



RAFFLES INSTITUTION
2018 Year 6 Preliminary Examination

MATHEMATICS

8865/01

Higher 1

12 September 2018

Total Marks: 100

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **7** printed pages.

[Turn over

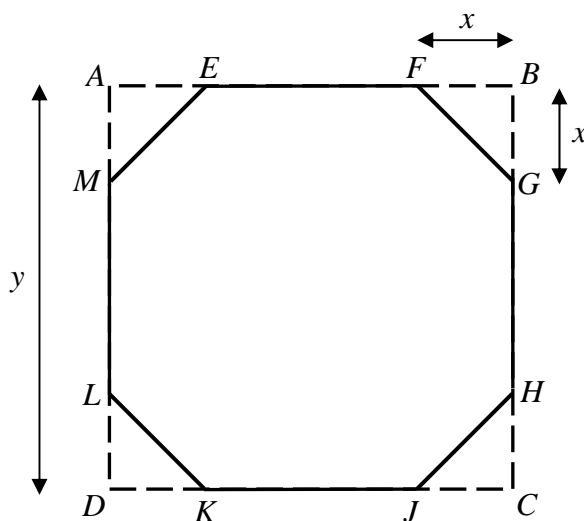
Section A: Pure Mathematics [40 marks]

1 Differentiate

(a) $-\frac{1}{4\sqrt{1+5x^2}}$. [2]

(b) $\ln\left(\frac{1}{\sqrt[3]{2-3x^2}}\right)$. [2]

2



The diagram shows a piece of paper $ABCD$ in the shape of a square of side y cm. An isosceles triangle with equal side x cm is removed from each corner of $ABCD$. The perimeter of the remaining shape $EFGHJKLM$ is 80 cm. Show that the area, A cm², of $EFGHJKLM$ is given by

$$A = 4 \left[100 + 10(2 - \sqrt{2})x + (1 - \sqrt{2})x^2 \right]. \quad [3]$$

Without using a calculator, find the maximum value of A as x varies, justifying that this value is a maximum. [3]

3 (a) Find the set of values of k for which the equation $3x^2 - (k - 1)x + 3 = 0$ has real roots. [4]

(b) Sketch the graphs of $y = \ln(3x + 1)$ and $y = 2 - x$ on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes.

Hence solve the inequality $\ln(3x + 1) \leq 2 - x$. [4]

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- 4** A team of zoologists is studying the population of wild bears in a large forest. They believe that the relation between the population size P , in hundreds, and the time t , in years, is given by

$$P = k - 6e^{-\frac{1}{10}t},$$

where k is a real constant.

It is known that there are 400 wild bears in the forest initially.

- (i) Show that $k = 10$. [1]
- (ii) Find the number of wild bears in the forest at the end of 2 years. [2]
- (iii) Find the time taken for the population of wild bears to reach 800. [3]
- (vi) Sketch the graph of P against t . [2]
- (v) Explain in simple terms what will eventually happen to the population of wild bears using this model. [2]
- 5** A company manufactures a flat beverage coaster which is made in the shape of the region enclosed by the curves C_1 and C_2 with equations $y = 4x^2$ and $y = -2x^2 + 4$ respectively. The curves intersect at the points P and Q , where the x -coordinate of P is smaller than the x -coordinate of Q .

- (i) Find the exact coordinates of P and Q . [3]
- (ii) Find the exact area of the beverage coaster. [4]
- (iii) As part of its design, a triangle with vertices P , Q and the origin O , is printed on the surface of the beverage coaster. Find the perimeter of the triangle. [2]

It is known that the manufacturing cost $\$C$, for producing n beverage coasters is given by the equation

$$C = 0.02n + 3\sqrt{n}$$

- (iv) Find the least selling price for an order of 500 if the company wants to make a profit of at least 20% for the order. [3]

[Profit = selling price – manufacturing cost]

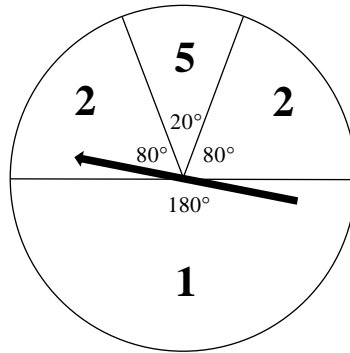
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Section B: Statistics [60 marks]

- 6** The lengths of leaves from a large tree have a normal distribution. It is found that 20% of the leaves have a length less than 14 cm and 15% of the leaves have a length greater than 17 cm.

Find the mean and standard deviation of the distribution. [4]

- 7** In a game of chance, Jonah has to throw a fair six-sided die once followed by spinning a fair spinner. The die has one face with the number 2 on it and the rest of the faces with the number 1 on them. The spinner is divided into 4 sections, each labelled with the numbers 1, 2 or 5 as shown in the diagram below.



If the die shows a 1, Jonah will spin the spinner once and his score for the game will be the number corresponding to the section which the spinner comes to rest over. If the die shows a 2, Jonah will spin the spinner twice and his score for the game will be the sum of the numbers resulted from the two spins. It is assumed that each spin is independent of the other.

The game is concluded after Jonah has obtained his score.

- (i) Find the probability that Jonah obtains a score of 2 in the game by spinning the spinner exactly once. [1]
- (ii) Find the probability that Jonah obtains a score of 2 or 6 in the game by spinning the spinner twice. [2]
- (iii) Find the probability that Jonah obtains an odd-numbered score in the game. [3]

8 A dental clinic sees n patients each day for a total of 6 days each week, and is closed on Sunday. On average, 30% of the patients are eligible for a dental subsidy after treatment, and the eligibility of a patient for dental subsidy is independent of another patient.

(i) It is known that on a randomly chosen day, there is a probability of 0.0308 that 10 patients are eligible for a dental subsidy. Find the value of n . [1]

Assume $n = 50$ for the rest of the question.

(ii) Find the probability that, on a randomly chosen day, at least 20 patients are eligible for the dental subsidy. [2]

(iii) Find the probability that, in a randomly chosen week, at least 85 and at most 95 patients are eligible for the dental subsidy. [3]

9 Clean Air Company develops and manufactures air purifiers for indoor use. To increase sales, it bought a fixed amount of advertisement time on Channel R each month. Its sales revenues, y thousand dollars in month x , are as follows.

Month x	1	2	3	4	5	6
Sales revenue y	20	23	37	29	33	38

(i) Give a sketch of the scatter diagram of the data. [2]

(ii) Suggest a possible reason why one of the observed sales revenue does not seem to follow the trend. [1]

The observed data in part (ii) is now omitted.

(iii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iv) Find the equation of the regression line of y on x in the form $y = mx + c$. [1]

(v) State, in context, the meaning of c . [1]

(vi) Use the equation of your regression line to calculate estimates of the sales revenue in month 3 and in month 9. Comment on the reliability of your estimates. [3]

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- 10** A manufacturer claims that the mean lifetime of his light bulbs is 10000 hours. To verify this claim, a random sample of 50 light bulbs is taken and their lifetimes (in hours) are summarised by

$$\sum(x - 10000) = -2018, \quad \sum(x - 10000)^2 = 1107124.$$

- (i) Find unbiased estimates of the population mean and variance. [3]
- (ii) Test, at the 4% significance level, whether the manufacturer's claim is valid. [4]

The manufacturing process is improved and the new population variance is k^2 hours². A new random sample of 50 light bulbs is chosen and the mean lifetime of this sample is 10025 hours. A test at the 4% significance level shows that there is sufficient evidence to suggest that the population mean lifetime of the light bulbs is more than 10000 hours.

- (iii) Find the range of values of k . [4]

- 11** The mass, in grams, of butter cookies and chocolate cookies baked by Heavenly Bakery have independent normal distributions. The means and standard deviations of these distributions are shown in the following table:

	Mean mass	Standard deviation
Butter cookies	15	0.4
Chocolate cookies	20	1.2

The cookies are sold by weight. Butter cookies cost \$6 per 100 gram and chocolate cookies cost \$10 per 100 gram.

- (i) Miss Lam bought 12 butter cookies as a gift for her best friend. Find, correct to the nearest integer, the least value of C such that there is a probability of at least 0.9 that these cookies weigh less than C grams. [3]
- (ii) Miss Harshini bought 12 butter cookies and 10 chocolate cookies for her family. Find the probability that Miss Harshini paid less than \$30 for these cookies. You should state the mean and variance of any distribution that you use. [4]

The waiting time, T minutes, before a customer is served at Heavenly Bakery during weekends has a mean of 30 minutes and a standard deviation of 10 minutes.

- (iii) Given that there are 50 customers who patronized Heavenly Bakery on a randomly chosen weekend, estimate the probability that the average waiting time of the 50 customers differs from the mean waiting time by at most 2 minutes. [4]

12 A team in a particular sport consists of 3 defenders, 1 centre and 3 attackers. A club has 6 defenders, 3 centres and 5 attackers to choose from to form a team

(i) Find the number of different teams that can be formed by the club. [2]

During a competition, one particular attacker was absent due to an illness. The club manager decides that one particular defender can play either as a defender or as an attacker.

(ii) Find the number of different teams that can be formed by the club now. [3]

After the competition, all the 13 players decided to watch a movie in a cinema. They are to sit randomly in a row of 13 seats. It is known that of the 6 defenders, 2 of them are sisters.

Find the probability that the

(iii) 3 centres are not all next to one another, [2]

(iv) 2 sisters are next to each other, [2]

(v) 4 attackers are all separated given that the 2 sisters are next to each other. [3]

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