Preliminary Examination 2016

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

4 Express / 5 Normal (Academic)

Additional materials : Writing Paper **2 hours**

Setter : Mrs Goh Heng Mei **12 August 2016**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 The function f is defined, for all values of x, by

$$f(x) = x(1-x)^2.$$

Find the values of x for which f is an increasing function. [4]

2 **(a)** Given that $\log_4 p = a$, $\log_{16} q = b$ and $\frac{p}{q} = 2^c$, [3]

express c in terms of a and b.

(b) On the same axes, sketch the graphs of $y = \log_4 x$ and $y = \log_{16} x$. [2]

3 The number of bacteria in a culture is given by $N = N_0 e^{kt}$, where N_0 is the number of bacteria at a particular time and N is the number of bacteria present t hours later. The number of bacteria in the culture triples every 2 hours.

Calculate the value of the constant k. [3]

4 **(a)** Show that the roots of the equation $x^2 + (a-2)x = 2a$ are real for all values of a. [3]

(b) Show that there are no values of b for which the curve $y = (b-3)x^2 - 2bx + (b-2)$ is always positive. [4]

5 The vertices of a parallelogram $ABCD$ are $A(5, 0)$, $B(-3, 4)$, $C(-2, 6)$ and $D(p, q)$ respectively.

(i) Find the mid-point of AC. [1]

(ii) Find the coordinates of D. [2]

[2]

Hence show that $ABCD$ is a rectangle.

6 The curve $y = a\sin bx + c$ is defined for $0 \le x \le 2\pi$, where a is a negative integer and b is a positive integer. Given that the amplitude of y is 4 and that the period of y is π,

(i) state the value of a and of b. [2]

Given that the maximum value of y is 6,

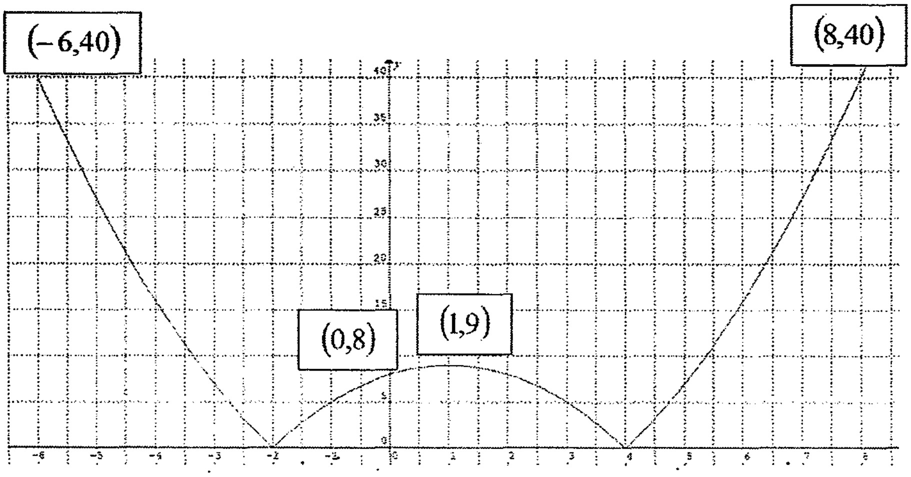
(ii) state the value of c, [1]

(iii) Sketch the graph of y, indicating the coordinates of any maximum or minimum points. [3]

7 **(a)** Show that $|x+5| = x-4$ has no solution. [2]

(b) **(i)** Sketch the graph of the function $y = |x^2 - 2x - 8|$ for $-6 \leq x \leq 8$, labelling the turning point and the intercepts of the graph. [4]

(ii) Hence, find the range of values of c if the graph of $y = c$ intersects the graph of $y = |x^2 - 2x - 8|$ at more than 2 points. [2]



(ii) $0 < c \leq 9$

8 **(i)** Find the value of each of the constants a and b for which $\sin 2x\left(5\tan x + 2\cot x\right) = a + b\sin^2 x$. [3]

(ii) Hence solve the equation $\sin 4\theta\left(5\tan 2\theta + 2\cot 2\theta\right) = 7$, stating the principal values of θ. [3]

9 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s^2, given by $a = 5 - pt$, where t seconds is the time since leaving O, and p is a real constant.

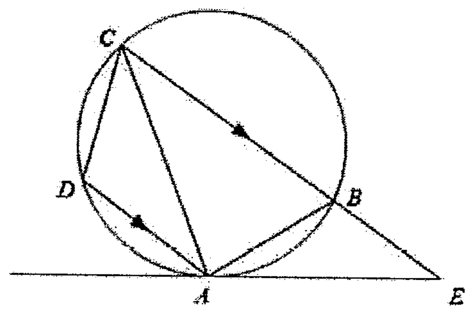
When $t = 3$, its velocity is 12 m/s.

(i) Find the value of p. [2]

(ii) When does the particle change its direction of motion? [2]

(iii) Show that the particle passes O again when $t = 22.5$. Hence find the total distance travelled by the particle between $t = 0$ and $t = 22.5$. [4]

10



The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point E lies on CB produced such that AE is a tangent to the circle.

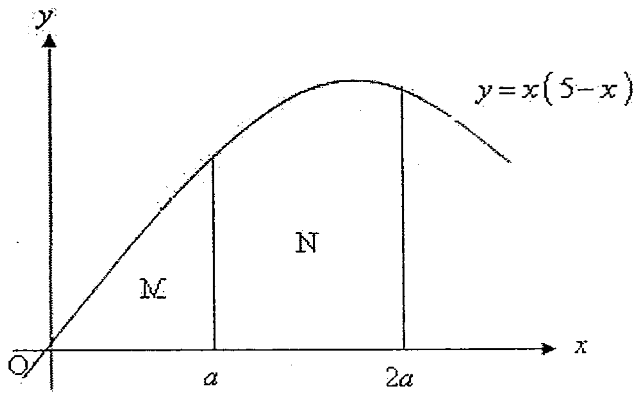
CE and AD are parallel.

(i) Show that angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [3]

(iii) Given that $AB = BE$, show that the line AC bisects the angle BCD. [2]

11 The diagram shows part of the curve $y = x(5 - x)$.



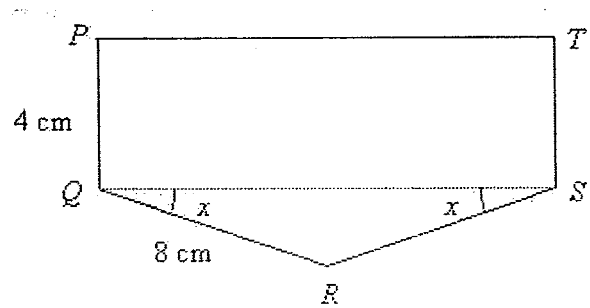
The region M is bounded by the curve $y = x(5 - x)$, the x-axis and the line $x = a$.
The region N is bounded by the curve $y = x(5 - x)$, the x-axis and the lines $x = a$ and $x = 2a$.
Given that the area of N is twice the area of M, find the value of a. [5]

12 It is given that $y=(x-1)\sqrt{4x+3}$.

(i) Express $\frac{dy}{dx}$ in the form $\frac{px+q}{\sqrt{4x+3}}$ where p and q are integers. [3]

(ii) Given that y is increasing at the rate of 2.5 units per second when $x=3$, find the rate of change of x at this instant. [2]

13



A piece of paper is cut into the shape *PQRST* as shown in the diagram. *PQST* is a rectangle with $PQ = 4$ cm and *QRS* is an isosceles triangle with $QR = 8$ cm.

(i) Given that angle SQR = angle $QSR = x$ radian, show that the area of the paper, A, is given by $A = 64\cos x(1+\sin x)$. [4]

(ii) Find the value of x, in terms of π, for which A has a stationary value. [4]

(iii) Find the exact value of A and determine whether it is a maximum or a minimum. [3]

Preliminary Examination 2016

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

4 Express/ 5 Normal (Academic)

Additional materials : Writing Paper
Graph Paper

2 hours 30 minutes

Setter : Mr Johney Joseph

05 Aug 2016

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
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Mathematical Formulae

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Binomial expansion

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\Delta = \frac{1}{2}ab\sin C$$

1 The curve $y = f(x)$ is such that $f'(x) = 3\sin x + 5$.

(i) Explain why the curve $y = f(x)$ has no stationary point. [2]

(ii) Given that the curve passes through the point (0, 5), find an expression for $f(x)$. [4]

2 (i) Differentiate $xe^{\frac{1}{2}x}$ with respect to x. [2]

(ii) Integrate $e^{\frac{1}{2}x}$ with respect to x. [2]

(iii) Using results from part **(i)** and **(ii)** show that $\int_0^4 xe^{\frac{1}{2}x}\,dx = 4e^2 + 4$. [4]

3 The equation of a curve is $y = (x+k)^2$.

(i) Show that the equation of the tangent to the curve where $x = 2k$ is
$y + 3k^2 = 6kx$. [5]

This tangent meets the x-axis at P and the y-axis at Q.
The mid-point of PQ is M.

(ii) Show that M lies on the curve $y + 24x^2 = 0$. [4]

4 (a) (i) Write down, and simplify, the expansion of $(2-p)^5$. [3]

(ii) Use the result from part **(i)** to find the expansion of $\left(2 - 2x + \frac{x^2}{2}\right)^5$ in ascending powers of x as far as the term in x^2. [3]

(b) (i) Write down the general term in the expansion of $\left(x^2 - \frac{1}{2x^6}\right)^{16}$. [1]

(ii) Hence, or otherwise, evaluate the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x^6}\right)^{16}$. [3]

[Turn over

5 Given that $k = 3 - 2\sqrt{2}$, express $k - \dfrac{1}{k^2}$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

6 (i) Prove that $x + 1$ is a factor of $2x^3 - 9x^2 + x + 12$. [2]

(ii) Factorise $2x^3 - 9x^2 + x + 12$ completely and hence solve the equation $2x^3 - 9x^2 + x + 12 = 0$. [4]

(iii) Express $\dfrac{25}{2x^3 - 9x^2 + x + 12}$ as the sum of three partial fractions. [4]

7 A curve has an equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{(x-3)^2}{x}$ for $x \neq 0$.

(i) Find an expression for $\mathrm{f}'(x)$ and obtain the coordinates of the stationary points on the curve. [4]

(ii) Showing full working, determine the nature of these stationary points. [4]

8 The roots of the quadratic equation $8x^2 - 11x + 67 = 0$ are $\alpha^3 + 1$ and $\beta^3 + 1$.

(i) Find the values of $\alpha^3 + \beta^3$ and $\alpha\beta$. [4]

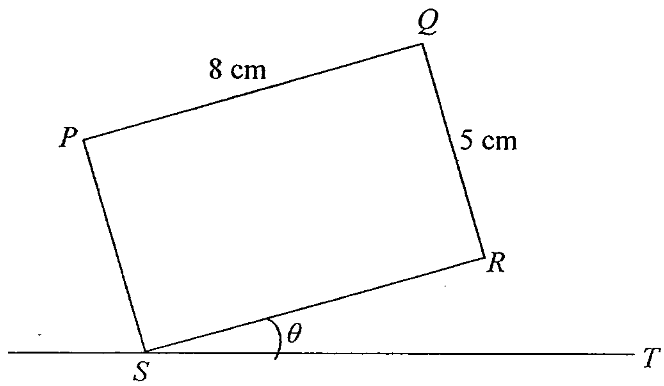
It is also given that the roots of the quadratic equation $4x^2 - 9x + 16 = 0$ are α^2 and β^2.

(ii) State the value of $\alpha^2 + \beta^2$. [1]

(iii) Use **all** results from **(i)** and **(ii)** to deduce the value of $\alpha + \beta$. [3]

(iv) Form a quadratic equation, with integer coefficients, whose roots are α and β. [2]

9



In the figure, $PQRS$ is a rectangle of length 8 cm and breadth 5 cm and $\angle RST = \theta$ radians, where θ is acute.

(i) Express h cm, the perpendicular distance from Q to the line ST, in the form $a\cos\theta + b\sin\theta$, where a and b are constants. [2]

(ii) Express h in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle in radians. [4]

(iii) Find the maximum value of h and the corresponding value of θ. [2]

(iv) Find the value of θ for which $h = 7.5$ cm. [3]

10 A circle passes through the points $A(4, 0)$ and $B(0, 6)$. Its centre lies on the line $y = x + 2$.

(i) Find the equation of the perpendicular bisector of AB and hence show that the centre of the circle is $(-1, 1)$. [6]

(ii) Find the equation of the circle. [3]

A second circle with equation $x^2 + y^2 + ax + by - 23 = 0$, has the same centre as the first circle.

(iii) Write down the value of a and of b. [1]

(iv) Show that the second circle lies inside the first circle. [2]

11 The table shows the experimental values of x and y.

x	1.5	2.0	2.5	3	3.5	4.0
y	1.8	2.1	2.4	2.6	2.9	3.1

It is known that x and y are related by the equation $y = kx^n$, where k and n are constants.

(i) Using suitable variables, draw on graph paper, a straight line graph and hence estimate the value of each of the constants k and n. [6]

(ii) Using your values of k and n, calculate the value of x for which $xy = 10$. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which $xy = 10$. Draw this line. [3]

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Preliminary Examination 2016

ANSWERS

ADDITIONAL MATHEMATICS **4047/01**

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Setter : Mrs Goh Heng Mei **12 August 2016**

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This document consists of 6 printed pages including the cover page.

[Turn over

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1 The function f is defined, for all values of x, by

$$f(x) = x(1-x)^2.$$

Find the values of x for which f is an increasing function. [4]

Solutions

$$f(x) = x(1-x)^2$$

$$= x(1-2x+x^2)$$

$$= x - 2x^2 + x^3$$

$$f'(x) = 1 - 4x + 3x^2$$

$$= (3x-1)(x-1)$$

Given that f is an increasing function.

$\Rightarrow \quad (3x-1)(x-1) > 0$

$\therefore \quad x < \frac{1}{3}$ **or** $x > 1$

2 **(a)** Given that $\log_4 p = a$, $\log_{16} q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [3]

(b) On the same axes, sketch the graphs of $y = \log_4 x$ and $y = \log_{16} x$. [2]

Solutions

(a) $\log_4 p = a$, $\log_{16} q = b$

$p = 4^a$, $q = 16^b$

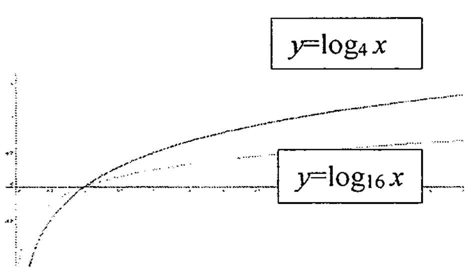
Given $\frac{p}{q} = 2^c$

$$\frac{4^a}{16^b} = 2^c$$

$$\frac{2^{2a}}{2^{4b}} = 2^c$$

$2^{2a-4b} = 2^c \quad \therefore \quad c = 2a - 4b$

(b)



3 The number of bacteria in a culture is given by $N = N_0\, e^{kt}$, where N_0 is the number of bacteria at a particular time and N is the number of bacteria present t hours later. The number of bacteria in the culture triples every 2 hours.

Calculate the value of the constant k. [3]

	Solutions $N = N_0\, e^{kt}$ When $t = 0$, $N = N_0$ When $t = 2$, $N = 3N_0$ When $t = 2$, $N = N_0\, e^{k(2)}$	$\therefore\ 3N_0 = N_0 e^{k(2)}$ $3 = e^{2k}$ $2k = \ln 3$ $k = \dfrac{\ln 3}{2}$ $= 0.5493$ ~ 0.549	

4 **(a)** Show that the roots of the equation $x^2 + (a-2)x = 2a$ are real for all values of a. [3]

(b) Show that there are no values of b for which the curve $y = (b-3)x^2 - 2bx + (b-2)$ is always positive. [4]

	Solutions **(a)** $x^2 + (a-2)x - 2a = 0$ Discriminant = $(a-2)^2 - 4(1)(-2a)$ $= a^2 - 4a + 4 + 8a$ $= a^2 + 4a + 4$ $= (a+2)^2$ Since $(a+2)^2 \geq 0$, The discriminant ≥ 0 $\therefore$ the roots are real for all real values of a	**(b)** If $y = (b-3)x^2 - 2bx + (b-2)$ is always positive, then $b - 3 > 0 \Rightarrow b > 3$ and discriminant < 0 $(-2b)^2 - 4(b-3)(b-2) < 0$ $4b^2 - 4(b^2 - 5b + 6) < 0$ $4b^2 - 4b^2 + 20b - 24 < 0$ $20b < 24$ $b < \dfrac{6}{5}$ But from above, $b > 3$. $\therefore$ there are no values of b for which y is always positive.	

5 The vertices of a parallelogram $ABCD$ are $A\,(5, 0)$, $B\,(-3, 4)$, $C\,(-2, 6)$ and $D\,(p, q)$ respectively.

(i) Find the mid-point of AC. [1]

(ii) Find the coordinates of D. [2]

Hence show that $ABCD$ is a rectangle. [2]

	Solutions (i) Mid-point of AC = $\left(\frac{5-2}{2}, \frac{0+6}{2}\right)$ $= \left(\frac{3}{2}, 3\right)$	**(ii)** Mid-point of $BD = \left(\frac{p-3}{2}, \frac{q+4}{2}\right)$ Mid-point of BD = mid-point of AC $\frac{p-3}{2} = \frac{3}{2}$ and $\frac{q+4}{2} = 3$ $p-3=3$, $q+4=6$ $p=6$, $q=2$ $\therefore$ D is $(6, 2)$	
	Given that $ABCD$ is a parallelogram. Therefore $AB = CD$ and $AB \,//\, CD$. Gradient of $AB = \frac{4-0}{-3-5}$ $= -\frac{1}{2}$ Gradient of $CD = \frac{2-0}{6-5}$ $= 2$ Gradient of $AB \times$ gradient of CD = -1 $\Rightarrow AB \perp CD$. $\Rightarrow ABCD$ is a rectangle.		

6 The curve $y = a\sin bx + c$ is defined for $0 \le x \le 2\pi$, where a is a negative integer and b is a positive integer. Given that the amplitude of y is 4 and that the period of y is π,

(i) state the value of a and of b. [2]

Given that the maximum value of y is 6,

(ii) state the value of c, [1]

(iii) Sketch the graph of y, indicating the coordinates of any maximum or minimum points. [3]

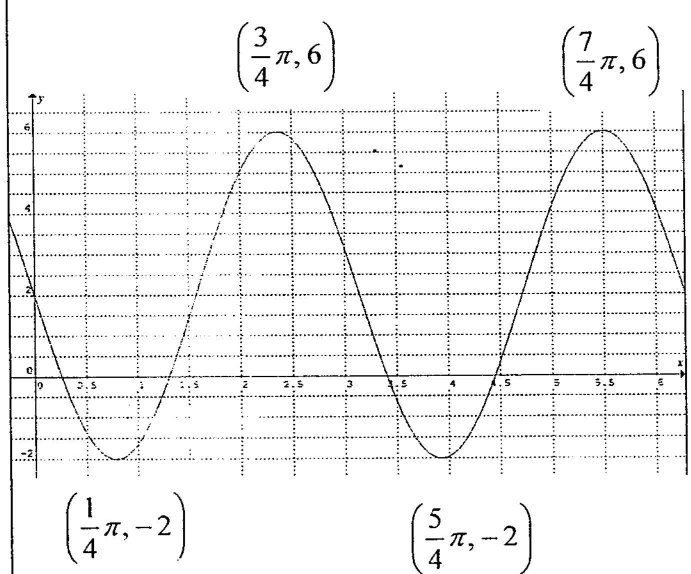
Solutions

(i) a is negative and amplitude is 4. Therefore $a = -4$.

Period of y is π. Therefore $b = 2$.

(ii)
$$y = a\sin bx + c$$
$$y = -4\sin 2x + c$$
When $\sin 2x = -1$,
$$6 = -4(-1) + c \quad \Rightarrow c = 2$$

(iii)



7	(a) Show that $\lvert x+5 \rvert = x-4$ has no solution.	[2]
	(b) (i) Sketch the graph of the function $y=\lvert x^2-2x-8 \rvert$ for $-6 \le x \le 8$, labelling the turning point and the intercepts of the graph.	[4]
	(ii) Hence, find the range of values of c if the graph of $y=c$ intersects the graph of $y=\lvert x^2-2x-8 \rvert$ at more than 2 points.	[2]

Solutions

(a) $\lvert x+5 \rvert = x-4$

$\Rightarrow \quad x+5=x-4 \quad$ or $\quad x+5=-(x-4)$

NA $\qquad x+5=-x+4$

$2x=-1$

$x=-\frac{1}{2}$

But when $x=-\frac{1}{2}$, $\quad \lvert x+5 \rvert = -\frac{1}{2}-4<0 \qquad$ NA

$\therefore \quad \lvert x+5 \rvert = x-4$ has no solution.

(b) $y=\lvert x^2-2x-8 \rvert$

$= \lvert (x-4)(x+2) \rvert$

When $x=-6$, $\quad y=\lvert (-10)(-4) \rvert$

$=40$

When $x=8$, $\quad y=\lvert (4)(10) \rvert$

$=40$

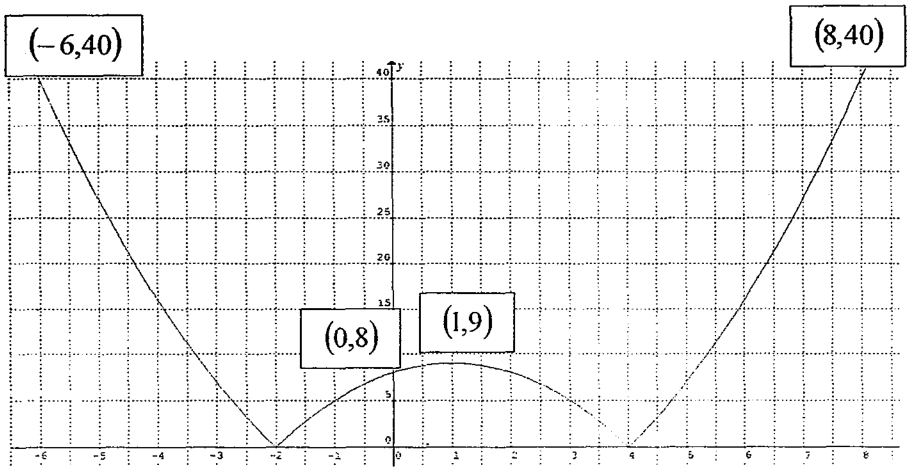
When $x=0$, $\quad y=\lvert -8 \rvert$

$=8$

Line of symmetry : $x=\frac{-2+4}{2}$

$=1$

When $x=1$, $\quad y=\lvert (-3)(3) \rvert = 9$



(ii) $0 < c \le 9$

8	**(i)**	Find the value of each of the constants a and b for which $\sin 2x\,(5\tan x + 2\cot x) = a + b\sin^2 x$.	[3]
	(ii)	Hence solve the equation $\sin 4\theta\,(5\tan 2\theta + 2\cot 2\theta) = 7$, stating the principal values of θ.	[3]

Solutions

(i) $\sin 2x\,(5\tan x + 2\cot x)$

$= 2\sin x\cos x\left(\dfrac{5\sin x}{\cos x} + \dfrac{2\cos x}{\sin x}\right)$

$= 10\sin^2 x + 4\cos^2 x$

$= 10\sin^2 x + 4(1 - \sin^2 x)$

$= 4 + 6\sin^2 x$

$a = 4$ and $b = 6$

(ii) Let $x = 2\theta$

$4 + 6\sin^2 2\theta = 7$

$6\sin^2 2\theta = 3$

$\sin^2 2\theta = \dfrac{1}{2}$

$\sin 2\theta = \pm\dfrac{1}{\sqrt{2}}$

Basic angle = 45^0

Principal value: $-90^0 \le \sin^{-1} x \le 90^0$

Principal values of $2\theta = -45^0, 45^0$

Principal values of $\theta = -22.5, 22.5$

9 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s^2, given by $a = 5 - pt$, where t seconds is the time since leaving O, and p is a real constant.

When $t = 3$, its velocity is 12 m/s.

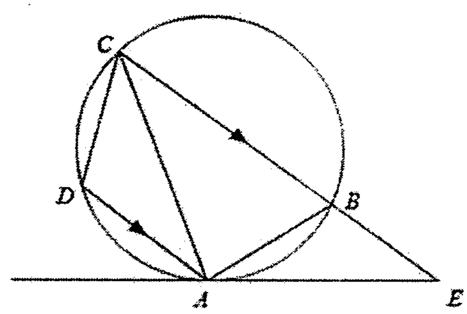
(i) Find the value of p. [2]

(ii) When does the particle change its direction of motion? [2]

(iii) Show that the particle passes O again when $t = 22.5$. Hence find the total distance travelled by the particle between $t = 0$ and $t = 22.5$. [4]

	Solutions **(i)** $a = 5 - pt$ $v = 5t - \frac{pt^2}{2} + c$ The particle starts from rest $\Rightarrow$ when $t = o, \quad v = 0 \quad \therefore c = 0$ $v = 5t - \frac{pt^2}{2}$ When $t = 3, \quad v = 12$ $5(3) - \frac{9p}{2} = 12$ $\frac{9p}{2} = 3$ $p = \frac{2}{3}$	**(ii)** When particle changes its direction, v =0 $5t - \frac{2}{3}\left(\frac{t^2}{2}\right) = 0$ $t\left(5 - \frac{1}{3}t\right) = 0$ $T = 0$ (NA) or $5 = \frac{1}{3}t$ t = 15 s	
	(iii) $v = 5t - \frac{t^2}{3}$ $s = \frac{5t^2}{2} - \frac{t^3}{9} + c$ When $t = 0, \quad s = 0 \quad \therefore c = 0$ $s = \frac{5t^2}{2} - \frac{t^3}{9}$	When $t = 22.5$, $s = \frac{5(22.5)^2}{2} - \frac{22.5^3}{9}$ $= 0$ $\Rightarrow$ the particle passes pt O again when t =22.5 s When $t = 15$, $s = \frac{5(15)^2}{2} - \frac{15^3}{9}$ = 187.5 Dist travelled = 187.5 × 2 = 375 m	

10



The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point E lies on CB produced such that AE is a tangent to the circle.

CE and AD are parallel.

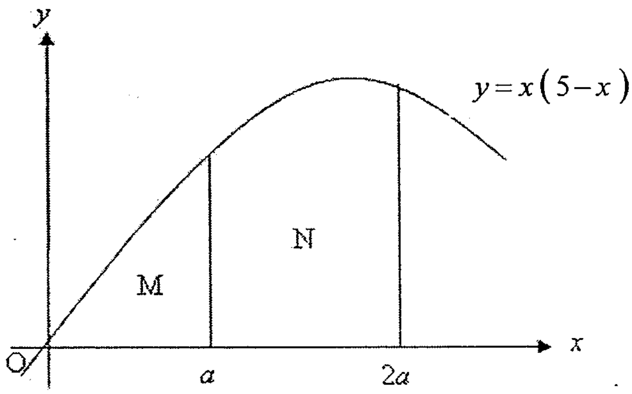
(i) Show that angle BAE = angle CAD. [2]

(ii) Show that triangles BAE and DAC are similar. [3]

(iii) Given that $AB = BE$, show that the line AC bisects the angle BCD. [2]

	Solutions **(i)** $\angle BAE = \angle BCA$ (tangent chord thm) $= \angle CAD$ (alt $\angle s$, CE // AD)	(iii) Given that $AB = BE$. BAE is an isosceles triangle. then DAC is also an isosceles Δ. $\angle DCA = \angle CAD$ $= \angle ACB$ $\therefore$ AC bisects the angle BCD.	
	(ii) $\angle ABC + \angle ABE = 180^0$ ($\angle s$ in Δ) $\angle ABC + \angle CDA = 180^0$ ($\angle s$ in opp seg) $\therefore$ $\angle ABE = \angle CDA$ From (i) $\angle BAE = \angle CAD$ $\therefore \Delta BAE$ is similar to ΔDAC (AA)		

11 The diagram shows part of the curve $y = x(5-x)$.



The region M is bounded by the curve $y = x(5-x)$, the x-axis and the line $x = a$.
The region N is bounded by the curve $y = x(5-x)$, the x-axis and the lines $x = a$ and $x = 2a$.
Given that the area of N is twice the area of M, find the value of a. [5]

	Solutions Area M = $\int_0^a x(5-x)\,dx$ $= \int_0^a (5x - x^2)\,dx$ $= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^a$ $= \frac{5a^2}{2} - \frac{a^3}{3}$ Area N = $\int_0^a x(5-x)\,dx$ $= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_a^{2a}$ $= \frac{5(4a^2)}{2} - \frac{8a^3}{3} - \left(\frac{5a^2}{2} - \frac{a^3}{3}\right)$ $= \frac{15a^2}{2} - \frac{7a^3}{3}$	Given N = 2 M $\frac{15a^2}{2} - \frac{7a^3}{3} = 2\left[\frac{5a^2}{2} - \frac{a^3}{3}\right]$ $= 5a^2 - \frac{2a^3}{3}$ $\frac{15a^2}{2} - \frac{7a^3}{3} - 5a^2 + \frac{2a^3}{3} = 0$ $\frac{5a^2}{2} - \frac{5a^3}{3} = 0$ $a^2\left(\frac{5}{2} - \frac{5a}{3}\right) = 0$ $a = 0$ (NA) or $\frac{5a}{3} = \frac{5}{2}$ $a = \frac{3}{2}$	

12 It is given that $y = (x-1)\sqrt{4x+3}$.

(i) Express $\frac{dy}{dx}$ in the form $\frac{px+q}{\sqrt{4x+3}}$ where p and q are integers. [3]

(ii) Given that y is increasing at the rate of 2.5 units per second when $x = 3$, find the rate of change of x at this instant. [2]

Solutions

(i) $y = (x-1)(4x+3)^{\frac{1}{2}}$

$$\frac{dy}{dx} = (x-1)\frac{1}{2}(4x+3)^{-\frac{1}{2}}(4) + (4x+3)^{\frac{1}{2}}(1)$$

$$= (4x+3)^{-\frac{1}{2}}[2(x-1) + (4x+3)]$$

$$= (4x+3)^{-\frac{1}{2}}(6x+1)$$

$$= \frac{6x+1}{\sqrt{4x+3}}$$

(ii) Given $\frac{dy}{dt} = 2.5$

$$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$$

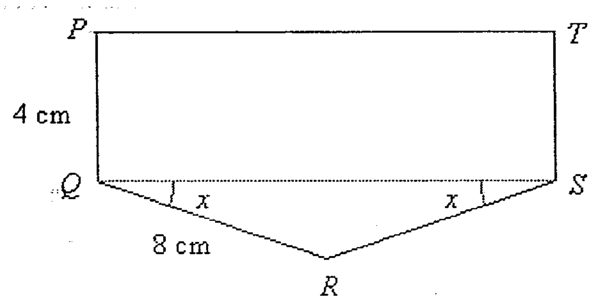
When $x = 3$,

$$\frac{dx}{dt} = \frac{\sqrt{15}}{19} \cdot (2.5)$$

$= 0.5096$

~ 0.510 units/s

13



A piece of paper is cut into the shape *PQRST* as shown in the diagram. *PQST* is a rectangle with $PQ = 4$ cm and *QRS* is an isosceles triangle with $QR = 8$ cm.

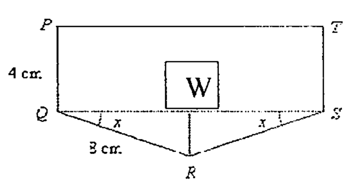
(i) Given that angle SQR = angle $QSR = x$ radian, show that the area of the paper, A, is given by $A = 64\cos x\,(1 + \sin x)$. [4]

(ii) Find the value of x, in terms of π, for which A has a stationary value. [4]

(iii) Find the exact value of A and determine whether it is a maximum or a minimum. [3]

Solutions

(i)



$RW = 8\sin x$ and $QW = 8\cos x$

Area of rect *PQST* = $4(16\cos x)$

Area of $\Delta QRS = \frac{1}{2}(16\cos x)(8\sin x)$

Area of A = $64\cos x + 64\cos x\sin x$

$= 64\cos x(1 + \sin x)$

(ii) $A = 64\cos x(1 + \sin x)$

$$\frac{dA}{dx} = 64\cos x(\cos x) + 64(1 + \sin x)(-\sin x)$$

$$= 64\cos^2 x - 64\sin^2 x - 64\sin x$$

A has stationary value $\Rightarrow \frac{dA}{dx} = 0$

$$64\cos^2 x - 64\sin^2 x - 64\sin x = 0$$

$$64 - 64\sin^2 x - 64\sin^2 x - 64\sin x = 0$$

$$1 - 2\sin^2 x - \sin x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$\sin x = \frac{1}{2}$ or $\sin x = -1$ (NA)

$$x = \frac{\pi}{6}$$

(iii) $A = 64\cos x(1+\sin x)$

$$A = 64\cos\frac{\pi}{6}\left(1+\sin\frac{\pi}{6}\right)$$

$$= 64\left(\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right)$$

$$= 48\sqrt{3}\ \text{cm}^2$$

$$\frac{dA}{dx} = 64 - 128\sin^2 x - 64\sin x$$

$$\frac{d^2A}{dx^2} = -256\sin x\cos x - 64\cos x$$

When $x = \frac{\pi}{6}$, $\frac{d^2A}{dx^2} < 0$

$\therefore$ A is maximum.

Answers

1	3.48 , 5.94	8	Proof
2	3	9(i)	$h = \frac{432}{r^2}$
3	$2 < p < 4$	9(ii)	Proof
4(i)	$\frac{1}{x-3} + \frac{2}{x+1} - \frac{5}{(x+1)^2}$	9(iii)	6 cm
4(ii)	$\ln(x-3) + 2\ln(x+1) + \frac{5}{x+1}$	10(a)	1 , −2
5(i)	$x = -0.8$; $x = 6$ (rej)	10(b)	Proof
5(ii)	graph	11(i)	$\frac{4}{3}$
6(i)	graph	11(ii)	$-\frac{143}{145}$
6(ii)	$x = e^{-\frac{2}{3}(\frac{1}{y}-2)}$	11(iii)	$-\frac{21}{20}$
7(i)	$B = (3 , 0)$	12(i)	Proof
7(ii)	45.5 units^2	12(ii)	0.24 m^2/s
7(iii)	Triangles BCD and BED		
7(iv)	$y = 10x - 76$		

ADDITIONAL MATHEMATICS Paper 2 (4047/02)

Marking Scheme

Qn	Solution	Marks		Remarks
1 (i)	$3\sin x + 5 = 0$			
	$\sin x = -\dfrac{5}{3}$ which is not possible as $-1 \le \sin x \le 1$			
	$f'(x) \neq 0$. $\therefore$ There is no stationary point	M1A1		
(ii)	$y = \int (3\sin x + 5)dx$			
	$= -3\cos x + 5x + c$	M1A1		
	$x = 0, y = 5 \Rightarrow c = 8$			
	$\therefore f(x) = -3\cos x + 5x + 8$	M1A1	[6]	
2 (i)	$\dfrac{d}{dx}\left(xe^{\frac{1}{2}x}\right) = \dfrac{1}{2}xe^{\frac{1}{2}x} + e^{\frac{1}{2}x}$	M1A1		
(ii)	$\int e^{\frac{1}{2}x}dx = 2e^{\frac{1}{2}x} + c$	B1A1		
(iii)	$\int_0^4 \dfrac{1}{2}xe^{\frac{1}{2}x}dx + \left[2e^2 - 2\right] = \left[4e^2 - 0\right]$	M1M1		
	$\int_0^4 \dfrac{1}{2}xe^{\frac{1}{2}x}dx = 2e^2 + 2$	M1		
	$\int_0^4 xe^{\frac{1}{2}x}dx = 4e^2 + 4$	A1	[8]	
3 (i)	$\dfrac{dy}{dx} = 2(x + k)$	B1		
	Gradient of the tangent $= 2(2k + k) = 6k$	M1		
	When $x = 2k$, $y = (k+2k)^2 = 9k^2$	M1		
	Equation of the tangent is			
	$y - 9k^2 = 6k(x - 2k)$			
	$y + 3k^2 = 6kx$	M1A1		
(ii)	$P\left(\dfrac{k}{2}, 0\right)$ and $Q\left(0, -3k^2\right)$	M1		
	Mid-point R is $\left(\dfrac{k}{4}, -\dfrac{3k^2}{2}\right)$	M1		
	Substituting in $y + 4x^2 = 0$,			
	$-\dfrac{3k^2}{2} + 24\left(\dfrac{k}{4}\right)^2 = 0$			
	$-\dfrac{3k^2}{2} + \dfrac{3k^2}{2} = 0$			
	$0 = 0$			
	$\therefore M$ lies on the curve $y + 24x^2 = 0$	M1A1	[9]	

4(a)(i)	$(2-p)^2 = 32 - 80p + 80p^2 - 40p^3 + 10p^4 - p^5$	M1A2		
(ii)	Let $p = 2x - \frac{x^2}{2}$	B1		
	$\left(2-2x+\frac{x^2}{2}\right) = 32 - 80(2x - \frac{x^2}{2}) + 80\,(2x - \frac{x^2}{2})^2 + ..$ $= 32 - 160x + 360x^2 + \ldots\ldots$	M1A1		
(b)(i)	$\binom{16}{r}\left(x^2\right)^{16-r}\left(-\frac{1}{2x^6}\right)^r$	B1		
(ii)	$\binom{16}{r}\left(x^2\right)^{16-r}\left(-\frac{1}{2x^6}\right)^r = \binom{16}{r}\left(-\frac{1}{2}\right)^r x^{32-2r}$	M1		
	$32 - 2r = 0 \Rightarrow r = 4$			
	Term independent of $x = \binom{16}{4}\left(-\frac{1}{2}\right)^4 = \frac{455}{4}$	M1A1	[10]	
5	$k^2 = \left(3-2\sqrt{2}\right)^2 = 17 - 12\sqrt{2}$	B1		
	$\frac{1}{k^2} = \frac{1}{17-12\sqrt{2}} = \frac{17+12\sqrt{2}}{\left(17-12\sqrt{2}\right)\left(17+12\sqrt{2}\right)} = 17+12\sqrt{2}$	M1A1		
	$k - \frac{1}{k^2} = 3 - 2\sqrt{2} - \left(17+12\sqrt{2}\right) = -14 - 14\sqrt{2}$	M1A1	[5]	
6(i)	$2(-1)^3 - 9(-1)^2 - (-1) + 12 = 0$ $\therefore$ x + 1 is a factor of $2x^3 - 9x^2 + x + 12$.	M1A1		
(ii)	$2x^3 - 9x^2 + x + 12 = (x+1)(2x^2 - 11x + 12)$	B1		
	$= (x+1)(2x-3)(x-4)$	A1		
	$(x+1)(2x-3)(x-4) = 0 \Rightarrow x = -1, \frac{3}{2}$ or 4.	A2		
(iii)	Let $\frac{25}{2x^3-9x^2+x+12} = \frac{A}{x+1} + \frac{B}{2x-3} + \frac{C}{x-4}$	M1		
	Evaluating A, B and C A=1, B = - 4, C = 1	M1A1		
	$\frac{25}{2x^3-9x^2+x+12} = \frac{1}{x+1} - \frac{4}{2x-3} + \frac{1}{x-4}$	A1	[10]	
7(i)	$f'(x) = \frac{2x(x-3) - (x-3)^2}{x^2}$ $= \frac{x^2-9}{x^2}$	M1		
	$\frac{x^2-9}{x^2} = 0 \Rightarrow x = \pm 3$	M1		
	The stationary points are (3, 0) and (– 3, – 12)	M1A1		

(ii)	$f''(x) = \dfrac{x^2(2x)-(x^2-9)(2x)}{x^4}$	M1		
	$= \dfrac{18}{x^3}$	M1		
	$f''(3) > 0$ and $f''(-3) < 0$ $\therefore (3, 0)$ Minimum point and $(-3, -12)$ Maximum point.	A1A1	[8]	
8(i)	$\alpha^3 + 1 + \beta^3 + 1 = \dfrac{11}{8}$	M1		
	$\alpha^3 + \beta^3 = -\dfrac{5}{8}$	A1		
	$(\alpha^3 + 1)(\beta^3 + 1) = \dfrac{67}{8}$	M1		
	$\alpha^3\beta^3 + \alpha^3 + \beta^3 + 1 = \dfrac{67}{8}$			
	$\alpha^3\beta^3 = \dfrac{67}{8} + \dfrac{5}{8} - 1 = 8$	A1		
	$\alpha\beta = 2$			
	$\alpha^2 + \beta^2 = \dfrac{9}{4}$	B1		
(ii)	$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$	B1		
(iii)	$-\dfrac{5}{8} = (\alpha+\beta)(\dfrac{9}{4} - 2)$			
	$(\alpha+\beta) = -\dfrac{5}{2}$	M1A1		
(iv)	The quadratic equation is $x^2 + \dfrac{5}{2}x + 2 = 0$ $2x^2 + 5x + 4 = 0$	M1A1	[10]	
9(i)	$h = 5\cos\vartheta + 8\sin\vartheta$	B1B1		
(ii)	$R = \sqrt{5^2+8^2} = \sqrt{89}$	B1		
	$\alpha = \tan^{-1}\left(\dfrac{8}{5}\right) = 1.012197$	M1A1		
	h = $\sqrt{89}\cos(\theta - 1.01)$	A1		
(iii)	Max value of h = 9.43	B1		
	$\theta = 1.01$	B1		
	$\sqrt{89}\cos(\theta - 1.012197) = 7.5$	M1		
	$\theta = 0.360$ (accept 0.360 to 0.361)	M1A1	[11]	

10(i)	Mid-point of AB is (2, 3) and Gradient of $AB = -\frac{3}{2}$	M1M1		
	Equation of the perpendicular bisector is			
	$y - 3 = \frac{2}{3}(x - 2)$			
	$3y = 2x + 5$	M1A1		
	Solving $y = x + 2$ and $3y = 2x + 5$, Centre is $(-1, 1)$	M1A1		
(ii)	Radius $= \sqrt{(-1-4)^2 + (1-0)^2} = \sqrt{26}$	M1		
	Equation of the circle is			
	$(x + 1)^2 + (y - 1)^2 = 26$	M1A1		
(iii)	$a = 2$, $b = -2$	B1		
(iv)	Radius of the second circle $= \sqrt{1^2 + (-1)^2 + 23} = 5$			
	$< \sqrt{26}$.			
	$\therefore$ The second circle lies inside the first circle.	M1A1	[12]	

11 (i)	$y = kx^n$			
	$\lg y = n \lg x + \lg k$			
	Plot $\lg y$ against $\lg x$ to obtain straight line graph	M2A1		
	Use graph to find $k \approx 1.43$ and $n \approx 0.563$	M1A2		
(ii)	$y = 1.43\, x^{0.563}$			
	$10 = 1.43\, x^{1.563}$			
	$x = 3.47$	M1A1		
(iii)	$xy = 10$			
	$\lg x + \lg y = 1$	B1		
	Plot this straight line using the same axes.	M1A1	[11]	