HUA YI SECONDARY SCHOOL

4E5N

Preliminary Examination 2022

4E5N

ADDITIONAL MATHEMATICS

Paper 1

MARKING SCHEME

1.	(a)		$p^2-4(1)(-4p+9)<0$ $p^2+16p-36<0$ $(p-2)(p+18)<0$ $-18<p<2$
	(b)		$x^2+4x-16+9=6x-8$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ Since there is only one solution, $y=6x-8$ is a tangent. Or use b^2-4ac.
2.	(a)	(i)	$f(-3)=2(-3)^3+a(-3)^2+b(-3)-6=0$ $-54+9a-3b-6=0$ $9a-3b=60$ $3a-b=20$ $f'(x)=6x^2+2ax+b$ $f'(1)=6(1)^2+2a(1)+b=1$ $2a+b=-5$ $5a=15$ $a=3$ $b=-11$
		(ii)	$2x^3+3x^2-11x-6=(x+3)(2x^2+px-2)$ $3p-2=-11$ $p=-3$ $2x^3+3x^2-11x-6=(x+3)(2x^2-3x-2)$ $2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)$

		(iii)	$2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)=0$ $x=-3,-\frac{1}{2},2$ $2+3y-11y^2-6y^3=0$ $\frac{1}{y}=-3,-\frac{1}{2},2$ $y=-\frac{1}{3},-2,\frac{1}{2}$
	(b)	$54x^6-16y^3$ $=2(27x^6-8y^3)$ $=2[(3x^2)^3-(2y)^3]$ $=2(3x^2-2y)(9x^4+6x^2y+4y^2)$	
3.	(a)	$V=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi-\frac{2}{3}\pi r^3=\pi r^2h$ $h=\frac{60}{r^2}-\frac{2}{3}r$	
	(b)	$SA=2\pi rh+\pi r^2+2\pi r^2$ $C=3(2\pi rh+\pi r^2)+4(2\pi r^2)$ $=6\pi r\left(\frac{60}{r^2}-\frac{2}{3}r\right)+11\pi r^2$ $=\frac{360\pi}{r}-4\pi r^2+11\pi r^2$ $=\frac{360\pi}{r}+7\pi r^2$	
	(c)	$\frac{dC}{dr}=-\frac{360\pi}{r^2}+14\pi r=0$ $\frac{360\pi}{r^2}=14\pi r$ $r^3=\frac{360}{14}$ $r=2.9516$	

	(d)	When $r = 2.9516$, $C = \dfrac{360\pi}{2.9516} + 7\pi(2.9516)^2 = 574.76$ $\dfrac{d^2C}{dr^2} = \dfrac{720\pi}{r^3} + 14\pi > 0$ \$5.75 is a minimum value which is > \$5.60. He should not continue to make this product.
4.	**(a)**	$2^{4p+1} + 20(4^{p-1}) = 3$ $2(4^p)^2 + 5(4^p) = 3$ $2k^2 + 5k - 3 = 0$ $(2k-1)(k+3) = 0$ $k = \frac{1}{2}, -3$ $4^p = \frac{1}{2}$ or -3 (rej) $p = -\frac{1}{2}$
	(b)	$2u^2 + 5u - k = 0$ If no solution, $b^2 - 4ac < 0$ $25 + 8k < 0$ $k < -\frac{25}{8}$ OR $2^{4p+1} + 20(4^{p-1}) = k$ $2(4^p)^2 + 5(4^p) = k$ Since $4^p > 0$ for all p, $2(4^p)^2 + 5(4^p)$ is always positive for all p. Therefore, no solution for $k < -3\frac{1}{8}$.

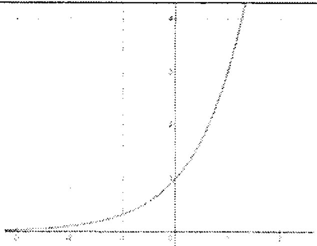
5	(a)	$\dfrac{(\log_y x)^2}{\log_x y} + 64 = 0$ $\dfrac{(\log_y x)^2}{\dfrac{\log_x x}{\log_x y}} = -64$ $(\log_y x)^3 = -64$ $\log_y x = -4$ $x = y^{-4}$ $y = x^{-\frac{1}{4}}$
	(b)	$2\log_2(x-1) - \log_2 x = 3$ $\log_2 \dfrac{(x-1)^2}{x} = 3$ $\dfrac{x^2 - 2x + 1}{x} = 8$ $x^2 - 10x + 1 = 0$ $x = 0.101$ (rej) or 9.90

6.	(a)	Max $= 9.9$ m Min $= 0.3$ m
	(b)	$k\pi$ cycles in 2π hours 1 cycle in $\dfrac{2}{k}$ hours 2 cycles in $\dfrac{4}{k}$ hours $\dfrac{4}{k} = 24$ $k = \dfrac{1}{6}$ Note: Do not accept $24k\pi = 4\pi$ as working.

	(c)		$4.8\sin\frac{\pi t}{6}+5.1>2$ $4.8\sin\frac{\pi t}{6}>-3.1$ $\sin\frac{\pi t}{6}>-\frac{31}{48}$ Basic Angle = 0.702 (Q3,4) $\frac{\pi t}{6}=\pi+0.702, 2\pi-0.702$ $t=7.34, 10.66$ $0<t<7.34, 10.66<t<12$
7.	(a)	(i)	$LHS=\frac{\cos x}{\sin x}-2\sin x\cos x$ $=\frac{\cos x-2\sin^2 x\cos x}{\sin x}$ $=\frac{\cos x(1-2\sin^2 x)}{\sin x}$ $=\cot x\cos 2x$
		(ii)	$4(\cot x-\sin 2x)=\cos 2x$ $4(\cot x\cos 2x)=\cos 2x$ $\cos 2x(4\cot x-1)=0$ $\cos 2x=0$ or $\cot x=\frac{1}{4}$ $\alpha=\pi$ (Q1,4) or $\tan x=4$ $2x=\frac{\pi}{2},\frac{3\pi}{2}$ or $\alpha=1.3258$ (Q1,3) $x=\frac{\pi}{4},\frac{3\pi}{4}$ or $x=1.33, 4.47$ (rej) Ans: $x=\frac{\pi}{4},\frac{3\pi}{4},1.33$
	(b)		$\sin A\cos B+\cos A\sin B=\frac{6}{7}$ $\sin A\cos B+\frac{2}{7}=\frac{6}{7}$ $\sin A\cos B=\frac{4}{7}$ $\frac{\sin A\cos B}{\cos A\sin B}=\frac{4}{7}\div\frac{2}{7}$ $\frac{\tan A}{\tan B}=2$

8.	**(a)**	$y=\frac{2}{\sqrt{3x+1}}=2(3x+1)^{-\frac{1}{2}}$ $\frac{dy}{dx}=-3(3x+1)^{-\frac{3}{2}}$ $\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$ $-3=-3(3x+1)^{-\frac{3}{2}}(0.125)$ $8=(3x+1)^{-\frac{3}{2}}$ $3x+1=\frac{1}{4}$ $x=-\frac{1}{4},\ y=4$	M1 M1 M1 M1 A1
	(b)	$y=2x\ln x$ $\frac{dy}{dx}=2x\left(\frac{1}{x}\right)+2\ln x=2+2\ln x$ For increasing functions, $1+\ln x>0$ $\ln x>-1$ $x>\frac{1}{e}$	M1 A1
9.	**(a)**	$y=(2x+1)^{-1}$ $\frac{dy}{dx}=-2(2x+1)^{-2}$ $x=0,\frac{dy}{dx}=-2$ Eqn of tangent: $y=-2x+c$ $1=-2(0)+c$ $c=1$ $y=-2x+1$	M1 M1 A1
	(b)	Where tangent intersects the x-axis, $y=0$ $0=-2x+1$ $x=\frac{1}{2}$ $Area=\int_0^1\frac{1}{2x+1}\ dx-\frac{1}{2}\left(\frac{1}{2}\right)1$ $=\left[\frac{1}{2}\ln(2x+1)\right]_0^1-\frac{1}{4}$ $=\frac{1}{2}\ln 3-\frac{1}{4}$	M1 M1 M1 A1

10.	**(a)**	Let $\angle FBC = x$, $\angle CAB = x$ (tangent chord theorem) $\angle FED = x$ (angles in same segment) By the converse of alternate angles, since $\angle CBA = \angle FED$, AB is parallel to DE.	M1 M1 A1
	(b)	ΔBCF and ΔEDF	B1 B1
	(b)	ΔABF is similar to ΔBCF. Therefore $\frac{AF}{BF} = \frac{BF}{CF}$ $AF \times CF = BF^2$ (shown)	M1 M1
11.	**(a)**	Grad $PR = \frac{10}{-5} = -2$ Grad $SQ = \frac{1}{2}$ Eqn SQ: $y = \frac{1}{2}x + c$ $7 = \frac{1}{2}(3) + c$ $c = \frac{11}{2}$ Eqn $SQ: y = \frac{1}{2}x + \frac{11}{2}$	M1 M1 A1
	(b)	Grad $PR = \frac{10}{-5} = -2$ Eqn: $y = -2x + c$ $7 = -2(3) + c$ $c = 13$ $y = -2x + 13$	M1 A1
	(c)	$Area\ PQR = \frac{1}{2}\begin{vmatrix} -3 & 2 & 3 & -3 \\ 9 & -1 & 7 & 9 \end{vmatrix}$ $= \frac{1}{2}[(3+14+27)-(18-3-21)]$ $= \frac{1}{2}(44-(-6))$ $= 25$	M1 A1

12	**(a)**		B1
	(b)	$\log_9(5-x)=\dfrac{x}{2}$ $(5-x)^2=3^{2x}$ $5-x=3^x$ Eqn straight line: $y=5-x$	 M1 A1
	(c)	1 solution	B1

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 4E5N

ADDITIONAL MATHEMATICS

Paper 2

MARKING SCHEME

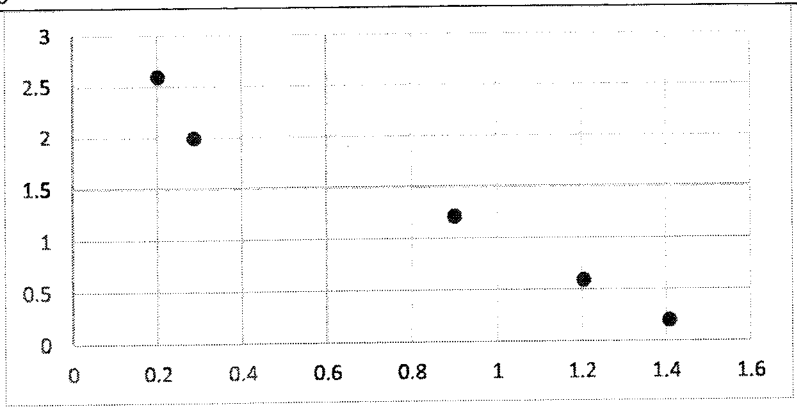
1	**(a)**		$b^2-4ac=(2a)^2-4(1)(2a^2-3b)$ $=4a^2-8a^2+12b$ $=12b-4a^2$ $=4(3b-a^2)$ Since $3b-a^2>0$, hence $b^2-ac>0$. Two real and distinct roots.
	(b)		Subst $y=5-2x$ into $y^2+y-3x=9$ $(5-2x)^2+5-2x-3x=9$ $25-20x+4x^2+5-5x=9$ $4x^2-25x+21=0$ $(4x-21)(x-1)=0$ $x=\frac{21}{4}$ or $x=1$
	(c)	**(i)**	$h=-\frac{1}{5}t^2+4t+2$ $=-\frac{1}{5}(t^2-20t-10)$ $=-\frac{1}{5}[(t-10)^2-100-10]$ $=-\frac{1}{5}[(t-10)^2-110]$ $=-\frac{1}{5}(t-10)^2+22$
		(ii)	Max height = 22 m
2			$(10+2\sqrt{3})\pi=\frac{1}{3}\pi r^2(3-\sqrt{3})$ $r^2=\frac{30+6\sqrt{3}}{3-\sqrt{3}}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}$ $=\frac{90+30\sqrt{3}+18\sqrt{3}+18}{9-3}$ $=\frac{108+48\sqrt{3}}{6}=18+8\sqrt{3}$ $l^2=18+8\sqrt{3}+(3-\sqrt{3})^2$ $=18+8\sqrt{3}+9-6\sqrt{3}+3$ $=30+2\sqrt{3}$

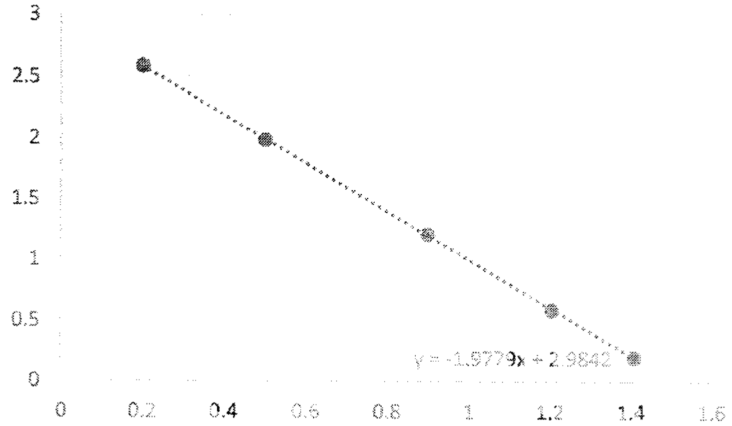
3	(a)	$\left(x^2-\frac{1}{3x}\right)^9$ $T_r=\binom{9}{r}(x^2)^{9-r}\left(-\frac{1}{3x}\right)^r$ Powers of $x=18-2r-r$ $=18-3r$ $=3(6-r)$ Multiple of 3
	(b)	$\left(x^2-\frac{1}{3x}\right)^9(1+6x^3+9x^6)$ Term with $x^{-6}, r=8\rightarrow\frac{9}{3^8x^6}$ Term with $x^{-3}, r=7\rightarrow-\frac{36}{3^7x^3}$ Term with $x^0, r=6\rightarrow\frac{84}{3^6}$ Product $=\frac{84}{3^6}(1)-\frac{36}{3^7x^3}(6x^3)+\frac{9}{3^8x^6}(9x^6)=\frac{28}{243}-\frac{8}{81}+\frac{1}{81}=\frac{7}{243}$
4	(a)	$MidptAB=\left(\frac{5-11}{2},\frac{7+15}{2}\right)=(-3,11)$ $GradAB=\frac{15-7}{-11-5}=-\frac{1}{2}$ $GradPB=2$ $EqnPB: y=2x+c$ $11=2(-3)+c$ $c=17$ $Eqn: y=2x+17$
	(b)	$y=2x+17------(1)$ $y=-2x-3------(2)$ $2x+17=-2x-3$ $x=-5$ $y=7$ $C(-5,7)$ $Radius=\sqrt{(5+5)^2+(0)}=10$ $EqnCircle: (x+5)^2+(y-7)^2=100$

	(c)	AD is diameter $C(-5,7)$ is midpoint of AD $\left(\frac{5+x}{2},\frac{7+y}{2}\right)=(-5,7)$ $x=-15$ $y=7$ $D(-15,7)$
	(d)	$-15<k<5$
	(e)	$(x-5)^2+(y-7)^2=100$
5	**(a)**	$y=\frac{1-\sin x}{\cos x}$ $\frac{dy}{dx}=\frac{\cos x(-\cos x)-(1-\sin x)(-\sin x)}{\cos^2 x}$ $=\frac{-\cos^2 x+\sin x-\sin^2 x}{\cos^2 x}$ $=\frac{\sin x-1}{\cos^2 x}$ $=\tan x\sec x-\sec^2 x$
	(b)	$\int \tan x\sec x-\sec^2 x\,dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx-\int \sec^2 x\,dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx-\tan x=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx=\frac{1-\sin x}{\cos x}+\tan x+c$ $\int_0^{\frac{\pi}{4}} \tan x\sec x\,dx=\left[\frac{1-\sin x}{\cos x}+\tan x\right]_0^{\frac{\pi}{4}}$ $=\left[\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}+1\right]-[1+0]$ $=\sqrt{2}-1+1-1=\sqrt{2}-1$

6	(a)		$\int_0^5 3f(x)\,dx + \int_5^3 x - kf(x)\,dx = 8$ $3(8) - \int_3^5 x - kf(x)\,dx = 8$ $24 - \int_3^5 x\,dx + \int_3^5 kf(x)\,dx = 8$ $24 - \left[\frac{x^2}{2}\right]_3^5 + k\int_3^5 f(x)\,dx = 8$ $24 - \left(\frac{25}{2} - \frac{9}{2}\right) + k(4) = 8$ $4k = -8$ $k = -2$
	(b)	(i)	$\frac{-14x^2+14x-3}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$ $-14x^2+14x-3 = A(2x-1)^2 + Bx(2x-1) + Cx$ Let $x = 0, -3 = A$ Let $x = \frac{1}{2}, -\frac{14}{4} + 7 - 3 = \frac{1}{2}C \to C = 1$ Let $x = 1, -14+14-3 = A+B+C$ $-3 = -2 + B$ $B = -1$ $\frac{-14x^2+14x-3}{x(2x-1)^2} = -\frac{3}{x} - \frac{1}{2x-1} + \frac{1}{(2x-1)^2}$
		(ii)	$\int \frac{-14x^2+14x-3}{x(2x-1)^2}\,dx$ $= \int -\frac{3}{x} - \frac{1}{2x-1} + \frac{1}{(2x-1)^2}\,dx$ $= -3\ln x - \frac{1}{2}\ln(2x-1) + \frac{(2x-1)^{-1}}{2(-1)} + c$ $= -3\ln x - \frac{1}{2}\ln(2x-1) - \frac{1}{2(2x-1)} + c$
7	(a)		$PQ = 100\sin\theta - 40\cos\theta$ $QR = 100\cos\theta + 40\sin\theta$ $P = 100 + 40 + 100\sin\theta - 40\cos\theta + 100\cos\theta + 40\sin\theta$ $= 140 + 140\sin\theta + 60\cos\theta$

	(b)	$P = 140 + 140\sin\theta + 60\cos\theta$ $60\cos\theta + 140\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{23200}$ $\alpha = 66.8°$ $P = 140 + \sqrt{23200}\cos(\theta - 66.8°)$
	(c)	$250 = 140 + \sqrt{23200}\cos(\theta - 66.8°)$ $\cos(\theta - 66.8°) = \dfrac{110}{\sqrt{23200}}$ Basic angle $= 43.764°$ (Q1, 4) $\theta - 66.8° = -43.764°$ or $43.764°$ $\theta = 23.0°$
8.	(a)	$\dfrac{d^2y}{dx^2} = 6e^{3x} - x$ $\dfrac{dy}{dx} = \dfrac{6e^{3x}}{3} - \dfrac{x^2}{2} + c$ $5 = 2 + c$ $c = 3$ $\dfrac{dy}{dx} = \dfrac{6e^{3x}}{3} - \dfrac{x^2}{2} + 3$ $y = \dfrac{2e^{3x}}{3} - \dfrac{x^3}{6} + 3x + c$ $\dfrac{2}{3}e^6 = \dfrac{2}{3}e^6 - \dfrac{4}{3} + 6 + c$ $c = -\dfrac{14}{3}$ $y = \dfrac{2e^{3x}}{3} - \dfrac{x^3}{6} + 3x - \dfrac{14}{3}$

	(b)		$\frac{dy}{dx}=2e^{3x}-\frac{x^2}{2}+3$ When $x=2$ $\frac{dy}{dx}=2e^6+1$ $y=(2e^6+1)x+c$ $\frac{2}{3}e^6=(2e^6+1)(2)+c$ $\frac{2}{3}e^6=4e^6+2+c$ $c=-\frac{10}{3}e^6-2$ $y=(2e^6+1)x-\frac{10}{3}e^6-2$
9.	(a)		$\lg y=\lg a+b\lg x$ $\lg y=b\lg x+\lg a$ $Y=\frac{1}{3}X+c$ $7=\frac{1}{3}(6)+c$ $c=5$ $b=\frac{1}{3}$ $\lg a=5$ $a=10^5$
	(b)	(i)	 Axis M1, Points and line M1 Incorrect value: $y=3.5$ A1

		(ii)	 $\frac{1}{y}=\frac{1}{2}$ Correct value of $y=2$
		(iii)	$y=\frac{p}{x^2+k}$ $x^2y+ky=p$ $x^2y=-ky+p$ $x^2=\frac{p}{y}-k$ Gradient $=p=-2$ Vertical intercept $=-k=3$ $k=-3$
10	**(a)**	$v=\frac{1}{2}t^2-t-4$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t+c$ When $s=0, t=0$ therefore $c=0$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ At max velocity, $\frac{dv}{dt}=0$ $t-1=0$ $t=1$ Displacement $=\frac{1}{6}-\frac{1}{2}-4=-\frac{13}{3}$	
	(b)	$t^2-2t-8=0$ $(t-4)(t+2)=0$ $t=4$ or -2	

	(c)	$s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ $t=0, s=0$ $t=4, s=-13\frac{1}{3}$ $t=6, s=-6$ Total distance travelled $=13\frac{1}{3}+7\frac{1}{3}=20\frac{2}{3}$ m

-End of Paper-