**ANGLO-CHINESE JUNIOR COLLEGE**
**JC2 PRELIMINARY EXAMINATION**

Higher 1

# MATHEMATICS 8865/01

Paper 1 **13 August 2018**

**3 hours**

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

**ANGLO-CHINESE JUNIOR COLLEGE**
**MATHEMATICS DEPARTMENT**
**JC2 Preliminary Examination 2018**

**MATHEMATICS 8865**
**Higher 1**
**Paper 1**

/ 100

Index No: | | | |    Form Class: ____________

Name: ____________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

| Question No. | Marks |
|---|---|
| 1 | /4 |
| 2 | /5 |
| 3 | /8 |
| 4 | /12 |
| 5 | /11 |
| 6 | /4 |
| 7 | /6 |
| 8 | /7 |
| 9 | /10 |
| 10 | /11 |
| 11 | /10 |
| 12 | /12 |

| Summary of Areas for Improvement | | | |
|---|---|---|---|
| Knowledge (K) | Careless Mistakes (C) | Read/Interpret Qn wrongly (R) | Presentation (P) |
| | | | |

## Section A: Pure Mathematics [40 marks]

**1** Find, algebraically, the set of exact values of $m$ for which $3mx^2 - 24x + 7m > 0$ for all real values of $x$. [4]

**2** Find

**(i)** $\int \frac{5}{\sqrt{3x-1}}\,dx$, [2] **(ii)** $\int \left(\frac{1}{\sqrt{x}} - x\right)^2 dx$. [3]

**3** A culvert is a tunnel structure constructed under roadways to provide cross drainage. One particular type is the low-profile arch culvert as shown in diagram 1. An engineer was tasked to build a low-profile arch culvert drain to improve the drainage system beneath a certain roadway.

Diagram 1: Low-profile arch culvert

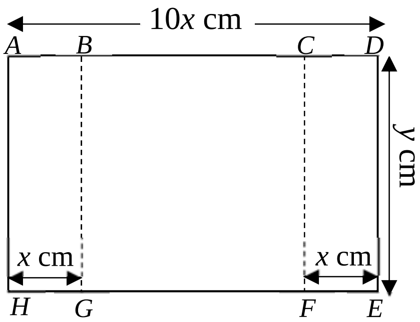



Diagram 2: Dimensions of cardboard

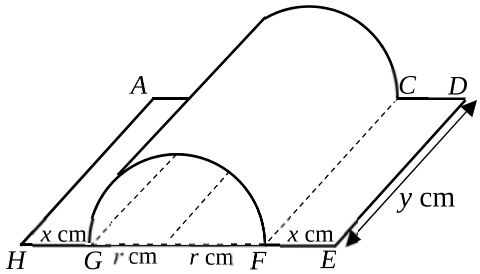



Diagram 3: Cardboard model of the culvert

The engineer decided to use a rectangular piece of cardboard to build a model of the culvert. The rectangular piece of cardboard used is shown in diagram 2 with length $AD = 10x$ cm and breadth $DE = y$ cm. The perimeter of this rectangular cardboard $ADEH$ is 54 cm. You may assume that the cardboard is of negligible thickness.

To build the culvert model, the engineer needs to fold the cardboard along the dotted lines $BG$ and $CF$ to create the model as shown in diagram 3. The arch $BC$ and $GF$ are the semicircles with radius $r$ cm.

**(i)** Using a non-calculator method, show that the radius of the culvert model in diagram 3 can be expressed as $r = \frac{8x}{\pi}$. [1]

**(ii)** Hence show that the volume of the space below the arch of the model is given by $V = \frac{32x^2}{\pi}(27 - 10x)$. [3]

**(iii)** Find the value of $x$ for which $V$ is a maximum. Justify that $V$ is maximum for this value of $x$. [4]

**4** **(i)** Find $\int_0^k 3x - e^{-x} + 2 \; dx$ in terms of $k$, where $k > 0$. [3]

The curve $C$ has equation $y = 3x - e^{-x} + 2$.

**(ii)** Sketch the graph of $C$, stating the coordinates of any points of intersection with the axes. [1]

**(iii)** Show that the equation of the tangent to the curve at $x = 1$ is $y = \left(3 + \frac{1}{e}\right)x - \frac{2}{e} + 2$. [4]

**(iv)** Hence find the exact area enclosed by curve $C$, the $y$-axis and the tangent to $C$ at the point where $x = 1$. [4]

**5** The top-selling bread from a baker are the Pretzel, the Baguette and the Ciabatta loaf. The cost of making 7 pretzels is equal to the cost of making 6 ciabatta loaves. The total cost to make 50 baguette is $16 less than the total cost to make 40 ciabatta loaves. The total cost to make 30 pretzels, 30 baguettes and 45 ciabatta loaves is $123.

**(i)** By writing down three linear equations, find the cost price of making each type of bread, correct to the nearest cent. [5]

A nearby café decided to make a daily order for baguette from the baker. The baker decided that he will only accept the order if there are at least 16 baguettes ordered. In a simple model, the total manufacturing cost for $x$ baguettes is given by this equation

$$C = \frac{x}{3} - \ln(2x - 30) + 20,$$

where $C$ is the manufacturing cost in dollars and $x$ is the number of baguette ordered daily.

**(ii)** Sketch the graph of $C$ against $x$ for $x > 15$. Estimate the minimum cost $C$ and state the number of baguettes for which this minimum value occurs. [3]

Suppose the baker wants to sell each baguette for $1, and we let $$P$ be the profit per baguette that the baker will earn.

**(iii)** Formulate an equation relating $P$ and $x$. [1]

**(iv)** Using your formula in **(iii)**, would you advise the baker to accept an order of 20 baguettes? Justify your answer. [2]

## Section B: Statistics [60 marks]

**6** Over a long period of time, a slimming centre found that at the end of a slimming programme, 20% of their clients lose more than 20 kg and 5% of their clients lose less than 10 kg. By modelling the weight loss of the slimming centre's clients to be a normal distribution, find the mean and variance of the distribution. [4]

**7** A candy factory manufactured a large amount of pastilles daily and their candies are randomly packed in boxes of 20. The probability of selecting an orange-flavoured pastille to be packed into a box is 0.25. The random variable $X$ is the number of orange-flavoured pastilles in a box of 20 pastilles.

**(i)** State an assumption that is needed for $X$ to be modelled by a binomial distribution. [1]

**(ii)** Find the probability that a box of pastilles contains at most 8 orange-flavoured pastilles. [1]

Jimmy likes the pastilles very much and would buy a box of pastilles each day from Monday to Friday. For a randomly selected week from Monday to Friday, find the probability that

**(iii)** John gets exactly 3 boxes of pastilles that contain at most 8 orange-flavoured pastilles; [2]

**(iv)** the box John gets on Friday is the third box of pastilles that contain at most 8 orange-flavoured pastilles. [2]

**8** **(a)** There are 8 girls and 10 boys in a class. Find the number of ways to form a class committee consisting of exactly 2 girls and 3 boys. [2]

**(b)** The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are arranged randomly to form 5-digit numbers. No digit is repeated. Find the number of 5-digit numbers

**(i)** greater than 50000; [2]

**(ii)** greater than 50000 and are odd. [3]

**9** The masses, in grammes, of oranges and tangerines are modelled as having independent normal distributions with means and standard deviations as shown in the table.

| | **Mean mass** | **Standard deviation** |
|---|---|---|
| **Oranges** | 181 g | 4.77 g |
| **Tangerines** | 165.8 g | 23.07 g |

Let $X$ represent the mass (in grammes) of a randomly selected orange.

Let $Y$ represent the mass (in grammes) of a randomly selected tangerine.

**(i)** Find the probability that a randomly selected orange weighs more than 183 g. [1]

**(ii)** Juices obtained from squeezing an orange is only 40% of the weight of the orange squeezed while juices obtained from squeezing a tangerine is only 35% of the weight of the tangerine squeezed.

Find $\mathrm{P}\left(0.4\left[X_1+X_2+X_3+X_4\right]>0.35\left[Y_1+Y_2+Y_3+Y_4+Y_5\right]\right)$, stating the mean and variance of the distribution. Explain, in the context of this question, what your answer represents. [5]

**(iii)** There are many benefits to orange skins. Mass of orange skins peeled, in grammes, have a mean of 90 g and standard deviation of 4 g. In order to collect a large amount of orange skins, a person peeled 55 oranges. Find the probability that the total mass of orange skins he gathered exceeds 5000 g. [4]

**10** The head of department for Mathematics believed that the students have done very well for a recent examination and claimed that the mean score for Mathematics for the entire Year 2 cohort is higher than 70. The mathematics score, $x$, of a random sample of 250 students are summarised as follows.

$$\sum(x-70)=305 \qquad \sum(x-70)^2=35565$$

**(i)** Calculate the unbiased estimates of the population mean and variance to the nearest 2 decimal places. [3]

**(ii)** Test at the 5% significant level whether the claim is justified. [4]

**(iii)** Another random sample of 300 students' scripts for Mathematics were marked. The sample mean score is 71.4 and the sample variance is $k$. Find the set of values of $k$ for which the same claim is justified at 5% significant level, giving your answer correct to 2 decimal places. [4]

**11** A marketing company wishes to investigate the relationship between the amount of time a person goes online while commuting back and forth between home and work. The marketing staff surveyed 10 people and the results are given in the following table.

| Amount of commuting time in a week, $x$ (hours) | 5 | 7 | 9 | 10 | 9 | 12 | 10 | 11 | 8 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|
| Amount of time spent online in a week, $y$ (hours) | 2.5 | 4.5 | 6 | 6.5 | 7 | 9.5 | 9.5 | 7 | 6.5 | 10 |

**(i)** Draw a sketch of the scatter diagram for the data, as shown on your calculator. [1]

**(ii)** Find the product moment correlation coefficient and comment on its value in the context of this question. [2]

**(iii)** Find the equation of the regression line of $y$ on $x$, in the form $y = mx + c$, giving the values of $m$ and $c$ correct to 4 decimal places. [1]

**(iv)** Explain the meaning of the value of $m$ in the context of this question. [1]

**(v)** Using the equation of the regression line of $y$ on $x$ found in **(iii)**, to estimate the number of hours per week for a person to be online if he/she has 8.5 hours of commuting time. Comment on the reliability of your answer. [3]

The staff decided to survey one more person who has commuted 25 hours in a week and spent 8 hours online while commuting.

**(vi)** Calculate the new product moment correlation coefficient when this person is included. [1]

**(vii)** State, with a reason, which of your answers to parts **(ii)** and **(vi)** is a better representation of the correlation between the amount of time spent commuting back and forth between home and work in a week and the amount of time spend online in a week. [1]

**12** In a particular year's O-level English results in Singapore, the candidates comprised Singaporeans, Malaysians and other foreigners. One of these candidates is chosen at random.

$E$ is the event that the candidate scored distinction for English.

$S$ is the event that the candidate is a Singaporean.

$M$ is the event that the candidate is a Malaysian.

Given that $P(E) = 0.3$, $P(S) = 0.8$ and $P(E|S) = 0.2$.

**(i)** Show that the events $E$ and $S$ are not independent. [1]

**(ii)** Find $P(E \cap S)$. [1]

**(iii)** Find $P(E' \cap S')$. [2]

**(iv)** If $P(M) = 0.15$ and $P(E'|M) = 0.2$, find $P(E \cap M)$. [2]

**(v)** It is given that no candidate can hold multiple citizenship. By drawing a Venn diagram, find $P(E' \cap M' \cap S')$. Explain in the context of this question, what your answer represents. [4]

**(vi)** Two candidates from that year were randomly chosen to be interviewed on their experience back in school. Find the probability that one candidate is a Singaporean and the other is a Malaysian. [2]

**~ End of Paper ~**

**Anglo-Chinese Junior College**
**2018 H1 Mathematics Prelim Solution**

<table>
<tr><th>Qn</th><th>Solution</th><th>Remarks</th></tr>
<tr><td>1</td><td>Since $3mx^2-24x+7m>0$,<br>$\therefore \quad m>0$ and $b^2-4ac<0$<br><br>Consider $b^2-4ac<0$<br>$(-24)^2-4(3m)(7m)<0$<br>$84m^2>576$<br>$84m^2-576>0$<br>$m^2-\frac{48}{7}>0$<br>$\left(m-\sqrt{\frac{48}{7}}\right)\left(m+\sqrt{\frac{48}{7}}\right)>0$<br>$m<-4\sqrt{\frac{3}{7}}$ or $m>4\sqrt{\frac{3}{7}}$<br>Since $m$>0,<br>$\therefore m>4\sqrt{\frac{3}{7}}$</td><td></td></tr>
<tr><td>2</td><td>(i) $\int \frac{5}{\sqrt{3\text{x}-1}}\,dx=\frac{5\sqrt{3x-1}}{3\left(\frac{1}{2}\right)}+c=\frac{10\sqrt{3x-1}}{3}+c$<br><br>(ii) $\int \left(\frac{1}{\sqrt{x}}-x\right)^2 dx=\int \frac{1}{x}-2\sqrt{x}+x^2\ dx$<br>$=\ln x-\frac{4x^{\frac{3}{2}}}{3}+\frac{x^3}{3}+c$</td><td></td></tr>
<tr><td>3</td><td>(i) With arch GF = 10x – 2x = 8x as the semicircle arch<br>$8x=\left(\frac{1}{2}\right)2(\pi r)$<br>$\therefore r=\frac{8x}{\pi}$ (Shown)<br>(ii) Since cardboard perimeter =54,<br>$20x+2y=54$<br>$\therefore y=27-10x$<br>$V=\frac{1}{2}\pi r^2 y$<br>$=\frac{1}{2}\pi\left(\frac{8x}{\pi}\right)^2(27-10x)$<br>$=\frac{32x^2}{\pi}(27-10x)$ shown</td><td></td></tr>
</table>

(iii) $V=\frac{32x^2}{\pi}(27-10x)=\frac{864x^2-320x^3}{\pi}$

$$\frac{dV}{dx}=\frac{1728x-960x^2}{\pi}$$

$$0=\frac{1728x-960x^2}{\pi}=\frac{192x(9-5x)}{\pi}$$

$x$ = 1.8 or 0 (rejected since $x$>0)

| $x$ | $x$ = 1.75 | $x$ = 1.8 | $x$ = 1.85 |
|---|---|---|---|
| $\frac{dV}{dx}$ | 26.7 | 0 | -28.3 |

Or $\frac{d^2V}{dx^2}=\frac{1728-1920(1.8)}{\pi}=-550\ (3s.f)<0$

Therefore, volume is maximum when $x$ = 1.8 cm.

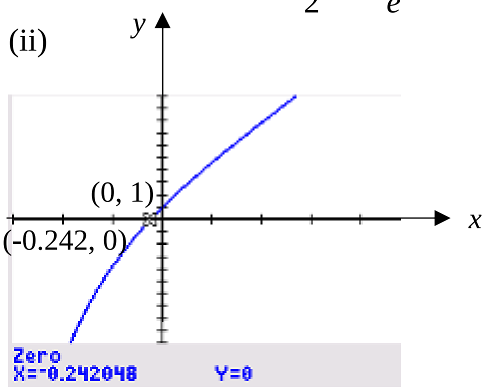
4

(i) $\int_0^k 3x-e^{-x}+2\ dx=\left[\frac{3}{2}x^2+\frac{1}{e^x}+2x\right]_0^k$

$$=\left(\frac{3}{2}k^2+\frac{1}{e^k}+2k\right)-\left(0+\frac{1}{e^0}\right)$$

$$=\frac{3}{2}k^2+\frac{1}{e^k}-1+2k$$

(ii)




(iii) When $x=1,\ y=3-\frac{1}{e}+2=5-\frac{1}{e}$

$$\frac{dy}{dx}=3+e^{-x}.$$

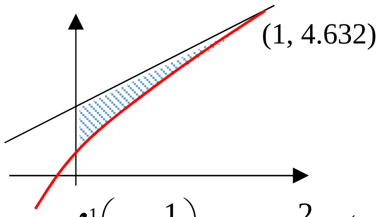
When $x=1,\ \frac{dy}{dx}=3+\frac{1}{e}$

Equation of tangent is

$$y-\left(5-\frac{1}{e}\right)=\left(3+\frac{1}{e}\right)(x-1)$$

$$y = 5 - \frac{1}{e} + \left(3 + \frac{1}{e}\right)x - 3 - \frac{1}{e}$$

$$y = \left(3 + \frac{1}{e}\right)x + 2 - \frac{2}{e}$$




(iv) $\int_0^1 \left(3 + \frac{1}{e}\right)x + 2 - \frac{2}{e} - \left(3x - e^{-x} + 2\right)\,dx$

$= \int_0^1 \left(3 + \frac{1}{e}\right)x + 2 - \frac{2}{e}\,dx - \left[\frac{3}{2}x^2 + \frac{1}{e^x} + 2x\right]_0^1$

$= \left[\left(3 + \frac{1}{e}\right)\frac{x^2}{2} + 2x - \frac{2}{e}x\right]_0^1 - \left(\frac{5}{2} + \frac{1}{e}\right)$

$= \left(\frac{7}{2} - \frac{3}{2e}\right) - \left(\frac{5}{2} + \frac{1}{e}\right)$

$= 1 - \frac{5}{2e}$

5

(i) Let B represent cost price of a Pretzel
Let T represent cost price of a Baguette
Let C represent cost price of a Ciabatta loaf

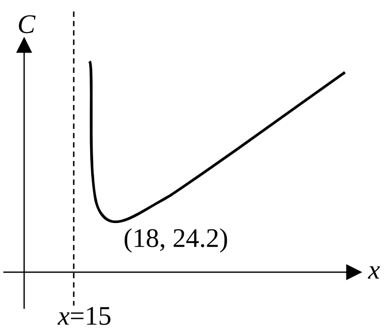
$7P = 6C$ ..............(1)

$50T - 40C = 16$ ..............(2)

$30P + 30T + 45C = 123$ ..........(3)

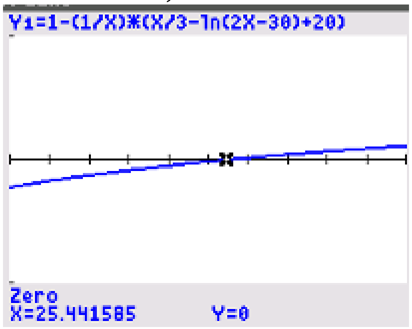
By GC,
Cost price for one Pretzel is \$1.2
Cost price for one Baguette is \$0.80
Cost price for one Ciabatta is \$1.40

(ii)




Estimated minimum cost is \$24.21 and that occurs when there 18 baguettes.

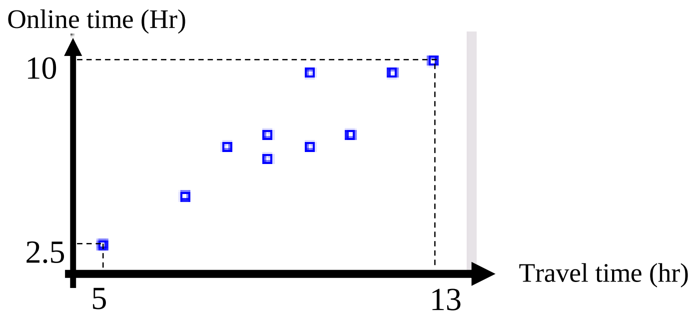
(iii) $P = 1 - \frac{C}{x} = 1 - \frac{1}{x}\left(\frac{x}{3} - \ln(2x - 30) + 20\right)$

| | | |
|---|---|---|
| | $P=\frac{2}{3}+\frac{1}{x}\ln(2x-30)-\frac{20}{x}$<br>(iv) From the GC,<br><br>As we can see from the graph, the profit is positive only when $x \geq 25.4$. Hence the baker will suffer a loss for an order of 20 baguettes, therefore not advisable to take the order. | |
| 6 | Let random variable $X$ be the amount of weight loss (in kg) of a customer at the end of a slimming programme.<br><br>Given $X \sim N(\mu, \sigma^2)$.<br><br>Given $P(X>20)=0.2$ and $P(X<10)=0.05$<br>$P(X>20)=0.2$<br>$\Leftrightarrow P(Z>\frac{20-\mu}{\sigma})=0.2$<br>$\Leftrightarrow \frac{20-\mu}{\sigma}=0.8416$<br>$\Leftrightarrow 20-\mu=0.8416\sigma$ (1)<br><br>$P(X<10)=0.05$<br>$\Leftrightarrow P(Z>\frac{10-\mu}{\sigma})=0.05$<br>$\Leftrightarrow \frac{10-\mu}{\sigma}=-1.645$<br>$\Leftrightarrow 10-\mu=-1.645\sigma$ (2)<br><br>(1) – (2), we have<br><br>$10=2.486\sigma$<br>$\sigma=4.02$ (3 sig. fig.)<br><br>Substitute $\sigma=4.02$ into (2), we have<br><br>$\mu=10+1.645\sigma=16.6$ (3 sig. fig.) | |

| | | |
|---|---|---|
| 7 | (i) The event that each pastilles packed into a box is orange-flavoured or not is independent of the flavour of any other pastille.<br><br>(ii) Let random variable $X$ be the number of orange-flavoured pastilles in a box of 20 pastilles.<br><br>$X \sim B(20, 0.25)$<br><br>$P(X \le 8) = 0.959$<br><br>(iii) Let random variable $Y$ be the number of boxes of pastilles that contain at most 8 orange-flavoured pastilles out of 5 boxes.<br><br>$Y \sim B(5, 0.959)$<br><br>$P(Y = 3) = 0.0148$<br><br>(iv) Let random variable $W$ be the number of boxes of pastilles that contain at most 8 orange-flavoured pastilles out of 4 boxes.<br><br>$W \sim B(4, 0.959)$<br>$A$ = {2 boxes out of 4 boxes John gets from Monday to Thursday contain at most 8 orange-flavoured pastilles}<br>$B$ = {the box John gets on Friday contains at most 8 orange-flavoured pastilles}<br>$P(A \cap B)$<br>$= P(A).P(B)$<br>$= P(W = 2)(0.959)$<br>$= 0.00887$ (3 sig. fig.) | |
| 8 | (a) Step 1: Choose 2 girls from 8 girls: ${}^{8}C_{2}$<br>Step 2: Choose 3 boys from 10 boys: ${}^{10}C_{3}$<br><br>Ans: ${}^{8}C_{2} \times {}^{10}C_{3} = 3360$<br><br>(b)(i) Step 1: Put one of the digits 5, 6, 7, 8, 9 as the leftmost digit: 5<br>Step 2: Arrange 4 digits from the remaining 8 digits to the right of the leftmost digit: ${}^{8}P_{4}$<br><br>Ans: $5 \times {}^{8}P_{4} = 8400$ | |

| | | |
|---|---|---|
| | (ii) Case 1: Leftmost digit is 5, 7 or 9<br><br>Step 1: Put one of the 3 digits 5, 7 or 9 as the leftmost digit: 3<br>Step 2: Put one of the remaining 4 odd digits as the rightmost digit: 4<br>Step 3: Arrange 3 digits from the remaining 7 digits between the leftmost and the rightmost digits: ${}^{7}P_{3}$<br>Case 2: Leftmost digit is 6 or 8<br><br>Step 1: Put one of the 2 digits 6 or 8 as the leftmost digit: 2<br>Step 2: Put one of the 5 odd digits (1, 3, 5, 7 or 9) as the rightmost digit: 5<br>Step 3: Arrange 3 digits from the remaining 7 digits between the leftmost and the rightmost digits: ${}^{7}P_{3}$<br><br>Ans: $3\times4\times{}^{7}P_{3}+2\times5\times{}^{7}P_{3}=4620$ | |
| 9 | Let random variable $X$ be the mass (in grammes) of an orange.<br>Let random variable $Y$ be the mass (in grammes) of an orange.<br>Given $X\sim(181,77^{2})$ and $Y\sim(165.8,23.07^{2})$<br><br>(i) $P(X>183)=0.338$ (3 sig. fig.)<br><br>(ii) $P(0.4(X_1+\ldots+X_4)>0.35(Y_1+\ldots+Y_5))$<br>$=P(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)>0)$<br>$E(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5))$<br>$=(0.4)(4)E(X)-(0.35)(5)E(Y)$<br>$=(0.4)(4)(181)-(0.35)(5)(165.8)$<br>$=-0.55$<br><br>$Var(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5))$<br>$=(0.4^{2})(4)Var(X)-(0.35^{2})(5)Var(Y)$<br>$=(0.4^{2})(4)(4.77^{2})-(0.35^{2})(5)(23.07^{2})$<br>$=340.5496073$<br>$0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)\sim N(-0.55,340.55)$<br>$P(0.4(X_1+\ldots+X_4)-0.35(Y_1+\ldots+Y_5)>0)=0.488$<br><br>The answer is the probability that the total weight of orange juice squeezed from 4 oranges is more than the total weight of tangerine juice squeezed from 5 tangerines. | |

| | | |
|---|---|---|
| | (iii) Let random variable $W$ be the mass of orange skins peeled (in grammes).<br>Given $E(W) = 90$ and $Var(W) = 4^2$.<br>Let $T = W_1 + K + W_5$<br>Since $n$ = 55 is large, by Central Limit Theorem, $T$ is approximately normally distributed.<br><br>$E(T) = 55E(W) = 55(90) = 4950$<br>$Var(T) = 55Var(W) = 55(4^2) = 880$<br>$T \sim N(4950, 880)$<br>$P(T > 5000) \approx 0.0459$ | |
| 10 | Let random variable $X$ be the Mathematics scores for Year 2 cohort.<br><br>Given $n$ = 250, $\sum(x-70) = 305$, $\sum(x-70)^2 = 35565$.<br><br>(i) $\bar{x} = \frac{\sum(x-70)}{n} + 70 = \frac{305}{250} + 70 = 71.22$ (2 dec. places)<br>Unbiased estimate of population variance:<br>$s^2 = \frac{n}{n-1}\left(\frac{\sum(x-70)^2}{n} - \left(\frac{\sum(x-70)}{n}\right)^2\right)$<br>$= \frac{250}{249}\left(\frac{35565}{250} - \left(\frac{305}{250}\right)^2\right)$<br>$= 141.3369478$<br>$\approx 141.34$ (correct to the nearest 2 decimal places)<br><br>(ii) Let $\bar{X} = \frac{X_1 + K + X_n}{n}$<br>Test $H_0$: $\mu = 70$<br>against $H_1$: $\mu > 70$<br>at 5% level of significance.<br>Critical Region: $z > 1.645$<br>Under $H_0$, $\bar{X} \sim N(70, \frac{141.34}{250})$ approximately by Central Limit Theorem, since $n$ = 250 is large.<br>Test Statistics: $Z = \frac{\bar{X} - 70}{\sqrt{\frac{141.34}{250}}}$<br>where $Z \sim N(0,1)$ approximately.<br>Test Value: $z = \frac{\bar{x} - 70}{\sqrt{\frac{141.34}{250}}} = \frac{71.22 - 70}{\sqrt{\frac{141.34}{250}}} = 1.62 < 1.645$<br>($z$ is not in critical region).<br>$p$-value: $p$ = 0.0523 > 0.05<br>Do not reject $H_0$. | |

| | | |
|---|---|---|
| | Conclusion: There is insufficient evidence at 5% level of significance to conclude that the mean scores of the Year 2 cohort (population) for Mathematics is more than 70.<br><br>(iii) Given $\bar{x} = 71.4$<br>Test $H_0$: $\mu = 70$<br>against $H_1$: $\mu > 70$<br>at 5% level of significance.<br>Critical Region: $z > 1.645$<br>Under $H_0$, $\bar{X} \sim N(70, \frac{k}{299})$ approximately by Central Limit Theorem, since $n$ = 250 is large.<br>Test Statistics: $Z = \frac{\bar{X} - 70}{\sqrt{\frac{k}{299}}}$<br>where $Z \sim N(0,1)$ approximately.<br>Test Value: $z = \frac{\bar{x} - 70}{\sqrt{\frac{k}{299}}} = \frac{71.4 - 70}{\sqrt{\frac{k}{299}}} = (1.4)\sqrt{\frac{299}{k}}$<br>To reject $H_0$, $z$ must be in critical region.<br>$z > 1.645$<br>$(1.4)\sqrt{\frac{299}{k}} > 1.645$<br>$\sqrt{k} < \frac{1.4\sqrt{299}}{1.645}$<br>$k < \left(\frac{1.4}{1.645}\right)^2 (299)$<br>$k < 216.6071293$<br>$k < 216.61$ | |
| 11 | (i) <br><br><br>(ii) $r$ = 0.909865 = 0.910 (3s.f)<br>There is a strong, postive linear correlation between the commuting time and the time spent online. As the amount of time spend commuting increases, the amount of time spent online increases | |

| | | |
|---|---|---|
| | (iii) Least square regression line is<br>$y = -1.5675 + 0.9008x$ (4d.p)<br><br>(iv) $m = 0.9008$ means that for a one hour increase in the travelling time there is an estimated 0.9008 hour increase in time spent online.<br><br>(v) $y = -1.5675 + 0.9008(8.5) = 6.09$ hr of online time. (3s.f)<br><br>Reliable because $x = 8.5$ is within the data range of $x$ (interpolation) and also $r = 0.910$ indicates that there is a strong and positive linear correlation between $x$ and $y$.<br><br>(vi) New r = 0.52082 = 0.521(3s.f)<br><br>(vii) The answer in (ii) is more likely to represent the amount of online time while commuting back and forth to work in a week because the r = 0.910 is closer to 1 and shows a stronger positive linear correlation between commuting time and online time. | |
| 12 | Let the events $E$, $S$ and $M$ be defined as follow:<br>$E$ = {a randomly selected candidate scored distinction for English}<br>$S$ = {a randomly selected candidate is a Singaporean}<br>$M$ = {a randomly selected candidate is a Malaysian}<br>Given $P(E) = 0.3$, $P(S) = 0.8$, $P(E\|S) = 0.2$.<br><br>(i) $P(E\|S) = 0.2$ and $P(E) = 0.3$<br>$P(E\|S) \neq P(E)$<br>Hence events $E$ and $S$ are not independent.<br><br>OR<br><br>$P(E \cap S) = P(E\|S)P(S) = (0.2)(0.8) = 0.16$<br>$P(E)P(S) = (0.3)(0.8) = 0.24 \neq 0.16$<br>$\therefore P(E \cap S) \neq P(E)P(S)$<br>Hence events $E$ and $S$ are not independent.<br><br>(ii) $P(E \cap S) = P(E\|S)P(S) = (0.2)(0.8) = 0.16$<br><br>(iii) $P(E' \cap S') = P[(E \cup S)'] = 1 - P(E \cup S)$<br><br>$P(E \cup S)$<br>$= P(E) + P(S) - P(E \cap S)$<br>$= 0.3 + 0.8 - 0.16$<br>$= 0.94$<br>$P(E' \cap S') = 1 - 0.94 = 0.06$ | |

(iv) $P(M) - 0.15$

$P(E'|M) = 0.2$

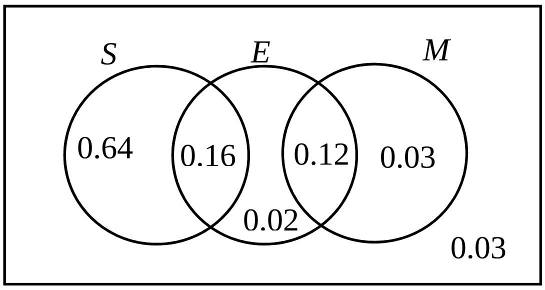
$P(E' \cap M) = P(E'|M)P(M) = 0.15 \times 0.2 = 0.03$

$P(E \cap M) = P(M) - P(E' \cap M)$

$= 0.15 - 0.03$

$= 0.12$

(v)




$P(E' \cap M' \cap S')$

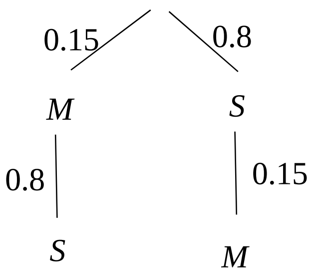
$= 1 - P(E \cup M \cup S)$

$= 1 - 0.97$

$= 0.03$

The answer represents the probability that a randomly selected candidate is a non-Malaysian foreigner (alternatively not a Singaporean nor a Malaysian) and did not score distinction for English.

(vi) $P(S) = 0.8$ and $P(M) = 0.15$




$P$(one randomly selected candidate is a Singaporean and the other randomly selected candidate is a Malaysian)

= (2)(0.8)(0.15)

= 0.24