. Better students were able to find the new mean and new variance but mistakes were made when calculate the probabilities, Mistakes: 1) $\text{P}(X \le 2.4)\,\text{P}(Y \le 2.4)$ 2) $\text{P}(X \le 1.2)\,\text{P}(Y \le 1.2)$ 3) $\text{P}(0 \le X \le 1.2)\,\text{P}(0 \le Y \le 1.2)$ Most students left this part blank.

<table>
<tr><th colspan="3">Q9. Hypothesis Testing</th></tr>
<tr><th>Assessment Objectives</th><th>Solution</th><th>Examiner’s Feedback</th></tr>
<tr><td>Provide reasoning to support the application of Central Limit Theorem in Hypothesis testing</td><td>(i) It is not necessary as the sample size is sufficiently large for Central Limit Theorem to apply.</td><td>This part is poorly attempted with many students discussing about the PARAMETERS of the distribution rather than the fact of whether it is a normal distribution.

Some students simply stated that it is necessary as the hypothesis testing requires the use of a normal distribution, showing clearly their lack of understanding for the Central Limit Theorem and Sampling Distributions in general.

There are also quite a number of students who either left out the fact that the sample size is large or that Central Limit Theorem is applicable, resulting in an incomplete explanation.</td></tr>
<tr><td>Conduct a z-test for a practical situation</td><td>(ii) $\bar{t} = 2384.5/50 = 47.69$ thousand hours
$s^2 = \frac{1}{50-1}\left(115885.23 - \frac{2384.5^2}{50}\right) = 44.25357143 = 44.3$
$\mathrm{H_0}: \mu = 50$
$\mathrm{H_1}: \mu \neq 50$
Under H_0, since <i>n</i> is large, by C.L.T.
$\bar{T} \sim \mathrm{N}\left(50, \frac{44.25357143}{50}\right)$ appx
p-value = 0.01407 OR test statistic = – 2.4554</td><td>Most candidates are successful with the unbiased estimates, but some left the answer as 47.7 for population mean, not realizing that it is an exact decimal. Quite a number of students also quoted the wrong formula for population variance or left their answers as the value before dividing by 49.

While most students with the correct estimates were successful</td></tr>
</table>

	Since p-value = 0.01407 > 0.01, we do not reject H_0 and conclude that there is insufficient evidence at 1% level of significance to claim that the mean number of hours before failure is not 50 thousand hour.	with the testing, there is also a significant number of students who lost all marks by simply stating an incorrect p-value based on their wrong parameters. P-values calculated based on 3 s.f. values of the parameters or a combination of 5 s.f. and 3 s.f. values were accepted. Correct p-values based on erroneous presentation of the sampling distributions were not penalized due to benefit of doubt given. Students who attempted to use critical values were less successful as they applied modulus to the the test statistic without doing so for the critical value resulting in erroneous comparisons. Students should state clearly the rejection region when using critical values. Most conclusions were not given in context, did not mention level of significance, or were not phrase in terms of the alternate hypothesis. Many students phrased the conclusion wrongly as "having sufficient evidence to claim that the mean is 50 thousand hours".
Provide reasoning to support the choice of alternate hypothesis based on the context of the practical situation	**(iii)** The reader would be more interested to test whether the mean is actually lower than the stated value which is not beneficial to them.	Most students simply stated that the test did not indicate whether the mean is more of less, but did not make any reference to why these

		cases would matter to the reader. Students who manage to make reference to higher mean being beneficial and/or lower mean being not beneficial, were given credit based on benefit of doubt.
Identifying alternate hypothesis relevant to the context and provide reasoning on the effect of the alternate hypothesis on the resulting p-value and final conclusion of a z-test	**(iv)** $\mathrm{H}_1 : \mu < 50$ Yes, the result will differ as the p-value will be halved when switching to a one-tail test.	Most students were successful with stating the correct alternate hypothesis, except some who used the left tail test in (ii). However, not all were able to provide an explanation to support the change in conclusion, especially those who used critical values in (ii). Students who did not identify that the p-value is exactly half of the value found in (ii) will have to state the actual value, simply mentioning that the p-value is smaller is in sufficient as 0.011 is also a smaller p-value, but it will not result in a change in the conclusion. Most students who re-did the test were most successful for this part, but many went on to write the full conclusion, which is not required by the question.
Identifying implicit assumptions made in a hypothesis test	**(v)** We need to assume that the usage hours before failure for hard drives are independent for all hard drives.	Many students were able to state the assumption needed, but some did not exhibit any understanding of the situation.

<table>
<tr><th colspan="3">Q10. Probability and Discrete Random Variables</th></tr>
<tr><th>Assessment Objectives</th><th>Solution</th><th>Examiner’s Feedback</th></tr>
<tr><td>Formulating an expression for the probability density function of a discrete random variable</td><td>(i) $P(S=6)$
$= P(RRBBB)+P(RGGGB)$
$= \frac{{}^5C_3(2!)\left[x(x-1)(x-2)\right]}{(x+5)(x+4)(x+3)(x+2)(x+1)} + \frac{{}^5C_3x(2)(2)(3!)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$
$= \frac{20x\left[x^2-3x+2\right]}{(x+5)(x+4)(x+3)(x+2)(x+1)} + \frac{20x(12)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$
$= \frac{20x(x^2-3x+14)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$</td><td>This part is usually well done. Lack of essential working is not acceptable.</td></tr>
<tr><td>Solving for an unknown parameter based on the expression for the probability density function of the discrete random variable</td><td>(ii) $\frac{5}{63} = \frac{20x(x^2-3x+14)}{(x+5)(x+4)(x+3)(x+2)(x+1)}$
$5(x+5)(x+4)(x+3)(x+2)(x+1) - 1260x(x^2-3x+14) = 0$
Solving, the only integer root is $x=5$</td><td>This part is well done.</td></tr>
<tr><td>Completing the probability distribution table of a discrete random variable</td><td>(iii)
<table>
<tr><td>s</td><td>1</td><td>4</td><td>7</td><td>25</td></tr>
<tr><td>$P(S=s)$</td><td>$\frac{5}{84}$ or $\frac{15}{252}$</td><td>$\frac{5}{21}$ or $\frac{60}{252}$</td><td>$\frac{5}{42}$ or $\frac{30}{252}$</td><td>$\frac{1}{252}$</td></tr>
</table>
$P(S=1) = P(\text{GBBBB})$
$= \left(\frac{3}{10}\right)\left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{2}{6}\right)\left(\frac{5!}{4!}\right)$
$= \frac{5}{84}$ or $\frac{15}{252}$
Note: $\frac{5!}{4!}$ is for arranging GBBBB with 4 repeated "B"s.</td><td>$P(S=4)$ and $P(S=7)$ proved to be quite challenging for most candidates.</td></tr>
</table>

	$\text{P}(S=4)=\text{P}(\text{RGBBB})$ $=\left(\frac{2}{10}\right)\left(\frac{3}{9}\right)\left(\frac{5}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{5!}{3!}\right)$ $=\frac{5}{21}$ or $\frac{60}{252}$ Note: $\frac{5!}{3!}$ is for arranging RGBBB with 3 repeated "B"s. $\text{P}(S=7)=\text{P}(\text{RRGBB})$ $=\left(\frac{2}{10}\right)\left(\frac{1}{9}\right)\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)\left(\frac{4}{6}\right)\left(\frac{5!}{2!2!}\right)$ $=\frac{5}{42}$ or $\frac{30}{252}$ Note: $\frac{5!}{2!2!}$ is for arranging RRGBB with 2 repeated "R"s and 2 repeated "B"s. $\text{P}(S=25)=\text{P}(\text{BBBBB})$ $=\left(\frac{5}{10}\right)\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)$ $=\frac{1}{252}$	
Calculating expectation of a discrete random variable and probabilities based on the probability distribution table	**(iv)** $\text{E}(S)=4.60$ or $\frac{1159}{252}$ $\text{P}(S>4.60)=\frac{127}{252}$ or 0.504	This part is poorly done as a result of errors from (iii).

Solving for conditional probabilities based on a discrete random variable	**(v)** $\mathrm{P}(\text{no}\, G \mid S > 4.60)$ $= \dfrac{\mathrm{P}(RRBBB) + \mathrm{P}(BBBBB)}{\frac{127}{252}}$ $= \dfrac{\frac{20(5)(4)(3)}{(10)(9)(8)(7)(6)} + \frac{1}{252}}{\frac{127}{252}}$ $= \dfrac{\frac{10}{252} + \frac{1}{252}}{\frac{127}{252}}$ $= \dfrac{11}{127}$	Common omission or error pertaining to the case $\mathrm{P}\left(RRBBB\right)$, led to wrong answer. This part is poorly done.