

**JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL (ACADEMIC)**

CANDIDATE NAME	
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CLASS	
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INDEX NUMBER	
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MATHEMATICS

4048/01

PAPER 1

**26 Aug 2022
2 hours**

Candidates answer on the Question paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of **23** printed pages, including this page.

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 Simplify $-2(-3x-4)+5$.

$$\begin{aligned} -2(-3x-4)+5 &= 6x+8+5 \\ &= 6x+13 \end{aligned}$$

B1

Answer [1]

2 The ratio of interior angle : exterior angle of a regular polygon is 5:1.

Calculate the number of sides that the polygon has.

$$\text{Exterior angle} = \frac{1}{1+5} \times 180^\circ = 30^\circ$$

$$\text{Number of sides} = \frac{360^\circ}{30^\circ} = 12$$

B1

Answer sides [1]

3 A photocopier can photocopy at a rate of 30 double-sided papers per minute.

Calculate

(a) in minutes, the time taken to print 600 pages on double-sided setting,

600 pages = 300 double sided papers

$$\text{Time taken} = \frac{300}{30} = 10 \text{ minutes}$$

B1

Answer minutes [1]

(b) the number of double-sided papers that can be photocopied in 1 hour.

1 hour = 60 minutes

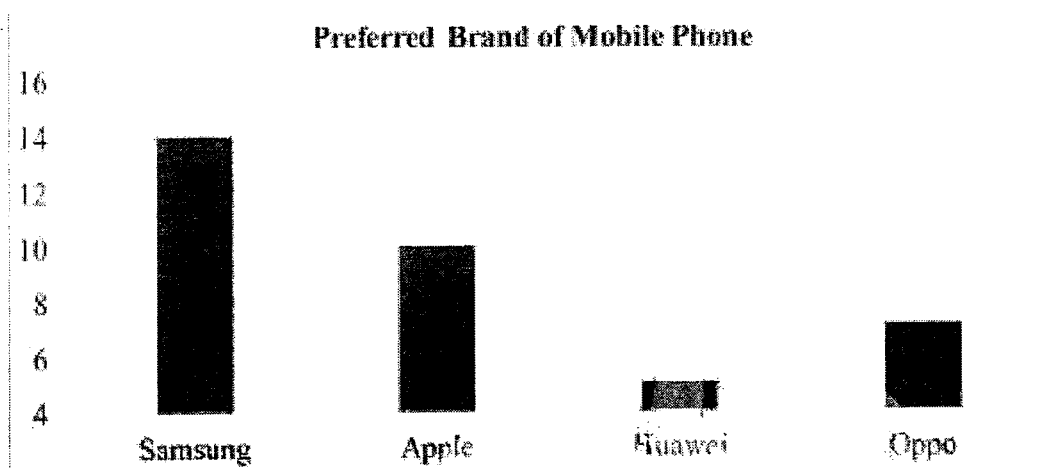
$$\text{No. of pages printed} = 30 \times 60 = 1800$$

B1

Answer double-sided papers [1]

4

- 4 A survey was conducted to find out the preferred brand of mobile phone that secondary school students like.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph

The y-axis does not start with 0. *MI*

Hence it looks like there are three times the number of students choosing Samsung than Oppo, which in fact is only two times. *BI* (14 students for Samsung, 7 students for Oppo)

[2]

- 5 Solve the inequality $-2+x \leq 2-3x < 9+4x$.

$$-2+x \leq 2-3x < 9+4x$$

MI: Split inequality

$$-2+x \leq 2-3x \text{ AND } 2-3x < 9+4x$$

$$4x \leq 4 \text{ AND } -7 < 7x$$

$$x \leq 1 \text{ AND } x > -1$$

$$\therefore -1 < x \leq 1$$

AI: (Don't give if there is no AND)

Answer [2]

4048/1/GE2/22

- 6 Find a possible set of integer values of a and b such that the lines $x+y=1$ and $ax+by=1$

(a) do not intersect,

$$a = 2$$

$$b = 2$$

B1: Any integer values of a and b such that $a = b$ but $a \neq 0, 1, b \neq 0, 1$

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$ [1]

(b) intersect at exactly one point.

$$a = 1$$

$$b = 2$$

B1: Any integer values of a and b such that $a \neq b$

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$ [1]

- 7 Factorise $4ax^2 - 4ay^2 + 2bx^2 - 2by^2$ completely.

$$4ax^2 - 4ay^2 + 2bx^2 - 2by^2$$

$$= 4a(x^2 - y^2) + 2b(x^2 - y^2)$$

$$= (4a + 2b)(x^2 - y^2)$$

$$= 2(2a + b)(x - y)(x + y)$$

B1: $(x + y)(x - y)$

B1

Answer $\dots\dots\dots$ [2]

8 Simplify $\left(\frac{a^{16}}{b^{-6}}\right)^{\frac{3}{2}}$.

$$\begin{aligned}\left(\frac{a^{16}}{b^{-6}}\right)^{\frac{3}{2}} &= \frac{(a^{16})^{\frac{3}{2}}}{(b^{-6})^{\frac{3}{2}}} \\ &= \frac{a^{24}}{b^{-9}} \\ &= a^{24}b^9\end{aligned}$$

M1: Correct multiplication of power to the indices respectively

A1: Must be in positive index notation

Answer[2]

9 Given that $25 \times 125^x = 5$, find the value of x .

$$25 \times 125^x = 5$$

$$5^2 \times (5^3)^x = 5$$

$$5^2 \times 5^{3x} = 5$$

$$5^{2+3x} = 5^1$$

$$\therefore 2 + 3x = 1$$

$$x = -\frac{1}{3}$$

M1: Convert all bases to base 5 correctly

A1

Answer $x =$ [2]

10 Solve $\frac{5}{(2-x)^2} + \frac{1}{x-2} = 4$.

$$\frac{5}{(2-x)^2} + \frac{1}{x-2} = 4$$

$$\frac{5}{(x-2)^2} + \frac{1}{x-2} = 4$$

$$\frac{5}{(x-2)^2} + \frac{x-2}{(x-2)^2} = 4$$

$$\frac{5+x-2}{(x-2)^2} = 4$$

$$\frac{3+x}{(x-2)^2} = 4$$

$$3+x = 4(x-2)^2$$

$$3+x = 4(x^2 - 4x + 4)$$

$$3+x = 4x^2 - 16x + 16$$

$$4x^2 - 17x + 13 = 0$$

$$(x-1)(4x-13) = 0$$

$$x = 1 \text{ or } x = 3.25$$

M1: Common denominator (only accept quadratic denominator)

M1: Factorisation or quadratic formula

A1

Answer $x = \dots\dots\dots$ [3]

11 (a) Express $1+x^2-5x$ in the form of $(x+a)^2 + b$.

$$1+x^2-5x = x^2 - 5x + 1$$

$$= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 1$$

$$= (x-2.5)^2 - 5.25$$

$$= [x + (-2.5)]^2 + (-5.25)$$

B1: Must be this form

Answer $\dots\dots\dots$ [1]

(b) Write down the equation of line of symmetry of the graph of $y = 1+x^2-5x$.

Eqn of line of symmetry: $x = 2.5$

B1

Answer $\dots\dots\dots$ [1]

- (c) Write down the coordinates of intersection between the line of symmetry and the graph of $y = 1 + x^2 - 5x$.

Point of intersection is at the turning point.

(2.5, -5.25)

B1

Answer (.....,) [1]

- 12 The area of triangle ABC is 7.5 cm^2 . Two of the sides are of length 5 cm and 6 cm respectively. Find the length of the third side.

Area of $\Delta = 7.5$

M1: Using Sine Rule correctly

$$\frac{1}{2}(5)(6)\sin\theta = 7.5$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\frac{1}{2} \text{ or } 180^\circ - \sin^{-1}\frac{1}{2}$$

$$\theta = 30^\circ \text{ or } 150^\circ$$

Length of third side

M1: Using Cosine Rule correctly (FT)

$$= \sqrt{5^2 + 6^2 - 2(5)(6)\cos 30^\circ}$$

OR

$$\sqrt{5^2 + 6^2 - 2(5)(6)\cos 150^\circ}$$

\therefore Length of third side = 3.01 cm or 10.6 cm

A1: Both correct

Answercm [3]

- 13 The Ideal Gas Law

$$PV = 8.3145nT$$

relates pressure P (measured in Pascal), volume V (measured in metres³), the number of moles of a gas n (measured in moles), and temperature T (measured in Kelvins).

One mole of gas is found to exert a pressure of 101 325 Pascal at room temperature 293 Kelvins. Find the volume of the gas in cm^3 .

$$V = \frac{8.3145(1)(293)}{101325}$$

$$= 0.0240 \text{ m}^3 \text{ (3 s.f)}$$

$$= 24000 \text{ cm}^3$$

M1: Correct substitution

A1

Answercm³ [2]

14 A map of the United States of America has a scale of 1 : 8 000 000 .

(a) The length of the Mississippi River is 3766 km.
Calculate the length, in centimetres, of Mississippi River on the map.

1 cm : 8000000 cm
 1 cm : 80 km
 80 km → 1 cm
 3766 km → $\frac{3766}{80} = 47.075$ cm

B1: 47.075

Answercm [1]

(b) The area of California on the map is 66.25 cm².
Calculate the actual area, in square kilometres, of California.

1 cm : 8000000 cm
 1 cm : 80 km
 ∴ 1 cm² : 6400 km²
 1 cm² → 6400 km²
 66.25 cm² → 66.25(6400) = 424000 km²

M1: Using proportion

A1

Answerkm² [2]

15 (a) n is a positive integer.

Show that, for all n , $6n^2 + 25n + 11$ is not a prime number.

$$6n^2 + 25n + 11 = (3n + 11)(2n + 1)$$

B1

For all positive integer n , $3n + 11$ and $2n + 1$ are also integers **more than 1**. Hence for all positive n , $6n^2 + 25n + 11$ is a composite

number since it can be factorised into smaller integers greater than 1. **B1:** With the idea of factors more than 1

.....

[2]

(b) Hence, determine whether 16261 is a prime number. Show your working clearly.

$$6n^2 + 25n + 11 = 16261$$

$$6n^2 + 25n - 16250 = 0$$

$$(n - 50)(6n + 325) = 0$$

$$n = 50 \text{ or } n = -\frac{325}{6} \text{ (NA, since } n \text{ is a positive integer)}$$

By previous part, 16261 is a composite number.

M1: Can also use previous part to guess $n = 50$ (No working required)

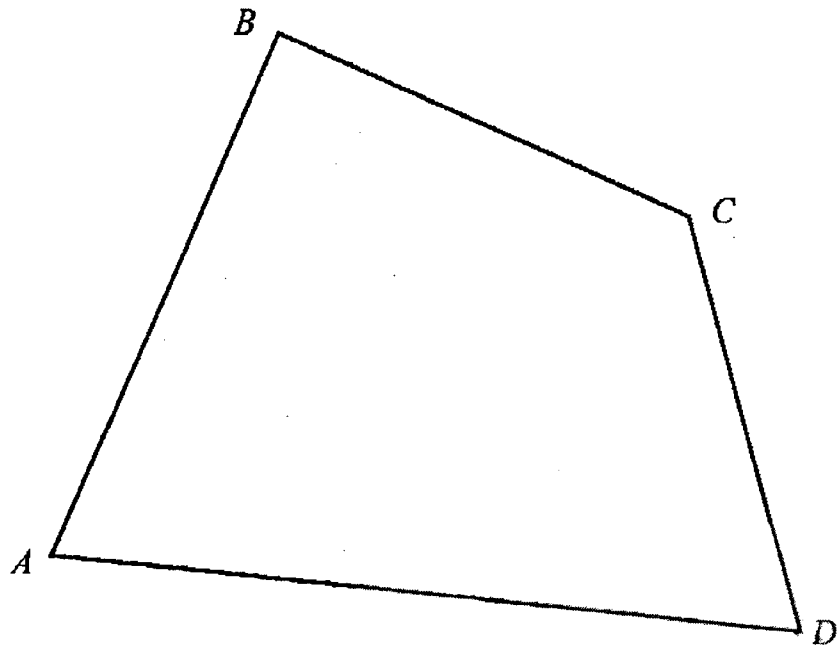
A1 (If solve equation never reject negative minus 1)

.....

[2]

16 The diagram represents a plot of land, $ABCD$. The police is looking for a robber hiding in the land.

Scale: 1 cm represents 1 km



Here are the accounts from three witnesses:

Witness 1: The robber is located more than 6 km away from A .

Witness 2: The robber is located nearer to B than A .

Witness 3: The robber is located nearer to AD than AB .

Shade the region where the robber is hiding.

[4]

- 17 $\xi = \{\text{integers } x : 1 < x < 21\}$
 $A = \{\text{integers that are perfect squares}\}$
 $B = \{\text{integers that are not prime numbers}\}$
 $C = \{\text{integers that are divisible by 5}\}$

(a) List down all the elements in A , B and C .

$$A = \{4, 9, 16\}$$

$$B = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$$

$$C = \{5, 10, 15, 20\}$$

B2

Minus one mark per mistake

Answer $A = \dots\dots\dots$

$B = \dots\dots\dots$

$C = \dots\dots\dots$ [2]

A number is chosen randomly from ξ .

(b) Find the number of elements in $B \cap C'$.

$$B = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$$

$$C = \{5, 10, 15, 20\}$$

$$B \cap C' = \{4, 6, 8, 9, 12, 14, 16, 18\}$$

$$\therefore \text{Required number} = 8$$

B1

Answer $\dots\dots\dots$ [1]

(c) Hence, find the probability that the chosen number is not a prime number and is not divisible by 5.

$$\text{Required probability} = \frac{8}{19}$$

B1: Follow through from part (b) answer, but denominator must be 19

Answer $\dots\dots\dots$ [1]

18 (a) The prime factorisation of 450 is $2 \times 3^2 \times 5^2$.
Express 648 as a product of its prime factors.

$648 = 2^3 \times 3^4$

B1

Answer 648 =[1]

(b) Using your answer to part (a), determine whether 583200 is a perfect square.
Note that $450 \times 648 = 291600$

$\therefore 583200 = 2 \times 450 \times 648 = 2^5 \times 3^6 \times 5^2$

B1

583200 is not a perfect square as the power (exponent) of base 2 in 583200 is not an even number.

B1

.....
.....
.....[2]

(c) Find the smallest integer n such that $450n$ is a multiple of 648.

$$n = \frac{LCM(450, 648)}{450}$$
$$= \frac{2^3 \times 3^4 \times 5^2}{2 \times 3^2 \times 5^2}$$
$$= 2^2 \times 3^2$$
$$= 36$$

B1

Answer $n =$ [1]

- 19 (a) The points $A(0,2)$ and $B(1,0)$ lie on the graph $y = x^2 + ax + b$.

Find the values of a and b .

When $x = 0, y = 2$:

$$2 = b \quad \text{B1}$$

When $x = 1, y = 0$:

$$0 = 1^2 + a + 2$$

$$a = -3 \quad \text{B1}$$

Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$ [2]

- (b) C is another point on the graph such that the gradient of line AB is twice the gradient of AC . Find the coordinates of point C .

Method 1

$$\text{Let } C = (c, c^2 - 3c + 2)$$

Gradient of $AB = 2 \times$ Gradient of AC

B1: Gradient of AB

$$\frac{2-0}{0-1} = 2 \left(\frac{c^2 - 3c + 2 - 2}{c} \right)$$

$$-2 = 2 \left(\frac{c^2 - 3c}{c} \right)$$

$$-1 = \frac{c^2 - 3c}{c}$$

$$c^2 - 3c = -c$$

$$c^2 - 2c = 0$$

$$c(c-2) = 0$$

$$c = 0 \text{ (NA, since will get back point } A) \text{ OR } c = 2 \quad \text{A1}$$

$$\therefore C = (2, 2^2 - 3(2) + 2) = (2, 0)$$

Method 2

Let $C = (x, y)$

Gradient of $AB = 2 \times$ Gradient of AC

$$\frac{2-0}{0-1} = 2 \left(\frac{y-2}{x-0} \right)$$

B1: Gradient of AB

$$\therefore y = -x + 2$$

Solve simultaneously with $y = x^2 - 3x + 2$:

M1: Formulating equations and solve simultaneously

$$x^2 - 3x + 2 = -x + 2$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$x = 0$ (NA, since will get back point A) OR $x = 2$

$$\therefore C = (2, 2^2 - 3(2) + 2) = (2, 0)$$

A1

Answer (.....,)[3]

20 It is given that $AB = \begin{pmatrix} -12 \\ a \end{pmatrix}$, $a > 0$ and $|BA| = 37$ units.

(a) Find the value of a .

$$|BA| = |AB| = 37$$

$$\left| \begin{pmatrix} -12 \\ a \end{pmatrix} \right| = 37$$

$$\sqrt{(-12)^2 + a^2} = 37$$

M1: Distance formula

$$144 + a^2 = 1369$$

$$a^2 = 1225$$

$$a = 35 \text{ or } a = -35 \text{ (NA)}$$

A1

Answer $a = \dots\dots\dots$ [2]

(b) C is a point $(0,10)$ and $\overline{AB} = \overline{DC}$.

Find the coordinates of D .

$$\overline{AB} = \overline{DC}$$

$$\begin{pmatrix} -12 \\ 35 \end{pmatrix} = \overline{OC} - \overline{OD}$$

M1: $OC - OD$

$$\overline{OD} = \overline{OC} - \begin{pmatrix} -12 \\ 35 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 10 \end{pmatrix} - \begin{pmatrix} -12 \\ 35 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ -25 \end{pmatrix}$$

\therefore Coordinates of $D = (12, -25)$

A1

Answer (.....,) [2]

(c) What type of quadrilateral is $ABCD$? Explain your answer.

$$AB = DC$$

There is a pair of parallel sides with equal length.

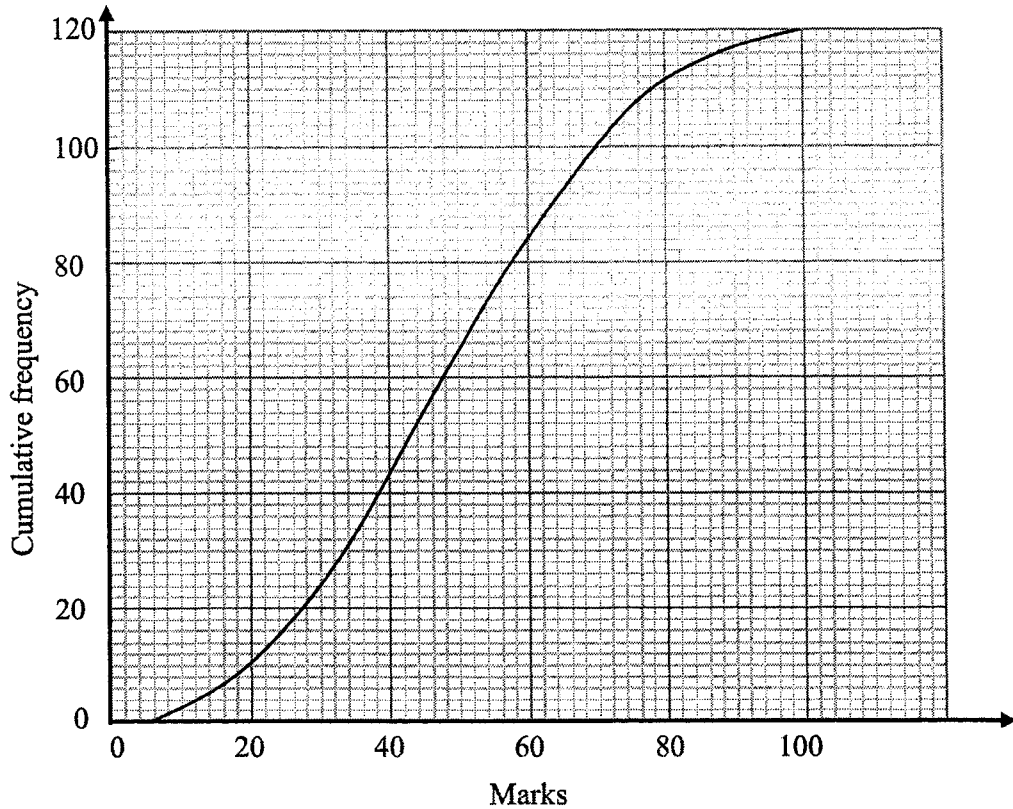
$\therefore ABCD$ is a parallelogram.

B1: Parallelogram

.....

 [1]

- 21 The cumulative frequency curve shows the marks obtained by the 120 students in a recent Mathematics Test. The maximum mark is 100.



- (a) Complete the grouped frequency table for the marks obtained.

Marks obtained by students, x	Frequency
$0 \leq x < 20$	10
$20 \leq x < 40$	34
$40 \leq x < 60$	40
$60 \leq x < 80$	28
$80 \leq x < 100$	8

[1]

- (b) Calculate an estimate of the mean mark.

Required estimate $x < 20$

$$= \frac{(10)(10) + (30)(34) + (50)(40) + (70)(28) + (90)(8)}{120}$$

$$= 48\frac{1}{3} \text{ marks}$$

B1: Accept 48.7 marks

Answermarks [1]

(c) Calculate an estimate of the standard deviation.

Required estimate

$$\begin{aligned}
 &= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \\
 &= \sqrt{\frac{(10)^2(10) + (30)^2(34) + (50)^2(40) + (70)^2(28) + (90)^2(8)}{120} - \left(48\frac{1}{3}\right)^2} && \text{M1: Using S.D} \\
 &= \sqrt{2780 - \left(48\frac{1}{3}\right)^2} && \text{formula} \\
 &= 21.1 \text{ marks (3 s.f)} && \text{A1}
 \end{aligned}$$

Answermarks [2]

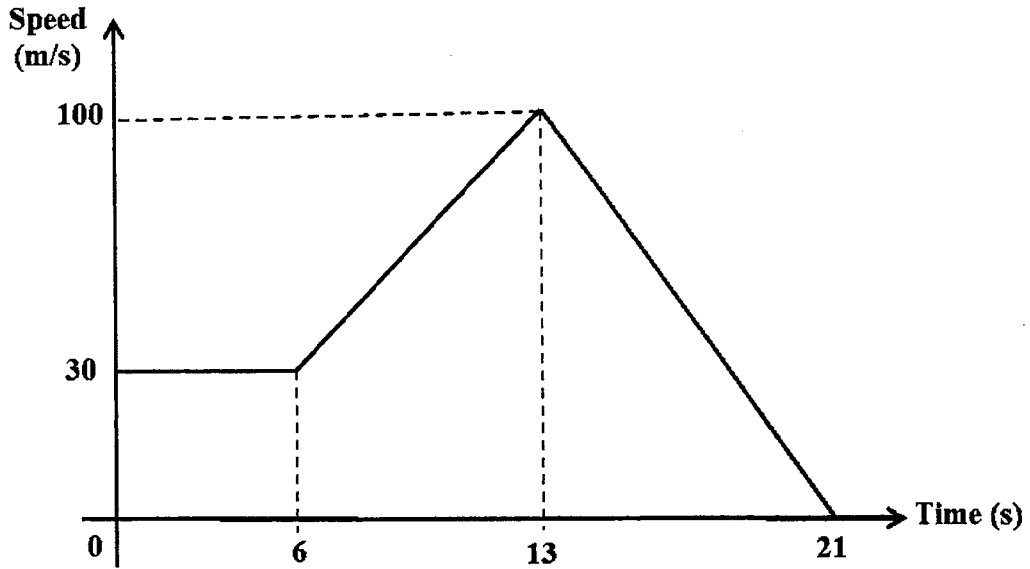
(d) The passing mark is 50. Two students are chosen at random.
Find the probability that both of them passed.

Within 10% of 50 means from 40 to 60.

$$\begin{aligned}
 \text{Required probability} &= \left(\frac{40}{120}\right)\left(\frac{39}{119}\right) \\
 &= \frac{13}{119} && \text{B1}
 \end{aligned}$$

Answer [1]

22 The diagram shows the speed-time graph of a particle.



(a) Find the speed of the particle when $t = 12$ s.

Let the speed of the particle when $t = 12$ be v M1: Equating gradients

$$\frac{v-30}{12-6} = \frac{100-30}{13-6}$$

$$\therefore v = 90 \text{ m/s}$$

A1

Answerm/s [2]

(b) Find the distance covered before the particle starts to slow down.

Distance = Area under the graph from 0s to 13s M1: Finding area under the graph using correct formula

$$= (6)(30) + \frac{1}{2}(30+100)(13-6)$$

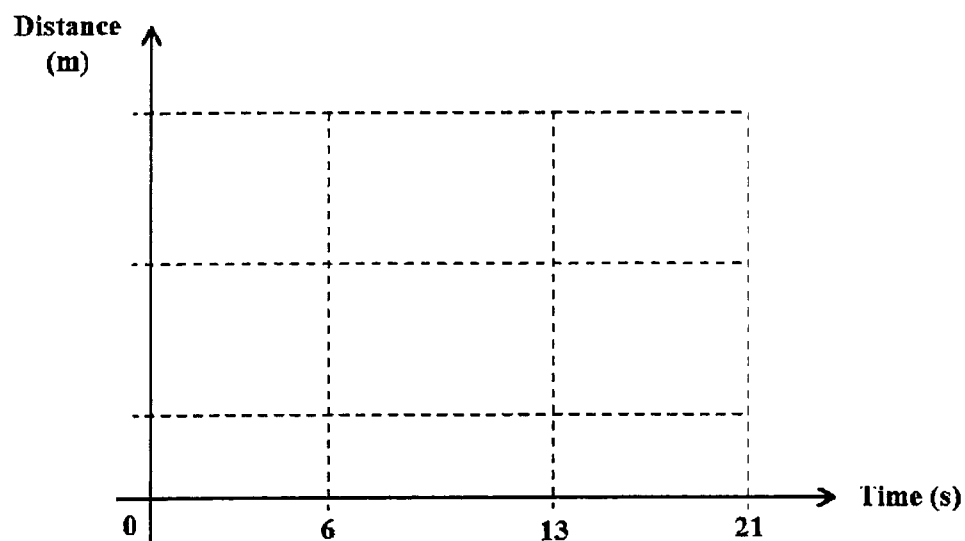
$$= 635 \text{ m}$$

A1

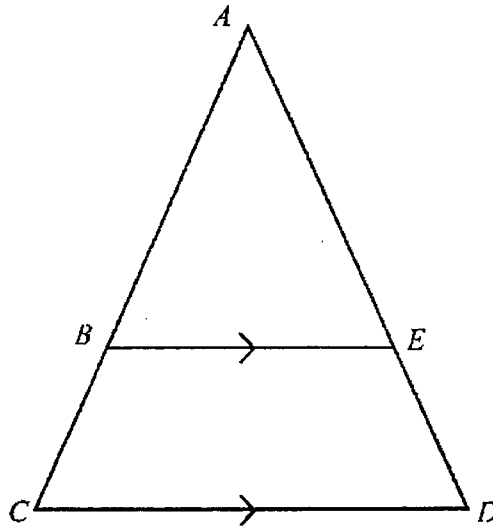
Answerm [2]

(c) Complete the distance-time graph of the particle.

[1]



23



In the diagram, ABC and AED are straight lines and BE is parallel to CD .

(a) Show that triangle ABE is similar to triangle ACD . Give a reason for each statement you make.

$\angle BAE = \angle CAD$ (Common angle)

$\angle ABE = \angle ACD$ (Corresponding angles, BE parallel to CD)

$\angle AEB = \angle ADC$ (Corresponding angles, BE parallel to CD)

\therefore By AA similarity test, $\triangle ABE$ is similar to $\triangle ACD$

M1: Any two correct statements with complete reasons

A1: Conclusion with test

.....

 [2]

(b) Show that $\frac{BC}{ED} = \frac{AB}{AE}$. [2]

By similar Δ s,

$$\frac{AC}{AB} = \frac{AD}{AE}$$

$$\frac{AB + BC}{AB} = \frac{AE + ED}{AE}$$

$$\frac{AB}{AB} + \frac{BC}{AB} = \frac{AE}{AE} + \frac{ED}{AE}$$

$$1 + \frac{BC}{AB} = 1 + \frac{ED}{AE}$$

$$\therefore \frac{BC}{AB} = \frac{ED}{AE}$$

$$\therefore \frac{BC}{ED} = \frac{AB}{AE}$$

M1: Correct ratio with corresponding sides

A1 (AG)

- (c) A circle is drawn with points B , C and D on its circumference. It is further given that $\angle BCD + \angle BED > 180^\circ$. Explain why point E is **inside** the circle.

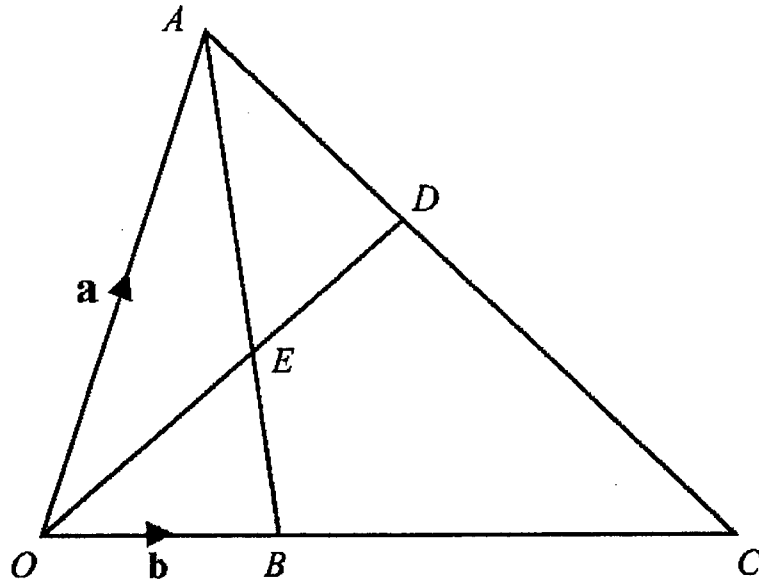
If point E is on the circumference of the circle, $BCDE$ will be a cyclic quadrilateral. Due to angles in opposite segments, $\angle BCD + \angle BED = 180^\circ$. **B1:** Accept drawing to aid explanation

Hence, E is not on circumference of the circle.

As E moves further away from the center of circle, $\angle BED$ will decrease in magnitude. Since $\angle BCD + \angle BED > 180^\circ$, E will be in the circle.

.....

[1]



OBC , ADC , AEB and OED are straight lines.

$OA = \mathbf{a}$ and $OB = \mathbf{b}$.

$OB = \frac{1}{3}OC$, $AE:EB = 3:2$ and $OE:ED = 1:1$.

(a) Find OE in terms of \mathbf{a} and/or \mathbf{b} as simply as possible.

$$OE = OA + AE$$

$$= OA + \frac{3}{5}AB$$

$$\text{M1: } OA + \frac{3}{5}AB$$

$$= OA + \frac{3}{5}(OB - OA)$$

$$= \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$$

A1

Answer: $OE = \dots\dots\dots$ [2]

(b) Find the value of $\frac{\text{area of } \triangle OBE}{\text{area of } BEDC}$.

Let area of $\triangle OBE$ be x

$$\therefore \text{area of } \triangle OAE = \frac{3}{2}x \text{ (Ratio of bases)}$$

$$\therefore \text{area of } \triangle OAB = \frac{5}{2}x \text{ and area of } \triangle AED = \frac{3}{2}x \text{ (Ratio of bases)}$$

$$\therefore \text{area of } \triangle ABC = 2\left(\frac{5}{2}x\right) = 5x \text{ (Ratio of bases)}$$

$$\therefore \frac{\text{area of } \triangle OBE}{\text{area of } BEDC} = \frac{x}{5x - \frac{3}{2}x} = \frac{2}{7}$$

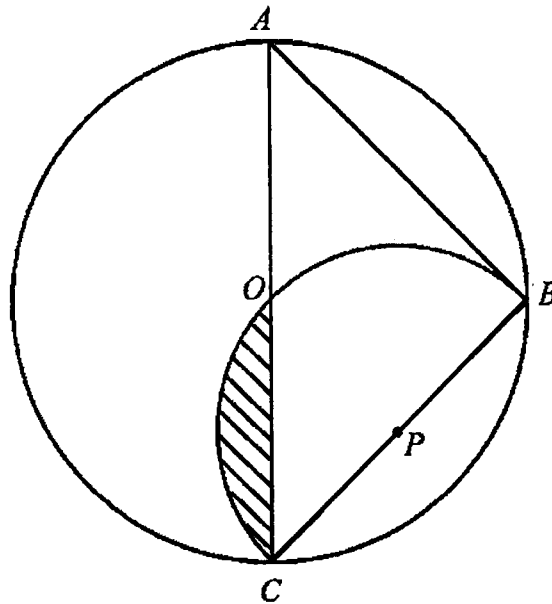
M1: Using ratio of bases to find area of $\triangle OAE$

OR

Using ratio of bases to find area of $\triangle AED$ or $\triangle ABC$

A1

Answer [2]



AC is a diameter of a circle with centre O . $AC = \sqrt{2}r$ and $AB = BC$.
 COB is a semi-circular arc, centre P .

What percentage of the circle is **not** shaded?

$\angle ABC = 90^\circ$ (Right angle in semicircle) **B1:** Need to show $\angle ABC = 90^\circ$

$$AB = BC,$$

$$(AB)^2 + (BC)^2 = (\sqrt{2}r)^2$$

$$2(BC)^2 = 2r^2$$

$$(BC)^2 = r^2$$

M1: Finding AB or BC

$$\therefore BC = r \text{ (Reject -ve)}$$

$$OC = \frac{1}{2}AC = \frac{\sqrt{2}}{2}r$$

$$CP = PO = \frac{1}{2}BC = \frac{r}{2}$$

$$OC^2 = CP^2 + PO^2$$

\therefore By converse of Pythagoras' Theorem,

Method 1: $\triangle OPC$ is a right angled Δ .

B1: Need to show $\triangle OPC$ is a right angled Δ

Method 2:

Note that P and O are midpoints
of BC and AC respectively.

\therefore By midpoint theorem,

OP is parallel to AB ,

and $\angle OPC = \angle ABC$

$\angle ABC = 90^\circ$ (Right angle in semicircle)

$\therefore \angle OPC = 90^\circ$

$\therefore \triangle OPC$ is a right angled Δ .

M1: Using midpoint theorem

A1: $\angle OPC = \angle ABC$

B1: Need to show
 $\triangle OPC$ is a right angled Δ

Percentage of circle shaded

$$= \frac{\text{Sector OPC} - \text{Triangle OPC}}{\text{Area of big circle}} \times 100\%$$

$$= \frac{\frac{1}{2} \left(\frac{r}{2}\right)^2 \left(\frac{\pi}{2}\right) - \frac{1}{2} \left(\frac{r}{2}\right)^2 \sin\left(\frac{\pi}{2}\right)}{\pi \left(\frac{\sqrt{2}}{2} r\right)^2} \times 100\%$$

$$= \frac{\frac{1}{16} \pi r^2 - \frac{1}{8} r^2}{\frac{1}{2} \pi r^2} \times 100\%$$

$$= \frac{\frac{1}{16} \pi - \frac{1}{8}}{\frac{1}{2} \pi} \times 100\%$$

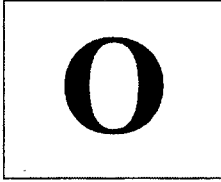
$$= 4.542252845\%$$

$$\begin{aligned} \text{Percentage of circle shaded} &= 100\% - 4.542252845\% \\ &= 95.5\% \text{ (3 s.f)} \end{aligned}$$

M1: Correct percentage formula with
the relevant shapes found using the
correct formula (FT)

A1

Answer% [5]



JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS/
SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	MARKING SCHEME
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CLASS	
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INDEX NUMBER	
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MATHEMATICS

4048/02

PAPER 2

24 August 2022
2 hour 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
 Write in dark blue or black pen on both sides of the paper.
 You may use an HB pencil for any diagrams or graphs.
 Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **26** printed pages including this page.

1	(a)	(i)	$\frac{3}{2}$	B1
		(ii)	$y = \frac{5x-2}{3x+1}$ $3xy + y = 5x - 2$ $5x - 3xy = y + 2$ $x(5 - 3y) = y + 2$ $x = \frac{y+2}{5-3y}$	M1 – simplify + group A1
	(b)		$\frac{x-5}{3} - \frac{x+3}{2} = -4$ $\frac{2(x-5) - 3(x+3)}{6} = -4$ $2x - 10 - 3x - 9 = -24$ $x = 5$	M1 – same denominator A1
	(c)		$2x + y = 23$ $y = 23 - 2x$ $3x - 2(23 - 2x) = 10$ $3x - 46 + 4x = 10$ $7x = 56$ $x = 8$ $y = 7$	M1 – substitution A1 A1
	(d)		$\frac{4x^2 - 9}{6x^2 - x - 12}$ $= \frac{(2x-3)(2x+3)}{(2x-3)(3x+4)}$ $= \frac{2x+3}{3x+4}$	M1 – numerator M1 – denominator A1
				Total: 11 marks

2	(a)	$\begin{pmatrix} 28 & 24 \\ 16 & 12 \end{pmatrix}$	B1
	(b)	$\begin{pmatrix} 70 \\ 80 \end{pmatrix}$	B1
	(c)	$\begin{pmatrix} 3880 \\ 2080 \end{pmatrix}$	B1
	(d)	The elements represent the total amount Mrs Teo collected in a month from the weekday and weekend sessions respectively.	B1
	(e)	<p>Total amount of money per session = $\begin{pmatrix} 10 & 8 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 70 \\ 80 \end{pmatrix} = \begin{pmatrix} 1340 \\ 370 \end{pmatrix}$</p> <p>Total after 10% increase = $(1 \ 1.1) \begin{pmatrix} 1340 \\ 370 \end{pmatrix} = (1747)$</p> <p>$1747 \times 4 \times 3 = \\20964</p>	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>Accept non-matrices working</p>
			Total: 8 marks

3	(a)	$\angle XCD = \frac{360^\circ}{18} = 20^\circ$	B1
	(b)	$\angle XCD = \angle XDC$ $\angle CXD = 180^\circ - 20^\circ - 20^\circ$ (isos. Δ) $= 140^\circ$	A1 (with reason)
	(c)	$BC = ED$ (sides of regular polygon) $XC = XD$ (given) $\therefore BC + CX = ED + DX$ $BX = EX$	M1 – evidence A1
	(d)	$\angle BXE = \angle CXD$ (common) $\angle XBE = \frac{180^\circ - 140^\circ}{2} = 20^\circ$ $\therefore \angle XCD = \angle XBE$ ΔXCD and ΔXBE are similar (AA)	M1 – evidence A1
	(e)	They are parallel.	B1
			Total: 7 marks

4	(a)	$T_5 = 9^2 - 34 = 47$	B1 (with working)
	(b)	$T_n = (n+4)^2 - 18 - 4(n-1)$ $= n^2 + 8n + 16 - 18 - 4n + 4$ $= n^2 + 4n + 2$	M1 - $n+4$ and $18+4(n-1)$ A1
	(c)	$n^2 + 4n + 2 = 962$ $n^2 + 4n - 960 = 0$ $n = \frac{-4 \pm \sqrt{4^2 - 4(1)(-960)}}{2(1)}$ $= 29.048 \text{ or } -33.048$ <p>Since n is not an integer, 962 is not a term of the sequence.</p>	M1 - find value of n A1
	(d)	$T_n - T_{n-1}$ $= (n^2 + 4n + 2) - [(n-1)^2 + 4(n-1) + 2]$ $= n^2 + 4n + 2 - (n^2 - 2n + 1 + 4n - 4 + 2)$ $= n^2 + 4n + 2 - n^2 + 2n - 1 - 4n + 4 - 2$ $= 2n + 3$	M1 - correct expression M1 - correct expansion A1
	(e)	<p>The difference between two consecutive terms is $2n + 3$.</p> <p>Since $2n$ is always even for all values of n, the sum of an even integer and an odd constant 3 will always give an odd integer.</p>	B1 - always even + add to odd
			Total: 8 marks

5	(a)	$p = 2.2$	B1	
	(b)	Refer to last page	G3	
	(c)	The graph of $y = x + \frac{5}{x^2} - 3$ does not meet the line $y = 0$ /does not have any x -intercepts/ the minimum y -value is , which is more than 0	B1	
	(d)	Draw tangent Gradient = -1.96 ± 0.7	B1 B1	
	(e)	(i)	Draw line with gradient 2 passing through (0.5, 2) (y -intercept = 1)	B1 - passes through (0.5, 2) B1 - correct gradient (& y -int)
		(ii)	$y = 2x + 1$	B1
		(iii)	$x + \frac{5}{x^2} - 3 = 2x + 1$ $\frac{5}{x^2} = x + 4$ $5 = x^3 + 4x^2$ $x^3 + 4x^2 - 5 = 0$	M1 - sim. eqn. A1 - final eqn
		(iv)	$x = 1$	B1 (ecf - accept answer from intersection of curve and line)
			Total: 13 marks	

6	(a)	$OS = OR$ (radii of smaller circle) $\angle POS = \angle QOR$ (vert. opp. \angle) $OP = OQ$ (radii of larger circle) $\therefore \triangle PSO$ is congruent to $\triangle QRO$ (SAS) Alternatives: RHS, AAS	M1 A1
	(b) (i)	$\angle ORQ = 90^\circ$ (tan \perp rad) $QR = \sqrt{8^2 - 4.36^2}$ $= 6.7075$ Area of $\triangle QRO = \frac{1}{2}(4.36)(6.7075)$ $= 14.62235$ $\approx 14.6 \text{ cm}^2$	M1 – with evidence M1 A1
	(ii)	Area of shaded minor sector $= \frac{1}{2}(4.36)^2(0.995)$ $= 9.457276$ Area of ring between the circles $= \pi(8)^2 - \pi(4.36)^2$ $= 141.3415$ Area of unshaded in $\triangle QRO$ $= 14.62235 - 9.457276$ $= 5.165074$ Shaded area $= 141.3415 + 2(9.457276) - 2(5.165074)$ $= 149.925904$ $\approx 150 \text{ cm}^2$	M1 M1 M1 A1
			Total: 9 marks

7	(a)	$\frac{5}{1+2+4+5} \times 300 \times \$148 = \$18500$	M1 A1
	(b)	(i) $\frac{300}{x}$ min	B1
		(ii) $\frac{300}{x-2}$ min	B1
		(iii) $\frac{300}{x-2} - \frac{300}{x} = 3$ $\frac{300x - 300(x-2)}{x(x-2)} = 3$ $x^2 - 2x - 200 = 0$ (shown)	M1 – form eqn M1 – combine fractions A1
		(iv) $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-200)}}{2(1)}$ $= 15.17744$ or -13.17744 ≈ 15.2 or -13.2	M1 A1 (3sf, no rej.)
		(v) Reject -13.2 $x = 15.2$ $15.17744 \times 60 = 910.6464$ ≈ 910	M1 A1 (round down) (Reject 3 x 60)
			Total: 11 marks

8	<p>(a) $\angle AOB = 120^\circ, \angle ABO = 30^\circ$</p> $\frac{OA}{\sin 30^\circ} = \frac{8}{\sin 120^\circ}$ $OA = \frac{8}{\sin 120^\circ} \times \sin 30^\circ$ $\approx 4.6188 \text{ cm}$ $= 4.619 \text{ cm (to 3 d.p.)}$ <p>OR</p> $OA = \frac{2}{3} \sqrt{8^2 - 4^2}$ $= 4.61880$ ≈ 4.619	<p>M1 – sine rule</p> <p>A1</p> <p>M1</p> <p>A1</p>
	<p>(b) Base area $\triangle ABC = \frac{1}{2}(8)(8)\sin 60^\circ$</p> $\approx 27.7128 \text{ cm}^2$ <p>Height of $\triangle AXB, l = \sqrt{18^2 - 4^2}$</p> $\approx 17.5499 \text{ cm}$ <p>Total surface area</p> $= 3 \times \frac{1}{2}(8)(17.5499) + 27.7128$ $\approx 238.31 \text{ cm}^2$ $= 238 \text{ cm}^2 \text{ (to 3 s.f.)}$	<p>M1 – base</p> <p>M1 – lateral height</p> <p>A1</p>
	<p>(c) $\cos \angle PQR = \frac{5^2 + 4^2 - 3.5^2}{2(5)(4)}$</p> $\angle PQR = \cos^{-1}\left(\frac{23}{32}\right)$ $= 44.0^\circ \text{ (to 1 d.p.)}$	<p>M1 – cosine rule</p> <p>A1</p>
	<p>(d) $AQ = 6 \text{ cm}$</p> $OQ = \sqrt{4.619^2 + 6^2 - 2(4.619)(6)\cos 30^\circ}$ $\approx 3.055 \text{ cm}$ $XO = \sqrt{18^2 - 4.619^2}$ $\approx 17.397 \text{ cm}$ $\tan \theta = \frac{17.397}{3.055}$ <p>Angle of elevation of X from Q</p> $= \tan^{-1}\left(\frac{17.397}{3.055}\right)$ $= 80.0^\circ \text{ (to 1 d.p.)}$	<p>M1 – find OQ</p> <p>M1 – Pythagoras' thm</p> <p>M1</p> <p>A1</p>
		Total: 11 marks

9	(a)	(i)	$100 < x \leq 120$	B1	
		(ii)	22%	B1	
		(iii)	$\text{S.D.} = \sqrt{\frac{809000}{50} - \left(\frac{6280}{50}\right)^2}$ $= 20.11566$ ≈ 20.1	M1 – working (show substitution) A1	
		(iv)	The blood pressures of the second group of patients is less consistent than that of the first group of patients, since it has a SD of 22.1 which is higher than the SD of the first group (20.1)	B1 – compare SD and comment on consistency	
	(bi)	<pre> graph LR A[] --- B["2/8"] A --- C["6/8"] B --- D[blue] B --- E[white] C --- F[blue] C --- G[white] D --- H[blue] D --- I[white] E --- J[blue] E --- K[white] F --- L[blue] F --- M[white] G --- N[blue] G --- O[white] </pre>		B2 -1 for each wrong pair of branches	
		(ii)	(a)	$\left(\frac{6}{8} \times \frac{5}{7}\right) + \left(\frac{2}{8} \times \frac{1}{7}\right) = \frac{4}{7}$	M1 – either both W or both B A1 FT from tree diagram
			(b)	$\left(\frac{2}{8} \times \frac{6}{7} \times \frac{1}{6}\right) + \left(\frac{6}{8} \times \frac{2}{7} \times \frac{5}{6}\right)$ $= \frac{3}{14}$	M1 – either WBW or all BWB A1
				Total: 11 marks	

10	(a)	$3.42 \times 2.7 = 9.234 \text{ m}^2$	B1 Reject 3s.f.
	(b)	<p>10 cm represents 168 cm 1 cm represent 16.8 cm Elbow to ground = 6.4 cm (accept 6.3cm and 6.5cm)</p> <p>$6.4 \times 16.8 = 107.52\text{cm}$</p> <p>Range of optimal kitchen cabinets height</p> <p>$107.52 - 10 = 97.52 \text{ cm to } 107.52 \text{ cm}$</p>	M1 – measure distance from elbow to ground A1 Measured 6.3 cm, 95.84 cm to 105.84 cm Measured 6.5 cm, 99.2 cm to 109.2 cm
	(c)	<p><u>Kitchen Cabinet</u></p> <p>Compulsory: Sink + corner cabinet + wire basket shelves Cabinet 1 + Cabinet 2 (must go at the junction of the wall) + Cabinet 5 Remaining length = $342 - 50 - 60 - 50 = 172 \text{ cm}$ Remaining width = $135 - 60 = 75 \text{ cm}$ Current total cost = $\\$156 + \\$166 + \\$166 = \\488</p> <p>Along the length – 1 Cabinet 3 + 1 Cabinet 4 (90 cm + 80 cm) Along the width – 1 Cabinet 6 (70 cm) in order to optimise storage</p> <p>Total cost of cabinets = $\\$488 + \\$193 + \\$260 + \\$290 = \\$1231$ Height customisation = $\\$25 \times 6 = \\150 Remaining budget = $\\$2200 - \\$1231 - \\$150 = \\819 \therefore Ceramic or acrylic for materials</p> <p>Mr Graham should choose the Ceramic counter top and his kitchen cabinet combination is one of cabinets 1, 2, 3, 4, 5, 6 with \$19 remaining in his budget.</p>	M1 (Cabinet 1+2+5) M1 (remaining space) M1 (combination of cabinets) M1 – total cost of cabinets M1 – height M1 – budget + conclusion for type of material A1 (relate to budget)
			Total: 10 marks

