NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME	

CT CLASS	2	1			Centre Number/ Index Number	/

MATHEMATICS — 9758/01

Paper 1

30 August 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	**Marks**
1	
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11	
Total	

This document consists of **5** printed pages and **0** blank page.

NANYANG JUNIOR COLLEGE
Internal Examinations

[Turn Over

1 A function f is defined by $f(x)=ax^3+bx^2+cx+d$. The graph of $y=f(x)$ has a minimum point at $(-1,-1)$ and $\int_0^1 f(x)\,dx=\frac{9}{4}$. When $f(x)$ is divided by $(x+2)$, the remainder is 3. Find the values of a, b, c and d. [5]

2 **(i)** Sketch the graph of $y=\left|\left(\frac{1}{3}\right)^{2x}-18\right|$, giving the exact values of any points where the curve meets the axes. [3]

(ii) Without using a calculator, and showing all your working, find the exact interval, or intervals, for which $\left|\left(\frac{1}{3}\right)^{2x}-18\right|\le 9$. Give your answer in its simplest form. [3]

3 **Do not use a calculator in answering this question.**

A curve C has equation $x^3+2y^3-3xy-40=0$.

(i) Find exact coordinates of the stationary points of C. [4]

(ii) For the stationary point with $x<0$, determine whether it is a maximum or minimum point. [2]

4 It is given that $y=\cos\left(1-e^{-x}\right)$.

(i) Show that $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$. [2]

(ii) Find the first 3 non-zero terms of the Maclaurin series for $\cos\left(1-e^{-x}\right)$. [3]

(iii) Use your series from part **(ii)** to estimate $\int_0^{0.05}\cos\left(1-e^{-x}\right)dx$, correct to 7 decimal places. [1]

(iv) It is now given that $\int_0^{0.05}\cos\left(1-e^{-x}\right)dx\approx 0.0499799$. Comment on the accuracy of your estimation in part **(iii)**, justifying your answer. [1]

5 **(i)** It is given that $-1+2i$ is a root of the equation $z^3+2(1+i)z^2+(5+4i)z+10i=0$. Explain why $-1-2i$ may not be a root. [1]

(ii) Without using a calculator, solve the equation $z^3+2(1+i)z^2+(5+4i)z+10i=0$, giving your answers in the form $a+ib$, where a and b are exact values. [5]

(iii) Hence solve $iz^3+2(1+i)z^2+(4-5i)z-10i=0$. [2]

6 **(a)** A sequence $\{u_r\}$ is defined by $u_r=u_{r-1}+\frac{2}{r^3-r}-\left(\frac{4}{5}\right)^{r-1}$, $r\ge 2$ and $u_1=3$.

(i) By expressing $\frac{2}{r^3-r}$ as $\frac{A}{r-1}+\frac{B}{r}+\frac{C}{r+1}$, where A, B and C are constants to be determined, find an expression for $\sum_{r=2}^{N}\left(\frac{2}{r^3-r}-\left(\frac{4}{5}\right)^{r-1}\right)$. [4]

(ii) Hence, by considering $\sum_{r=2}^{N}(u_r-u_{r-1})$, express u_N in terms of N. [2]

(b) Another sequence $\{b_n\}$ is such that $1-\left(\frac{1}{2}\right)^n < b_n < 1+\left(\frac{1}{3}\right)^n$ for all positive integers $n \ge 1$.

The squeeze theorem states that if $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are sequences of real numbers such that $a_n \le b_n \le c_n$ for all positive integers $n \ge 1$ and that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then b_n is convergent and $\lim_{n\to\infty} b_n = L$. Use the squeeze theorem to show that b_n is convergent and state its limit. [3]

7 A curve is defined parametrically by

$$x=\frac{at}{t+1},\ y=\frac{at^2}{t+1},$$

where a is a non-zero constant and $t\in\mathbb{R}, t\ne -1$.

(i) Show that the equation of the normal to the curve at the point T with parameter t is given by

$$t(t+1)(t+2)y+(t+1)x=at(t^3+2t^2+1).$$ [4]

(ii) The normal to the curve at the point $P\left(-a,\frac{1}{2}a\right)$ meets the curve again at point Q. Find the exact coordinates of Q in terms of a. [4]

(iii) Let M be the mid-point of PQ. Find a cartesian equation of the curve traced by M if a varies. [2]

8 A curve C has parametric equations

$$x=2\sin 2t+3,\ y=\cos 2t,\quad \text{for } \frac{\pi}{2}\le t\le \pi.$$

(i) Sketch the graph of C. Give the coordinates of the point(s) where C meets the x-axis and the end points on the curve. [2]

(ii) Find by integration, the exact area of the region R bounded by C, the line $y = 1$ and the axes. [4]

(iii) Show that the cartesian equation of C is $y^2=1-\frac{(x-3)^2}{4}$. [1]

(iv) Write down the cartesian equation of the curve if C is translated in the direction of the negative y-axis by 1 unit. Hence find the volume of the solid of revolution when R is rotated about the line $y = 1$. [4]

9 An arithmetic progression has first term a and common difference d, and a geometric progression has first term b and common ratio r, where a, b and d are non-zero. The second, fourth and sixth term of the arithmetic progression are equal to the first, fourth and eighth term of the geometric progression respectively.

(i) Show that $r^7 - 2r^3 + 1 = 0$. [2]

It is given that the geometric progression has positive terms.

(ii) Find the value of r and justify that this is the only answer. Deduce whether the geometric progression is convergent. [3]

(iii) Another arithmetic progression has first term k and common difference $3k$, where $k > 0$. The difference between the sum of the first $2n$ terms of this arithmetic progression and the nth term of the geometric progression with the common ratio found in part **(ii)** is at most $1000k$. Given that $b = 4k$, write down an inequality satisfied by n, and hence find the largest possible value of n. [5]

(iv) Let u_n denote the nth term of the geometric progression. Show that a new sequence with nth term $\ln\left(\frac{1}{u_n}\right)$ is an arithmetic progression. [2]

10 Two charged particles, U and V, are confined to the planes F_1 and F_2 with position vectors given by

$$(3+6p)\mathbf{i}+(1+4p+q)\mathbf{j}+(6+2p-4q)\mathbf{k} \text{ and } (-9+3p)\mathbf{i}+(1+p-2q)\mathbf{j}+(3-p+8q)\mathbf{k}$$

respectively, where $p, q \in \mathbb{R}$.

(i) Obtain the equation of F_1 in scalar product form. [2]

(ii) Find the acute angle between F_1 and F_2. [3]

The forces of the two particles U and V allow another charged particle W to remain suspended between them, such that $2UW = WV$.

(iii) As the positions of U and V vary, show that the set of points described by the path of W is a line l, whose vector equation is to be determined. [3]

(iv) An uncharged particle A is fired along a path described by a line $\mathbf{r} = s\mathbf{k}$, where $s \in \mathbb{R}$ and crosses F_1 at some instant in time. Find the shortest distance of A from l at this instant. [4]

11 In a chemical reaction, two substances A and B are combined to form a new substance C. The initial masses of A, B and C are 8, 6 and 0 units respectively. After time t seconds, the masses of A and B are each reduced by x units, and the mass of C increases by $2x$ units. The rate of change of the mass of C with respect to t is proportional to the product of the masses of A and B at any time t.

(i) Write down a differential equation relating x and t. [1]

(ii) It is observed that when $t = 5$, $x = 5$. By solving the differential equation in part **(i)**, show that

$$x = \frac{24\left(\frac{9}{4}\right)^{-\frac{t}{5}} - 24}{3\left(\frac{9}{4}\right)^{-\frac{t}{5}} - 4}.$$ [7]

(iii) Sketch the graph of x against t. [2]

In another experiment with the same initial masses of substances A and B as before, the increase in the mass of C is modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-8\mathrm{e}^{-2t}-\frac{16}{9}(t+1)^{-\frac{7}{3}}.$$

Given that x eventually stabilises at 6 units after a long time, find x in terms of t. [4]

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JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME	

CT CLASS	2	1			Centre Number/ Index Number					/					

MATHEMATICS

9758/02

Paper 2

16 September 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

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NANYANG JUNIOR COLLEGE
Internal Examinations

[Turn Over

Section A: Pure Mathematics [40 marks]

1 The origin O and the points A and B lie in the same plane, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\mathbf{a}$ and $\mathbf{b}$ are non-parallel constant vectors.

(i) Interpret geometrically the vector equation $\mathbf{r} = \lambda\mathbf{a} + \mu\mathbf{b}$, where λ and μ are parameters. [1]

(ii) The point P, with position vector $\mathbf{p}$, does not lie in the same plane as O, A and B. Interpret geometrically $|\mathbf{p} \times \mathbf{u}|$, where $\mathbf{u}$ is a unit vector parallel to $\mathbf{a} \times \mathbf{b}$. [1]

(iii) The point C with position vector $\mathbf{c}$ lies on AB, between A and B, such that $10AC = AB$. OC is perpendicular to AB and angle AOB is 90°. Find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and the ratio $OB : OA$. [4]

2 **Do not use a calculator in answering this question.**

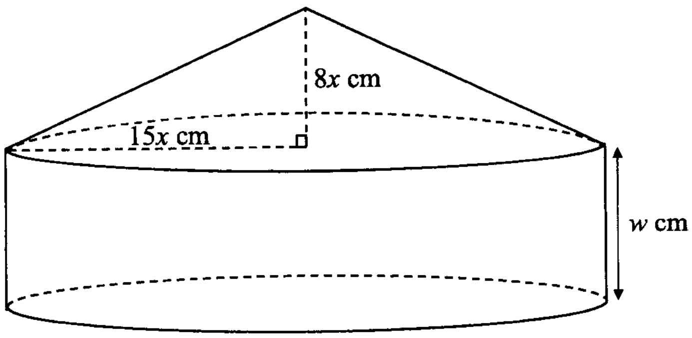
The complex numbers z and w satisfy the following equations

$$w - z = 1 - \sqrt{3},$$

$$\mathrm{i}z + w = (\sqrt{3} + 1)\mathrm{i}.$$

Find w in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give r and θ in exact form. Hence find the three smallest positive whole number values of n for which $(\mathrm{i}w)^n$ is a real number. [7]

3 [It is given that a right circular cone with base radius r and height h has volume $\frac{1}{3}\pi r^2 h$ and curved surface area $\pi r l$, where l is the slanted height of the cone.]



The model of a house is made up of three parts.

- The roof is modelled by the curved surface of a right circular cone with base radius $15x$ cm and height $8x$ cm.
- The walls are modelled by the curved surface of a cylinder of radius $15x$ cm and height w cm.
- The floor is modelled by a circular disc of radius $15x$ cm.

The three parts are joined together as shown in the diagram. The model is made of material of negligible thickness. It is given that the volume of the model is a fixed value k cm^3, and the external surface area is a minimum. Use differentiation to find the exact values of x and w in terms of k. Simplify your answers. [8]

4 The function f is defined by $f: x \mapsto x^2 - 6x$ for $x \in \mathbb{R}, -1 < x < 1$.

(i) Find $f^{-1}(x)$ and state its domain. [3]

(ii) Sketch on the same diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$. Hence, determine the value of x for which $f(x) = f^{-1}(x)$. [3]

The functions g and h are defined by

$$g: x \mapsto 1 + \frac{1}{x+5} \text{ for } x \in \mathbb{R}, x \neq -5,$$

$$h: x \mapsto gf(x) \text{ for } x \in \mathbb{R}, -1 < x < 1.$$

(iii) Using differentiation, show that h(x) increases as x increases. [3]

5 **(a)** **(i)** Differentiate $\sqrt{x^2+4}$ with respect to x. [1]

(ii) Hence find $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$. [3]

(b) Find

(i) $\int \sin mx \cos mx \, dx$, where $m > 0$, [2]

(ii) $\int \cos 3x \cos x \sin 2x \, dx$. [4]

Section B: Probability and Statistics [60 marks]

6 Jean has forgotten the six-character login password for her laptop. She remembers that the password consists of four distinct letters from the twenty-six letters of the alphabet A – Z and two distinct digits from the ten digits 0 – 9.

(i) Assuming that Jean keys in a six-character password for all her login attempts and she never repeats the same incorrect password, find the largest number of unsuccessful login attempts. [2]

(ii) Find the number of possible six-character passwords if the first four characters are distinct letters in alphabetical order. [2]

(iii) Given that the first four characters are distinct letters, and the last two characters are distinct digits, find the probability that exactly one of the four letters is a vowel. [3]

7 An experiment was conducted to study the relationship between the rate of enzymatic reactions, y, and the concentration of a reactant, x, in a chemical reaction. Six readings, in appropriate units, were obtained.

x	0.1125	0.225	0.45	0.9	1.8	3.6
y	1.081	1.756	2.733	3.735	4.462	4.717

Source: Adapted from https://techvidvan.com/tutorials/nonlinear-regression

(i) Sketch a scatter diagram of the data. [1]

(ii) Find the product moment correlation coefficient between x and y and comment on its value based on the scatter diagram in part **(i)**. [2]

(iii) It is suspected that the data is modelled by the regression equation $y = \dfrac{ax}{b+x}$. To find the values of a and b, x and y are transformed to $\dfrac{1}{x}$ and $\dfrac{1}{y}$ respectively. State the product moment correlation coefficient between $\dfrac{1}{x}$ and $\dfrac{1}{y}$, giving your answer correct to 4 decimal places. Find the values of a and b. [3]

(iv) Use the model in part **(iii)** to estimate the value of x when $y = 5$. Comment on the reliability of the estimate found. [2]

8 A shooter can hit the bullseye of his target 7 out of 10 attempts on average. In a particular training, he made 35 attempts on the target.

(i) State two assumptions for the number of bullseyes achieved in a training to be well modelled by a binomial distribution. [2]

(ii) Explain why one of these assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that the number of bullseyes the shooter achieved in a training follows a binomial distribution.

(iii) Given that m is the most probable number of bullseyes the shooter can achieve in a training, find the value of m and state its corresponding probability. [2]

(iv) Find the probability that the shooter achieves at least 25 bullseyes in a training. [2]

(v) The shooter attended 40 trainings. Using a suitable approximation, find the probability that he achieved an average of at least 25 bullseyes per training. [3]

9 A bag contains three yellow marbles, one blue marble and x red marbles, where $x > 1$. In a game, Lily takes 2 marbles at random from the bag, without replacement. She scores 2 points for each red marble taken and 1 point for each yellow marble taken. If a blue marble is taken, she loses 2 points. The random variable S is the sum of the points of the two marbles taken.

(i) Find $\mathrm{P}(S = s)$ for all possible values of s. [4]

(ii) Show that $\mathrm{E}(S) = \dfrac{2(2x+1)}{x+4}$ and find an expression for $\mathrm{Var}(S)$. [5]

(iii) Find the least value of x if $\mathrm{E}(S) > 2$. [2]

10 The manager of a bank claims that the mean waiting time for customers to be served by a bank consultant is 15 minutes. The bank director suspects that the waiting time is longer than 15 minutes and decides to carry out a hypothesis test on a sample of these customers.

The waiting times, x minutes, of a random sample of 40 customers are summarised below.

$$\Sigma x = 650 \qquad \Sigma(x-\bar{x})^2 = 944$$

(i) Calculate the unbiased estimates of the population mean and variance of the waiting times of the customers. [2]

(ii) Carry out the test, at the 5% level of significance, for the bank director. You should state your hypotheses and define any symbols that you use. [5]

(iii) Explain why there is no need for the bank director to know anything about the population distribution of the waiting times of the customers. [1]

It is given instead that the population variance of the waiting times is 30.25 minutes^2. Another random sample of 32 customers is taken and the bank director now carries out the same test at the 5% level of significance.

(iv) Find the range of possible values of the mean waiting time of this random sample of 32 customers if the bank director's suspicion is confirmed. [4]

11 In this question you should state the parameters of any normal distributions you use.

In strength training, repetitions are the number of times a person completes a single exercise before taking a rest.

Randy goes to a gym for his workout. At the gym, he does three different exercises, namely bench press, hack squat and leg press. The time he takes, in seconds, to do **one repetition** of bench press, hack squat and leg press are independent normal distributions with means and standard deviations given in the table below.

Exercise	Mean	Standard deviation
Bench press	5	0.5
Hack squat	3	0.1
Leg press	μ	σ

The probability that he takes more than 5.1 seconds to do one repetition of leg press is 0.15. The probability that he takes less than 2.9 seconds to do one repetition of leg press is also 0.15.

(i) State the mean time, μ, to do one repetition of leg press and show, with clear working, that the standard deviation, σ, is 1.06 seconds. [3]

A circuit is completed when Randy completes the three different exercises, each exercise consists of ten repetitions. After each exercise, he takes a 60-second break such that in total, a complete circuit comprises two breaks.

(ii) Show that the expected time taken to complete one circuit is 240 seconds. [1]

(iii) Find the probability that the first circuit takes more than 5 seconds longer than the second circuit. [2]

Between any two circuits, Randy takes a 120-second break.

(iv) Find the probability that the total time taken for any two consecutive circuits is longer than twice the time taken for any one circuit by more than 150 seconds. [3]

Randy decides to modify his training plan for a circuit such that each 60-second break is k-second instead.

(v) Find the largest integer k such that the probability that the time taken for one circuit is longer than 3 minutes is less than 0.01. [3]

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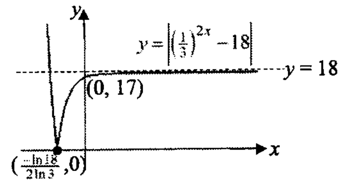
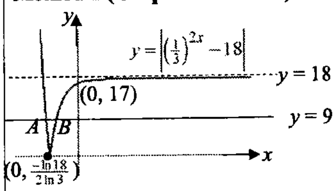
Q1	Suggested Answers
(i)	$\mathrm{f}(x)=ax^3+bx^2+cx+d$ At $(-1,-1)$, $-a+b-c+d=-1$ ----------------------------(1) $\mathrm{f}'(x)=3ax^2+2bx+c$ At minimum point $(-1,-1)$, $3a-2b+c=0$ ---------------(2) $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x=\frac{9}{4}$ $\int_0^1 \left(ax^3+bx^2+cx+d\right)\mathrm{d}x=\frac{9}{4}$ $\left[\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}+dx\right]_0^1=\frac{9}{4}$ $\frac{a}{4}+\frac{b}{3}+\frac{c}{2}+d=\frac{9}{4}$ --(3) When f(x) is divided by (x + 2), the remainder is 3 $\mathrm{f}(-2)=a(-2)^3+b(-2)^2+c(-2)+d=3$ $-8a+4b+-2c+d=3$ -------------------------------------(4) Solving, $a=-1$, $b=0$, $c=3$ and $d=1$

BP~326

Q2	Suggested Answers
(i)	 $\left(\frac{1}{3}\right)^{2x}-18=0$ $2x=\frac{\ln 18}{\ln\left(\frac{1}{3}\right)} \Rightarrow x=\frac{\ln 18}{-2\ln 3}$
(ii)	**Method 1 (Graphical method):**  To find x-coordinates of point A, $\left(\frac{1}{3}\right)^{2x}-18=9 \Rightarrow 3^{-2x}=3^3$

	$x=-\frac{3}{2}$ To find x-coordinates of point B, $-\left[\left(\frac{1}{3}\right)^{2x}-18\right]=9 \Rightarrow 3^{-2x}=3^2$ $x=-1$ $\therefore -\frac{3}{2}\le x\le -1$
(ii)	**Method 2 (Algebraic method):** $\left\|\left(\frac{1}{3}\right)^{2x}-18\right\|\le 9$ $-9\le\left(\frac{1}{3}\right)^{2x}-18\le 9$ $9\le\left(\frac{1}{3}\right)^{2x}\le 27$ $3^2\le 3^{-2x}\le 3^3 \Rightarrow 2\le -2x\le 3$ $\therefore -\frac{3}{2}\le x\le -1$ Alternatively, $\left(\frac{1}{3}\right)^{-2}\le\left(\frac{1}{3}\right)^{2x}\le\left(\frac{1}{3}\right)^{-3}$ $\Rightarrow -2\ge 2x\ge -3$ $\therefore -\frac{3}{2}\le x\le -1$
(ii)	**Method 3 (Algebraic method):** $\left\|\left(\frac{1}{3}\right)^{2x}-18\right\|\le 9$ $-9\le\left(\frac{1}{3}\right)^{2x}-18\le 9$ $9\le\left(\frac{1}{3}\right)^{2x}\le 27$

	$\left(\frac{1}{3}\right)^{2x}=9 \Rightarrow x=\frac{\ln 9}{2\ln\frac{1}{3}}=-1$ $\left(\frac{1}{3}\right)^{2x}=27 \Rightarrow x=\frac{\ln 27}{2\ln\frac{1}{3}}=-\frac{3}{2}$ $\therefore -\frac{3}{2}\le x\le -1$

<table>
<tr><th>Q3</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>$x^3+2y^3-3xy-40=0$
Differentiate with respect to x,
$3x^2+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}-3y=0\cdots(1)$
At stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$,
Thus $y=x^2$

Sub $y=x^2$ into $x^3+2y^3-3xy-40=0$, we get
$x^3+2x^6-3x^3-40=0$
$2x^6-2x^3-40=0$
$x^6-x^3-20=0$
$\left(x^3-5\right)\left(x^3+4\right)=0$
Thus $x=5^{\frac{1}{3}},-2^{\frac{2}{3}}$.
When $x=5^{\frac{1}{3}}, y=5^{\frac{2}{3}}$.
When $x=-2^{\frac{2}{3}}, y=2^{\frac{4}{3}}$
The coordinates of the stationary points are $(5^{\frac{1}{3}},5^{\frac{2}{3}})$ and $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$</td></tr>
<tr><td>(ii)</td><td>Differentiating equation (1) wrt x
$6x+12y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+6y^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)-3x\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)-3\frac{\mathrm{d}y}{\mathrm{d}x}-3\frac{\mathrm{d}y}{\mathrm{d}x}=0$
At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.
At $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$, $6\left(-2^{\frac{2}{3}}\right)+6\left(2^{\frac{8}{3}}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+3\left(2^{\frac{2}{3}}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=0$</td></tr>
</table>

	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\frac{6\left(2^{\frac{2}{3}}\right)}{6\left(2^{\frac{8}{3}}\right)+3\left(2^{\frac{2}{3}}\right)}>0$ Thus $(-2^{\frac{2}{3}},2^{\frac{4}{3}})$ is minimum point.

2022 NYJC J2 H2 Mathematics Preliminary Exam 9758/1 Marking Guide

Q4	Suggested Answers
(i)	**Method 1:** $y=\cos\left(1-e^{-x}\right)$ $\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)\times e^{-x}$ $e^{x}\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)$ $e^{x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=-\cos\left(1-e^{-x}\right)\times e^{-x}$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=-y$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ (shown)
(i)	**Method 2:** $y=\cos\left(1-e^{-x}\right)$ $\frac{dy}{dx}=-\sin\left(1-e^{-x}\right)\times e^{-x}$-----------------------------(1) $\frac{d^2y}{dx^2}=e^{-x}\times\sin\left(1-e^{-x}\right)-e^{-x}\cos\left(1-e^{-x}\right)\times e^{-x}$----(2) (1) + (2): $\frac{d^2y}{dx^2}+\frac{dy}{dx}=-e^{-2x}\cos\left(1-e^{-x}\right)$ $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)=e^{2x}\times\left[-e^{-2x}\cos\left(1-e^{-x}\right)\right]=-y$

	$e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ (shown)
(ii)	Differentiate $e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+y=0$ again, $e^{2x}\left(\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+2e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)+\frac{dy}{dx}=0$ When $x=0,\ y=1,\ \frac{dy}{dx}=0,\ \frac{d^2y}{dx^2}=-1,\ \frac{d^3y}{dx^3}=3$ $\cos\left(1-e^{-x}\right)=1-\frac{1}{2}x^2+\frac{1}{2}x^3+\ldots$
(iii)	$\int_0^{0.05}\cos\left(1-e^{-x}\right)dx \approx \int_0^{0.05}\left(1-\frac{1}{2}x^2+\frac{1}{2}x^3\right)dx$ = 0.0499799 (to 7 dec. pl.)
(iv)	The answer from part **(iii)** is the same as the given result in the question up to 7 decimal places. This is because the values of x are between 0 and 0.05, which are small, such that x^4 and higher powers of x can be neglected. Hence the estimation in part **(iii)** is accurate.

Q5	Suggested Answers
(i)	Since the equation does not have all real coefficients, therefore Conjugate Root Theorem will not be applicable. Hence $-1-2i$ may not be a root.
(ii)	Let $z^3+2(1+i)z^2+(5+4i)z+10i=(z+1-2i)(z^2+Az+B)$ Comparing constant, $10i=(1-2i)B$ $B=\dfrac{10i}{(1-2i)}=-4+2i$ Comparing z^2 term, $2(1+i)=A+(1-2i)$ $A=1+4i$ $\therefore\ z^3+2(1+i)z^2+(5+4i)z+10i=0$ $(z+1-2i)(z^2+(1+4i)z+(-4+2i))=0$ $(z+1-2i)=0$ or $z^2+(1+4i)z+(-4+2i)=0$ $z=-1+2i$ or $z=\dfrac{-1-4i\pm\sqrt{(1+4i)^2-4(-4+2i)}}{2}$ $z=\dfrac{-1-4i\pm\sqrt{1}}{2}$ $z=\dfrac{-1-4i\pm1}{2}$ $z=\dfrac{-2-4i}{2}$ or $z=\dfrac{-4i}{2}$ Therefore, the roots are $z=-1+2i$ or $z=-1-2i$ or $z=-2i$
(iii)	From $z^3+2(1+i)z^2+(5+4i)z+10i=0$, if we replace z with iz, $(iz)^3+2(1+i)(iz)^2+(5+4i)(iz)+10i=0$

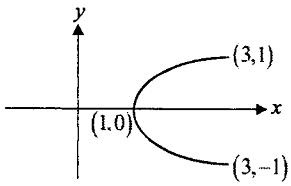
	$-\mathrm{i}z^3-2(1+\mathrm{i})z^2+(5\mathrm{i}-4)z+10\mathrm{i}=0$ $\mathrm{i}z^3+2(1+\mathrm{i})z^2+(4-5\mathrm{i})z-10\mathrm{i}=0$ So $\mathrm{i}z=-1+2\mathrm{i}$ or $\mathrm{i}z=-1-2\mathrm{i}$ or $\mathrm{i}z=-2\mathrm{i}$ $z=2+\mathrm{i}$ or $z=\mathrm{i}-2$ or $z=-2$

Q6	Suggested Answers
(a)(i)	$\frac{2}{r^3-r}=\frac{2}{(r-1)r(r+1)}=\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}$ $\sum_{r=2}^{N}\frac{2}{r^3-r}=\sum_{r=2}^{N}\left[\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right]=\frac{1}{1}-\frac{2}{2}+\cancel{\frac{1}{3}}$ $+\frac{1}{2}-\cancel{\frac{2}{3}}+\cancel{\frac{1}{4}}$ $+\cancel{\frac{1}{3}}-\cancel{\frac{2}{4}}+\cancel{\frac{1}{5}}$ $+\ldots$ $+\cancel{\frac{1}{N-3}}-\cancel{\frac{2}{N-2}}+\cancel{\frac{1}{N-1}}$ $+\cancel{\frac{1}{N-2}}-\cancel{\frac{2}{N-1}}+\frac{1}{N}$ $+\cancel{\frac{1}{N-1}}-\frac{2}{N}+\frac{1}{N+1}$ $=\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}$ $\sum_{r=2}^{N}\left(\frac{4}{5}\right)^{r-1}=\left(\frac{4}{5}\right)+\left(\frac{4}{5}\right)^2+\ldots+\left(\frac{4}{5}\right)^{N-1}=\frac{\frac{4}{5}\left[1-\left(\frac{4}{5}\right)^{N-1}\right]}{1-\frac{4}{5}}=4\left[1-\left(\frac{4}{5}\right)^{N-1}\right]$

	$\sum_{r=2}^{N}\left(\frac{2}{r^3-r}-\left(\frac{4}{5}\right)^{r-1}\right)=\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}-4\left[1-\left(\frac{4}{5}\right)^{N-1}\right]$ $=-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$
(a)(ii)	$\sum_{r=2}^{N}\left(u_r-u_{r-1}\right)=u_N-u_1$ Hence, $u_N-u_1=-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$ $u_N=3-\frac{7}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$ $=-\frac{1}{2}-\frac{1}{N}+\frac{1}{N+1}+4\left(\frac{4}{5}\right)^{N-1}$
(b)	$1-\left(\frac{1}{2}\right)^n<b_n<1+\left(\frac{1}{3}\right)^n$ Let $\mathrm{a}_n=1-\left(\frac{1}{2}\right)^n$ and $\lim_{n\to\infty}\mathrm{a}_n=1$ Let $\mathrm{c}_n=1+\left(\frac{1}{3}\right)^n$ and $\lim_{n\to\infty}\mathrm{c}_n=1$ Since $\lim_{n\to\infty}a_n=\lim_{n\to\infty}c_n=1$ By squeeze theorem, b_n is convergent and $\lim_{n\to\infty}b_n=1$.

Q7	Suggested Answers
(i)	$x=\frac{at}{t+1}, y=\frac{at^2}{t+1}$ $\frac{dx}{dt}=\frac{a(t+1)-at}{(t+1)^2}=\frac{a}{(t+1)^2}$, $\frac{dy}{dt}=\frac{2at(t+1)-at^2}{(t+1)^2}=\frac{at^2+2at}{(t+1)^2}$ $\frac{dy}{dx}=t^2+2t$ Equation of normal at point T: $y-\frac{at^2}{t+1}=-\frac{1}{t(t+2)}\left[x-\frac{at}{t+1}\right]$ $t(t+2)y-\frac{at^2}{t+1}\times t(t+2)=-x+\frac{at}{t+1}$ $t(t+1)(t+2)y-at^3(t+2)=-(t+1)x+at$ $t(t+1)(t+2)y+(t+1)x=at(t^3+2t^2+1)$ (shown)
(ii)	At $P\left(-a,\frac{1}{2}a\right)$, $\frac{at}{t+1}=-a \Rightarrow t=-\frac{1}{2}$ Equation of normal at P is $-\frac{3}{8}y+\frac{1}{2}x=-\frac{11}{16}a$ $\Rightarrow -6y+8x=-11a$ Alternatively, when $t=-\frac{1}{2}$, $x=-a, y=\frac{1}{2}a$ and $\frac{dy}{dx}=-\frac{3}{4}$ Equation of normal at P is $y-\frac{1}{2}a=\frac{4}{3}(x+a)$ $\Rightarrow y=\frac{4}{3}x+\frac{11}{6}a$

	Given that this normal meets the curve again at Q, $-6\left(\dfrac{at^2}{t+1}\right)+8\left(\dfrac{at}{t+1}\right)=-11a$ $\Rightarrow 6t^2-8t=11(t+1)$ $\Rightarrow 6t^2-19t-11=0$ $\Rightarrow (2t+1)(3t-11)=0 \Rightarrow t=-\dfrac{1}{2}$ or $\dfrac{11}{3}$ When $t=\dfrac{11}{3}$, $x=\dfrac{a\left(\frac{11}{3}\right)}{\left(\frac{11}{3}\right)+1}=\dfrac{11}{14}a$, $y=\dfrac{a\left(\frac{11}{3}\right)^2}{\left(\frac{11}{3}\right)+1}=\dfrac{121}{42}a$ The coordinates of Q are $\left(\dfrac{11}{14}a,\dfrac{121}{42}a\right)$
(iii)	Mid-point of PQ, M is $\left(-\dfrac{3}{28}a,\dfrac{71}{42}a\right)$ $x=-\dfrac{3}{28}a, y=\dfrac{71}{42}a \Rightarrow y=-\dfrac{142}{9}x$

Q8	Suggested Answers
(i)	
(ii)	**Method 1:** $3-\int_1^3 y\,\mathrm{d}x = 3-\int_{\frac{3\pi}{4}}^{\pi}\cos 2t\cdot 4\cos 2t\,\mathrm{d}t$ $=3-4\int_{\frac{3\pi}{4}}^{\pi}\cos^2 2t\,\mathrm{d}t$ $=3-4\int_{\frac{3\pi}{4}}^{\pi}\frac{1+\cos 4t}{2}\,\mathrm{d}t$ $=3-2\int_{\frac{3\pi}{4}}^{\pi}1+\cos 4t\,\mathrm{d}t$ $=3-2\left[t+\frac{1}{4}\sin 4t\right]_{\frac{3\pi}{4}}^{\pi}$ $=3-2\left[\pi-\frac{3\pi}{4}\right]$ $=3-\frac{\pi}{2}$ units^2

(ii)	**Method 2:** $\int_0^1 x\,\mathrm{d}y = \int_{\frac{3\pi}{4}}^{\pi}(2\sin 2t+3)\cdot(-2\sin 2t)\,\mathrm{d}t$ $= -2\int_{\frac{3\pi}{4}}^{\pi} 2\sin^2 2t + 3\sin 2t\,\mathrm{d}t$ $= -2\int_{\frac{3\pi}{4}}^{\pi} 1-\cos 4t + 3\sin 2t\,\mathrm{d}t$ $= -2\left[t - \frac{1}{4}\sin 4t - \frac{3}{2}\cos 2t\right]_{\frac{3\pi}{4}}^{\pi}$ $= -2\left[\left(\pi - \frac{3}{2}\right) - \frac{3\pi}{4}\right]$ $= -2\left[\frac{\pi}{4} - \frac{3}{2}\right]$ $= 3 - \frac{\pi}{2}$ units^2
(iii)	$x = 2\sin 2t + 3 \qquad y = \cos 2t$ $\sin 2t = \frac{x-3}{2}$ Since $\sin^2 2t + \cos^2 2t = 1$, $\frac{(x-3)^2}{4} + y^2 = 1 \Rightarrow y^2 = 1 - \frac{(x-3)^2}{4}$

(iv)	$(y+1)^2 = 1-\frac{(x-3)^2}{4}$ $y=-1+\sqrt{1-\frac{(x-3)^2}{4}}$ (Since $y \geq -1$) Volume $=\pi(1^2)\cdot 1+\pi\int_1^3 y^2 \, \mathrm{d}x$ $=\pi(1^2)\cdot 1+\pi\int_1^3 \left(-1+\sqrt{1-\frac{(x-3)^2}{4}}\right)^2 \mathrm{d}x$ ≈ 3.7440 units^3 ≈ 3.74 units^3

Q9	Suggested Answers
(i)	$a+d=b \quad -(1)$ $a+3d=br^3 \quad -(2)$ $a+5d=br^7 \quad -(3)$ $(3)-(2)=(2)-(1)$ $br^7-br^3=br^3-b$ $r^7-2r^3+1=0$ (since b is non-zero)
(ii)	Using G.C. $r=1,\ r=-1.2578,\ r=0.92057$ Since d is non-zero, $r\neq 1$. The geometric progression has positive terms, so r must be positive. Hence $r=0.921$ is the only answer. Since $\lvert r\rvert<1$, the geometric progression is convergent.
(iii)	$\left\lvert\frac{2n}{2}\left[2k+(2n-1)(3k)\right]-4k(0.92057)^{n-1}\right\rvert\leq 1000k$ $\left\lvert n(6n-1)-4(0.92057)^{n-1}\right\rvert-1000\leq 0$ <table><tr><td>n</td><td>$\left\lvert n(6n-1)-4(0.92057)^{n-1}\right\rvert-1000$</td></tr><tr><td>13</td><td>−0.482</td></tr><tr><td>14</td><td>160.64</td></tr></table>Largest value of n is 13

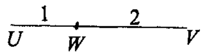
(iv)	Since $\ln\left(\frac{1}{u_n}\right)-\ln\left(\frac{1}{u_{n-1}}\right)=-\ln u_n+\ln u_{n-1}$ $=\ln\frac{u_{n-1}}{u_n}=\ln\frac{1}{r}$ is a constant, the sequence is an arithmetic progression. Alternatively, $\ln\left(\frac{1}{u_n}\right)-\ln\left(\frac{1}{u_{n-1}}\right)=-\ln u_n+\ln u_{n-1}$ $=-\ln br^{n-1}+\ln br^{n-2}$ $=\ln\frac{br^{n-2}}{br^{n-1}}=\ln r^{-1}=-\ln r$ is a constant, the sequence is an arithmetic progression.

Q10	Suggested Answers
(i)	$F_1:\ \mathbf{r}=\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}+p\begin{pmatrix}6\\4\\2\end{pmatrix}+q\begin{pmatrix}0\\1\\-4\end{pmatrix}$ $\mathbf{n}_1=\begin{pmatrix}6\\4\\2\end{pmatrix}\times\begin{pmatrix}0\\1\\-4\end{pmatrix}=\begin{pmatrix}-18\\24\\6\end{pmatrix}=6\begin{pmatrix}-3\\4\\1\end{pmatrix}$ $F_1:\mathbf{r}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=1$
(ii)	$F_2:\ \mathbf{r}=\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}=\begin{pmatrix}-9\\1\\3\end{pmatrix}+p\begin{pmatrix}3\\1\\-1\end{pmatrix}+q\begin{pmatrix}0\\-2\\8\end{pmatrix}$ $\mathbf{n}_2=\begin{pmatrix}3\\1\\-1\end{pmatrix}\times\begin{pmatrix}0\\-2\\8\end{pmatrix}=\begin{pmatrix}6\\-24\\-6\end{pmatrix}=-6\begin{pmatrix}-1\\4\\1\end{pmatrix}$ $\theta=\cos^{-1}\left\|\dfrac{\begin{pmatrix}-3\\4\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\4\\1\end{pmatrix}}{\sqrt{9+16+1}\sqrt{1+16+1}}\right\|=\cos^{-1}\left\|\dfrac{20}{\sqrt{26}\sqrt{18}}\right\|\approx 22.4^{\circ}$

(iii)

$$\overrightarrow{OU}=\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix} \qquad \overrightarrow{OV}=\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}$$

Given $2UW = WV$



$$\frac{UW}{WV}=\frac{1}{2}$$

$$\overrightarrow{OW}=\frac{1}{3}\left(\overrightarrow{OV}+2\overrightarrow{OU}\right)$$

$$=\frac{1}{3}\left[\begin{pmatrix}-9+3p\\1+p-2q\\3-p+8q\end{pmatrix}+2\begin{pmatrix}3+6p\\1+4p+q\\6+2p-4q\end{pmatrix}\right]$$

$$=\frac{1}{3}\begin{pmatrix}-3+15p\\3+9p\\15+3p\end{pmatrix}=\begin{pmatrix}-1+5p\\1+3p\\5+1p\end{pmatrix}=\begin{pmatrix}-1\\1\\5\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix}$$

$$l:\ \mathbf{r}=\begin{pmatrix}-1\\1\\5\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix},\ p\in\mathbb{R}$$

(iv)	Method 1: Since point A is on F_1 and on the line $\mathbf{r} = s\mathbf{k} = s\begin{pmatrix}0\\0\\1\end{pmatrix}$,we have $s\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=1$. Therefore, $s = 1$ and $\overrightarrow{OA}=\begin{pmatrix}0\\0\\1\end{pmatrix}$
	Method 2: Since point A is on F_1 and on the line $\mathbf{r} = s\mathbf{k} = s\begin{pmatrix}0\\0\\1\end{pmatrix}$,we have $\mathbf{s}\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}+p\begin{pmatrix}6\\4\\2\end{pmatrix}+q\begin{pmatrix}0\\1\\-4\end{pmatrix}---(*)$ $3+6p=0$ and $1+4p+q=0$ $p=-\frac{1}{2}$ and $1+4(-\frac{1}{2})+q=0$ $q=1$ Sub back (*), $\overrightarrow{OA}=\begin{pmatrix}3\\1\\6\end{pmatrix}-\frac{1}{2}\begin{pmatrix}6\\4\\2\end{pmatrix}+\begin{pmatrix}0\\1\\-4\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$

	To find perpendicular distance between A and line l: Method 1: Distance of A to $l = \left\lvert \dfrac{\left(\begin{pmatrix}-1\\1\\5\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}\right)\times\begin{pmatrix}5\\3\\1\end{pmatrix}}{\sqrt{25+9+1}} \right\rvert = \left\lvert \dfrac{\begin{pmatrix}-1\\1\\4\end{pmatrix}\times\begin{pmatrix}5\\3\\1\end{pmatrix}}{\sqrt{25+9+1}} \right\rvert$ $= \dfrac{\left\lvert\begin{pmatrix}-11\\21\\-8\end{pmatrix}\right\rvert}{\sqrt{25+9+1}} = \dfrac{\sqrt{626}}{\sqrt{35}} = 4.23$ (to 3 s.f.)
(iv)	To find perpendicular distance between A and line l: Method 2: Let P be a point on l $\overrightarrow{OP} = \begin{pmatrix}-1\\1\\5\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix}$ $\overrightarrow{AP} = \begin{pmatrix}-1\\1\\5\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\4\end{pmatrix} + p\begin{pmatrix}5\\3\\1\end{pmatrix}$

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	Since $\overrightarrow{AP}$ is perpendicular to $\begin{pmatrix}5\\3\\1\end{pmatrix}$, $\left[\begin{pmatrix}-1\\1\\4\end{pmatrix}+p\begin{pmatrix}5\\3\\1\end{pmatrix}\right]\cdot\begin{pmatrix}5\\3\\1\end{pmatrix}=0$ $2+(25+9+1)p=0$ $p=-\frac{2}{35}$ $\overrightarrow{AP}=\begin{pmatrix}-1\\1\\4\end{pmatrix}-\frac{2}{35}\begin{pmatrix}5\\3\\1\end{pmatrix}=\frac{1}{35}\begin{pmatrix}-45\\29\\138\end{pmatrix}$ Distance of A to $l=\left\|\overrightarrow{AP}\right\|=\frac{\sqrt{(-45)^2+(29)^2+(138)^2}}{35}\approx 4.23$

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Q11	Suggested Answers
(i)	$\frac{\mathrm{d}(2x)}{\mathrm{d}t}=m(8-x)(6-x)$ $2\frac{\mathrm{d}x}{\mathrm{d}t}=m(8-x)(6-x)$ $\frac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$, where $k=\frac{m}{2}$ Equivalently, $\frac{\mathrm{d}x}{\mathrm{d}t}=k(x-8)(x-6)$
(ii)	**Method 1:** $\frac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$ $\int\frac{1}{(8-x)(6-x)}\mathrm{d}x=\int k\,\mathrm{d}t$ $\int\frac{1}{(x-7)^2-1}\mathrm{d}x=\int k\,\mathrm{d}t$ $\frac{1}{2}\ln\left\|\frac{(x-7)-1}{(x-7)+1}\right\|=kt+C$ $\frac{1}{2}\ln\left\|\frac{x-8}{x-6}\right\|=kt+C$ Since $0\le x\le 6$, $\left\|\frac{x-8}{x-6}\right\|=\frac{x-8}{x-6}$ So, $\frac{1}{2}\ln\left(\frac{x-8}{x-6}\right)=kt+C$

	$\frac{x-8}{x-6}=e^{2kt+2C}$ $\frac{x-8}{x-6}=Ae^{2kt}$ where $A=e^{2C}$ When $t=0$, $x=0$, $A=\frac{4}{3}$ $\frac{x-8}{x-6}=\frac{4}{3}e^{2kt}$ When $t=5$, $x=5$, $3=\frac{4}{3}e^{10k}$ $\left(e^{k}\right)^{10}=\frac{9}{4}$ $e^{k}=\left(\frac{9}{4}\right)^{\frac{1}{10}}$ $\frac{x-8}{x-6}=\frac{4}{3}e^{2kt}$ $3x-24=(4x-24)e^{2kt}$ $3x-4xe^{2kt}=24-24e^{2kt}$ $x\left(3-4e^{2kt}\right)=24-24e^{2kt}$ $x=\frac{24-24e^{2kt}}{3-4e^{2kt}}$

	$x=\dfrac{24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}}{3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}}$ $x=\dfrac{24\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-24}{3\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-4}$
(ii)	**Method 2:** $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$ $\int\dfrac{1}{(8-x)(6-x)}\mathrm{d}x=\int k\,\mathrm{d}t$ $\int\dfrac{1}{(x-7)^2-1}\mathrm{d}x=\int k\,\mathrm{d}t$ $\dfrac{1}{2}\ln\left\|\dfrac{(x-7)-1}{(x-7)+1}\right\|=kt+C$ $\dfrac{1}{2}\ln\left\|\dfrac{x-8}{x-6}\right\|=kt+C$ $\left\|\dfrac{x-8}{x-6}\right\|=\mathrm{e}^{2kt+C}$ $\dfrac{x-8}{x-6}=\pm\mathrm{e}^{2kt+2C}$ $\dfrac{x-8}{x-6}=A\mathrm{e}^{2kt}$ where $A=\pm\mathrm{e}^{2C}$

When $t=0$, $x=0$,

$$A=\frac{4}{3}$$

$$\frac{x-8}{x-6}=\frac{4}{3}e^{2kt}$$

When $t=5$, $x=5$,

$$3=\frac{4}{3}e^{10k}$$

$$e^{10k}=\frac{9}{4}$$

$$10k=\ln\frac{9}{4}$$

$$k=\frac{1}{10}\ln\frac{9}{4}$$

$$\frac{x-8}{x-6}=\frac{4}{3}e^{\left(\frac{1}{5}\ln\frac{9}{4}\right)t}$$

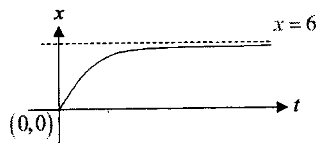
$$3x-24=(4x-24)e^{\left(\ln\left(\frac{9}{4}\right)^{\frac{1}{5}}\right)t}$$

$$3x-24=(4x-24)\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$

$$3x-4x\left(\frac{9}{4}\right)^{\frac{1}{5}t}=24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$

$$x\left(3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}\right)=24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}$$

	$x=\dfrac{24-24\left(\frac{9}{4}\right)^{\frac{1}{5}t}}{3-4\left(\frac{9}{4}\right)^{\frac{1}{5}t}}$ $x=\dfrac{24\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-24}{3\left(\frac{9}{4}\right)^{-\frac{1}{5}t}-4}$
(ii)	**Alternative method (use of partial fractions):** $\dfrac{\mathrm{d}x}{\mathrm{d}t}=k(8-x)(6-x)$ $\int\dfrac{1}{(8-x)(6-x)}\mathrm{d}x=\int k\,\mathrm{d}t$ $\int\dfrac{-\frac{1}{2}}{8-x}+\dfrac{\frac{1}{2}}{6-x}\mathrm{d}x=\int k\,\mathrm{d}t$ $\frac{1}{2}\ln\lvert 8-x\rvert-\frac{1}{2}\ln\lvert 6-x\rvert=kt+C$ $\frac{1}{2}\ln\left\lvert\dfrac{x-8}{x-6}\right\rvert=kt+C$ $\vdots$

(iii)	
(iv)	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -8\mathrm{e}^{-2t} - \frac{16}{9}(t+1)^{-\frac{7}{3}}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 4\mathrm{e}^{-2t} + \frac{4}{3}(t+1)^{-\frac{4}{3}} + A$ $x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + At + B$ As $t \to \infty$, $x \to 6$, $A = 0$ (else $At \to \pm\infty$), $B = 6$ $x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + 6$ <u>Alternatively,</u> When $t = 0$, $x = 0$, $0 = -2 - 4 + B \Rightarrow B = 6$ $x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + At + 6$ As $t \to \infty$, $x \to 6$, $A = 0$ $x = -2\mathrm{e}^{-2t} - 4(t+1)^{-\frac{1}{3}} + 6$

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Q1	Suggested Answers
(i)	A plane containing the vectors **a** and **b**, and the origin O. OR A plane containing O, A and B.
(ii)	It is the length of projection of $\overrightarrow{OP}$ onto the plane containing the vectors **a** and **b**, and the origin O. It is the length of projection of $\overrightarrow{OP}$ onto the plane containing O, A and B.
(iii)	$\mathbf{c}=\dfrac{\mathbf{b}+9\mathbf{a}}{10}=\dfrac{1}{10}\mathbf{b}+\dfrac{9}{10}\mathbf{a}$ $\overrightarrow{OC}\cdot\overrightarrow{AB}=0$ $(\dfrac{1}{10}\mathbf{b}+\dfrac{9}{10}\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})=0$ $\dfrac{1}{10}\mathbf{b}\cdot\mathbf{b}-\dfrac{1}{10}\mathbf{b}\cdot\mathbf{a}+\dfrac{9}{10}\mathbf{a}\cdot\mathbf{b}-\dfrac{9}{10}\mathbf{a}\cdot\mathbf{a}=0$ Since **a** is perpendicular to **b**, $\mathbf{a}\cdot\mathbf{b}=0$ $\dfrac{1}{10}\|\mathbf{b}\|^2-\dfrac{9}{10}\|\mathbf{a}\|^2=0$ $\|\mathbf{b}\|^2=9\|\mathbf{a}\|^2\Rightarrow\dfrac{\|\mathbf{b}\|^2}{\|\mathbf{a}\|^2}=9$ $\therefore OB:OA=3:1$

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Q2	Suggested Answers
	$w-z=1-\sqrt{3}$ ----(1) $\mathrm{i}z+w=(\sqrt{3}+1)\mathrm{i}$ ----(2) From (1), $z=w-1+\sqrt{3}$ sub into (2) $\mathrm{i}(w-1+\sqrt{3})+w=(\sqrt{3}+1)\mathrm{i}$ $\mathrm{i}w-\mathrm{i}+\sqrt{3}\mathrm{i}+w=\sqrt{3}\mathrm{i}+\mathrm{i}$ $(1+\mathrm{i})w=\sqrt{3}\mathrm{i}+\mathrm{i}+\mathrm{i}-\sqrt{3}\mathrm{i}$ $(1+\mathrm{i})w=2\mathrm{i}$ $w=\frac{2\mathrm{i}}{1+\mathrm{i}}=1+\mathrm{i}$ $w=1+\mathrm{i}=\sqrt{2}\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$ $\mathrm{i}w=\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}\times\sqrt{2}\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}=\sqrt{2}\mathrm{e}^{\frac{3\pi}{4}\mathrm{i}}$ $(\mathrm{i}w)^n=(\sqrt{2})^n\mathrm{e}^{\frac{3n\pi}{4}\mathrm{i}}$ For $(\mathrm{i}w)^n$ to be real, $\frac{3n\pi}{4}=k\pi,\quad k\in\mathbb{Z}$ $n=\frac{4k}{3},\quad k\in\mathbb{Z}$ $\therefore$ 3 smallest positive whole number values $n=4, 8, 12$

Q3	Suggested Answers
	Volume $= k = \pi(15x)^2 w + \frac{1}{3}\pi(15x)^2(8x)$ ---(1) $A = 2\pi(15x)w + \pi(15x)^2 + \pi(15x)\sqrt{(8x)^2+(15x)^2}$ $= 2\pi(15x)w + \pi(15x)^2 + \pi(15x)(17x)$ $= 30\pi x w + 480\pi x^2$ ----(2) From (1), $w = \frac{k-600\pi x^3}{225\pi x^2}$ Subst. into (2) $A = 30\pi x\left[\frac{k-600\pi x^3}{225\pi x^2}\right] + 480\pi x^2$ $= \frac{2k}{15x} + 400\pi x^2$ $\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{2k}{15x^2} + 800\pi x$ Let $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$, thus $x^3 = \frac{k}{6000\pi}$, or $x = \frac{1}{10}\left(\frac{k}{6\pi}\right)^{\frac{1}{3}}$ $w = \frac{1}{225\pi\frac{k^{\frac{2}{3}}}{100(6^{\frac{2}{3}})(\pi^{\frac{2}{3}})}}\left[k - 600\pi \times \frac{k}{6000\pi}\right]$ $= \frac{100(6^{\frac{2}{3}})}{225\pi^{\frac{1}{3}}k^{\frac{2}{3}}}\left[\frac{9}{10}k\right] = \frac{12}{5}\left(\frac{k}{6\pi}\right)^{\frac{1}{3}}$

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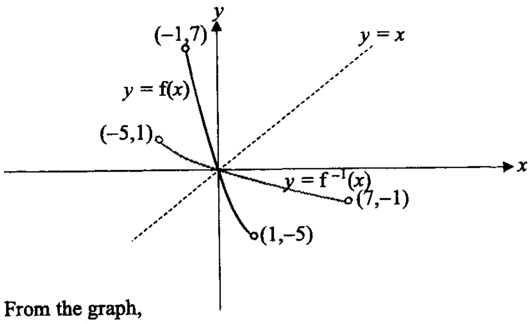
Q4	Suggested Answers
(i)	Let $y=x^2-6x$ $y=x^2-6x=(x-3)^2-9$ $(x-3)^2=y+9$ $x=3\pm\sqrt{y+9}$ Alternatively, $x^2-6x-y=0$ $x=\frac{-(-6)\pm\sqrt{36-4(-y)}}{2}=3\pm\sqrt{9+y}$ Since $-1<x<1, x=3-\sqrt{y+9}$ $f^{-1}(x)=3-\sqrt{x+9}$ $D_{f^{-1}}=(-5,7)$
(ii)	 From the graph, $f(x)=f^{-1}(x)\Rightarrow x=0$

(iii)	$\mathrm{h}(x)=\mathrm{gf}(x)=1+\dfrac{1}{x^2-6x+5}$ $\mathrm{h}'(x)=\dfrac{-(2x-6)}{\left(x^2-6x+5\right)^2}$ $-1<x<1 \Rightarrow -8<2x-6<-4$ $4<-(2x-6)<8$ Since $-(2x-6)>0$ and $\left(x^2-6x+5\right)^2>0$, $\therefore \mathrm{h}'(x)>0$ for $-1<x<1$. Hence $\mathrm{h}(x)$ increases as x increases.

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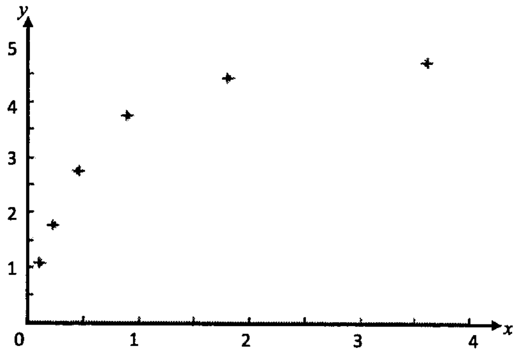
Q5	Suggested Answers
(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+4}\right)=\frac{2x}{2\sqrt{x^2+4}}=\frac{x}{\sqrt{x^2+4}}$
(a)(ii)	$u=x^2 \qquad \frac{\mathrm{d}v}{\mathrm{d}x}=\frac{x}{\sqrt{x^2+4}}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=2x \qquad v=\sqrt{x^2+4}$ $\int\frac{x^3}{\sqrt{x^2+4}}\,\mathrm{d}x=x^2\sqrt{x^2+4}-\int 2x\sqrt{x^2+4}\,\mathrm{d}x$ $=x^2\sqrt{x^2+4}-\frac{2\left(x^2+4\right)^{\frac{3}{2}}}{3}+C$
(b)(i)	$\int\sin mx\cos mx\,\mathrm{d}x=\frac{1}{2}\int\sin 2mx\,\mathrm{d}x$ $=-\frac{1}{4m}\cos 2mx+C$ OR $\int\sin mx\cos mx\,\mathrm{d}x=-\frac{1}{m}\int(-m\sin mx)\cos mx\,\mathrm{d}x$ $=-\frac{1}{2m}\cos^2 mx+C$ OR $\int\sin mx\cos mx\,\mathrm{d}x=\frac{1}{m}\int(m\cos mx)\sin mx\,\mathrm{d}x$ $=\frac{1}{2m}\sin^2 mx+C$

(ii)	Method 1: $\int(\cos 3x\cos x)\sin 2x\,\mathrm{d}x=\int\frac{1}{2}(\cos 4x+\cos 2x)\sin 2x\,\mathrm{d}x$ $=\frac{1}{2}\int\cos 4x\sin 2x+\cos 2x\sin 2x\,\mathrm{d}x$ $=\frac{1}{4}\int(\sin 6x-\sin 2x)\,\mathrm{d}x+\frac{1}{2}\int\cos 2x\sin 2x\,\mathrm{d}x$ $=-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x-\frac{1}{16}\cos 4x+C$ Equivalent answers: $-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x-\frac{1}{8}\cos^2 2x+C$ Or $-\frac{1}{24}\cos 6x+\frac{1}{8}\cos 2x+\frac{1}{8}\sin^2 2x+C$
(ii)	Method 2: $\int(\cos 3x\sin 2x)\cos x\,\mathrm{d}x=\int\frac{1}{2}(\sin 5x-\sin x)\cos x\,\mathrm{d}x$ $=\frac{1}{2}\int\sin 5x\cos x-\sin x\cos x\,\mathrm{d}x$ $=\frac{1}{2}\int\frac{1}{2}(\sin 6x+\sin 4x)-\sin x\cos x\,\mathrm{d}x$ $=\frac{1}{4}\int\sin 6x+\sin 4x-\sin 2x\,\mathrm{d}x$ $=\frac{1}{4}\left(-\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x+\frac{1}{2}\cos 2x\right)+C$

(ii)	Method 3: $\int(\sin 2x\cos x)\cos 3x\,\mathrm{d}x=\int\frac{1}{2}(\sin 3x+\sin x)\cos 3x\,\mathrm{d}x$ $=\frac{1}{2}\int\sin 3x\cos 3x+\sin x\cos 3x\,\mathrm{d}x$ $=\frac{1}{2}\int\frac{1}{2}\sin 6x+\frac{1}{2}(\sin 4x+\sin(-2x))\,\mathrm{d}x$ $=\frac{1}{4}\int\sin 6x+\sin 4x-\sin 2x\,\mathrm{d}x$ $=\frac{1}{4}\left(-\frac{1}{6}\cos 6x-\frac{1}{4}\cos 4x+\frac{1}{2}\cos 2x\right)+C$

Q6	Suggested Answers
(i)	Largest number of unsuccessful login attempts $= {}^{26}C_4 \times {}^{10}C_2 \times 6! - 1 = 484379999$
(ii)	Number of passwords $= {}^{26}C_4 \times {}^{10}C_2 \times 2!$ $= 1345500$
(iii)	Method 1: Number of passwords with exactly one vowel in the first four distinct letters and last two digits are distinct $= {}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!$ Number of passwords such that the first four letters are distinct and the last two digits are distinct $= {}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!$ Required probability $= \dfrac{{}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!}{{}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!} = \dfrac{133}{299}$ Alternative 1: Required probability $= \dfrac{{}^{21}C_3 \times {}^{5}C_1 \times 4! \times {}^{10}C_2 \times 2!}{{}^{26}C_4 \times 4! \times {}^{10}C_2 \times 2!}$ $= \dfrac{{}^{21}C_3 \times {}^{5}C_1}{{}^{26}C_4} = \dfrac{133}{299}$ Alternative 2:

	Required probability $=\dfrac{{}^{21}C_3\,{}^{5}C_1\,4!\,{}^{10}C_2\,2!/{}^{26}C_4\,{}^{10}C_2\,6!}{{}^{26}C_4\,4!\,{}^{10}C_2\,2!/{}^{26}C_4\,{}^{10}C_2\,6!}$ $=\dfrac{{}^{21}C_3\,{}^{5}C_1}{{}^{26}C_4}=\dfrac{133}{299}$

7	Suggested Answers
(i)	
(ii)	Using GC, product moment correlation coefficient, $r = 0.84162$, i.e. $r = 0.842$, suggesting a positive linear correlation between the concentration of a reactant, x, and rate of enzymatic reactions, y. However, the scatter diagram in **(i)** shows that this linear relation is not strong since as x increases, y increases at a decreasing rate. **Or** The value of r is not close to 1, indicating the linear correlation between x and y is not strong as indicated by / agreeing with the scatter diagram in **(i)**, showing that as x increases, y increases at a decreasing rate.

(iii)	**Method 1** $y=\frac{ax}{b+x}$ $\frac{1}{y}=\frac{b+x}{ax}=\frac{b}{a}\left(\frac{1}{x}\right)+\frac{1}{a}$ Using GC, $r_{new}=0.99956\approx 0.9996$ $\frac{b}{a}=0.084164$ and $\frac{1}{a}=0.18190$ Hence $a = 5.4974 = 5.50$ and $b = 0.462695 = 0.463$ (3 sig fig) **Method 2** $\frac{1}{y}=A\left(\frac{1}{x}\right)+B$ Using GC, $r_{new}=0.99956\approx 0.9996$ $A=0.084164$ and $B=0.18190$ $\frac{1}{y}=0.084164\left(\frac{1}{x}\right)+0.18190$ $=\frac{0.084164+0.18190x}{x}$ $y=\frac{x}{0.18190(0.46269+x)}$ $=\frac{5.4975x}{0.46269+x}$ Hence $a = 5.4975 = 5.50$ and $b = 0.46269 = 0.463$ (3 sig fig)

(iv)	When $y=5$, $5=\dfrac{5.4974x}{0.462695+x}$ Hence $x=\dfrac{5\times0.462695}{0.4974}=4.6511=4.65$ Since $y=5$ is outside the data range [1.081, 4.717], estimate is unreliable.

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8	Suggested Answers
(i)	Whether the shooter achieves / hits bullseye in an attempt is independent of whether the shooter achieves / hits bullseye in any of the other attempts. The shooter has the same probability of achieving / hitting bullseye for each / every attempt made.
(ii)	The shooter achieving / hitting bullseye in any attempt may <u>not</u> be <u>independent:</u> Or The shooter may <u>not</u> have <u>the same probability</u> of achieving / hitting bullseye for each / every attempt made: • as the shooter may be tired from the previous attempts made • his accuracy may improve with more attempts • his accuracy may be affected by the changing weather conditions
(iii)	Let X be the random variable denoting the number of bullseyes achieved out of 35 attempts. $X \sim \mathrm{B}(35, 0.7)$ Using GC, <table><tr><td>x</td><td>$\mathrm{P}(X=x)$</td></tr><tr><td>24</td><td>0.14160</td></tr><tr><td>25</td><td>0.14537</td></tr><tr><td>26</td><td>0.13046</td></tr></table>Hence most probable number of bullseye $m = 25$ $\mathrm{P}(X=25) = 0.14537 \approx 0.145$
(iv)	$\mathrm{P}(X \geq 25) = 1 - \mathrm{P}(X \leq 24)$ $= 0.50996 \approx 0.510$
(v)	<u>Method 1:</u> $X \sim \mathrm{B}(35, 0.7)$

	$\mathrm{E}(X)=24.5$ $\mathrm{Var}(X)=35(0.7)(0.3)=7.35$ Since number of trainings (= 40) is large, by Central Limit Theorem, $\overline{X}\sim \mathrm{N}\left(24.5,\dfrac{7.35}{40}\right)$ approximately $\mathrm{P}\left(\overline{X}\geq 25\right)=0.12172\approx 0.122$
(v)	Method 2: $X\sim \mathrm{B}(35,0.7)$ $\mathrm{E}(X)=24.5$ $\mathrm{Var}(X)=35(0.7)(0.3)=7.35$ Let $T=X_1+X_2+\ldots+X_{40}$ Since number of trainings (= 40) is large, by Central Limit Theorem, $T\sim \mathrm{N}(24.5\times 40, 7.35\times 40)$ ie $T\sim \mathrm{N}(980, 294)$ approximately $\mathrm{P}(T\geq 25\times 40)=\mathrm{P}(T\geq 1000)=0.12172\approx 0.122$

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Q9	Suggested Answers
(i)	$\mathrm{P}(S=-1)=\mathrm{P}(\{\mathrm{Y,B}\})=\dfrac{2(3\times1)}{(x+4)(x+3)}=\dfrac{6}{(x+4)(x+3)}$ $\mathrm{P}(S=0)=\mathrm{P}(\{\mathrm{B,R}\})=\dfrac{2x}{(x+4)(x+3)}$ $\mathrm{P}(S=2)=\mathrm{P}(\{\mathrm{Y,Y}\})=\dfrac{3\times2}{(x+4)(x+3)}=\dfrac{6}{(x+4)(x+3)}$ $\mathrm{P}(S=3)=\mathrm{P}(\{\mathrm{Y,R}\})=\dfrac{2(3x)}{(x+4)(x+3)}=\dfrac{6x}{(x+4)(x+3)}$ $\mathrm{P}(S=4)=\mathrm{P}(\{\mathrm{R,R}\})=\dfrac{x(x-1)}{(x+4)(x+3)}$
(ii)	

s	-1	0	2	3	4
$\mathrm{P}(S=s)$	$\dfrac{6}{(x+4)(x+3)}$	$\dfrac{2x}{(x+4)(x+3)}$	$\dfrac{6}{(x+4)(x+3)}$	$\dfrac{6x}{(x+4)(x+3)}$	$\dfrac{x(x-1)}{(x+4)(x+3)}$

	$\mathrm{E}(S)$ $=-1\cdot\frac{6}{(x+4)(x+3)}+0\cdot\frac{2x}{(x+4)(x+3)}+2\cdot\frac{6}{(x+4)(x+3)}+3\cdot\frac{6x}{(x+4)(x+3)}$ $+4\cdot\frac{x(x-1)}{(x+4)(x+3)}$ $=\frac{-6+12+18x+4x(x-1)}{(x+4)(x+3)}$ $=\frac{4x^2+14x+6}{(x+4)(x+3)}$ $=\frac{2(2x+1)(x+3)}{(x+4)(x+3)}$ $=\frac{2(2x+1)}{x+4}$ $\mathrm{Var}(S)$ $=(-1)^2\cdot\frac{6}{(x+4)(x+3)}+0\cdot\frac{2x}{(x+4)(x+3)}+2^2\cdot\frac{6}{(x+4)(x+3)}+3^2\cdot\frac{6x}{(x+4)(x+3)}$ $+4^2\cdot\frac{x(x-1)}{(x+4)(x+3)}-\left(\frac{2(2x+1)}{x+4}\right)^2$ $=\frac{30+38x+16x^2}{(x+4)(x+3)}-\frac{4(2x+1)^2}{(x+4)^2}$ $=\frac{(30+38x+16x^2)(x+4)-4(2x+1)^2(x+3)}{(x+4)^2(x+3)}$

	$=\dfrac{2\left(19x^2+65x+54\right)}{(x+4)^2(x+3)}$ $=\dfrac{2(19x+27)(x+2)}{(x+4)^2(x+3)}$
(iii)	Method 1: $\mathrm{E}(S)=\dfrac{2(2x+1)}{x+4}>2$ Since $x>1$, $2(2x+1)>2x+8$ $x>3$ Least $x=4$ Method 2: $\mathrm{E}(S)=\dfrac{2(2x+1)}{x+4}>2$ Using GC, <table><tr><td>x</td><td>$\mathrm{E}(S)$</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>2.25</td></tr></table> Least $x=4$

Q10	Suggested Answers
(i)	Unbiased estimates of the population mean and variance are: $\bar{x} = \frac{650}{40} = 16.25$ $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$ $= \frac{944}{39} \approx 24.205 \approx 24.2$
(ii)	Let X be the waiting time of a randomly chosen customer and μ be the population mean waiting time of a randomly chosen customer. To test $H_0 : \mu = 15$ $H_1 : \mu > 15$ at 5% level of significance. Under H_0, since $n = 40$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(15, \frac{944/39}{40}\right)$ approximately and $Z = \frac{\bar{X} - 15}{\sqrt{\frac{944/39}{40}}} \sim \mathrm{N}(0,1)$ approximately. Reject H_0 if p-value ≤ 0.05 or $z_{\text{cal}} \geq 1.64485$ $z_{\text{cal}} = \frac{16.25 - 15}{\sqrt{\frac{944/39}{40}}} = 1.60689$ Calculations: p-value $= 0.05404$

	Since p-value > 0.05 (or z_{cal} < 1.64485), we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance to conclude that the population mean waiting time is longer than 15 minutes.
(iii)	There is no need to know the distribution since the sample size $n = 40$ is large, by Central Limit Theorem, the sample mean waiting time, $\overline{X}$, is approximately normally distributed, ie $\overline{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ approximately and z-test can be used.
(iv)	Let Y be the waiting time of a randomly chosen customer. To test $H_0 : \mu = 15$ $H_1 : \mu > 15$ at 5% level of significance. Under H_0, since $n = 32$ is large, by Central Limit Theorem, $\overline{Y} \sim N\left(15, \frac{30.25}{32}\right)$ approximately and $Z = \frac{\overline{Y} - 15}{\sqrt{30.25/32}} \sim N(0,1)$ approximately. Reject H_0 if $z_{cal} \geq 1.64485$ $z_{cal} = \frac{\overline{y} - 15}{\sqrt{30.25/32}} \geq 1.64485$ $\overline{y} \geq 16.599$ Thus if bank director's suspicion is confirmed, we need $\overline{y} \geq 16.6$

11	Suggested Answers
(i)	Let B, H and L be random variables denoting the time taken to do a repetition of bench press, a hack squat and a leg press respectively. $L \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\mu = \dfrac{2.9+5.1}{2} = 4$
	Method 1: $\mathrm{P}(L < 2.9) = 0.15$ $\mathrm{P}\left(Z < \dfrac{2.9-4}{\sigma}\right) = 0.15$ Using GC, $\dfrac{2.9-4}{\sigma} = -1.0364$ $\sigma = 1.0613 \approx 1.06$
	Method 2: $\mathrm{P}(L > 5.1) = 0.15$ $\mathrm{P}\left(Z > \dfrac{5.1-4}{\sigma}\right) = 0.15$ Using GC, $\dfrac{5.1-4}{\sigma} = 1.0364$ $\sigma = 1.0613 \approx 1.06$
	Method 3: $\mathrm{P}(2.9 \le L \le 5.1) = 0.7$

	$\mathrm{P}\left(\frac{2.9-4}{\sigma} \le Z \le \frac{5.1-4}{\sigma}\right)=0.7$ Using GC, $\mathrm{P}(-1.0364 \le Z \le 1.0364)=0.7$ $\frac{2.9-4}{\sigma}=-1.0364$ or $\frac{5.1-4}{\sigma}=1.0364$ $\sigma=1.0613 \approx 1.06$
(ii)	Let C be random variable denoting the time taken for a circuit. $C=B_1+\cdots+B_{10}+H_1+\cdots+H_{10}+L_1+\cdots+L_{10}+120$ Expected time taken for a circuit, $\mathrm{E}(C)=10(5)+10(3)+10(4)+120=240$ (shown)
(iii)	$\mathrm{Var}(C)=10(0.5^2)+10(0.1^2)+10(1.0613^2)=13.8635769$ $C \sim \mathrm{N}(240, 13.8635769)$ $C_1-C_2 \sim \mathrm{N}(0, 27.7271538)$ $\mathrm{P}(C_1>C_2+5)=\mathrm{P}(C_1-C_2>5)=0.1711 \approx 0.171$
(iv)	Let R be the random variable denoting the time taken for two circuits. $R=C_1+C_2+120$ $\mathrm{E}(R)=2(240)+120=600$ $\mathrm{Var}(R)=2(13.8635769)=27.7271538$ $R-2C \sim \mathrm{N}(600-2(240), 27.7271538+4(13.8635769))$ i.e. $R-2C \sim \mathrm{N}(120, 83.1814614)$ $\mathrm{P}(R-2C>150)=0.00050218 \approx 0.000502$
(v)	Let $D=B_1+\cdots+B_{10}+H_1+\cdots+H_{10}+L_1+\cdots+L_{10}+2k$ $D \sim \mathrm{N}(120+2k, 13.8635769)$ $\mathrm{P}(D>180)<0.01$

Method 1

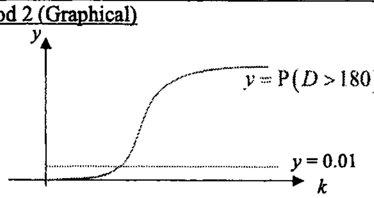
$\mathrm{P}\left(Z > \dfrac{180-120-2k}{\sqrt{13.8635769}}\right) < 0.01$

$\dfrac{60-2k}{\sqrt{13.8635769}} > 2.3263$

$k < 25.669$

Hence largest integer k is 25 seconds

Method 2 (Graphical)



Using GC, $k = 25.669$

Hence $0 < k < 25.669$

Largest integer k is 25 seconds

Method 3

$\mathrm{P}(D > 180) - 0.01 < 0$

k	$\mathrm{P}(D>180)-0.01$
24	$-0.0093654 < 0$
25	$-0.0063814 < 0$
26	$0.0058336 > 0$

Largest integer k is 25 seconds