H2

TAMPINES JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

MATHEMATICS 9758/01

Paper 1 Monday, 10 September 2018

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in, including the Cover Page.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

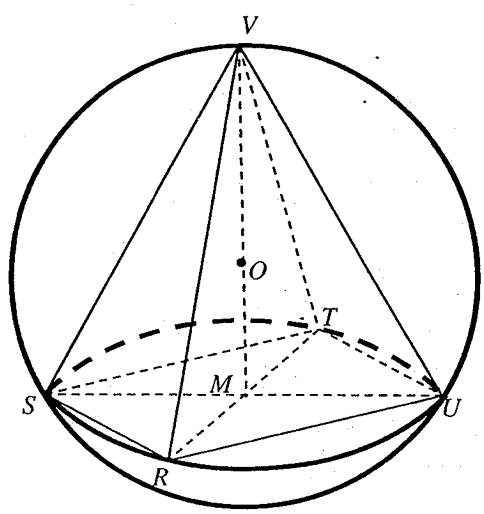
1 (i) Expand $\dfrac{1}{\sqrt[3]{8+12x}}$ in ascending powers of x up to and including the term in x^3. [3]

(ii) State the set of values of x for which the series expansion is valid. [1]

2 Without using a calculator, solve the inequality

$$\frac{3x^2+2x-2}{2x^2+3x-2} \leq 1.$$ [5]

3



A right pyramid toy is placed inside a spherical container of fixed radius r. The toy has a square base $RSTU$ and vertical height VM of length $(r+h)$ where $0<h<r$. M is the point where the diagonals SU and RT of the square meet. The vertices R, S, T, U and V of the toy just touch the interior of the container with the vertical height VM passing through the centre O of the container.

(i) Show that the length of the side of the square base $RSTU$ is $\sqrt{2(r^2-h^2)}$. [2]

(ii) Hence, find the maximum volume of the toy in terms of r. [5]

[It is given that the volume of a pyramid is $\frac{1}{3} \times \text{base area} \times \text{height}$.]

4 **(i)** It is given that

$$y=\frac{x^2-3x+18}{x+10}, \qquad x \in \mathbb{R},\ x \neq -10.$$

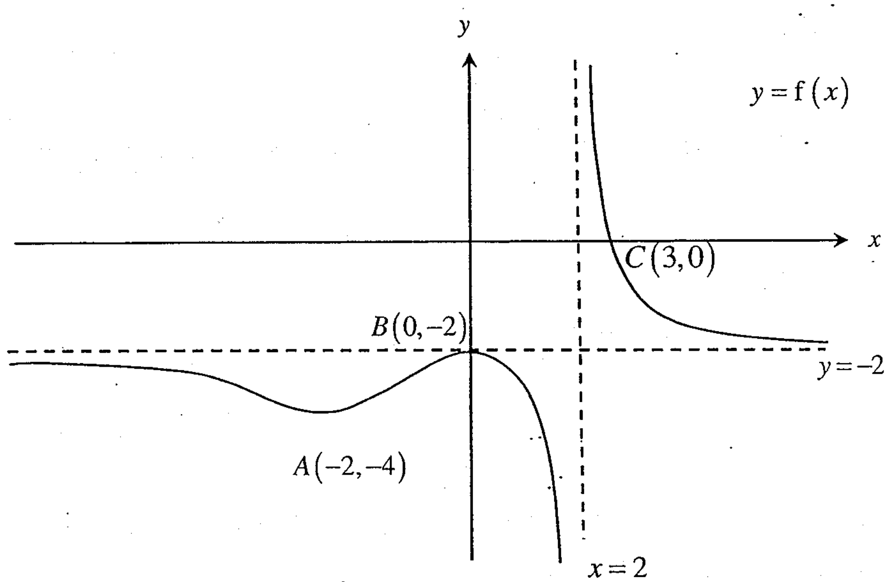
Without using a calculator, show that the range of values that y can take satisfies the inequality

$$y^2+46y-63 \geq 0.$$

Find the range of y. [5]

(ii) Hence find the exact range of values of y for $e^{2y}+46e^{y}-63 \geq 0$. [2]

5



The diagram shows the graph of $y=f(x)$. The curve has turning points at $A(-2,-4)$ and $B(0,-2)$ and crosses the x-axis at point $C(3,0)$. The equations of the asymptotes of the curve are $x=2$ and $y=-2$.

On separate diagrams, sketch the graphs of

(i) $y=f\left(\frac{1-x}{2}\right)$, [3]

(ii) $y=\frac{1}{f(x)}$, [3]

(iii) $y=f'(x)$, [3]

labelling, where applicable, the exact coordinates of the points corresponding to A, B and C and the equations of any asymptotes.

6 Referred to the origin O, points A and B have position vectors given by $\mathbf{a} = -p\mathbf{i} + 2p\mathbf{j} + 2p\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$ respectively, where $p > 0$.

(i) Given that $\mathbf{a}$ is a unit vector, find the exact value of p. [2]

(ii) Give a geometrical interpretation of $|\mathbf{a} \square \mathbf{b}|$. [1]

Point C lies on AB, between A and B, such that $AC : CB = 3 : 2$.

(iii) Find the position vector of C. [2]

(iv) Find the exact area of triangle OBC. [3]

7 **(i)** Prove that $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\operatorname{cosec} x^2 + \cot x^2\right)\right] = -2x \operatorname{cosec} x^2$. [3]

(ii) Find $\int x \cos x^2 \, \mathrm{d}x$. [2]

(iii) Hence find the exact value of $\int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}} \left[x \cos x^2 \ln\left(\operatorname{cosec} x^2 + \cot x^2\right) \right] \mathrm{d}x$. [5]

8 **Do not use a calculator in answering this question.**

The complex numbers z and w are given by $1 - \mathrm{i}$ and $-1 + \mathrm{i}\sqrt{3}$ respectively.

(i) Express each of z and w in polar form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give r and θ in exact form. [2]

(ii) Find zw and $\frac{z}{w}$ in exact polar form. [2]

(iii) Hence, by finding zw in exact cartesian form $x + \mathrm{i}y$, show that

$$\sin\left(\frac{5\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}.$$ [2]

(iv) Sketch the points A and B representing the complex numbers w and $\frac{z}{w}$ respectively on an Argand diagram. You should identify the modulus and argument of the points A and B. [2]

(v) Use part **(iii)** to find the exact area of triangle OAB, where O is the origin. [2]

9 **(i)** Show that $\dfrac{r}{(r+2)(r+3)(r+4)}$ can be expressed as $\dfrac{A}{r+2}+\dfrac{B}{r+3}+\dfrac{C}{r+4}$, where A, B and C are constants to be determined. [2]

The sum $\displaystyle\sum_{r=1}^{n}\frac{r}{(r+2)(r+3)(r+4)}$, is denoted by S_n.

(ii) Find an expression for S_n in terms of n. (There is no need to express your answer as a single algebraic fraction.) [3]

(iii) Explain why S_∞ is a convergent series, and write down its value. [2]

(iv) Find the smallest value of n for which $S_\infty - S_n < 0.05$. [2]

(v) Using results in parts **(ii)** and **(iii)**, show that

$$\sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\frac{3}{20}.$$ [4]

10 On 1 January 2018, Amy deposits \$200 into a bank account. On the first day of each subsequent month, she deposits \$20 more than in the previous month. Thus on 1 February, she deposits \$220 into the account and on 1 March, she deposits \$240 into the account, and so on. The account pays no interest.

(i) On what date will the value of Amy's account first exceed \$6000? [5]

On 1 January 2018, Benjamin deposits \$200 into a savings account which pays compound interest at a rate of 0.4% per month on the last day of each month. He puts a further \$200 into the account on the first day of each subsequent month.

(ii) Show that the value of Benjamin's savings account, in dollars, on the last day of the n^{th} month is given by

$$50200(1.004^n - 1).$$ [2]

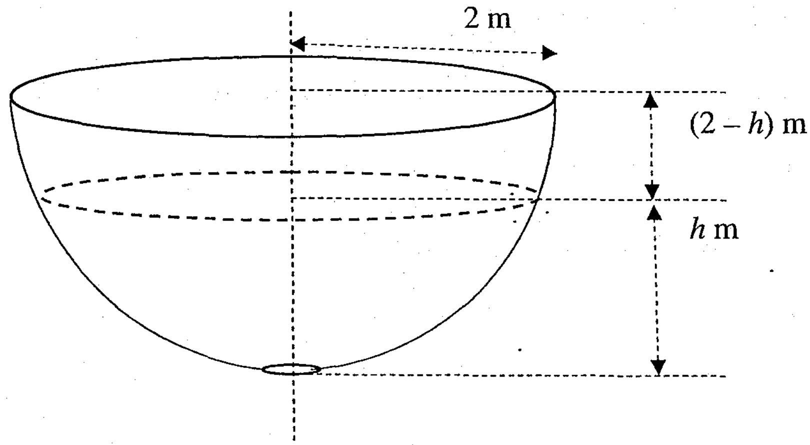
(iii) After how many complete months will the value of Benjamin's savings account first exceed \$6000? [3]

(iv) Benjamin would like the value of his savings account to be \$6000 on 2 December 2019. What interest rate per month, applied from January 2018, would achieve this? [3]

11 **Torricelli's Law** states that water will flow from an opening at the bottom of a tank with the same speed that it would attain falling from the surface of the water to the opening. One of the forms of Torricelli's Law is

$$A\frac{\mathrm{d}h}{\mathrm{d}t}=-k\sqrt{2gh}$$

where h is the height of the water in the tank, k is the area of the opening at the bottom of the tank, A is the horizontal cross-sectional area at height h, and g is the acceleration due to gravity.



A hemispherical water tank has a radius of 2 m. When the tank is full, a circular valve with a radius of 1 cm is opened at the bottom, as shown in the diagram.

Take $g = 10\ \mathrm{m/s^2}$.

(i) By expressing A in terms of h and finding the value of k, show that the rate of change of h metres, with respect to time, t seconds, satisfies the differential equation

$$(4h-h^2)\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{10000}\sqrt{20h}.$$ [4]

(ii) By solving the differential equation in part **(i)**, show that

$$t=\frac{400\sqrt{5}}{3}(ah^2\sqrt{h}-bh\sqrt{h}+28\sqrt{2}),$$

where a and b are constants to be determined. [7]

(iii) How long will it take for the tank to drain completely? Give your answer to the nearest second. [1]

(iv) Sketch the graph of h against t. [2]

End of Paper

H2

TAMPINES JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

MATHEMATICS 9758/02

Paper 2

Wednesday, 12 September 2018

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

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Section A: Pure Mathematics [40 marks]

1 Functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{-x^2+9}{x^2-4}, \qquad x \in \mathbb{R}, x \leq 0, x \neq -2,$$

$$\mathrm{g}: x \mapsto -\frac{3x+1}{x+1}, \qquad x \in \mathbb{R}, x \neq -1.$$

(i) Find $\mathrm{f}^{-1}(x)$. [2]

(ii) Explain why the composite function fg does not exist. [2]

(iii) Find $\mathrm{gf}(x)$ and the range of gf. [3]

2 A curve C has parametric equations

$$x = \cos 2t, \qquad y = \frac{1}{2}\sin 4t,$$

where $0 \leq t \leq \pi$.

(i) Find the equation of the normal to C at the point P with parameter p. [3]

(ii) The normal to C at the point where $t = \frac{\pi}{3}$ meets the curve again. Find the coordinates of the point of intersection. [3]

(iii) Find the cartesian equation of C. [2]

3 The line l has equation $\frac{x+9}{3} = \frac{y+5}{1}$, $z = 1$, and the plane p_1 has equation $-x+2y+z=6$.

(i) Find the acute angle between l and p_1. [3]

Referred to the origin O, the point A has position vector $2\mathbf{i}+\mathbf{j}-6\mathbf{k}$.

(ii) Find the position vector of F, the foot of the perpendicular from A to p_1. [3]

(iii) Find the perpendicular distance from A to p_1, in exact form. [2]

(iv) Given that l is the line of intersection of the planes p_2 and p_3 with equations $x-3y-z=a$ and $x+by+z=7$ respectively, where a and b are real constants, find the values of a and b. [4]

4 (i) Express $\dfrac{18x}{\cos 9x^2 \cos\dfrac{\pi}{6}+\sin 9x^2 \sin\dfrac{\pi}{6}}$ in the form $18x\sec\left(ax^2+b\right)$, where a and b are constants to be determined. Show your workings clearly. [2]

(ii) It is given that $\mathrm{f}(x)=\dfrac{18x}{\cos 9x^2 \cos\dfrac{\pi}{6}+\sin 9x^2 \sin\dfrac{\pi}{6}}$. Use the substitution $\theta=9x^2-\dfrac{\pi}{6}$ to find the exact area bounded by the curve $y=\mathrm{f}(x)$, the x-axis and lines $x=\sqrt{\dfrac{\pi}{54}}$ and $x=\sqrt{\dfrac{\pi}{18}}$. [7]

(iii) The region bounded by the curve $y=\mathrm{f}(x)$, the x-axis and lines $x=-\sqrt{\dfrac{\pi}{18}}$ and $x=\sqrt{\dfrac{\pi}{18}}$ is rotated through 2π radians about the x-axis. Show that $\mathrm{f}(-x)=-\mathrm{f}(x)$. Hence, or otherwise, find the volume of the solid obtained, giving your answer correct to 2 decimal places. [4]

Section B: Probability and Statistics [60 marks]

5 An unbiased die has three faces painted red, two faces painted green and one face painted blue. A red face has a score of 1 point, a green face has a score of 2 points and a blue face has a score of 3 points.

Two such dice are thrown and the sum of their scores is denoted by X.

(i) Show that $\mathrm{P}(X=4)=\dfrac{5}{18}$ and find the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and show that $\mathrm{Var}(X)=\dfrac{10}{9}$. [2]

Suppose now a red face has a score of 3 points, a green face has a score of 2 points and a blue face has a score of 1 point.

(iii) Deduce the expectation and variance of the sum of scores obtained from a throw of two such dice. [2]

6 Find the number of ways in which the letters of the word DIGITISE can be arranged if

(i) there are no restrictions, [1]

(ii) G and S must not be next to each other, [2]

(iii) consonants (D, G, T, S) and vowels (I, E) must alternate, [3]

(iv) between any two Is there must be at least 2 other letters. [3]

7 A food processor produces large batches of jars of jam. The production manager wishes to take a random sample of the jars of jam produced in one day, for quality control purposes. He wishes to check that the mean mass of the jars of jam is 502 grams, as stated on the jars.

(i) State what it means for a sample to be random in this context. [1]

The masses, x grams, of a random sample of 50 jars of jam are summarised as follows.

$$n=50 \qquad \sum(x-502)=-81 \qquad \sum(x-502)^2=1138$$

(ii) Calculate unbiased estimates of the population mean and variance of the mass of jars of jam. [2]

(iii) Test, at the 1% level of significance, the claim that the mean mass of jars of jam is 502 grams. You should state your hypothesis and define any symbols you use. [5]

(iv) Explain why there is no need for the production manager to know anything about the population distribution of the masses of the jars of jam. [2]

8 (a) Draw separate scatter diagrams, each with 6 points, all in the first quadrant, which represents the situation where the product moment correlation coefficient between x and y is

(i) between -0.9 and -0.5,

(ii) 1. [2]

(b) The age of students in years (x) and the median amount a month, in dollars (y), spent on tuition of a random sample of students are given in the table.

Age, x	6	7	9	11	13	15	18
Amount, y	155	170	211	230	248	260	265

(i) Draw a scatter diagram for these values, labelling the axes. [1]

It is thought that the median amount a month spent on tuition can be modelled by one of the formulae

$$y=a+bx \qquad \text{or} \qquad y=c+d\ln x$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

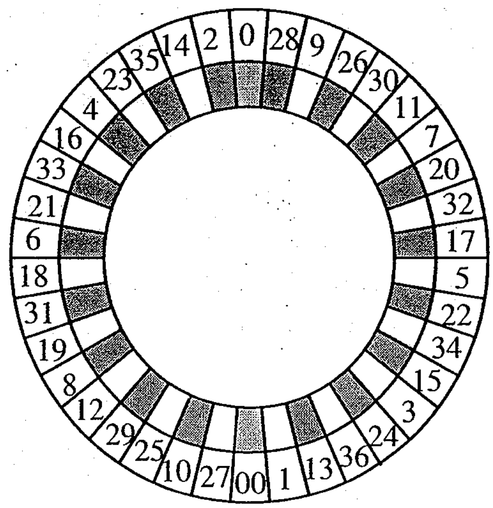
(A) x and y,

(B) $\ln x$ and y. [2]

(iii) Explain which of $y=a+bx$ and $y=c+d\ln x$ is the better model and find the equation of a suitable regression line for this model. [3]

(iv) Use the equation of your regression line to estimate the median amount a month spent on tuition by a student who is 16 years old. Comment on the reliability of your estimate. [2]

9



An American roulette wheel has 38 pockets numbered 00, 0, 1, 2, 3, …, 35 and 36. The 0 and 00 pockets are coloured "Green". 18 pockets are coloured "Black". The remaining pockets are coloured "Red". A typical round of roulette would involve spinning the roulette wheel in one direction and spinning a ball in the other direction, and the number of the pocket the ball eventually lands in would be the winning number and colour.

A player decides to play roulette for 10 rounds, and only makes the bet that "Black" will win for each round he plays in.

(i) State, in context, two assumptions needed for the number of wins with "Black" in 10 rounds to be well modelled by a binomial distribution. [2]

Assume now that the number of wins with "Black" in 10 rounds has a binomial distribution.

(ii) Find the expected number of rounds of roulette the player will win with "Black". [1]

(iii) Find the probability that the player will win at least 6 rounds of roulette. [2]

(iv) Find the probability that the player will win the fifth round of roulette given that the player has won two of the previous rounds. [2]

The player decides to visit the casino n times, playing 10 rounds of roulette each time. The player considers the casino visit "good" if he wins with "Black" for at least 6 rounds of roulette.

(v) Find the most probable number of "good" casino visits when $n = 20$. [3]

(vi) The player wants the probability that at least 5 casino visits are "good" in n casino visits to be more than 0.5. Find the range of values that n can take. [2]

10 **(a)** A and B are independent random variables with the distributions $N(25, 20)$ and $N(\mu, \sigma^2)$ respectively. It is known that $P(B<12) = P(B>19)$ and $P(A > B) = 0.68$. State the value of μ and calculate the value of σ. [4]

(b) There are two vets Jerry and Mary who attend to customers with sick pets in a small vet clinic. The time taken for Jerry to attend to a customer follows a normal distribution with mean 10.1 minutes and standard deviation 0.8 minutes and the time taken for Mary to attend to a customer follows an independent normal distribution with mean 10.3 minutes and standard deviation 0.75 minutes.

(i) Find the probability that among three randomly chosen customers attended to by Jerry, one took less than 10 minutes while the other two each took more than 10.5 minutes. [2]

(ii) Find the probability that the total time Jerry took to attend to two randomly chosen customers is less than twice the time Mary took to attend to one randomly chosen customer by at least 3 minutes. State the distribution you use and its parameters. [3]

(iii) Jerry and Mary each attended to two randomly chosen customers. Find the probability that the difference in the total time taken by Jerry and Mary to attend to their two customers is more than 2 minutes. State the parameters of any distribution you use. [3]

End of Paper

2018 JC2 Preliminary Examination
H2 Mathematics Paper 1
Solution

1(i)

$$\frac{1}{\sqrt[3]{8+12x}}$$

$$=(8+12x)^{-\frac{1}{3}}$$

$$=(8)^{-\frac{1}{3}}\left(1+\frac{3}{2}x\right)^{-\frac{1}{3}}$$

$$=\frac{1}{2}\left(1+\left(-\frac{1}{3}\right)\left(\frac{3}{2}x\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{3}{2}x\right)^2+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(\frac{3}{2}x\right)^3+\cdots\right)$$

$$=\frac{1}{2}\left(1-\frac{1}{2}x+\frac{1}{2}x^2-\frac{7}{12}x^3+\cdots\right)$$

$$\approx\frac{1}{2}-\frac{1}{4}x+\frac{1}{4}x^2-\frac{7}{24}x^3$$

(ii)

$$\left\{x\in\mathbb{R}:-\frac{2}{3}<x<\frac{2}{3}\right\}$$

2

$$\frac{3x^2+2x-2}{2x^2+3x-2}\le 1$$

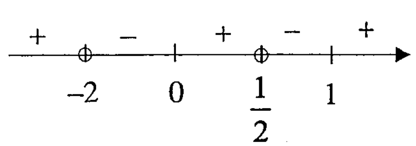
$$\frac{3x^2+2x-2}{2x^2+3x-2}-1\le 0$$

$$\frac{3x^2+2x-2-\left(2x^2+3x-2\right)}{2x^2+3x-2}\le 0$$

$$\frac{3x^2+2x-2-2x^2-3x+2}{(2x-1)(x+2)}\le 0$$

$$\frac{x^2-x}{(2x-1)(x+2)}\le 0$$

$$\frac{x(x-1)}{(2x-1)(x+2)}\le 0$$



$-2<x\le 0$ or $\frac{1}{2}<x\le 1$

3(i)	$MU=\sqrt{r^2-h^2}$ $SU=2\sqrt{r^2-h^2}$ Length of square base $=\sin\left(\frac{\pi}{4}\right)2\sqrt{r^2-h^2}$ $=\frac{\sqrt{2}}{2}2\sqrt{r^2-h^2}$ $=\sqrt{2\left(r^2-h^2\right)}$ (shown)	
(ii)	Volume of toy, $V=\frac{2}{3}\left(r^2-h^2\right)\left(r+h\right)$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{2}{3}\left[\left(r^2-h^2\right)+\left(r+h\right)\left(-2h\right)\right]$ $=\frac{2}{3}\left(r^2-2rh-3h^2\right)$ For stationary point, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$ $\frac{2}{3}\left(r^2-2rh-3h^2\right)=0$ $\left(r+h\right)\left(r-3h\right)=0$ $h=-r$ (reject since $h>0$) or $h=\frac{1}{3}r$ $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=\frac{2}{3}(-2r-6h)$ When $h=\frac{1}{3}r$, $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-\frac{8}{3}r<0$ $h=\frac{1}{3}r$ gives maximum V. Maximum volume of toy, $V=\frac{2}{3}\left(r^2-\left(\frac{r}{3}\right)^2\right)\left(r+\left(\frac{r}{3}\right)\right)$ $=\frac{64r^3}{81}$ units^3	

4(i)

$$y=\frac{x^2-3x+18}{x+10}$$

$$(x+10)y=x^2-3x+18$$

$$0=x^2-(3+y)x+(18-10y)$$

For quadratic equations to have real roots,
discriminant ≥ 0

$$\left[-(3+y)\right]^2-4(1)(18-10y)\geq 0$$

$$9+6y+y^2-72+40y\geq 0$$

$$y^2+46y-63\geq 0$$

Consider $y^2+46y-63=0$

$$y=\frac{-46\pm\sqrt{46^2-4(-63)}}{2}$$

$$=-23\pm 4\sqrt{37}$$

$+$ | $-$ | $+$ (sign diagram with critical values $-23-4\sqrt{37}$ and $-23+4\sqrt{37}$)

$y\leq -23-4\sqrt{37}$ or $y\geq -23+4\sqrt{37}$

Set of values is $\left\{y\in □ : y\leq -23-4\sqrt{37} \text{ or } y\geq -23+4\sqrt{37}\right\}$

(ii)

$y^2+46y-63\geq 0$

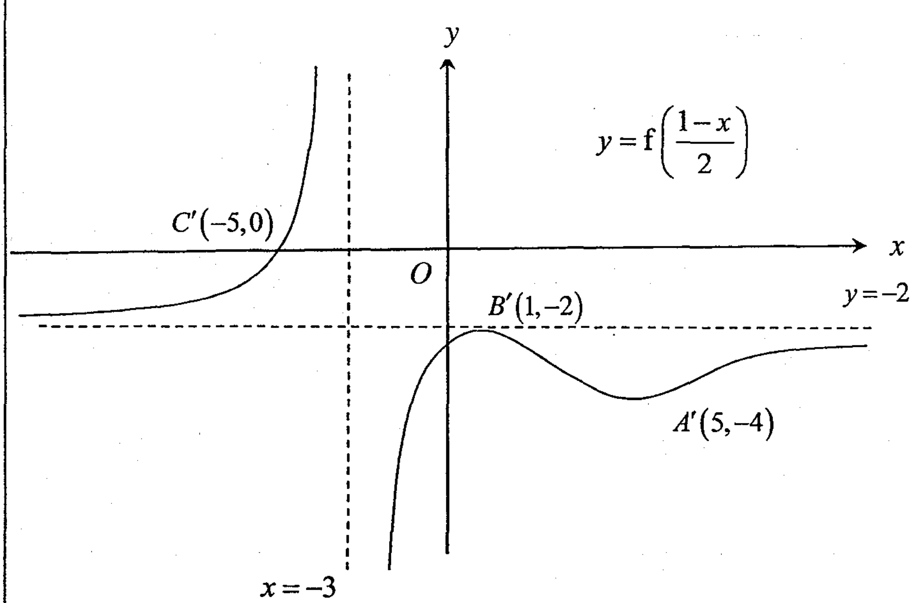
Replace y with e^y

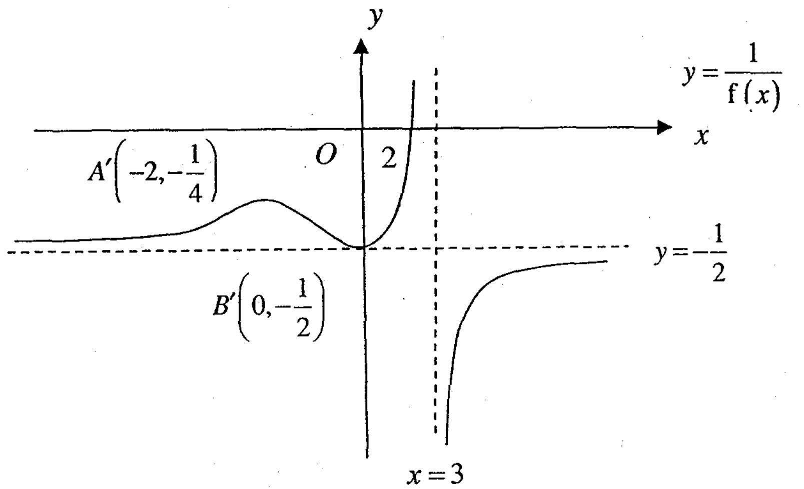
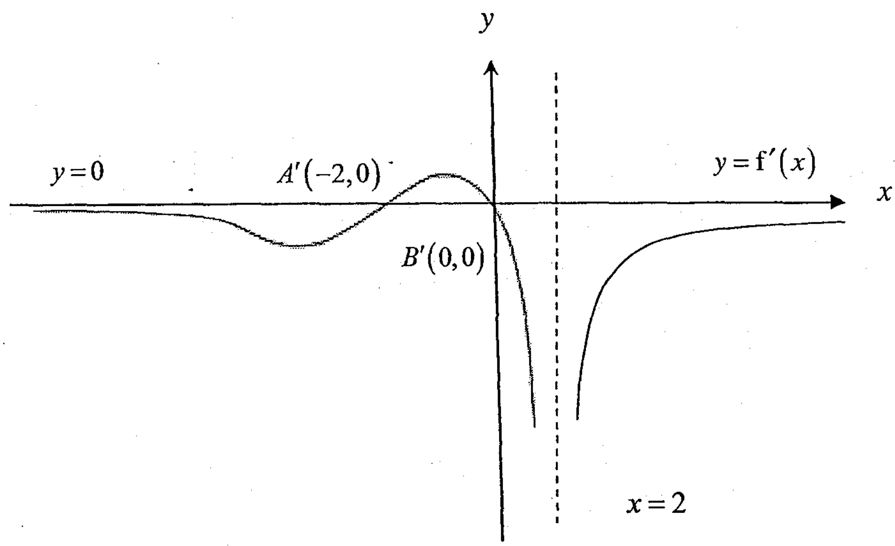
$e^{2y}+46e^y-63\geq 0$

$e^y\leq -23-4\sqrt{37}$ or $e^y\geq -23+4\sqrt{37}$

$(\text{rej, } e^y>0)$ $\quad y\geq \ln\left(-23+4\sqrt{37}\right)$

5(i)

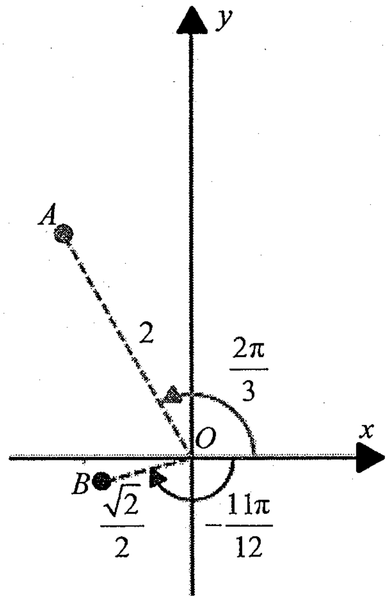


(ii)		
(iii)		
6(i)	Since **a** is a unit vector, $\lvert\mathbf{a}\rvert = 1$. $\sqrt{\left(p^2+4p^2+4p^2\right)}=1$ $\Rightarrow \lvert 3p\rvert = 1$ (Accept: $9p^2=1$) $\Rightarrow p=\frac{1}{3}$ (since $p>0$)	
(ii)	It is the length of projection of OB along OA.	
(iii)	By ratio theorem, $\overrightarrow{OC}=\frac{2\overrightarrow{OA}+3\overrightarrow{OB}}{5}=\frac{1}{5}\left[2\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix}+3\begin{pmatrix}4\\3\\0\end{pmatrix}\right]$	

	$=\frac{1}{15}\begin{pmatrix}34\\31\\4\end{pmatrix}$	
(iv)	Area of triangle *OBC* $=\frac{1}{2}\left\|\begin{pmatrix}4\\3\\0\end{pmatrix}\times\frac{1}{15}\begin{pmatrix}34\\31\\4\end{pmatrix}\right\|$ $=\frac{1}{2}\frac{1}{15}\left\|\begin{pmatrix}12\\-16\\22\end{pmatrix}\right\|$ $=\frac{1}{15}\left\|\begin{pmatrix}6\\-8\\11\end{pmatrix}\right\|$ $=\frac{\sqrt{221}}{15}$	
7(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\operatorname{cosec} x^2+\cot x^2\right)\right]$ $=\frac{2x\left(-\operatorname{cosec} x^2\cot x^2\right)+2x\left(-\operatorname{cosec}^2 x^2\right)}{\operatorname{cosec} x^2+\cot x^2}$ $=-2x\operatorname{cosec} x^2$	
(ii)	$\int x\cos x^2\,\mathrm{d}x$ $=\frac{1}{2}\int 2x\cos x^2\mathrm{d}x$ $=\frac{1}{2}\sin x^2+C$	
(iii)	$\int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}}\left[\ln\left(\operatorname{cosec} x^2+\cot x^2\right)\cdot x\cos x^2\right]\mathrm{d}x$ $=\left[\ln\left(\operatorname{cosec} x^2+\cot x^2\right)\left(\frac{1}{2}\sin x^2\right)\right]_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}}-\int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}}\left(\frac{1}{2}\sin x^2\right)\cdot\left(-2x\operatorname{cosec} x^2\right)\mathrm{d}x$ $=\ln\left(\operatorname{cosec}\frac{\pi}{2}+\cot\frac{\pi}{2}\right)\left(\frac{1}{2}\sin\frac{\pi}{2}\right)-\ln\left(\operatorname{cosec}\frac{\pi}{6}+\cot\frac{\pi}{6}\right)\left(\frac{1}{2}\sin\frac{\pi}{6}\right)+\int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}}x\,\mathrm{d}x$ $=\ln\left(\frac{1}{1}+\frac{0}{1}\right)\left(\frac{1}{2}(1)\right)-\ln\left(\frac{1}{1/2}+\frac{\sqrt{3}/2}{1/2}\right)\left(\frac{1}{2}\left(\frac{1}{2}\right)\right)+\left[\frac{x^2}{2}\right]_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{2}}}$	

	$=-\frac{1}{4}\ln\left(2+\sqrt{3}\right)+\frac{\pi}{4}-\frac{\pi}{12}$ $=\frac{\pi}{6}-\frac{1}{4}\ln\left(2+\sqrt{3}\right)$	
8(i)	$z=1-\mathrm{i}=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+\mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right]$ $w=-1+\mathrm{i}\sqrt{3}=2\left[\cos\left(\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(\frac{2\pi}{3}\right)\right]$	
(ii)	$zw=2\sqrt{2}\left[\cos\left(-\frac{\pi}{4}+\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{4}+\frac{2\pi}{3}\right)\right]$ $=2\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right)+\mathrm{i}\sin\left(\frac{5\pi}{12}\right)\right]$ $\frac{z}{w}=\frac{\sqrt{2}}{2}\left[\cos\left(-\frac{\pi}{4}-\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{\pi}{4}-\frac{2\pi}{3}\right)\right]$ $=\frac{\sqrt{2}}{2}\left[\cos\left(-\frac{11\pi}{12}\right)+\mathrm{i}\sin\left(-\frac{11\pi}{12}\right)\right]$	
(iii)	$zw=(1-\mathrm{i})\left(-1+\mathrm{i}\sqrt{3}\right)=\left(-1+\sqrt{3}\right)+\mathrm{i}\left(1+\sqrt{3}\right)$ $zw=\left(-1+\sqrt{3}\right)+\mathrm{i}\left(1+\sqrt{3}\right)$ $=2\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right)+\mathrm{i}\sin\left(\frac{5\pi}{12}\right)\right]$ $=2\sqrt{2}\cos\left(\frac{5\pi}{12}\right)+\mathrm{i}2\sqrt{2}\sin\left(\frac{5\pi}{12}\right)$ Equating imaginary parts, $1+\sqrt{3}=2\sqrt{2}\sin\left(\frac{5\pi}{12}\right)$ $\Rightarrow\sin\left(\frac{5\pi}{12}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}}$. (shown)	

(iv)



(v)

$$\angle AOB = 2\pi - \frac{2\pi}{3} - \frac{11\pi}{12}$$
$$= \frac{5\pi}{12}$$

area of triangle $OAB = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$$= \frac{1}{2}(2)\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{5\pi}{12}\right)$$
$$= \frac{\sqrt{2}}{2}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)$$
$$= \frac{1+\sqrt{3}}{4} \text{ units}^2$$

9(i)

$$\frac{r}{(r+2)(r+3)(r+4)} = \frac{A}{r+2} + \frac{B}{r+3} + \frac{C}{r+4}$$
$$r = A(r+3)(r+4) + B(r+2)(r+4) + C(r+2)(r+3)$$

Sub $r=-2$: $A=-1$

Sub $r=-3$: $B=3$

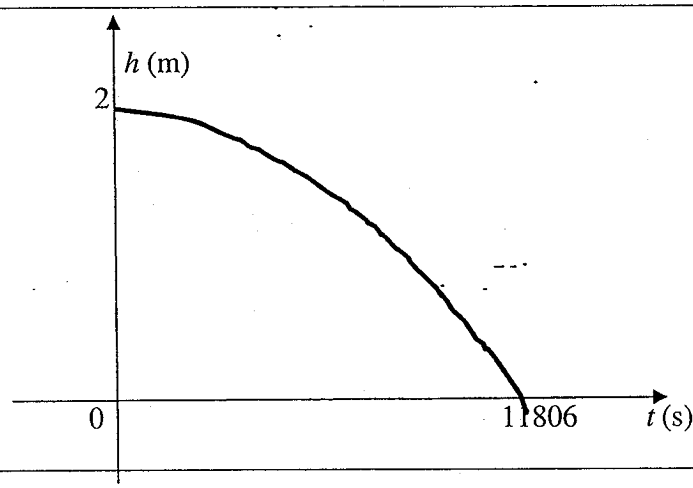
Sub $r=-4$: $C=-2$

$$\frac{r}{(r+2)(r+3)(r+4)} = -\frac{1}{r+2} + \frac{3}{r+3} - \frac{2}{r+4}$$

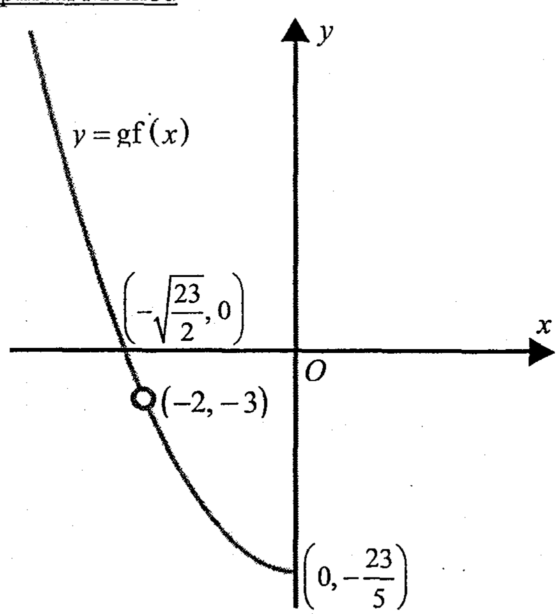
(ii)	$S_n = \sum_{r=1}^{n} \frac{r}{(r+2)(r+3)(r+4)}$ $= \sum_{r=1}^{n} \left[-\frac{1}{r+2} + \frac{3}{r+3} - \frac{2}{r+4} \right]$ $= [-\frac{1}{3} + \frac{3}{4} - \frac{2}{5}$ $-\frac{1}{4} + \frac{3}{5} - \frac{2}{6}$ $-\frac{1}{5} + \frac{3}{6} - \frac{2}{7}$ . . $-\frac{1}{n} + \frac{3}{n+1} - \frac{2}{n+2}$ $-\frac{1}{n+1} + \frac{3}{n+2} - \frac{2}{n+3}$ $-\frac{1}{n+2} + \frac{3}{n+3} - \frac{2}{n+4}]$ $S_n = \frac{1}{6} + \frac{1}{n+3} - \frac{2}{n+4}$	
(iii)	As $n \to \infty, \frac{1}{n+3} \to 0$ and $\frac{2}{n+4} \to 0$ $\therefore S_\infty$ is a convergent series and $S_\infty = \frac{1}{6}$	
(iv)	$S_\infty - S_n < 0.05$ $\frac{1}{6} - \left(\frac{1}{6} + \frac{1}{n+3} - \frac{2}{n+4} \right) < 0.05$ $-\frac{1}{n+3} + \frac{2}{n+4} < 0.05$ $\frac{n+2}{(n+3)(n+4)} < 0.05$ Since $n \in \mathbb{Z}^+$, Using G.C, Smallest value of n =15.	

(v)	$\sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\sum_{r=1}^{\infty}\frac{r+1}{(r+3)(r+4)(r+5)}$ $\left(\because (r+5)^2>(r+3)(r+4)\right)$ $\sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\sum_{r=2}^{\infty}\frac{r}{(r+2)(r+3)(r+4)}$ $\sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\sum_{r=1}^{\infty}\frac{r}{(r+2)(r+3)(r+4)}-\frac{1}{3(4)(5)}$ $\sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\frac{1}{6}-\frac{1}{60}$ $\therefore \sum_{r=1}^{\infty}\frac{r+1}{(r+5)^3}<\frac{3}{20}$ (shown)	
10 (i)	AP $a=200$ and $d=20$ $S_n>6000$ $\frac{n}{2}[2(200)+(n-1)20]>6000$ $200n+10n^2-10n>6000$ $n^2+19n-600>0$ $(n-16.77)(n+35.77)>0$ $n<-35.77$ (reject, n is positive integer) or $n>16.77$ Least $n=17$ Amy's account first exceeds \$6000 on 1 May 2019.	
(ii)	Amount at the end of first month = 200 (1.004) Amount at the end of second month = (200 (1.004) + 200) 1.004 $=200(1.004^2)+200(1.004)$ Amount at the end of n months $=200(1.004^n+1.004^{n-1}+...+1.004^2+1.004)$ $=200(1.004)\frac{1.004^n-1}{1.004-1}$ $=50200(1.004^n-1)$	
(iii)	$50200(1.004^n-1)>6000$ $1.004^n-1>\frac{30}{251}$ $n\ln(1.004)>\ln\left(\frac{281}{251}\right)$ $n>28.282$ Benjamin's account will exceed \$6000 after 29 months.	

(iv)	At the end of Nov 2019, $n = 23$ $\frac{200r(r^{23}-1)}{r-1} + 200 > 6000$ $\frac{r(r^{23}-1)}{r-1} > 29$ Using GC, $r > 1.01885$ Interest rate is 1.89% per month.	
11 (i)	Let r metre be the radius of the water surface when the height of water is h metre. By Pythagoras Theorem, $(2-h)^2 + r^2 = 2^2$ $4 - 4h + h^2 + r^2 = 4$ $r^2 = 4h - h^2$ $A = \pi r^2$ $= \pi(4h - h^2)$ $k = \pi(0.01)^2 = \frac{\pi}{10000}$ Hence, $A\frac{dh}{dt} = -k\sqrt{2gh}$ where $g = 10\text{m/s}^2$ $\Rightarrow \pi(4h-h^2)\frac{dh}{dt} = -\frac{1}{10000}\pi\sqrt{20h}$ $\Rightarrow (4h-h^2)\frac{dh}{dt} = -\frac{1}{10000}\sqrt{20h}$	
(ii)	$(4h-h^2)\frac{dh}{dt} = -\frac{1}{10000}\sqrt{20h}$ $\int \frac{(4h-h^2)}{\sqrt{h}}\,dh = \int -\frac{\sqrt{20}}{10000}\,dt$ $\Rightarrow \int 4h^{\frac{1}{2}} - h^{\frac{3}{2}}\,dh = \int -\frac{2\sqrt{5}}{10000}\,dt$ $\Rightarrow \frac{8}{3}h^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}} = -\frac{\sqrt{5}}{5000}t + C$ When $t = 0$, $h = 2$, $\frac{8}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}} = C$ $C = \frac{16\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{56\sqrt{2}}{15}$ $\therefore \frac{8}{3}h^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}} = -\frac{\sqrt{5}}{5000}t + \frac{56\sqrt{2}}{15}$ Hence,	

	$t=\left(\frac{8}{3}h^{\frac{3}{2}}-\frac{2}{5}h^{\frac{5}{2}}-\frac{56\sqrt{2}}{15}\right)\left(\frac{-5000}{\sqrt{5}}\right)$ $=\frac{5000\sqrt{5}}{5(15)}\left(-40h^{\frac{3}{2}}+6h^{\frac{5}{2}}+56\sqrt{2}\right)$ $=\frac{400\sqrt{5}}{3}(3h^{2}\sqrt{h}-20h\sqrt{h}+28\sqrt{2})$ $\therefore\ a=3,\ b=20$	
(iii)	When $h=0$, $t=\frac{400\sqrt{5}}{3}(28\sqrt{2})=11805.8366\approx 11806$ (nearest second)	
(iv)		

2018 JC2 Preliminary Examination
H2 Mathematics Paper 2
Solution

1(i)	Let $y=\mathrm{f}(x)=\dfrac{-x^2+9}{x^2-4}$ Then $y(x^2-4)=-x^2+9$ $\Rightarrow yx^2+x^2=4y+9$ $\Rightarrow x^2(y+1)=4y+9$ $\Rightarrow x^2=\dfrac{4y+9}{y+1}$ $\Rightarrow x=-\sqrt{\dfrac{4y+9}{y+1}}$ or $x=\sqrt{\dfrac{4y+9}{y+1}}$ (reject, $x\le 0$). Hence $\mathrm{f}^{-1}(x)=-\sqrt{\dfrac{4x+9}{x+1}}$	
(ii)	$\mathrm{R_g}=(-\infty,-3)\cup(-3,\infty)$, $\mathrm{D_f}=(-\infty,-2)\cup(-2,0]$ Since $\mathrm{R_g}\not\subseteq\mathrm{D_f}$, fg does not exist.	
(iii)	$\mathrm{gf}(x)=-\dfrac{3\left(\dfrac{-x^2+9}{x^2-4}\right)+1}{\left(\dfrac{-x^2+9}{x^2-4}\right)+1}=-\dfrac{3(-x^2+9)+(x^2-4)}{(-x^2+9)+(x^2-4)}=\dfrac{2x^2-23}{5}$ <u>Method 1: Graphical Method</u>  From the graph of $y=\mathrm{gf}(x)$, the range of gf can be seen to be	

	$\left[-\frac{23}{5},-3\right)\cup(-3,\infty)$. <u>Method 2: Analytical Method</u> $(-\infty,-2)\cup(-2,0]\xrightarrow{f}\left(-\infty,-\frac{9}{4}\right]\cup(-1,\infty)\xrightarrow{g}\left[-\frac{23}{5},-3\right)\cup(-3,\infty)$ Hence, the range of gf is $\left[-\frac{23}{5},-3\right)\cup(-3,\infty)$.	
2(i)	$x=\cos 2t$ $\qquad y=\frac{1}{2}\sin 4t$ $\frac{dx}{dt}=-2\sin 2t$ $\qquad \frac{dy}{dt}=\frac{1}{2}(4\cos 4t)=2\cos 4t$ $\frac{dy}{dx}=\frac{2\cos 4t}{-2\sin 2t}=-\frac{\cos 4t}{\sin 2t}$ At $t=p$, gradient of normal $=\frac{\sin 2p}{\cos 4p}$ Equation of normal at P is $y-\frac{1}{2}\sin 4p=\frac{\sin 2p}{\cos 4p}(x-\cos 2p)$ $y=\frac{\sin 2p}{\cos 4p}x-\frac{\sin 2p\cos 2p}{\cos 4p}+\frac{1}{2}\sin 4p$ $y=\frac{\sin 2p}{\cos 4p}x-\frac{1}{2}\tan 4p+\frac{1}{2}\sin 4p$	
(ii)	When $t=\frac{\pi}{3}$, equation of normal is $y=-\sqrt{3}x-\frac{1}{2}\sqrt{3}+\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)$ $y=-\sqrt{3}x-\frac{3}{4}\sqrt{3}$ Since the normal meets the curve, $\frac{1}{2}\sin 4t=-\sqrt{3}\cos 2t-\frac{3}{4}\sqrt{3}$ From GC, $t=\frac{\pi}{3}$ (reject since it is the given point) or 1.74796 Point of intersection is $(-0.938, 0.325)$.	
(iii)	$x^2=\cos^2 2t$ $y=\frac{1}{2}\sin 4t$ $=\frac{1}{2}(2\sin 2t\cos 2t)$	

	$= \sin 2t \cos 2t$ $y^2 = \sin^2 2t \cos^2 2t$ $= (1-\cos^2 2t)\cos^2 2t$ $y^2 = (1-x^2)x^2$	
3(i)	$\theta = \sin^{-1}\left\lvert \frac{\begin{pmatrix}3\\1\\0\end{pmatrix} \cdot \begin{pmatrix}-1\\2\\1\end{pmatrix}}{\sqrt{10}\sqrt{6}} \right\rvert$ $= \sin^{-1}\left\lvert \frac{1}{2\sqrt{15}} \right\rvert$ $= 7.4°$ (1 d.p.) Accept: If student uses $\cos^{-1}$ and subtract their answer from $90°$.	
(ii)	$\overrightarrow{OF} = \begin{pmatrix}2-\lambda\\1+2\lambda\\-6+\lambda\end{pmatrix}$ for some $\lambda \in \mathbb{R}$. Since F lies on p_1, $-(2-\lambda)+2(1+2\lambda)+(-6+\lambda)=6$ $6\lambda = 12$ $\lambda = 2$ $\therefore \overrightarrow{OF} = \begin{pmatrix}0\\5\\-4\end{pmatrix}$	
(iii)	$\overrightarrow{AF} = \begin{pmatrix}0\\5\\-4\end{pmatrix} - \begin{pmatrix}2\\1\\-6\end{pmatrix} = \begin{pmatrix}-2\\4\\2\end{pmatrix}$ Perpendicular distance $= 2\left\lvert \begin{pmatrix}-1\\2\\1\end{pmatrix} \right\rvert$ $= 2\sqrt{6}$ units	
(iv)	Since l is perpendicular to normal vector of p_3: $\begin{pmatrix}3\\1\\0\end{pmatrix} \cdot \begin{pmatrix}1\\b\\1\end{pmatrix} = 0$ $\therefore b = -3$ Since l lies in p_2: $(-9)-3(-5)-(1) = a$ $\therefore a = 5$	

4(i)	Using MF26, $\dfrac{18x}{\cos 9x^2 \cos\frac{\pi}{6} + \sin 9x^2 \sin\frac{\pi}{6}}$ $= \dfrac{18x}{\cos\left(9x^2 - \frac{\pi}{6}\right)}$ $= 18x \sec\left(9x^2 - \frac{\pi}{6}\right)$ $a = 9, b = -\frac{\pi}{6}$ OR Using R-formula, $\dfrac{18x}{\cos 9x^2 \cos\frac{\pi}{6} + \sin 9x^2 \sin\frac{\pi}{6}}$ $= \dfrac{18x}{\frac{\sqrt{3}}{2}\cos 9x^2 + \frac{1}{2}\sin 9x^2}$ $= \dfrac{36x}{\sqrt{3+1}\cos\left(9x^2 - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)}$ $= 18x \sec\left(9x^2 - \frac{\pi}{6}\right)$	
(ii)	Area required $= \displaystyle\int_{\sqrt{\frac{\pi}{54}}}^{\sqrt{\frac{\pi}{18}}} \frac{18x}{\cos 9x^2 \cos\frac{\pi}{6} + \sin 9x^2 \sin\frac{\pi}{6}}\,dx$ $= \displaystyle\int_{\sqrt{\frac{\pi}{54}}}^{\sqrt{\frac{\pi}{18}}} 18x\sec\left(9x^2 - \frac{\pi}{6}\right)dx$ $= \displaystyle\int_0^{\frac{\pi}{3}} 6\sqrt{\theta + \frac{\pi}{6}}\sec\theta \cdot \left(\frac{1}{6(\theta+\frac{\pi}{6})^{\frac{1}{2}}}\right)d\theta$ $= \int_0^{\frac{\pi}{3}} \sec\theta\, d\theta$ $= \left[\ln\lvert\sec\theta + \tan\theta\rvert\right]_0^{\frac{\pi}{3}}$ $= \ln\left\lvert\frac{1}{\frac{1}{2}} + \sqrt{3}\right\rvert - \ln\left\lvert\frac{1}{1} + 0\right\rvert$ $= \ln(2+\sqrt{3})$ units² $\theta = 9x^2 - \frac{\pi}{6}$ $x = \dfrac{\sqrt{\theta + \frac{\pi}{6}}}{3}$ $\dfrac{dx}{d\theta} = \frac{1}{2}\left(\frac{1}{3}\right)\left(\theta + \frac{\pi}{6}\right)^{-\frac{1}{2}}(1)$ $= \dfrac{1}{6\left(\theta + \frac{\pi}{6}\right)^{\frac{1}{2}}}$ When $x = \sqrt{\frac{\pi}{18}}$, $\theta = \frac{\pi}{3}$ When $x = \sqrt{\frac{\pi}{54}}$, $\theta = 0$	

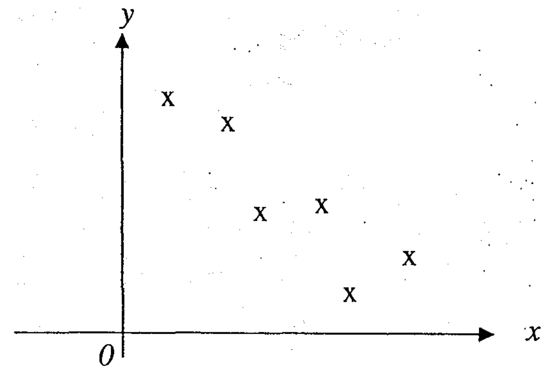
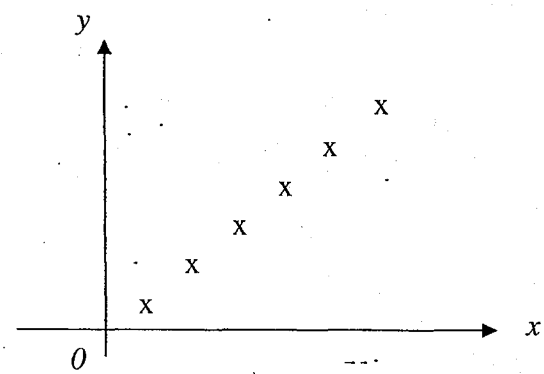
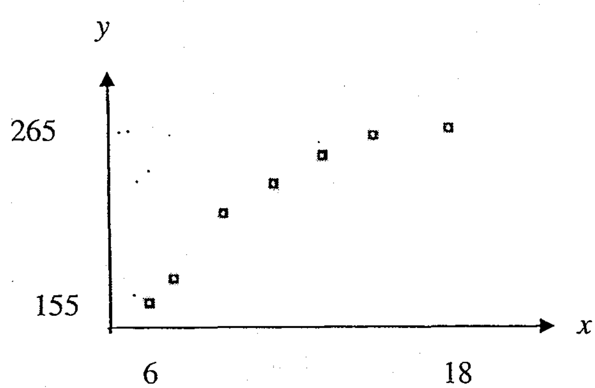
(iii)	$\mathrm{f}(-x)$ $=\dfrac{18(-x)}{\cos\left(9(-x)^2-\dfrac{\pi}{6}\right)}$ $=-\dfrac{18x}{\cos\left(9x^2-\dfrac{\pi}{6}\right)}$ $=-\mathrm{f}(x)$ (shown) Volume required $=2\pi\int_0^{\sqrt{\frac{\pi}{18}}}\left(\dfrac{18x}{\cos\left(9x^2-\dfrac{\pi}{6}\right)}\right)^2 \mathrm{d}x$ $\approx 80.16 \text{ units}^3$	
5(i)	$\mathrm{P}(X=2)=\mathrm{P}(\text{red})\times\mathrm{P}(\text{red})=\frac{3}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ $\mathrm{P}(X=3)=\mathrm{P}(\text{red})\times\mathrm{P}(\text{green})+\mathrm{P}(\text{green})\times\mathrm{P}(\text{red})$ $=\frac{3}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{3}\times2=\frac{1}{3}$ $\mathrm{P}(X=4)=\mathrm{P}(\text{red})\times\mathrm{P}(\text{blue})+\mathrm{P}(\text{blue})\times\mathrm{P}(\text{red})+\mathrm{P}(\text{green})\times\mathrm{P}(\text{green})$ $=\frac{3}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{3}{6}+\frac{2}{6}\times\frac{2}{6}=\frac{1}{2}\times\frac{1}{6}\times2+\frac{1}{3}\times\frac{1}{3}=\frac{5}{18}$ $\mathrm{P}(X=5)=\mathrm{P}(\text{green})\times\mathrm{P}(\text{blue})+\mathrm{P}(\text{blue})\times\mathrm{P}(\text{green})$ $=\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}=\frac{1}{3}\times\frac{1}{6}\times2=\frac{1}{9}$ $\mathrm{P}(X=6)=\mathrm{P}(\text{blue})\times\mathrm{P}(\text{blue})$ $=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$	
(ii)	$\mathrm{E}(X)=\sum_{x=2}^{6}x\cdot\mathrm{P}(X=x)$ $=2\times\frac{1}{4}+3\times\frac{1}{3}+4\times\frac{5}{18}+5\times\frac{1}{9}+6\times\frac{1}{36}$ $=\frac{10}{3}$	

Table within 5(i):

x	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{1}{9}$	$\frac{1}{36}$

	$\mathrm{E}(X^2)=\sum_{x=2}^{6} x^2 \cdot \mathrm{P}(X=x)$ $=2^2\times\frac{1}{4}+3^2\times\frac{1}{3}+4^2\times\frac{5}{18}+5^2\times\frac{1}{9}+6^2\times\frac{1}{36}$ $=\frac{110}{9}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ $=\frac{110}{9}-\left(\frac{10}{3}\right)^2$ $=\frac{10}{9}$	
(iii)	Let Y be the sum of scores obtained from a throw of two dice where a red face has a score of 3 points, a green face has a score of 2 points and a blue face has a score of 1 point. Then $Y=8-X$. $\mathrm{E}(Y)=\mathrm{E}(8-X)=8-\mathrm{E}(X)=8-\frac{10}{3}=\frac{14}{3}$ $\mathrm{Var}(Y)=\mathrm{Var}(8-X)=\mathrm{Var}(8)+\mathrm{Var}(X)=\mathrm{Var}(X)=\frac{10}{9}$	
6(i)	Number of arrangements without restriction $=\frac{8!}{3!}=6720$	
(ii)	Number of arrangements with G and S grouped together $=\frac{7!}{3!}\times 2!=1680$ Number of arrangements with G and S not next to each other $= 6720-1680$ $= 5040$	
(iii)	Two possible cases D_G_T_S_ _D_G_T_S Number of ways to arrange 4 consonants = 4! Number of ways to arrange 4 vowels (I, I, I, E) = 4 Total number of arrangements = $4!\times4\times2!$ = 192	
(iv)	Four possible cases I_ _I_ _ _I I_ _ _I_ _I _I_ _I_ _I I_ _I_ _I_	

	Number of ways to arrange D, G, T, S, E = 5! = 120 Total number of arrangements = $5!\times 4 = 480$	
7(i)	A random sample is a sample drawn from the jars of jam produced in one day such that every jar of jam has an **equal chance** of being in the sample, and the selections of the jars of jam are made **independently.**	
(ii)	$\bar{x} = \frac{-81}{50} + 502 = 500.38$ $s^2 = \frac{1}{49}\left[1138 - \frac{(-81)^2}{50}\right] = 20.54653 = 20.5$ (3s.f)	
(iii)	Let X denote the mass of jars of jam, in grams. Let μ be the population mean mass of jars of jam, in grams. To test $H_0 : \mu = 502$ against $H_1 : \mu \neq 502$ at 1% level of significance. Since n = 50 is large, by Central Limit Theorem, $\bar{X} \sim N\left(502, \frac{20.54653}{50}\right)$ approximately under H_0 Test statistic: $Z = \frac{\bar{X} - 502}{\sqrt{\frac{20.54653}{50}}} \sim N(0,1)$ approximately under H_0 $z_{test} = \frac{500.38 - 502}{\sqrt{\frac{20.54653}{50}}} = -2.5271$ Using GC, $n = 50$, $\bar{x} = 500.38$, $s^2 = 20.54653$, $z_{test} = -2.5271$, p-value = 0.011499 Since $p-$value $= 0.011499 > 0.01$, we do not reject H_0 and conclude that there is insufficient evidence, at the 1% significance level, that the mean mass of jars of jam differs from 502g.	
(iv)	Since n = 50, sample size is large, by Central Limit Theorem, the sample mean of mass of jars of jam follows a normal distribution approximately.	

8(a) **(i)**		
(ii)		
(b) **(i)**		
(ii) **(A)**	Correlation coefficient between x and y is 0.9483	
(B)	Correlation coefficient between $\ln x$ and y is 0.9849	
(iii)	$y = c + d\ln x$ is a better model. Scatter diagram shows a better fit. The correlation coefficient between $\ln x$ and y is closer to 1. As x increases, y increases at a decreasing rate. Equation of regression line of y on $\ln x$ is $y = -31.2643 + 106.56117\ln x$ $y = -31.3 + 107\ln x$	
(iv)	When $x = 16$, $y = -31.2643 + 106.56117\ln 16 = 264.1859$ Median amount a month spent on tuition by student who 16 years old is \$264. Estimate is reliable. Correlation coefficient is close to 1. $x = 16$ is within the range of values of x. Interpolation is a good practice.	

9(i)	Each round has the same probability of winning with "Black". Winning with "Black" in a round is independent of winning with "Black" in another round.	
(ii)	Let X be the number of wins with "Black" in 10 rounds. $X \sim \mathrm{B}\left(10, \frac{18}{38}\right)$ or $X \sim B\left(10, \frac{9}{19}\right)$ Expected number of wins $= \mathrm{E}(X) = 10 \times \frac{9}{19} = \frac{90}{19} = 4.74$ (3 s.f.)	
(iii)	Probability required $= 1 - \mathrm{P}(X \le 5)$ $= 0.31412$ (5 s.f.) $= 0.314$ (3 s.f.)	
(iv)	Each round of roulette is independent of any other round of roulette. Hence, the probability that the player will win the fifth round of roulette given that the player has won two of the previous rounds $= \frac{9}{19}$. <u>Alternatively,</u> Probability required $= \mathrm{P}(\text{win fifth round} \mid \text{won two previous rounds})$ $= \frac{\mathrm{P}(\text{win fifth round and won two previous rounds})}{\mathrm{P}(\text{won two rounds out of four})}$ $= \frac{\binom{4}{2}\left(\frac{9}{19}\right)^2\left(1-\frac{9}{19}\right)^2 \times \frac{9}{19}}{\binom{4}{2}\left(\frac{9}{19}\right)^2\left(1-\frac{9}{19}\right)^2}$ $= \frac{9}{19}$	
(v)	Let Y be the number of "good" casino visits out of 20 visits. $Y \sim \mathrm{B}(20, 0.31412)$ Consider $\mathrm{P}(X = k)$, where $0 \le k \le 20$. When $k = 5$, $\mathrm{P}(X = 5) = 0.16582$ When $k = 6$, $\mathrm{P}(X = 6) = 0.18986$ When $k = 7$, $\mathrm{P}(X = 7) = 0.17390$ Hence, $\mathrm{P}(X = k)$ is the largest when $k = 6$. The most probable number of "good" casino visits when $n = 20$ is 6.	
(vi)	Let W be the number of casino visits that are "good" out of n casino visits. $W \sim \mathrm{B}(n, 0.31412)$ From GC, when $n = 14$, $\mathrm{P}(W \ge 5) = 1 - \mathrm{P}(W \le 4) = 0.46206 < 0.5$ When $n = 15$, $\mathrm{P}(W \ge 5) = 0.53259 > 0.5$ Hence, $n \ge 15$	

10(a)	$A \sim \mathrm{N}(25, 20)$ $B \sim \mathrm{N}(\mu, \sigma^2)$ Since $\mathrm{P}(B<12)=\mathrm{P}(B>19)$, $\mu = \frac{12+19}{2} = 15.5$ $B \sim \mathrm{N}(15.5, \sigma^2)$ $A-B \sim \mathrm{N}(9.5, 20+\sigma^2)$ $\mathrm{P}(A>B)=0.68$ $\mathrm{P}(A-B>0)=0.68$ $\mathrm{P}(Z>-\frac{9.5}{\sqrt{20+\sigma^2}})=0.68$ $-\frac{9.5}{\sqrt{20+\sigma^2}}=-0.467699$ $\sigma=19.814=19.8$ (3 sf)	
(b)(i)	Let X and Y be the time taken by Jerry and Mary to attend to a customer in minutes respectively. $X \sim \mathrm{N}(10.1, 0.8^2)$ $Y \sim \mathrm{N}(10.3, 0.75^2)$ Required probability = $\binom{3}{1} \cdot \mathrm{P}(X<10) \cdot \left[\mathrm{P}(X>10.5)\right]^2$ $= 0.12859 = 0.129$ (3 sf)	
(ii)	$\mathrm{E}\left(2Y-(X_1+X_2)\right)=2(10.3)-2(10.1)=0.4$ $\mathrm{Var}\left(2Y-(X_1+X_2)\right)=2^2(0.75^2)+2(0.8^2)=3.53$ $2Y-(X_1+X_2) \sim \mathrm{N}(0.4, 3.53)$ $\mathrm{P}\left(2Y-(X_1+X_2)\geq 3\right)=0.0832$ (3 sf)	
(iii)	$(X_1+X_2)-(Y_1+Y_2) \sim \mathrm{N}\left(2(10.1)-2(10.3),\ 2(0.8^2)+2(0.75^2)\right)$ $(X_1+X_2)-(Y_1+Y_2) \sim \mathrm{N}(-0.4,\ 2.405)$ $\mathrm{P}\left(\lvert(X_1+X_2)-(Y_1+Y_2)\rvert>2\right)=1-\mathrm{P}\left(-2\leq(X_1+X_2)-(Y_1+Y_2)\leq 2\right)$ $=0.21196$ $=0.212$ (3 sf)	