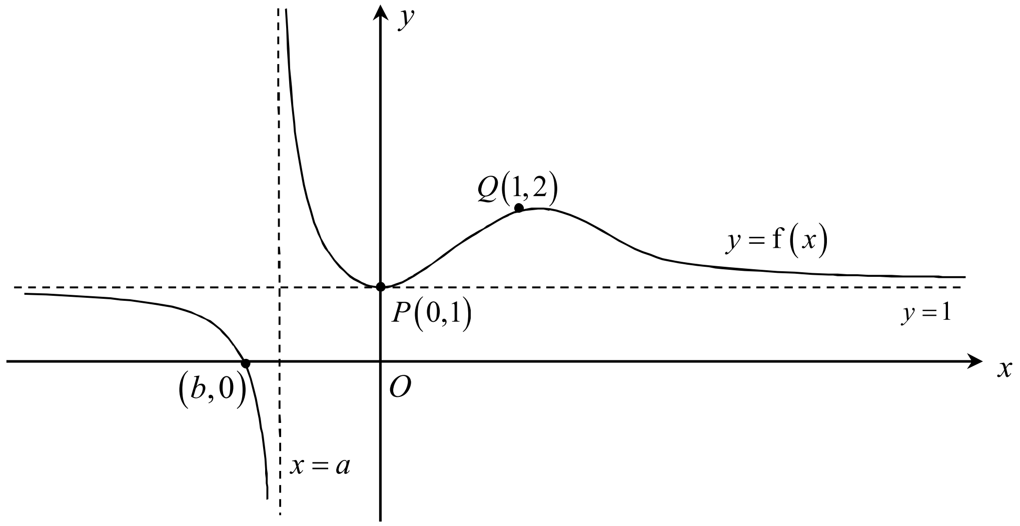
1	The graph of $y=\dfrac{x-1}{ax^2+bx+c}$, where a, b and c are non-zero constants, has a turning point at $(-1,1)$, and an asymptote with equation $x=-\dfrac{1}{3}$. Find the values of a, b and c. [5]
2	The diagram below shows the graph of $y=\mathrm{f}(x)$.  The graph passes through the point $(b,0)$ and has turning points at $P(0,1)$ and $Q(1,2)$. The lines $y=1$ and $x=a$, where $b<a<-\dfrac{1}{2}$, are asymptotes to the curve. On separate diagrams, sketch the graphs of **(i)** $y=\mathrm{f}\left(\dfrac{x-1}{2}\right)$, [3] **(ii)** $y=\mathrm{f}'(x)$, [3] labelling, in terms of a and b where applicable, the exact coordinates of the points corresponding to P and Q, and the equations of any asymptotes.
3	Solve the inequality $\dfrac{1}{x+a}\le\dfrac{2a}{x^2-a^2}$, leaving your answer in terms of a, where a is a positive real number. [3] Hence or otherwise, find $\displaystyle\int_{2a}^{4a}\left\lvert\frac{1}{x+a}-\frac{2a}{x^2-a^2}\right\rvert\,\mathrm{d}x$ exactly. [4]
4	**(i)** Expand $(k+x)^n$, in ascending powers of x, up to and including the term in x^2, where k is a non-zero real constant and n is a negative integer. [3] **(ii)** State the range of values of x for which the expansion is valid. [1] **(iii)** In the expansion of $\left(k+y+3y^2\right)^{-3}$, the coefficient of y^2 is 2. By using the expansion in **(i)**, find the value of k. [3]
5	The points O, A and B are on a plane such that relative to the point O, the points A and B have non-parallel position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point C with position vector $\mathbf{c}$ is on the plane OAB such that OC bisects the angle AOB. Show that $\left(\dfrac{\mathbf{a}}{\lvert\mathbf{a}\rvert}-\dfrac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)\cdot\mathbf{c}=0$. [2]

The lines AB and OC intersect at P. By first verifying that $\overrightarrow{OC}$ is parallel to $\dfrac{\mathbf{a}}{|\mathbf{a}|}+\dfrac{\mathbf{b}}{|\mathbf{b}|}$, show that the ratio of $AP:PB=|\mathbf{a}|:|\mathbf{b}|$. [6]

6 It is given that $\mathrm{e}^{y}=(1+\sin x)^{2}$.

(i) Show that

$$\mathrm{e}^{y}\left[\frac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^{2}\right]=2(\cos 2x-\sin x).$$

By repeated differentiation, find the series expansion of y in ascending powers of x, up to and including the term in x^{3}, simplifying your answer. [5]

(ii) Show how you can use the standard series expansion(s) to verify that the terms up to x^{3} for your series expansion of y in **(i)** are correct. [3]

7 **(a)** Given that $2z+1=|w|$ and $2w-z=4+8\mathrm{i}$, solve for w and z. [5]

(b) Find the exact values of x and y, where $x,y\in\mathbb{R}$, such that $2\mathrm{e}^{-\left(\frac{3+x+\mathrm{i}y}{\mathrm{i}}\right)}=1-\mathrm{i}$. [4]

8 The curve C and the line L have equations $y=x^{2}$ and $y=\dfrac{1}{2}x-2$ respectively.

(i) The point A on C and the point B on L are such that they have the same x-coordinate. Find the coordinates of A and B that gives the shortest distance AB. [3]

(ii) The point P on C and the point Q on L are such that they have the same y-coordinate. Find the coordinates of P and Q that gives the shortest distance PQ. [3]

(iii) Find the exact area of the polygon formed by joining the points found in **(i)** and **(ii)**. [2]

(iv) A variable point on the curve C with coordinates (s,s^{2}) starts from the origin O and moves along the curve with s increasing at a rate of 2 units/s. Find the rate of change of the area bounded by the curve, the y-axis and the line $y=s^{2}$, at the instant when $s=\sqrt{2}$. [4]

9 **(a)** By writing

$$\sin\left(x+\frac{1}{4}\right)\pi-\sin\left(x-\frac{3}{4}\right)\pi$$

in terms of a single trigonometric function, find $\displaystyle\sum_{x=1}^{n}\cos\left(x-\frac{1}{4}\right)\pi$, leaving your answer in terms of n. [4]

(b) The function f is defined by

$$\mathrm{f}:x\mapsto\sin\left(x+\frac{1}{4}\right)\pi-\sin\left(x-\frac{3}{4}\right)\pi,\ x\in\mathbb{R},\ a\le x\le 1.$$

(i) State the range of f and sketch the curve when $a=-1$, labelling the exact coordinates of the points where the curve crosses the x- and y- axes. [3]

(ii) State the least value of a such that f^{-1} exists, and define f^{-1} in similar form. [3]

The function g is defined by

$$\mathrm{g}:x\mapsto\frac{2x}{1-x},\ x\in\mathbb{R},\ x\ge\frac{13}{5}.$$

	Given that fg exists, find the greatest value of a, and the corresponding range of fg. [3]
10	Abbie and Benny each take a \$50 000 study loan for their 3-year undergraduate program, disbursed on the first day of the program. The terms of the loan are such that during the 3-year period of their studies, interest is charged at 0.1% of the outstanding amount at the end of each month. Upon graduation, interest is charged at 0.375% of the outstanding amount at the end of each month. **(a)** Since the interest rate is lower during her studies, Abbie decides that she will make a constant payment at the beginning of each month from the start of the program for its entire duration. **(i)** Find the amount, correct to the nearest cent, Abbie needs to pay at the beginning of each month so that the outstanding amount after interest is charged remains at \$50 000 at the end of every month. [2] **(ii)** After graduating, Abbie intends to increase her payment to a constant \$$k$ at the beginning of every month. Show that the outstanding amount Abbie owes the bank at the end of n months after graduation, and after interest is charged, is $$\$\left[1.00375^n(50000)-\frac{803}{3}k\left(1.00375^n-1\right)\right].$$ [2] **(iii)** Abbie plans to repay her loan within 10 years after graduation. Determine if she can do this with a monthly instalment of \$500, justifying your answer. [1] Find the amount she needs to pay so that she fully repays her loan at the end of exactly 10 years after graduation, leaving your answer to the nearest cent. [2] **(b)** Benny wishes to begin his loan repayment only after graduation. Like Abbie, he aims to repay the loan at the end of exactly 10 years after graduation. Leaving your answer to the nearest cent, find **(i)** the constant amount Benny needs to pay each month in order to do this, [3] **(ii)** the amount of interest Benny pays altogether. [2]
11	**(i)** Show that for any real constant k, $$\int t^2 e^{-kt}\,dt = -e^{-kt}\left(\frac{a}{k}t^2+\frac{b}{k^2}t+\frac{c}{k^3}\right)+D,$$ where D is an arbitrary constant, and a, b, and c are constants to be determined. [3] On the day of the launch of a new mobile game, there were 100,000 players. After t months, the number of players on the game is x, in hundred thousands, where x and t are continuous quantities. It is known that, on average, one player recruits 0.75 players into the game per month, while the number of players who leave the game per month is proportional to t^2. **(ii)** Write down a differential equation relating x and t. [1] **(iii)** Using the substitution $x = ue^{\frac{3}{4}t}$, show that the differential equation in **(ii)** can be reduced to $$\frac{du}{dt} = -pt^2 e^{-\frac{3}{4}t},$$ where p is a positive constant. Hence solve the differential equation in **(ii)**, leaving your answer in terms of p. [5] **(iv)** For $p=\frac{1}{3}$, find the maximum number of players on the game, and determine if there will be a time when there are no players on the game. [2] **(v)** Find the range of values of p such that the game will have no more players after some time. [2]

ANNEX B

ACJC H2 Math JC2 Preliminary Examination Paper 1

QN	Topic Set	Answers
1	Graphs and Transformation	$a=3,\ b=7$ and $c=2$.
2	Graphs and Transformation	(i) $P(1, 1), Q(3, 2), x = 2a+1, y = 1$; (ii) $P(0, 0), Q(1, 0), x = a, y = 0$.
3	Integration techniques	$x<-a$ or $a<x\le 3a$; $\ln\frac{75}{64}$.
4	Binomial Expansion	(i) $k^n\left(1+\frac{n}{k}x+\frac{(n)(n-1)}{2k^2}x^2+\dots\right)$; (ii) $-\lvert k\rvert<x<\lvert k\rvert$; (iii) 0.642.
5	Vectors	
6	Maclaurin series	(i) $y=2x-x^2+\frac{1}{3}x^3+\dots$;
7	Complex numbers	(a) $z=2,\ \ w=3+4\mathrm{i}$; (b) $x=-\frac{\pi}{4}-3,\ y=\frac{1}{2}\ln 2$.
8	Differentiation & Applications	(i) $\mathrm{A}\left(\frac{1}{4},\frac{1}{16}\right)$ & $\mathrm{B}\left(\frac{1}{4},-\frac{15}{8}\right)$; (ii) $\mathrm{P}\left(\frac{1}{4},\frac{1}{16}\right)$ & $\mathrm{Q}\left(\frac{33}{8},\frac{1}{16}\right)$; (iii) $\frac{961}{256}$; (iv) 8 units^2/s
9	Functions	(a) $\frac{1}{2}\sin\left(n+\frac{1}{4}\right)\pi-\frac{1}{2\sqrt{2}}$; (b)(i) $R_{\mathrm{f}}=[-2,2]$, $(-\frac{1}{4},0)$, $(\frac{3}{4},0)$, $(0,\sqrt{2})$; (b)(ii) $a=\frac{1}{4}$, $\mathrm{f}^{-1}:x\mapsto\frac{1}{\pi}\cos^{-1}\left(\frac{x}{2}\right)+\frac{1}{4}$, $x\in\left[-\sqrt{2},2\right]$; (b)(iii) greatest value of a is $-\frac{13}{4}$, $R_{\mathrm{fg}}=\left[-2,\sqrt{2}\right)$.
10	AP and GP	(a)(i) \$49.95; (iii) No, \$516.26 per month; (b)(i) \$535.17 per month; (ii) \$14220.43
11	Differential Equations	(i) $-\mathrm{e}^{-kt}\left(\frac{1}{k}t^2+\frac{2}{k^2}t+\frac{2}{k^3}\right)+D$ (ii) $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{3}{4}x-pt^2$

		(iii) $x = p\left(\frac{4}{3}t^2 + \frac{32}{9}t + \frac{128}{27}\right) + De^{\frac{3}{4}t}$; (iv) max no of players on the game = 365 000; yes, $x = 0$ when $t = 4.35$ months; (v) $p > \frac{27}{128} = 0.211$.

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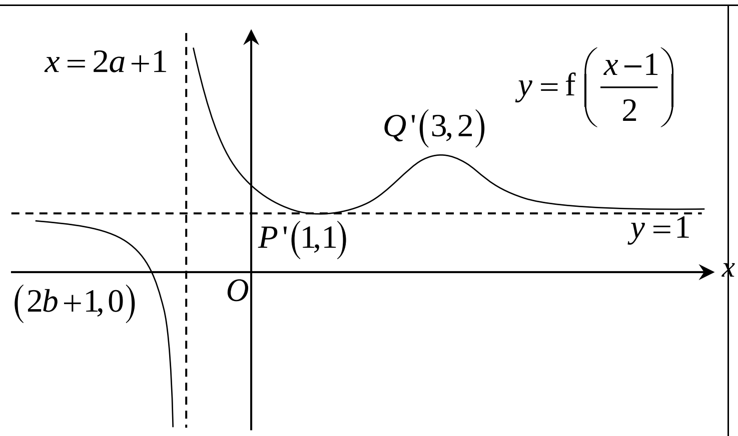
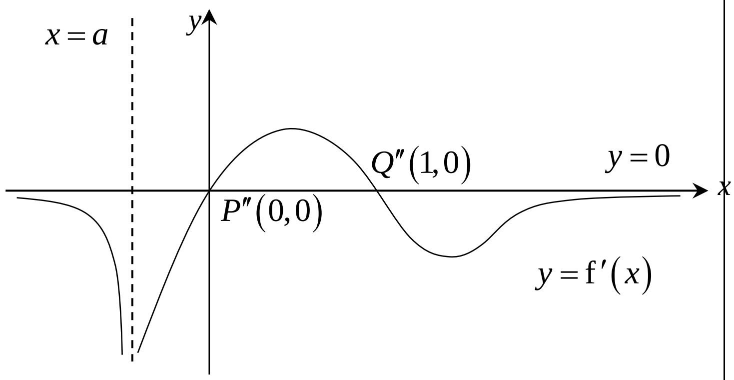
Preliminary Examination Paper 1 Markers Report

Qns	**Solutions**	**Remarks**
1	Passes through $\left(-1,1\right)$: $1=\dfrac{-2}{a-b+c}\ \Rightarrow\ a-b+c=-2$ ……………………..(1) Turning point at $\left(-1,1\right)$: $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=-1}=0$ now $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\left(ax^2+bx+c\right)-\left(x-1\right)\left(2ax+b\right)}{\left(ax^2+bx+c\right)^2}$ Hence $\dfrac{\left(a-b+c\right)-\left(-2\right)\left(-2a+b\right)}{\left(a-b+c\right)^2}=0$ $\Rightarrow\ \left(a-b+c\right)-\left(-2\right)\left(-2a+b\right)=0$ $\Rightarrow\ -3a+b+c=0$ …………………………………(2) When $x=-\frac{1}{3}$, $ax^2+bx+c=0$: Hence $\dfrac{a}{9}-\dfrac{b}{3}+c=0$ ………………………………...(3) Solving (1), (2) and (3) simultaneously, we get $a=3,\ b=7$ and $c=2$. y	Some students forgot that the turning point (-1,1) lies on the curve and failed to substitute the point into the given equation to get an essential equation required for solving the unknowns. Some students made mistakes when differentiating using the product or quotient rule, or incorrectly rewrote y as $y=(x-1)(ax^2+bx+c)$ instead of $y=(x-1)(ax^2+bx+c)^{-1}$ which also resulted in an incorrect derivative. Some students did not know how to handle the information given on the asymptote. Some completed the square or did long division (both not necessary) and came up with an incorrect equation/conclusion. Some wrongly assumed that since $x=-\frac{1}{3}$ is an asymptote, therefore, $\rightarrow ax^2+bx+c=\left(x+\dfrac{1}{3}\right)\left(x-c\right)$ $\rightarrow ax^2+bx+c=\left(3x+1\right)\left(x-c\right)$ $\rightarrow ax^2+bx+c=\left(x+\dfrac{1}{3}\right)^2$ $\rightarrow ax^2+bx+c=\left(3x+1\right)^2$ which made assumptions on the values of *a*; those who assumed *a*=3 might have obtained the same final answer because *a* happened to be 3 in this case, but the method was incorrect.

<table>
<tr>
<td>2(i)</td>
<td>
</td>
<td>Almost the whole cohort gets either full marks or 1 mark (shape of the curve) for this question. Students has difficulty in handling $\frac{x-1}{2}$. Most students failed to read it as $\frac{x}{2} - \frac{1}{2}$. Thus the common mistake majority did was a translation of 1 unit in the positive x – direction followed by a stretching of factor 2 parallel to the x – axis.</td>
</tr>
<tr>
<td>2(ii)</td>
<td>
</td>
<td>About 80% of the students are able to identify the asymptotes $x = a$, $y = 0$ and the x-axis intercepts $P''(0,0)$, $Q''(1,0)$. Of these students, about 70% got full marks as some students couldn't get the shape of the curve. Most students remembered to write the points in coordinates.</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{x+a} \le \frac{2a}{x^2-a^2} \Rightarrow \frac{1}{x+a} - \frac{2a}{x^2-a^2} \le 0$
$\Rightarrow \frac{x-3a}{(x+a)(x-a)} \le 0$

$-$ $+$ $-$ $+$
$-a$ a $3a$

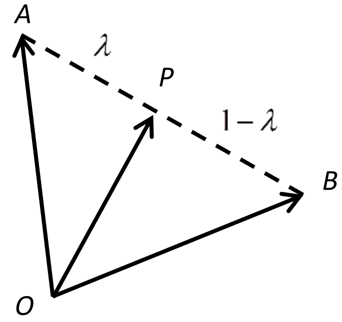
$\therefore x < -a$ or $a < x \le 3a$

$\int \frac{1}{x+a} - \frac{2a}{x^2-a^2}\,\mathrm{d}x = \ln(x+a) - \ln\left(\frac{x-a}{x+a}\right) = \ln\frac{(x+a)^2}{x-a}$
OR
$\int \frac{x-3a}{(x+a)(x-a)}\,\mathrm{d}x = \int \frac{2}{x+a} - \frac{1}{x-a}\,\mathrm{d}x = \ln\frac{(x+a)^2}{x-a}$

$\int_{2a}^{4a} \left|\frac{1}{x+a} - \frac{2a}{x^2-a^2}\right| \mathrm{d}\,x$
$= \int_{2a}^{3a} -\left(\frac{1}{x+a} - \frac{2a}{x^2-a^2}\right)\mathrm{d}\,x + \int_{3a}^{4a} \frac{1}{x+a} - \frac{2a}{x^2-a^2}\,\mathrm{d}\,x$</td>
<td>Many students still are unfamiliar with the basics of solving inequalities and lack the basic skills of factorisation:
(1) Do not know how to find the lowest common multiple of the denominators. Many gave $(x^2-a^2)(x+a)$ as the denominator instead of $(x-a)(x+a)$. Those who did so made a mess out of the numerator and could not factorise the numerator properly.
(2) Many did not even know how to factorise x^2-a^2.
(3) Many insisted on removing the denominator and change the inequality to an inequality involving polynomial only. However they could not do it properly and made a mess out of the polynomial and could not factorise.
(4) For those using graphical method, they attempted to draw the graph of $y = \frac{1}{x+a} - \frac{2a}{x^2-a^2}$ and did not do it properly. They most likely just copied the graph from G.C. without drawing the horizontal asymptote.
(5) Whether by using the sign test with number line or using the graphical method, students still could not obtain the answer correctly, giving the wrong range of values of x.</td>
</tr>
</table>

	$\int_{2a}^{3a} -\left(\frac{1}{x+a}-\frac{2a}{x^2-a^2}\right) \mathrm{d}x + \int_{3a}^{4a} \frac{1}{x+a}-\frac{2a}{x^2-a^2} \mathrm{d}x$ $= -\left[\ln\frac{(x+a)^2}{x-a}\right]_{2a}^{3a} + \left[\ln\frac{(x+a)^2}{x-a}\right]_{3a}^{4a}$ $= -\left(\ln\frac{16a^2}{2a}-\ln\frac{9a^2}{a}\right)+\left(\ln\frac{25a^2}{3a}-\ln\frac{16a^2}{2a}\right)$ $= -\ln\frac{8}{9}+\ln\frac{25}{24}=\ln\left(\frac{25}{24}\times\frac{9}{8}\right)=\ln\frac{75}{64}$	Even some of the values for the x-intercept and vertical asymptotes, $x=-a, x=a, x=3a$ were incorrect particularly, $x=3a.$ Even for those who did almost everything correct included $x=-a, x=a$ as part of the answer. For integration, very few students use Partial Fractions but used the formula in MF26 to integrate directly and most people applied the formula correctly. Most people could carry out the integration properly but could not obtain the final simplified answer $\ln\frac{75}{64}$. There were quite a number of students who apply the formula $\int \frac{1}{x^2-a^2}\,dx=\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)+c$ to $\int\left\|\frac{1}{x^2-a^2}\right\|dx=\left\|\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)\right\|+c$. Some even carried forward the polynomial obtained in the earlier portion for the question on inequality to replace fractions $\frac{1}{x+a}-\frac{2a}{x^2-a^2}$ as the integrand.
4(i)	$(k+x)^n = k^n\left(1+\frac{x}{k}\right)^n$ $= k^n\left(1+n\left(\frac{x}{k}\right)+\frac{(n)(n-1)}{2!}\left(\frac{x}{k}\right)^2+\ldots\right)$ $= k^n\left(1+\frac{n}{k}x+\frac{(n)(n-1)}{2k^2}x^2+\ldots\right)$	(i) Most candidates knew more or less what to do, although mistakes were common; the most common were $(k+x)^n = k\left(1+\frac{x}{k}\right)^n$ or $(k+x)^n=\left(\frac{1}{k}\right)^n(1+kx)^n$ $=\left(\frac{1}{k}\right)^n\left(1+nkx+\frac{(n)(n-1)}{2!}(kx)^2+\ldots\right)$ Some left answer as $(k+x)^n = k^n\left(1+n\left(\frac{x}{k}\right)+\frac{(n)(n-1)}{2!}\left(\frac{x}{k}\right)^2+\ldots\right)$ Did not simplify $\left(\frac{x}{k}\right)^2=\frac{x^2}{k^2}$ No marks was awarded for $(k+x)^n \cong \left(k^n+nk^{n-1}x+\frac{(n)(n-1)}{2}k^{n-2}x^2\right)$. And $(k+x)^n=\left(k^n+\binom{n}{1}k^{n-1}x+\binom{n}{2}k^{n-2}x^2+\ldots..\right)$

		$= \left(k^n + nk^{n-1}x + \frac{(n)(n-1)}{2} k^{n-2}x^2 \right)$
4(ii)	$\left\|\frac{x}{k}\right\| < 1 \Rightarrow \|x\| < \|k\|$ $\therefore -\|k\| < x < \|k\|$	Very badly done . Do not know how to proceed after $\left\|\frac{x}{k}\right\| < 1$ and left answers like $\|x\| < \|k\|$ or $-k < x < k$ or $-1 < x < 1$ Candidates who used Maclaurin series to find the binomial expansion of $(k+x)^n$ have problems finding region of validity. Gave answers like $\|x\| < 1$ or $x \in R$
4(iii)	Let $x = y + 3y^2$ and $n = -3$: $\left(k + y + 3y^2\right)^{-3}$ $= k^{-3}\left(1 + \frac{(-3)}{k}\left(y+3y^2\right) + \frac{(-3)(-4)}{2k^2}\left(y+3y^2\right)^2 + \ldots\right)$ $= k^{-3}\left(1 - \frac{3}{k}y - \frac{9}{k}y^2 + \frac{6}{k^2}y^2 + \ldots\right)$ $\Rightarrow k^{-3}\left(-\frac{9}{k} + \frac{6}{k^2}\right) = 2 \Rightarrow 2k^5 + 9k - 6 = 0$ $\therefore k = 0.642$ (to 3 sf)	Surprisingly quite a number of students do not know how to solve $-\frac{9}{k^4} + \frac{6}{k^5} = 2$ or $2k^5 + 9k - 6 = 0$
5	$\frac{\overrightarrow{OC} \cdot \overrightarrow{OA}}{\left\|\overrightarrow{OC}\right\|\left\|\overrightarrow{OA}\right\|} = \frac{\overrightarrow{OC} \cdot \overrightarrow{OB}}{\left\|\overrightarrow{OC}\right\|\left\|\overrightarrow{OB}\right\|}$ $\frac{\mathbf{c}\cdot\mathbf{a}}{\|\mathbf{a}\|} = \frac{\mathbf{c}\cdot\mathbf{b}}{\|\mathbf{b}\|} \Rightarrow \frac{\mathbf{c}\cdot\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{c}\cdot\mathbf{b}}{\|\mathbf{b}\|} = 0 \Rightarrow \mathbf{c}\cdot\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right) = 0$ Alternatively $\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)\cdot\mathbf{c} = \frac{\mathbf{a}\cdot\mathbf{c}}{\|\mathbf{a}\|} - \frac{\mathbf{b}\cdot\mathbf{c}}{\|\mathbf{b}\|}$ $= \frac{\|\mathbf{a}\|\|\mathbf{c}\|\cos\theta}{\|\mathbf{a}\|} - \frac{\|\mathbf{b}\|\|\mathbf{c}\|\cos\theta}{\|\mathbf{b}\|} = 0$	This question was not well done with a significant number of students not attempting the question at all. Among those who attempted the questions, very few students managed to show that $AP : PB = \|\mathbf{a}\| : \|\mathbf{b}\|$. Many students **wrongly assumed** that $\|\mathbf{a}\| = \|\mathbf{b}\|$. Students need to know that for this question, ⇨ *OC* bisecting angle *AOB* **doesn't mean** that *AP*=*PB*. ⇨ $\overrightarrow{OP}$ & $\overrightarrow{OC}$ may **NOT** be perpendicular to $\overrightarrow{AB}$. ⇨ $\mathbf{c}$ **may not** be parallel to $\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}$ since $\|\mathbf{a}\|$ **may not** be equal to $\|\mathbf{b}\|$. ⇨ $\mathbf{a} + \mathbf{b} \neq \frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}$
	$\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)\cdot\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right) = \left(\frac{\mathbf{a}\cdot\mathbf{a}}{\|\mathbf{a}\|^2} - \frac{\mathbf{b}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\right)$ $= \left(\frac{\|\mathbf{a}\|^2}{\|\mathbf{a}\|^2} - \frac{\|\mathbf{b}\|^2}{\|\mathbf{b}\|^2}\right) = 1 - 1 = 0$	
	P is on $l_{AB} \Rightarrow \overrightarrow{OP} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) = \lambda\mathbf{b} + (1-\lambda)\mathbf{a}$ P is on $l_{OC} \Rightarrow \overrightarrow{OP} = \mu\overrightarrow{OC} = \mu\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)$ Equating $\lambda\mathbf{b} + (1-\lambda)\mathbf{a} = \mu\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)$ Comparing coefficients of **a** and **b**	

	$\lambda = \dfrac{\mu}{\lvert\mathbf{b}\rvert}$ and $1-\lambda = \dfrac{\mu}{\lvert\mathbf{a}\rvert}$ Note that $AP:PB = \lambda : 1-\lambda$, therefore $AP:PB = \dfrac{\mu}{\lvert\mathbf{b}\rvert} : \dfrac{\mu}{\lvert\mathbf{a}\rvert} = \lvert\mathbf{a}\rvert : \lvert\mathbf{b}\rvert$. 	⇨ $\dfrac{\mathbf{a}\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert^2} \neq \dfrac{\mathbf{a}^2}{\lvert\mathbf{a}\rvert^2}$ There was also poor usage of notation. For example many students wrote "a" instead of "$\underset{\sim}{a}$" and also $\overrightarrow{AB}$ instead of $\dfrac{\overrightarrow{AB}}{\lvert\overrightarrow{AB}\rvert}$.
6(i)	$e^y = (1+\sin x)^2$ Differentiating w.r.t. x, $e^y \dfrac{dy}{dx} = 2(1+\sin x)\cos x$ $e^y \dfrac{dy}{dx} = 2\cos x + \sin 2x$ Differentiating w.r.t. x again, $e^y \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} e^y \dfrac{dy}{dx} = -2\sin x + 2\cos 2x$ $e^y \left[\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2\right] = 2(\cos 2x - \sin x)$ (shown) Differentiating w.r.t. x: $e^y \left[\dfrac{d^3y}{dx^3} + 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}\right] + \left[\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2\right] e^y \dfrac{dy}{dx} = 2(-2\sin 2x - \cos x)$ Substituting $x = 0$, $y = 0$; $\dfrac{dy}{dx} = 2$; $\dfrac{d^2y}{dx^2} = -2$; $\dfrac{d^3y}{dx^3} = 2$ $\Rightarrow y = 0 + 2x + \dfrac{-2}{2!}x^2 + \dfrac{2}{3!}x^3 + \ldots$ $\therefore y = 2x - x^2 + \dfrac{1}{3}x^3 + \ldots$	Most students can do the proof in the first part quite well although some have longer methods. Shorter method is to differentiate implicitly to get $e^y \dfrac{dy}{dx} = 2(1+\sin x)\cos x$ Most students could differentiate correctly $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2$ to get $\dfrac{d^3y}{dx^3} + 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ A few fail to use the product rule to differentiate and got this part wrong.
6(ii)	**<u>Method 1</u>**:	Common mistake made is to assume x is a small angle and use the small angle approximation. Correct approximation is

$$e^y = (1+\sin x)^2$$

$$\Rightarrow y = \ln(1+\sin x)^2$$

$$= 2\ln(1+\sin x)$$

$$= 2\ln\left(1+\left(x-\frac{x^3}{3!}\right)+\ldots\right)$$

$$= 2\left(\left(x-\frac{x^3}{3!}\right)-\frac{\left(x-\frac{x^3}{3!}\right)^2}{2}+\frac{\left(x-\frac{x^3}{3!}\right)^3}{3}+\ldots\right)$$

$$= 2\left(x-\frac{x^3}{6}-\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)$$

$$= 2x - x^2 + \frac{1}{3}x^3 + \ldots$$

which is same as the expansion for y found in (i), up to and including the term in $x^3 \Rightarrow$ verified.

<u>Method 2</u>:

$$\text{RHS} = (1+\sin x)^2$$

$$= \left(1+x-\frac{x^3}{3!}\right)^2$$

$$= 1+x-\frac{x^3}{6}+x+x^2-\frac{x^3}{6}+\ldots$$

$$= 1+2x+x^2-\frac{x^3}{3}+\ldots$$

$$\text{LHS} = e^y$$

$$= e^{\left(2x-x^2+\frac{1}{3}x^3+\ldots\right)}$$ (using expansion for y in (i))

$$= 1+\left(2x-x^2+\frac{1}{3}x^3\right)+\frac{\left(2x-x^2+\frac{1}{3}x^3\right)^2}{2!}+\frac{\left(2x-x^2+\frac{1}{3}x^3\right)^3}{3!}+\ldots$$

$$= 1+2x-x^2+\frac{1}{3}x^3+\frac{4x^2-2x^3-2x^3}{2}+\frac{8x^3}{6}+\ldots$$

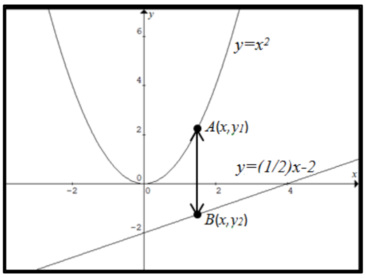
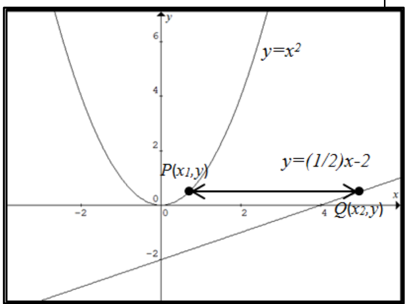
$$= 1+2x+x^2-\frac{1}{3}x^3+\ldots$$

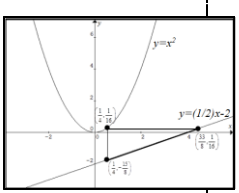
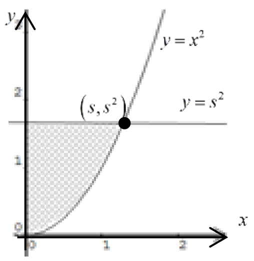
LHS = RHS $\Rightarrow$ verified.

$\sin x = x - \frac{x^3}{3!}$.

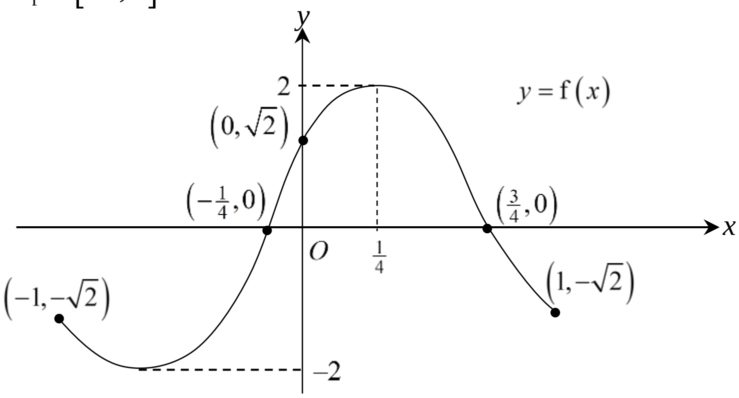
In some answers, detailed workings were not shown clearly.

7(a)	$2z+1=\|w\|$ ....(1) $2w-z=4+8\mathrm{i}$.......(2) $2z+1=$ a positive real number $\Rightarrow$ Let $z=x$ and $w=a+b\mathrm{i}$ From (2): $2(a+bi)-x=4+8\mathrm{i}$ $\Rightarrow$ Comparing Re and Im parts, $2a-x=4$ $2b=8\Rightarrow b=4$ From (1): $2x+1=\sqrt{a^2+b^2}$....(3) Substitute $b=4$ and $x=2a-4$ into (3): $2(2a-4)+1=\sqrt{a^2+16}\Rightarrow(4a-7)^2=a^2+16$ $16a^2-56a+49=a^2+16\Rightarrow 15a^2-56a+33=0$ $\Rightarrow a=\frac{11}{15}$ or $a=3$ $\Rightarrow x=-\frac{98}{15}$ or $x=2$ but $2z+1=$ a positive real number $\Rightarrow$ when $x=-\frac{98}{15}$, $2z+1=2\left(-\frac{98}{15}\right)+1<0$ $\Rightarrow$ reject $x=-\frac{98}{15}$ and $a=\frac{11}{15}$ $\Rightarrow x=2,\ \ a=3, b=4$ $\Rightarrow z=2,\ \ w=3+4\mathrm{i}$	Many students failed to see that z is a real number from eqn (1), resulting in solving simultaneous eqns with many unknown, which most failed to simplify and continue to solve correctly. Some common mistakes: 1. $\|w\|=w$ 2. $\|w\|=\pm w$ 3. $\|w\|=\sqrt{a^2+(ib)^2}=\sqrt{a^2-b^2}$
7(b)	$2\mathrm{e}^{-\left(\frac{3+x+\mathrm{i}y}{\mathrm{i}}\right)}=1-\mathrm{i}$ $2\mathrm{e}^{3\mathrm{i}+x\mathrm{i}-y}=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)}$ $2\mathrm{e}^{-y}\mathrm{e}^{\mathrm{i}(3+x)}=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4}\right)}$ $\Rightarrow$ By comparing modulus and args: $2\mathrm{e}^{-y}=\sqrt{2}$ and $3+x=-\frac{\pi}{4}$ $-y=\ln\left(\frac{\sqrt{2}}{2}\right)$ $\Rightarrow x=-\frac{\pi}{4}-3$ $\Rightarrow y=-\ln\left(\frac{\sqrt{2}}{2}\right)$ (or $\ln\sqrt{2}$ or $\frac{1}{2}\ln 2$)	It's a surprise to see that many students didn't write $1-\mathrm{i}$ in $r\mathrm{e}^{\mathrm{i}\theta}$ form to solve the problem. Even if some did it, they made a mistake in the value of $\theta=\frac{3}{4}\pi$ or $\frac{1}{4}\pi$. In general, students have good idea how to manipulate $-\left(\frac{3+x+\mathrm{i}y}{\mathrm{i}}\right)$ to get $-y+3\mathrm{i}+x\mathrm{i}$ and they also have clear idea of comparing the modulus and argument terms.

8(i)	Let V be the distance AB.  $V=y_1-y_2$ $=x^2-\left(\frac{1}{2}x-2\right)$ $=x^2-\frac{1}{2}x+2$ $\frac{\mathrm{d}V}{\mathrm{d}x}=2x-\frac{1}{2}$ when $\frac{\mathrm{d}V}{\mathrm{d}x}=0,\quad x=\frac{1}{4}$ $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=2>0\Rightarrow$ min. value when $x=\frac{1}{4}$ when $x=\frac{1}{4}$, $y=\left(\frac{1}{4}\right)^2=\frac{1}{16}$ $y=\frac{1}{2}\left(\frac{1}{4}\right)-2=-\frac{15}{8}$ $\therefore$ coords on C (Pt A): $\left(\frac{1}{4},\frac{1}{16}\right)$ & coords on L (Pt B): $\left(\frac{1}{4},-\frac{15}{8}\right)$.	For many, distance was not even considered, instead look at gradients of L and C. Those who used distance, some were penalised for not checking nature of stationary value. Many students made slips in simple calculations such as $2x-\frac{1}{2}\Rightarrow x=1,$ $y^{-\frac{1}{2}}=\frac{1}{4}\Rightarrow y=\pm\frac{1}{2}$ etc.
8(ii)	Let H be the distance PQ.  $H=x_2-x_1=2(y+2)-\sqrt{y}$ $\frac{\mathrm{d}H}{\mathrm{d}y}=2-\frac{1}{2}y^{-\frac{1}{2}}$ when $\frac{\mathrm{d}H}{\mathrm{d}y}=0,$ $2-\frac{1}{2}y^{-\frac{1}{2}}=0\Rightarrow 2=\frac{1}{2}y^{-\frac{1}{2}}$ $\Rightarrow y=4^{-2}=\frac{1}{16}$ $\frac{\mathrm{d}^2H}{\mathrm{d}y^2}=\frac{1}{4}y^{-\frac{3}{2}}$ $\Rightarrow$ when $y=\frac{1}{16},\ \frac{\mathrm{d}^2H}{\mathrm{d}y^2}=\frac{1}{4}\left(\frac{1}{16}\right)^{-\frac{3}{2}}=16>0$ $\Rightarrow$ min. value when $y=\frac{1}{16}$	

	when $y=\frac{1}{16}$, $x=\sqrt{\frac{1}{16}}=\frac{1}{4}$ $x=2\left(\frac{1}{16}\right)+2=\frac{33}{8}$ $\therefore$ coords on C (Pt P): $\left(\frac{1}{4},\frac{1}{16}\right)$ & coords on L (Pt Q): $\left(\frac{33}{8},\frac{1}{16}\right)$.	
8(iii)	Area of polygon = Area of triangle  Minimum distance $AB=\frac{1}{16}-\left(-\frac{15}{8}\right)=\frac{31}{16}$ Minimum distance $PQ=\frac{33}{8}-\left(\frac{1}{4}\right)=\frac{31}{8}$ $\therefore$ Area of polygon $=\frac{1}{2}\times\frac{31}{16}\times\frac{31}{8}=\frac{961}{256}$ sq units	
8(iv)	 $\frac{ds}{dt}=2$ Method 1: Area $=A=\int_0^{s^2} x\ dy=\int_0^{s^2}\sqrt{y}\ dy=\left[\frac{y^{\frac{3}{2}}}{3/2}\right]_0^{s^2}=\frac{2}{3}s^3$ $\frac{dA}{dt}=\frac{dA}{ds}\times\frac{ds}{dt}=2s^2\times 2=4s^2$ $\therefore$ when $s=\sqrt{2}$, $\frac{dA}{dt}=(4)\left(\sqrt{2}\right)^2=8$ units²/s	Well answered except those who treated area bounded as a constant instead of a variable, hence were clueless as to how to get $\frac{dA}{ds}$. When finding area, confused by the variable point, many students did not use definite integral.

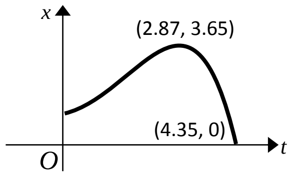
	Method 2: Area = A = Area of rectangle – Area bounded by curve, x-axis and $x = s$ $= s \times s^2 - \int_0^s y \, dx = s^3 - \int_0^s x^2 \, dx = s^3 - \left[\frac{x^3}{3}\right]_0^s = \frac{2}{3}s^3$ $\frac{dA}{dt} = \frac{dA}{ds} \times \frac{ds}{dt} = 2s^2 \times 2 = 4s^2$ $\therefore$ when $s = \sqrt{2}, \Rightarrow \frac{dA}{dt} = 4\left(\sqrt{2}\right)^2 = 8$ units^2/s	
9(a)	By factor formula, $\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi = 2\cos\left[\frac{1}{2}\left(2x-\frac{1}{2}\right)\pi\right]\sin\left(\frac{1}{2}\pi\right)$ $= 2\cos\left(x-\frac{1}{4}\right)\pi.$ Hence $\sum_{x=1}^{n} 2\cos\left(x-\frac{1}{4}\right)\pi$ $= \sum_{x=1}^{n}\left[\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi\right]$ $= \left[\sin\frac{5}{4}\pi - \sin\frac{1}{4}\pi\right] + \left[\sin\frac{9}{4}\pi - \sin\frac{5}{4}\pi\right] + \ldots$ $+ \left[\sin\left(n-\frac{3}{4}\right)\pi - \sin\left(n-\frac{7}{4}\right)\pi\right] + \left[\sin\left(n+\frac{1}{4}\right)\pi - \sin\left(n-\frac{3}{4}\right)\pi\right]$ $= \sin\left(n+\frac{1}{4}\right)\pi - \sin\frac{1}{4}\pi$ $= \sin\left(n+\frac{1}{4}\right)\pi - \frac{1}{\sqrt{2}}$ Therefore, $\sum_{x=1}^{n} \cos\left(x-\frac{1}{4}\right)\pi = \frac{1}{2}\sin\left(n+\frac{1}{4}\right)\pi - \frac{1}{2\sqrt{2}}$.	Many students expanded each term using compound angle formula then tried to collapse the terms back into one trig function, mostly without success. The **most common error** was to first factorise π out of the expression then use factor formula: $\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi$ $= \pi\left[\sin\left(x+\frac{1}{4}\right) - \sin\left(x-\frac{3}{4}\right)\right]$ which is ridiculous. Students need to realise that this is a 1-mark question which should not require page-long working. Those who couldn't do the first part naturally were not able to do this part accurately. Amongst those who did, some evaluated the value of each trigo expression and hence could not see which terms cancelled out using the method of difference: $\sum_{x=1}^{n}\left[\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi\right]$ $= \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] + \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] + \ldots$ $+ \left[\sin\left(n-\frac{3}{4}\right)\pi - \sin\left(n-\frac{7}{4}\right)\pi\right] + \left[\sin\left(n+\frac{1}{4}\right)\pi - \sin\left(n-\frac{3}{4}\right)\pi\right]$

9(b)(i)	$R_{\mathrm{f}} = [-2, 2]$  	Biggest problem for the plot is students keying in to G.C. wrongly. Plotting $Y = \sin\left(X + \frac{1}{4}\right)\pi - \sin\left(X - \frac{3}{4}\right)\pi$ instead of $Y = \sin\left[\left(X + \frac{1}{4}\right)\pi\right] - \sin\left[\left(X - \frac{3}{4}\right)\pi\right]$ Students should be careful, using brackets when appropriate. Once the graph is correctly plotted in the G.C. with the correct domain, they should notice that one full period is plotted, and that the range is easily read off the G.C.
(b)(ii)	Least value of *a* is $\frac{1}{4}$. Let $y = 2\cos\left(x - \frac{1}{4}\right)\pi$. Then $\cos^{-1}\left(\frac{y}{2}\right) = \left(x - \frac{1}{4}\right)\pi \quad \Rightarrow \quad x = \dfrac{\cos^{-1}\left(\frac{y}{2}\right)}{\pi} + \dfrac{1}{4}$. $\therefore \quad \mathrm{f}^{-1} : x \mapsto \dfrac{1}{\pi}\cos^{-1}\left(\dfrac{x}{2}\right) + \dfrac{1}{4}, \qquad x \in \left[-\sqrt{2}, 2\right]$	If graph is correctly sketched, least value of *a* is easily found. Method mark for making *x* the subject of $y = 2\cos\left(x - \frac{1}{4}\right)\pi$ is awarded for any attempt to find the inverse function, regardless of whether students' graphs are sketched correctly. Many students were careless in either not quoting the domain of f^{-1} or, for those who did, quoted it forgetting that domain of f is now restricted so that its inverse exsits.
(b)(iii)	fg exists $\quad \Rightarrow \quad R_{\mathrm{g}} \subseteq D_{\mathrm{f}}$ now $R_{\mathrm{g}} = \left[-\dfrac{13}{4}, -2\right)$ and $D_{\mathrm{f}} = [a, 1]$ since fg exists, $a \le -\dfrac{13}{4}$. Hence the greatest value of *a* is $-\frac{13}{4}$. $R_{\mathrm{fg}} = \mathrm{f}\left(R_{\mathrm{g}}\right) = \mathrm{f}\left[-\dfrac{13}{4}, -2\right) = \left[-2, \sqrt{2}\right]$.	Students were not tenacious enough to find R_{g} properly, perhaps discouraged from the earlier parts. g is a straight forward function that can be sketched with the G.C., bearing in mind that there is a horizontal asymptote at $y = -2$.
10(a)(i)	After one month, if she pays $*x* at the beginning of the month, she will owe the bank	Many students were confused about the interest rate, and hence multiplied by 1.1 or

	$\$(50000-x)\times(1.001)$ Hence $(50000-x)\times(1.001)=50000 \quad \Rightarrow \quad x=49.95$ Abbie needs to pay \$49.95 (to the nearest cent) a month.	1.01. Some merely took 0.1% of \$50,000.
(a)(ii)	One month after graduating, she owes $(50000-k)\times(1.00375)$. *n* months after graduating, she will owe $1.00375^n(50000-k)-1.00375^{n-1}k-\ldots-1.00375k$ $=1.00375^n(50000)-k\left(1.00375^n+1.00375^{n-1}+\ldots+1.00375\right)$ $=1.00375^n(50000)-k\left[\frac{1.00375\left(1.00375^n-1\right)}{1.00375-1}\right]$ $=1.00375^n(50000)-\frac{803}{3}k\left(1.00375^n-1\right)$ (shown).	While many students were able to deduce that this was the sum of a GP, a common mistake was thinking that the last/first term of the GP was 1 instead of 1.00375.
(a)(iii)	Sub n = 120, and k = 500: $1.00375^{120}(50000)-\frac{803}{3}(500)\left(1.00375^{120}-1\right)=2467.11>0$. No, she cannot. A monthly payment of \$500 is not enough. When n = 120, $1.00375^{120}(50000)-\frac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=516.26$ (nearest cent) She needs to pay \$516.26 per month.	Many students did not realise *n* was in months, and used *n* = 10.
(b)(i)	Oustanding amount upon graduation $=1.001^{36}(50000)$ $=51831.86$ Using Abbie's formula, but with a starting outstanding amount of \$51831.86, $1.00375^{120}(51831.86)-\frac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=535.17$ (nearest cent) He needs to pay \$535.17 per month.	Some students used 1.00375^{36}. Some took the 35th power. Many students did not realise they could use the same formula as (a)(iii) but with a different starting amount. As with the previous parts, some interpreted the interest rate wrongly and used 1.1 or 1.01, and some thought *n* was in years.
(b)(ii)	$120\times535.17-50000=14220.43$ (to 2 d.p.) He paid \$14220.43 in interest altogether.	Some students had very involved ways of calculating the interest, including summing the GP all over again. Many students did not subtract

		50,000.
11(i)	$\int t^2 e^{-kt}\, dt = -\frac{1}{k}e^{-kt}(t^2) - \int -\frac{1}{k}e^{-kt}(2t)\, dt$ $= -\frac{1}{k}t^2 e^{-kt} + \frac{2}{k}\left[-\frac{1}{k}e^{-kt}(t) - \int -\frac{1}{k}e^{-kt}(1)\, dt\right]$ $= -\frac{1}{k}t^2 e^{-kt} - \frac{2}{k^2}t e^{-kt} - \frac{2}{k^3}e^{-kt} + D$ $= -e^{-kt}\left(\frac{1}{k}t^2 + \frac{2}{k^2}t + \frac{2}{k^3}\right) + D$	Some students were careless in the first step and could only be awarded the subsequent method mark if they proceeded to integrate by parts a second time. Some students integrated the terms incorrectly or made wrong choices for the terms. Students should remember that the aim of integration by parts is to obtain a simpler integral which can then be integrated (unless it requires the "loop" technique which is not the case for this question) and realise that something is wrong if they ended up with one which looks even more complicated. Few students left this part blank or did not proceed to do integration by parts a second time. Quite a number of students did not put the final expression in the required form and lost marks. Students are reminded to take note of the requirements of the questions.
(ii)	$\frac{dx}{dt} = \frac{3}{4}x - pt^2$	Majority could not get this expression or even gave an expression for x in terms of t instead ($\frac{dx}{dt}$ was not even seen) which should not be the case since the question asked for a "differential equation". Some students also made mistakes in the unit for x (in hundred thousands) or missed out the "x" in the "$0.75x$" term (or incorrectly wrote it as $0.75t$) or missed out the constant of proportionality "p".

(iii)	$x = u\,e^{\frac{3}{4}t} \Rightarrow \frac{dx}{dt} = \frac{3}{4}u\,e^{\frac{3}{4}t} + e^{\frac{3}{4}t}\frac{du}{dt}$ $\frac{3}{4}u\,e^{\frac{3}{4}t} + e^{\frac{3}{4}t}\frac{du}{dt} = \frac{3}{4}u\,e^{\frac{3}{4}t} - pt^2 \Rightarrow \frac{du}{dt} = -pt^2\,e^{-\frac{3}{4}t}$ $u = p\,e^{-\frac{3}{4}t}\left(\frac{1}{\frac{3}{4}}t^2 + \frac{2}{\left(\frac{3}{4}\right)^2}t + \frac{2}{\left(\frac{3}{4}\right)^3}\right) + D$ $= p\,e^{-\frac{3}{4}t}\left(\frac{4}{3}t^2 + \frac{32}{9}t + \frac{128}{27}\right) + D$ $\Rightarrow \frac{x}{e^{\frac{3}{4}t}} = p\,e^{-\frac{3}{4}t}\left(\frac{4}{3}t^2 + \frac{32}{9}t + \frac{128}{27}\right) + D$ $\therefore x = p\left(\frac{4}{3}t^2 + \frac{32}{9}t + \frac{128}{27}\right) + D\,e^{\frac{3}{4}t}$ When $t = 0, x = 1$, $1 = p\left(\frac{128}{27}\right) + D \Rightarrow D = 1 - \frac{128}{27}p$ $x = p\left(\frac{4}{3}t^2 + \frac{32}{9}t + \frac{128}{27}\right) + \left(1 - \frac{128}{27}p\right)e^{\frac{3}{4}t}$	Students would not be able to show the given differential equation if the expression in (i) was incorrect. Some students were not able to correctly differentiate $u\,e^{\frac{3}{4}t}$. Students should read the question carefully and if they are not able to show the required DE, students should still proceed to solve the given DE, and not solve their own incorrect DE, which was what many students did. Many students incorrectly used $k = -\frac{3}{4}$ and were penalised. A few students failed to see the link to part (i) and redid the integration without using the results obtained in (i). Many students failed to substitute "*x*" back into the solution and of those who did, majority forgot the arbitrary constant D or forgot to multiply $e^{\frac{3}{4}t}$ to D – some even labelled $D\,e^{\frac{3}{4}t}$ as another constant $E = D\,e^{\frac{3}{4}t}$ which is incorrect since it now contains the variable *t* and is not just a product of constants. Many also failed to sub in the initial conditions, which was required to obtain the arbitrary constant in terms of *p*. Some did so in the next part but no credit was awarded since it was the requirement in (iii). Some students used the wrong units or failed to show the link from *x* to *u* when using the initial conditions.
(iv)	When $p = \frac{1}{3}$,	Parts (iv) and (v) were badly

	$x=\frac{1}{3}\left(\frac{4}{3}t^2+\frac{32}{9}t+\frac{128}{27}\right)+\left(-\frac{47}{81}\right)e^{\frac{3}{4}t}$  Maximum number of players on the game = 365 000. Yes, x = 0 when t = 4.35 months.	done as students were not likely to obtain the answers to these parts if their expression for *x* was incorrectly in (iii) – only a handful of students could obtain the correct final expression for *x* in (iii). Students were expected to use the GC (graph) for this part and not expected to differentiate, solve the equation, etc to find the maximum value or the *t*-value which gave 0 players - only 2 marks are awarded for the two required answers and students can get the hint from the marks allocation that they were not expected to manually find these answers on their own.
(v)	For $x=0$ after some time, $1-\frac{128}{27}p<0 \Rightarrow p>\frac{27}{128}=0.211$	This required students to see that the coefficient of the exponential term, $\left(1-\frac{128}{27}p\right)e^{\frac{3}{4}t}$, in the expression for *x* found in (iii) had to be negative in order for there to be no players after some time. However, as mentioned in (iii), only a handful of students had the exponential term in their solution for *x*, thus this part was not well done.

Section A: Pure Mathematics [40 marks]

1 Given that $1+\mathrm{i}$ is a root of the equation $z^3-4(1+\mathrm{i})z^2+(-2+9\mathrm{i})z+5-\mathrm{i}=0$, find the other roots of the equation. [4]

2 A curve C has parametric equations

$$x=\cos t$$
$$y=\frac{1}{2}\sin 2t$$

where $\frac{\pi}{2}\le t\le\frac{3\pi}{2}$.

(i) Find the equation of the normal to C at the point P with parameter p. [2]

The normal to C at the point when $t=\frac{2\pi}{3}$ cuts the curve again. Find the coordinates of the point of intersection. [2]

(ii) Sketch C, clearly labelling the coordinates of the points where the curve crosses the x- and y- axes. [1]

(iii) Find the cartesian equation of C. [2]

The region bounded by C is rotated through π radians about the x-axis. Find the exact volume of the solid formed. [3]

3

(i) Find $\displaystyle\int \frac{x}{\left(1+x^2\right)^2}\,\mathrm{d}x$. [2]

(ii) By using the substitution $x=\tan\theta$, show that

$$\int \frac{1}{\left(1+x^2\right)^2}\,\mathrm{d}x=k\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c,$$

where c is an arbitrary constant, and k is a constant to be determined. [5]

(iii) Hence find $\displaystyle\int \frac{x^2}{\left(1+x^2\right)^2}\,\mathrm{d}x$. [3]

(iv) Using all of the above, find $\displaystyle\int \frac{x^2+2x+5}{\left(1+x^2\right)^2}\,\mathrm{d}x$, simplifying your answer. [2]

4 **(a)** **(i)** The unit vector $\mathbf{d}$ makes angles of $60°$ with both the x- and y-axes, and θ with the z-axis, where $0°\le\theta\le 90°$. Show that $\mathbf{d}$ is parallel to $\mathbf{i}+\mathbf{j}+\sqrt{2}\mathbf{k}$. [3]

(ii) The line m is parallel to $\mathbf{d}$ and passes through the point with coordinates $(2,-1,0)$. Find the coordinates of the point on m that is closest to the point with coordinates $(3,2,0)$. [3]

(b) The plane p_1 has equation $\mathbf{r}\cdot(\mathbf{i}+2\mathbf{j}+2\mathbf{k})=5$, and the line l has equation $\frac{x-a}{2}=\frac{y-1}{b}=-\frac{z}{2}$, where a and b are constants.

Given that l lies on p_1, show that $b=1$ and find the value of a. [2]

(i) The plane p_2 contains l and is perpendicular to p_1. Find the equation of p_2 in the form $\mathbf{r}\cdot\mathbf{n}=c$, where c is a constant to be determined. [3]

	(ii) The variable point $P(x, y, z)$ is equidistant from p_1 and p_2. Find the cartesian equation(s) of the locus of P. [3]

Section B: Statistics [60 marks]

5	A group of 12 students consists of 5 bowlers, 4 canoeists and 3 footballers. **(i)** The group sits at a round table with 12 seats. In how many different ways can they sit so that all the players of the same sport sit together? [2] **(ii)** The group stands in a line. In how many different ways can they stand so that *either* the bowlers are all next to one another *or* the canoeists are all next to one another *or* both? [2] **(iii)** Find the number of ways in which a delegation of 8 can be selected from this group if it must include at least 1 student from each of the 3 sports. [2]
6	Alex and his friend stand randomly in a queue with 3 other people. The random variable X is the number of people standing between Alex and his friend. **(i)** Show that $\text{P}(X = 2) = 0.2$. [2] **(ii)** Tabulate the probability distribution of X. [2] **(iii)** Find $\text{E}(X)$ and $\text{E}(X-1)^2$. Hence find $\text{Var}(X)$. [3]
7	It has been suggested that the optimal pH value for shampoo should be 5.5, to match the pH level of healthy scalp. Any pH value that is too low or too high may have undesirable effects on the user's hair and scalp. A shampoo manufacturer wants to investigate if the pH level of his shampoo is at the optimal value, by carrying out a hypothesis test at the 10% significance level. He measures the pH value, x, of n randomly chosen bottles of shampoo, where n is large. **(a)** In the case where $n = 30$, it is found that $\sum x = 178.2$ and $\sum x^2 = 1238.622$. **(i)** Find unbiased estimates of the population mean and variance, and carry out the test at the 10% significance level. [6] **(ii)** Explain if it is necessary for the manufacturer to assume that the pH value of a bottle of shampoo follows a normal distribution. [1] **(b)** In the case where n is unknown, assume that the sample mean is the same as that found in **(a)**. **(i)** State the critical region for the test. [1] **(ii)** Given that n is large and that the population variance is found to be 6.5, find the greatest value of n that will result in a favourable outcome for the manufacturer at the 10% significance level. [3]
8	A swim school takes in both male and female primary school students for competitive swimming lessons. The school assesses its students' progress each year by recording the time, t seconds, each student takes to swim a 50-metre lap in breaststroke, and the number of months, m, that he or she has been at the school. The records for 8 randomly chosen students are shown in the following table.

m	6	7	10	12	15	19	21	24
t	92.32	87.11	66.12	59.41	53.94	43.82	42.07	41.45

(i) Labelling the axes clearly, draw a scatter diagram for the data and explain, in context, why a linear model would not be suitable to predict the time taken by a student to swim a lap of breaststroke given the number of months that he or she has been at the school. [2]

It is desired to fit a model of the form $\ln(t-C)=a+bm$, where C is a suitable constant. The product moment correlation coefficient r between m and $\ln(t-C)$ for some possible values of C are shown in the table below.

C	36	37	38	39
r	–0.992114		–0.992681	–0.992192

(ii) Calculate the value of r for $C=37$, giving your answer correct to 6 decimal places. [1]

(iii) Use the table and your answer to **(ii)** to choose the most appropriate value for C. Explain your choice. [2]

For the remainder of this question, use the value of C that you have chosen in **(iii)**.

(iv) Find the equation of the least squares regression line of $\ln(t-C)$ on m. Give an interpretation of C in the context of the question. [2]

(v) Another student who has been swimming at the school for 9 months clocked a time of 60.33 seconds for a lap of breaststroke. Using your regression line, comment on the student's swimming ability. [2]

(vi) Suggest an improvement to the data collection process so that the results could provide a fairer gauge of the expected outcome for the students in the first 2 years of lessons. [1]

9 **(i)** A procedure for accepting or rejecting a large batch of manufactured articles is such that an inspector first selects and examines a random sample of 10 articles from the batch. If the sample contains at least 2 defective articles, the batch is rejected.

It is known that the proportion of articles that are defective is 0.065. Show that the probability that a batch of articles is accepted is 0.866, correct to three significant figures. [1]

To confirm the decision, another inspector follows the same procedure with another random sample of 10 articles from the batch. If the conclusion of both inspectors are the same, the batch will be accepted or rejected as the case may be. Otherwise, one of the inspectors will select a further random sample of 10 from the same batch to examine. The batch is then rejected if there are at least 2 defective articles. Otherwise, it is accepted. Find

(a) the probability that a batch is eventually accepted, [3]

(b) the expected number of articles examined per batch. [4]

(ii) In order to cut labour cost, an alternative procedure is introduced. A random sample of 10 articles is taken from the batch and if the sample contains not more than 1 defective article then the batch is accepted. If the sample contains more than 2 defective articles, the batch is rejected. If the sample contains exactly 2 defective articles, a second sample of 10 articles is taken and if this contains no defective article then the batch is accepted. Otherwise, the batch is rejected. Given that the proportion of defective articles in the batch is p, show that the probability that the batch is accepted is A where

$$A=(1+9p)(1-p)^9+45p^2(1-p)^{18}.$$ [2]

If the probability that, of 100 batches inspected, more than 80 of them will be accepted is 0.98, find the value of p. [3]

10 **(a)** An examination taken by a large number of students is marked out of a total score of 100. It is found that the mean is 73 marks and that the standard deviation is 15 marks.

(i) Give a reason why the normal distribution is not a good model for the distribution of marks for the examination. [1]

(ii) The marks for a random sample of 50 students is recorded. Find the probability that the mean mark of this sample lies between 70 and 75. [2]

	(b)	The interquartile range of a distribution is the difference between the upper and lower quartile values for the distribution. The lower quartile value, l, of a distribution X, is such that $\mathrm{P}(X < l) = 0.25$. The upper quartile value, u, of the same distribution is such that $\mathrm{P}(X < u) = 0.75$. The marks of another examination is known to follow a normal distribution. If a student who scores 51 marks is at the 80th percentile, and the interquartile range is found to be 10.8 marks, find the mean mark and the standard deviation of the marks scored by students who took the examination. [5]
	(c)	In a third examination, the marks scored by students are normally distributed with a mean of 52 marks and a standard deviation of 13 marks. **(i)** If 50 is the passing mark and 289 students are expected to pass, how many candidates are there? [2] **(ii)** Find the smallest integer value of m such that more than 90% of the candidates will score within m marks of the mean. [3]

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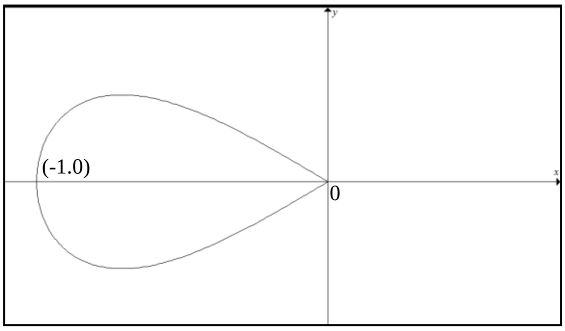
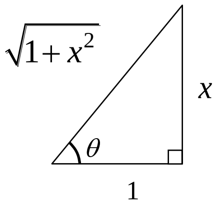
Preliminary Examination Paper 2 Markers Report

Qns	Solutions	Remarks
1	$z^3-4(1+\mathrm{i})z^2+(-2+9\mathrm{i})z+5-\mathrm{i}=0$ $\left(z-(1+\mathrm{i})\right)\left(Az^2+Bz+C\right)=0$ By comparing coefficients, $z^3: A=1$ $z^0: -(1+\mathrm{i})C=5-\mathrm{i}$ $\Rightarrow C=\dfrac{5-\mathrm{i}}{-(1+\mathrm{i})}=-2+3\mathrm{i}$ $z^2: B-(1+\mathrm{i})=-4(1+\mathrm{i})$ $\Rightarrow B=-3(1+\mathrm{i})$ $\Rightarrow \left(z-(1+\mathrm{i})\right)\left(z^2-3(1+\mathrm{i})z-2+3\mathrm{i}\right)=0$ Solving $\left(z^2-3(1+\mathrm{i})z-2+3\mathrm{i}\right)=0$: $z=\dfrac{-\left(-3(1+\mathrm{i})\right)\pm\sqrt{\left(-3(1+\mathrm{i})\right)^2-4(1)\left(-2+3\mathrm{i}\right)}}{2(1)}$ $=\dfrac{3+3\mathrm{i}\pm\sqrt{8+6\mathrm{i}}}{2}$ $=\dfrac{3+3\mathrm{i}\pm(3+\mathrm{i})}{2}=3+2\mathrm{i}$ or i $\therefore$ other 2 roots are $z=3+2\mathrm{i}$ or $z=\mathrm{i}$	Quite a large number of students say that $1-\mathrm{i}$ is another root, which is wrong because not all the coefficients are real. Students who did this gets a 0. When comparing coefficients, many students use $a+ib, c+\mathrm{i}d$ as the two other roots which resulted in unnecessarily tedious and complicated working. About half who used the quadratic formula had problem evaluating $\sqrt{8+6\mathrm{i}}$, which can be done using GC.
2(i)	$x=\cos t$ $y=\dfrac{1}{2}\sin 2t$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{\cos 2t}{-\sin t}$ $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=p}=\dfrac{\cos 2p}{-\sin p}\Rightarrow$ gradient of normal $=\dfrac{\sin p}{\cos 2p}$ $\Rightarrow$ equation of normal at $\left(\cos p, \dfrac{1}{2}\sin 2p\right)$: $y-\dfrac{1}{2}\sin 2p=\dfrac{\sin p}{\cos 2p}\left(x-\cos p\right)$ $y=\dfrac{\sin p}{\cos 2p}x+\dfrac{1}{2}\left(\sin 2p-\tan 2p\right)$	Generally students were able to write down the eqn of normal at point with parameter *p*. However, some wrote $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}p}\times\dfrac{\mathrm{d}p}{\mathrm{d}x}$. Although no mark is deducted here, students should realize that *p* in most cases is a constant (though not specified by question) and $\dfrac{\mathrm{d}y}{\mathrm{d}p}=0$. A minority wrote the eqn of normal as $y-\dfrac{1}{2}\sin 2p=\dfrac{\sin t}{\cos 2t}\left(x-\cos p\right)$ without putting $t=p$. Many careless mistakes in evaluating the cosine and sine values when $t=\dfrac{2\pi}{3}$, resulting in wrong eqns of

	$\Rightarrow$ equation of normal at $t=\frac{2\pi}{3}$: $y=\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}x+\frac{1}{2}\left(-\frac{\sqrt{3}}{2}-\sqrt{3}\right)\Rightarrow y=-\sqrt{3}x-\frac{1}{4}\left(3\sqrt{3}\right)$....(1) To find point of intersection of normal and C (when the normal cuts C again), Substitute $x=\cos t$ and $y=\frac{1}{2}\sin 2t$ into (1): $\frac{1}{2}\sin 2t=-\sqrt{3}\left(\cos t\right)-\frac{1}{4}\left(3\sqrt{3}\right)$ $\frac{1}{2}\sin 2t+\sqrt{3}\left(\cos t\right)+\frac{1}{4}\left(3\sqrt{3}\right)=0$ From GC, $t=2.094395$ (corresponds to $t=\frac{2\pi}{3}$) or $t=3.495928$ $\Rightarrow$ point normal meets C again: $\left(\cos\left(3.495928\right),\frac{1}{2}\sin\left(2\left(3.495928\right)\right)\right)=\left(-0.938,0.325\right)$	normal, such as $y=-\sqrt{3}x-\frac{\sqrt{3}}{4}$, $y=\sqrt{3}x-\frac{3\sqrt{3}}{4}$ etc Many did not understand that the question is asking for point of intersection **between the curve and the normal at** $t=\frac{2\pi}{3}$ and simply sub $t=\frac{2\pi}{3}$ to find the point. Those who correctly sub $x=\cos t$ and $y=\frac{1}{2}\sin 2t$ into (1) often did not use GC to solve the eqn, and simply stopped at this step.
2(ii)		Many did not note the range of values of *t* and sketched 2 loops. A number of students did not give the coordinates of the *x*-intercept.
2(iii)	Method 1: $x=\cos t\Rightarrow x^2=\cos^2 t$ $y=\frac{1}{2}\sin 2t\Rightarrow y=\sin t\cos t$ $\Rightarrow y^2=\sin^2 t\cos^2 t=\left(1-\cos^2 t\right)\cos^2 t=\left(1-x^2\right)x^2$ $\therefore$ Cartesian equation: $y^2=\left(1-x^2\right)x^2$ Method 2: $x=\cos t\Rightarrow\cos t=\frac{x}{1},\sin t=\frac{\pm\sqrt{1-x^2}}{1}$ $\left(\because\frac{\pi}{2}\le t\le\frac{3\pi}{2}\right)$ $y=\frac{1}{2}\sin 2t\Rightarrow y=\sin t\cos t=\pm\sqrt{1-x^2}\left(x\right)$ $\therefore$ Cartesian equation: $y=\pm x\sqrt{1-x^2}$	Many simply wrote the eqn as $y=\sin 2(\cos^{-1}x)$ and did not go on to simplify. Those who used method 2 often omitted the negative sign.

	Method 3: $x=\cos t \Rightarrow x^2=\cos^2 t \Rightarrow \cos 2t = 2\cos^2 t - 1 = 2x^2 - 1$ $y=\frac{1}{2}\sin 2t \Rightarrow \sin 2t = 2y$ Using $\sin^2 2t+\cos^2 2t=1$, $(2y)^2+(2x^2-1)^2=1$ $\therefore$ Cartesian equation: $4y^2+(2x^2-1)^2=1$	
	Method 1: $\int_{-1}^{0}\pi y^2 \mathrm{d}x$ $=\pi\int_{-1}^{0}(1-x^2)x^2\mathrm{d}x$ $=\pi\int_{-1}^{0}x^2-x^4\mathrm{d}x=\pi\left[\frac{x^3}{3}-\frac{x^5}{5}\right]_{-1}^{0}=\frac{2}{15}\pi$ units^3 Method 2 (not advised): $x=\cos t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=-\sin t$ when $x=0,\ t=\frac{\pi}{2},\frac{3\pi}{2}$ (can use either) when $x=-1,\ t=\pi$ $\int_{-1}^{0}\pi y^2\mathrm{d}x$ $=\pi\int_{\pi}^{\frac{3\pi}{2}}\left(\frac{1}{2}\sin 2t\right)^2(-\sin t)\ \mathrm{d}t$ $=-\pi\int_{\pi}^{\frac{3\pi}{2}}(\sin t\cos t)^2(\sin t)\ \mathrm{d}t$ $=-\pi\int_{\pi}^{\frac{3\pi}{2}}\sin^2 t\cos^2 t(\sin t)\ \mathrm{d}t$ $=-\pi\int_{\pi}^{\frac{3\pi}{2}}(1-\cos^2 t)\cos^2 t(\sin t)\ \mathrm{d}t$ $=-\pi\int_{\pi}^{\frac{3\pi}{2}}(\cos^2 t-\cos^4 t)(\sin t)\ \mathrm{d}t$ $=-\pi\left(-\int_{\pi}^{\frac{3\pi}{2}}(\cos t)^2(-\sin t)\ \mathrm{d}t+\int_{\pi}^{\frac{3\pi}{2}}(\cos t)^4(-\sin t)\ \mathrm{d}t\right)$ $=-\pi\left(-\left[\frac{(\cos t)^3}{3}\right]_{\pi}^{\frac{3\pi}{2}}+\left[\frac{(\cos t)^5}{5}\right]_{\pi}^{\frac{3\pi}{2}}\right)$ $=-\pi\left(-0-\frac{1}{3}+0+\frac{1}{5}\right)=\frac{2}{15}\pi$ units^3	Many did not realize that method 1 is the desired method and were stucked with method 2 as they did not know how to integrate the integrand. For method 2, common mistakes include wrong limits, or writing volume as $2\int_{-1}^{0}\pi y^2\mathrm{d}x$.
3(i)	$\int\frac{x}{(1+x^2)^2}\,\mathrm{d}x=\frac{1}{2}\int\frac{2x}{(1+x^2)^2}\,\mathrm{d}x=-\frac{1}{2(1+x^2)}+c$	This is a simple question. No one should be getting this wrong.

3(ii)	$x=\tan\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$ $\displaystyle\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{1}{\left(1+\tan^2\theta\right)^2}\sec^2\theta\,\mathrm{d}\theta$ $\displaystyle=\int \frac{1}{\sec^2\theta}\,\mathrm{d}\theta=\int\cos^2\theta\,\mathrm{d}\theta$ $\displaystyle=\int \frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta=\frac{1}{2}\left(\frac{\sin 2\theta}{2}+\theta\right)$ $\displaystyle=\frac{1}{2}\left(\sin\theta\cos\theta+\theta\right)+c$ $\displaystyle=\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c$ $x=\tan\theta$ $\sin\theta=\dfrac{x}{\sqrt{1+x^2}}$ $\cos\theta=\dfrac{1}{\sqrt{1+x^2}}$  	(ii) was done better than (i) in general. A significant minority did not know that $1+\tan^2\theta=\sec^2\theta$ though, and either got stuck or used very long methods to get to a integrand they could work with. As this is a **show** question, students have to present the way they substitute the variable *x* back into the integral clearly, either using the triangle or with identities. This was quite poorly done though a lot of leeway was given in the awarding of marks.
3(iii)	$\displaystyle\int \frac{x^2}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{x^2+1-1}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{1}{1+x^2}-\frac{1}{\left(1+x^2\right)^2}\,\mathrm{d}x$ $\displaystyle=\tan^{-1}x-\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c$ $\displaystyle=\frac{1}{2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)+c$	There were many different methods available here, the splitting (shown on the left). Other easy methods include: (1) using the substitution provided in (ii). (2) by parts with parts $\dfrac{x}{\left(1+x^2\right)^2}$ and *x* and using (i). A long method uses the parts $\dfrac{1}{\left(1+x^2\right)^2}$ and x^2. Many careless mistakes surfaced in this part (although they were prevalent throughout the question as well), such as confusing $\dfrac{1}{\left(1+x^2\right)^2}$ with $\dfrac{1}{1+x^2}$ or $\dfrac{1}{\left(1+x\right)^2}$.
3(iv)	$\displaystyle\int \frac{x^2+2x+5}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{x^2}{(1+x^2)^2}+\frac{2x}{(1+x^2)^2}+\frac{5}{(1+x^2)^2}\,\mathrm{d}x$ $\displaystyle=\frac{1}{2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)+2\left(-\frac{1}{2(1+x^2)}\right)+5\left(\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)\right)+c$ $\displaystyle=3\tan^{-1}x+\frac{2x-1}{1+x^2}+c$	This was generally well done, as students could use (i)-(iii). Working mark was given even if their integrals were wrong, as long as they were based on their answers in the earlier part. The simplification of the answer was not done by a significant minority.
4(a) (i)	$\mathbf{d}=\cos 60^\circ\mathbf{i}+\cos 60^\circ\mathbf{j}+\cos\gamma\mathbf{k}$ $\cos^2 60^\circ+\cos^2 60^\circ+\cos^2\gamma=1$ $\Rightarrow \cos^2\gamma=1-\frac{1}{4}-\frac{1}{4}=\frac{1}{2}$ $\Rightarrow \cos\gamma=\frac{1}{\sqrt{2}}\ (\because \gamma \text{ is acute})$	This part was rather poorly done, though most students can apply the geometrical definition of the scalar product and get 1 or 2 marks. Common errors include: (1) Not reading that **d** is a **unit** vector.

	$\mathbf{d}=\frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{1}{\sqrt{2}}\mathbf{k}\,//\,\mathbf{i}+\mathbf{j}+\sqrt{2}\mathbf{k}$	(2) poor presentation with regard to the treatment of vectors and scalars, for e.g. **d** = 0.5. In addition, the **<u>show</u>**ing part needs to be worked on. Students have to present steps logically and quote relevant information from the question as part of their reasoning.
(a)(ii)	$m:\mathbf{r}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$ $\left(\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}-\begin{pmatrix}3\\2\\0\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=0\Rightarrow\left(\begin{pmatrix}-1\\-3\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=0$ $\therefore(-1-3)+\lambda(1^2+1^2+\sqrt{2}^2)=0\Rightarrow\lambda=1$ Therefore position vector of point is $\begin{pmatrix}2\\-1\\0\end{pmatrix}+\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=\begin{pmatrix}3\\0\\\sqrt{2}\end{pmatrix}$ Coordinates = $\left(3,0,\sqrt{2}\right)$ <u>OR</u> $\overrightarrow{AN}=\left(\overrightarrow{AP}\cdot\mathbf{d}\right)\mathbf{d}=\frac{\left(\begin{pmatrix}3\\2\\0\end{pmatrix}-\begin{pmatrix}2\\-1\\0\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}}{\sqrt{1+1+2}}\frac{\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}}{\sqrt{1+1+2}}=\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$ $\therefore\overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AN}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=\begin{pmatrix}3\\0\\\sqrt{2}\end{pmatrix}$ Coordinates = $\left(3,0,\sqrt{2}\right)$	This is a simple part. No one should be getting this wrong. There were still students who upon not being able to show (a)(i), decided that (a)(ii) was not doable and had no attempt on it. A variety of methods were applied, though the easiest one is shown first on the left. Students who applied the vector of the projection with modulus sign instead of brackets could arrive at the answer as well, but they were not awarded the full marks due to a conceptual error. Of those who could do this part, around 50% of them lost the answer mark for not expressing in coordinates form.
4(b)	$l:\frac{x-a}{2}=\frac{y-1}{b}=-\frac{z}{2}\Rightarrow l:\mathbf{r}=\begin{pmatrix}a\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\b\\-2\end{pmatrix}$ $\begin{pmatrix}2\\b\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=0\Rightarrow2+2b-4=0\Rightarrow b=1$ $\begin{pmatrix}a\\1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=5\Rightarrow a+2=5\Rightarrow a=3$	This was generally well-done, though a minority wrote $\begin{pmatrix}a+2\lambda\\1+b\lambda\\-2\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=5$ $\Rightarrow a+2\lambda+2+2b\lambda-4\lambda=5$ but obviously did not understand why $2\lambda+2b\lambda-4\lambda=0$.
(b)(i)	p_2 perpendicular to p_1 $\Rightarrow\mathbf{n}_1\,//\,p_2$ $p_2:\mathbf{r}=\begin{pmatrix}3\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\2\end{pmatrix}$	Some used longer method where they solved $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix}=0$ and $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=0$ Some remembered that the direction vector of line of intersection is $\mathbf{n}_1\times\mathbf{n}_2$ and

<table>
<tr>
<td></td>
<td>$\begin{pmatrix}2\\1\\-2\end{pmatrix}\times\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}6\\-6\\3\end{pmatrix}//\begin{pmatrix}2\\-2\\1\end{pmatrix}$
$p_2:\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}3\\1\\0\end{pmatrix}\begin{pmatrix}2\\-2\\1\end{pmatrix}=4$</td>
<td>wrote $\begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ but failed to include $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=0$ as another condition.
A significant minority made careless mistakes while computing the vector product. They should remind themselves how to check for correctness of the vector product.</td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>$\frac{1}{\sqrt{9}}\left\lvert\begin{pmatrix}x-3\\y-1\\z\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}\right\rvert=\frac{1}{\sqrt{9}}\left\lvert\begin{pmatrix}x-3\\y-1\\z\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}\right\rvert$
$\lvert x-3+2(y-1)+2z\rvert=\lvert 2(x-3)-2(y-1)+z\rvert$
$\Rightarrow x+2y+2z-5=2x-2y+z-4$
$\Rightarrow x-4y-z=-1$
or
$\Rightarrow x+2y+2z-5=-(2x-2y+z-4)$
$\Rightarrow 3x+3z=9\Rightarrow x+z=3$</td>
<td>Only less than 30 students are able to do this part.
A handful gave good solutions, obtaining n as $\begin{pmatrix}1\\2\\2\end{pmatrix}+\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}3\\0\\3\end{pmatrix}$ or $\begin{pmatrix}1\\2\\2\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}-1\\4\\1\end{pmatrix}$.</td>
</tr>
<tr>
<td>5 (i)</td>
<td>Number of ways = (3 – 1)!·5!·4!·3! = 34 560</td>
<td>Generally well done</td>
</tr>
<tr>
<td>5 (ii)</td>
<td>Number of ways
= N(5 bowlers together) + N(4 canoeists together)
– N(5 bowlers together & 4 canoeists together)
= 8!·5! + 9!·4! – 5!·5!·4!
= 4 838 400 + 8 709 120 – 345 600
= 13 201 920</td>
<td>Most students added the three numbers instead of subtracting the case for intersection:
8!·5! + 9!·4! + 5!·5!·4!.
If students had drawn a venn diagram, the correct operation would have been clearer.</td>
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<td>5 (iii)</td>
<td>Number of ways
= N(Total) – N(0 bowlers) – N(0 canoeists) – N(0 footballers)
= ${}^{12}C_8 - 0 - {}^{8}C_8 - {}^{9}C_8 = 485$</td>
<td>Very badly done, although there is a question in Tutorial 20 Q9.
Many did
${}^{5}C_1 * {}^{4}C_1 * {}^{3}C_1 * {}^{9}C_1$
which is a gross overcount.</td>
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<td>6 (i)</td>
<td>$\mathrm{P}(X=2)=\mathrm{P}(A{*}{*}F{*},{*}A{*}{*}F)=2\left(\frac{2\times3!}{5!}\right)=\frac{1}{5}=0.2$ (shown)</td>
<td rowspan="2">6(i) and (ii) were very crucial parts to this question. Students who were unable to start finding the pdf of X, or did it wrongly, would not have been able to answer (iii).

Some students lost marks for (i) because they lacked sufficient elaboration, e.g. writing simply 4/20 or 2/10 without justifying how they arrived at these numbers. They would have gotten the</td>
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<td>6 (ii)</td>
<td>$\mathrm{P}(X=0)=\mathrm{P}(AF{*}{*}{*},{*}AF{*}{*},{*}{*}AF{*},{*}{*}{*}AF)=4\left(\frac{2\times3!}{5!}\right)=\frac{2}{5}$
$=0.4$
$\mathrm{P}(X=1)=\mathrm{P}(A{*}F{*}{*},{*}A{*}F{*},{*}{*}A{*}F)=3\left(\frac{2\times3!}{5!}\right)=\frac{3}{10}=0.3$
$\mathrm{P}(X=3)=\mathrm{P}(A{*}{*}{*}F)=\left(\frac{2\times3!}{5!}\right)=\frac{1}{10}=0.1$
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr></table></td>
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	| $P(X=x)$ | 0.4 | 0.3 | 0.2 | 0.1 |	mark if they had drawn some diagram of how there are 4 ways of arranging Alex and his friend 2 persons apart (ignoring the arrangement of the other 3 people). A significant number of students assumed X was a binomial random variable. Students are also reminded to present sufficient working for the other probabilities in the table.
6 (iii)	$\mathrm{E}(X)=\sum_{all\ x} x\mathrm{P}(X=x)=0(0.4)+1(0.3)+2(0.2)+3(0.1)=1$ $\mathrm{E}(X-1)^2=\sum_{all\ x}(x-1)^2\mathrm{P}(X=x)=1(0.4)+0(0.3)+1(0.2)+4(0.1)=1$ $\mathrm{Var}(X)=\mathrm{E}(X-\mu)^2=\mathrm{E}(X-1)^2=1$	This part was generally well done. Most of the errors came from the formula for $\mathrm{E}(X-1)^2$. A variety of methods were seen for calculating $\mathrm{Var}(X)$, but very few students figured out the shortest method: by the definition of $\mathrm{Var}(X)$, which is $\mathrm{E}[(X-\mu)^2]$. One way for students to check if their answer for $\mathrm{Var}(X)$ is correct is to know that variance cannot be a negative number.
7(a) (i)	Let X be the random variable "pH value of a randomly chosen bottle of shampoo". Unbiased estimate of population mean $=\bar{x}$ $=\dfrac{178.2}{30}$ $=\underline{5.94}$ Unbiased estimate of population variance $=s^2$ $=\dfrac{1}{29}\left(1238.622-\dfrac{178.2^2}{30}\right)$ $=6.21083$ $=\underline{6.21}$ (3 s.f.) To test $\mathrm{H_0}:\ \mu=5.5$ against $\mathrm{H_1}:\ \mu\neq 5.5$ at 10% significance level Under $\mathrm{H_0}$, since $n=30>20$ is large, $\bar{X}\sim \mathrm{N}\left(5.5,\dfrac{6.21083}{30}\right)$ approx. by Central Limit Theorem Test statistic $Z=\dfrac{\bar{X}-5.5}{\sqrt{\dfrac{6.21083}{30}}}\sim \mathrm{N}(0,1)$ approx.	

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Value of test statistic $z = \dfrac{5.94-5.5}{\sqrt{\dfrac{6.21083}{30}}} = 0.967$ (3 s.f.)

Either Since $-1.64 < 0.967 < 1.64$, z lies <u>outside</u> the critical region
$\Rightarrow$ Do <u>not</u> reject H_0
Or $p\text{-value} = 0.334 > 0.1 \Rightarrow$ Do <u>not</u> reject H_0

$\therefore$ There is insufficient evidence at 10% significance level to conclude that the mean pH value of the shampoo is not 5.5.

<u>Comments</u>
The best solutions for this question are a result of careful attention to the way students phrase their working and calculate the required values. If students take some time to understand the rationale for writing things a certain way, they would be able to appreciate the principles behind a statistical hypothesis test.

Students are encouraged to spell out “unbiased estimate of” rather than just writing $\bar{x}$ or s^2 . Some students even wrote “pop. mean/variance” or μ and σ^2 instead of the unbiased estimates.

The correct alternative hypothesis has been hinted in the question (“…too high or too low…”). Presentation wise, a number of students wrote subscripts on μ , which is not necessary.

Many students are still writing the wrong mean in the distribution for $\bar{X}$. The phrase “Under H_0 ” implies that we’re assuming that the population mean $\mu = 5.5$, therefore $E(\bar{X}) = 5.5$. Students should also be aware of whether CLT is used.

An alarming number of students attempted to write down the formula of the p-value, and then seemed to calculate the p-value using normalcdf instead of the Z-test. Students should only attempt to do this if they’re very sure of the correct formula for the p-value in the respective tests; otherwise, they’re better off using the Z-test function in the GC and letting it do its work.

Some students keyed in the wrong σ into the GC, which resulted in an extremely low p-value.

The final part of comparing p-value to significance level and the conclusion was also horribly done. Students generally made some permutation of the following mistakes:
1. Dividing the p-value by 2, or using the p-value for the one-tail test
2. Comparing p-value to 0.05 instead of 0.1
3. Comparing wrongly (e.g. $0.334 < 0.1$)
4. Mixing up the results of the test (e.g. $0.334 > 0.1$, hence reject H_0)
5. Mixing up “sufficient/insufficient evidence” and “ H_0 / H_1 is true/not true”.
In particular, students should learn that the purpose of the test is to use the evidence to try and prove that H_1 is true, and hence the final conclusion must reflect this (i.e. is there sufficient evidence to conclude that H_1 is true?).
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<td>(a)(ii)</td>
<td>It is not necessary to assume X is normally distributed. As the sample size is large, by Central Limit Theorem, $\bar{X}$ is approximately normally distributed.</td>
<td>This question has highlighted a fundamental conceptual error that many students have about CLT: that CLT allows us to approximate X as a normal distribution. It therefore results in answers</td>
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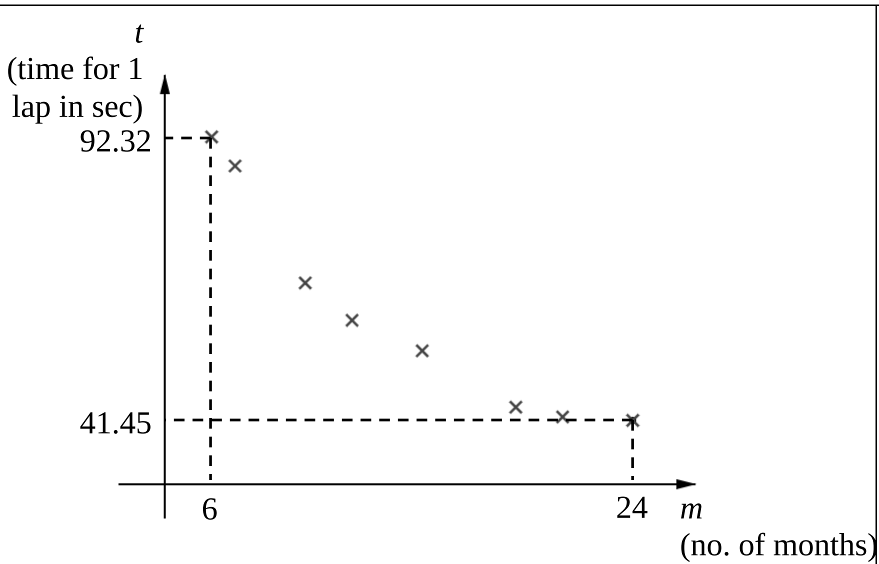
		ranging from "No, CLT says X is normal" to "Yes, since CLT says X is normal". Because it is very easy for students to simply give the correct answer "No" with a superficial explanation, the marking of this part is very much stricter. Many students simply said "It is not necessary, since n is large, **it** is approximately normal by CLT". These are important concepts that need to be corrected so students can have a better picture of how CLT is used.
(b)(i)	Critical region of the test is $z < -1.64485$ or $z > 1.64485$ $\Rightarrow$ $\underline{z < -1.64 \text{ or } z > 1.64}$ (3 s.f.)	The phrases "critical value" and "critical region" are added into the new syllabus, so students must know and distinguish between them. A number of students gave just the critical values. Also, critical region is usually expressed in terms of the test statistic (in our case, z). Finally, there are also students who gave the non-critical region as the critical region. One way to rectify this is to reinforce the fact that the critical region is also known as the rejection region (i.e. rejection of H_0).
(b)(ii)	Value of test statistic $z = \dfrac{5.94 - 5.5}{\sqrt{\dfrac{6.5}{n}}} = \dfrac{0.44\sqrt{n}}{\sqrt{6.5}}$ For a favourable outcome at 10% significance level, do not reject H_0 $\Rightarrow$ z lies <u>outside</u> the critical region $\Rightarrow -1.64485 < \dfrac{0.44\sqrt{n}}{\sqrt{6.5}} < 1.64485$ $\Rightarrow \dfrac{-1.64485\sqrt{6.5}}{0.44} < \sqrt{n} < \dfrac{1.64485\sqrt{6.5}}{0.44}$ $\Rightarrow n < \left(\dfrac{1.64485\sqrt{6.5}}{0.44}\right)^2$ $\Rightarrow n < 90.837$ Hence largest $n = \underline{90}$	Students who are careless with reading the questions would have used either $\frac{178.2}{n}$ as the sample mean or 6.2108 as s^2 . Some students were confused about what the "favourable outcome" meant about the rejection of H_0 . This involves understanding the context of the problem. A significant portion of students only wrote down $z < 1.64485$ and not the full non-critical region. Credit was only given if the correct inequality with the p-value was given earlier; the assumption is that with the correct inequality, students would be able to use invNorm to find the correct critical value. Otherwise, the

		full region should be written down. It is actually also possible to obtain the correct answer with $z > -1.64485$, but the earlier inequality would have been more appropriate since the test statistic here is positive. Students were also generally very careless with solving inequalities.

8(i)	 A linear model would imply that in the long run, the time taken to swim a lap would be <u>negative</u>, which is unrealistic. *(Note: Extrapolation is not accepted as a reason, as the question isn't looking for a reason based on the data obtained.)*	3 important points to note for scatter diagram: 1) axes *t* and *m* labelled 2) extreme values labelled 3) 8 points in total Acceptable answers include: - negative time - zero time
8(ii)	Using GC, for $C = 37$, $r = \underline{-0.992555}$	**R**: 6 d.p.
8(iii)	The most appropriate value for C is <u>38</u>, as the magnitude of its corresponding value of r is closest to 1.	Acceptable answers include: - $\lvert r \rvert \approx 1$ - $r \approx -1$ Quite a number of scripts had "closet" instead of "closest"!
8(iv)	From GC, least squares regression line of $\ln(t-38)$ on m is $\ln(t-38) = 5.01236 - 0.16349m$ $\Rightarrow \underline{\ln(t-38) = 5.01 - 0.163m}$ (3 s.f.) $C = 38$ is the <u>fastest time</u> that a student can expect to complete a lap of breaststroke <u>after spending a long time</u> at the swim school. (Making t the subject in the equation of the regression line gives us $t = 38 + e^{5.01-0.163m}$, so as $m \to \infty$, $t \to 38$.)	**R**: use C =38 **R**: $\ln(t-38)$ on m 3 s.f. for final answer Please note that C is NOT the gradient; C is NOT the *y*-intercept Acceptable answers include: - fastest time after a long period - shortest time after a long period
8(v)	When $m = 9$, $t = 38 + e^{5.01236-0.16349(9)}$ $= 72.50$ (2 d.p.) A timing of 60.33 seconds is well below the expected timing of 72.50 seconds. Therefore, we can say that the student is <u>exceptionally strong</u> in his/her swimming ability.	Acceptable answers include: - very strong - very talented - way above average
8(vi)	The 8 randomly selected students might have been of different genders and ages. To make the results fairer, data could be collected separately based on <u>genders</u> and <u>age ranges</u>.	The following may not give **fairer results**: - increase sample size - increase frequency - group by ability (beginner, intermediate, advanced) is subjective

9 (a)	Let X be the random variable 'number of defective articles in sample of 10'. $X \sim \text{B}(10, 0.065)$ $\text{P(accepting a batch)} = \text{P}(X \le 1) = 0.86563 = 0.866$	Although most people are able to do this part, there are quite a number of students who doesn't know how to do this basic question. Or some calculated this manually instead of using Binomial distribution.
(i)	P(batch eventually accepted) $= (0.86563)^2 + 2(0.86563)(1-0.86563)(0.86563)$ $= 0.95069$ $= 0.951$	Most students who got this wrong did not multiply by 2 for the second case. Some did not understand the question and interpret it as a geometric series question.
(ii)	Let N be the number of articles examined per batch. $N = \begin{cases} 20 & \text{if both findings agree} \\ 30 & \text{otherwise} \end{cases}$ $\text{P}(N = 20) = (0.86563)^2 + (1-0.86563)^2 = 0.76737$ $\text{P}(N = 30) = 1 - 0.76737 = 0.23263$ $\therefore\ \text{E}(N) = 20(0.76737) + 30(0.23263) = 22.3$	About 30% have no clue how to do this part. 40% of those who attempted missed out some cases, such as RR or did not multiply by 2 to account for AR and RA.
9 (b)	Let Y be the random variable 'number of defective articles in a sample of 10'. $Y \sim \text{B}(10, p)$ $A = \text{P}(Y \le 1) + \text{P}(Y = 2) \cdot \text{P}(Y = 0)$ $= {}^{10}C_0 p^0 (1-p)^{10} + {}^{10}C_1 p^1 (1-p)^9 + {}^{10}C_2 p^2 (1-p)^8 \cdot {}^{10}C_0 p^0 (1-p)^{10}$ $= (1-p)^{10} + 10p(1-p)^9 + 45p^2(1-p)^{18}$ $= (1+9p)(1-p)^9 + 45p^2(1-p)^{18}$ (shown)	Except for some who did not interpret the question properly, this part is quite well done for those who attempted it. Except for those who did not use the formula and thus left out ${}^{10}C_1$ or ${}^{10}C_2$.
9 (b)	Let W be the random variable 'number of acceptable batches, out of 100 inspected'. $W \sim \text{B}(100, A)$ $\text{P}(W > 80) = 0.98 \Rightarrow \text{P}(W \le 80) = 0.02$ By GC, A = 0.876235 $\therefore\ A = (1+9p)(1-p)^9 + 45p^2(1-p)^{18} = 0.87624$ By GC, p = 0.08	There are a good number students who have problem dealing with complement. $\text{P}(W > 80) = 0.98 \Rightarrow 1 - \text{P}(W \le 79) = 0.98$ A large number of students applied (CLT) erroneously or normal approximation to this qn, and took invNorm. Students should also be advised not to use table to solve for A as A is not an integer value.
10 (a) (i)	Let X be the random variable 'marks of an examination'. By GC, $\text{P}(X > 100) = 0.0359$ if $X \sim \text{N}(73, 15^2)$	Common wrong answers are: The

	i.e., there are 3.59% of the students scoring more than the maximum mark of 100, which is impossible.	students marks are not independent of one another / the mean should be around 50 / mark is a discrete random variable / mark cannot take negative values or values above 100. Students need to understand that normal distribution is a model to help analyze the data and can be applied as long the population is large and the values that it cannot take have negligible probabilities.
10 (a) (ii)	Since $n = 50 \geq 20$ is large, by Central Limit Theorem, $\overline{X} \sim \text{N}(73, \frac{15^2}{50})$ approximately. $\therefore\ \text{P}(70 < \overline{X} < 75) = 0.748$	Majority assumed X is normal and then applied CLT for $\overline{X}$. This questions shows that most people do not understand the meaning of $\overline{X}$.
10 (b)	Let Y be the random variable 'marks of a school examination'. $Y \sim \text{N}(\mu, \sigma^2)$ $\text{P}(Y < 51) = 0.8$ $\text{P}(Z < \frac{51-\mu}{\sigma}) = 0.8$ $\frac{51-\mu}{\sigma} = 0.84162$ $\mu + 0.84162\sigma = 51$ $\text{P}(\mu - 5.4 < Y < \mu + 5.4) = 0.5$ $\text{P}(\frac{-5.4}{\sigma} < Z < \frac{5.4}{\sigma}) = 0.5$ $\text{P}(Z < -\frac{5.4}{\sigma}) = 0.25$ $-\frac{5.4}{\sigma} = -0.67449$ $\therefore\ \sigma = 8.01$ $\therefore\ \mu = 51 - 0.84162(8.0061) = 44.3$	Quite a number had problem with 80th percentile: P(Y > 51) = 0.8 & P(Y = 51) are WRONG! Standardisation should be $\boxed{Z = \frac{X-\mu}{\sigma}}$ Note that $\text{InvNorm }(0.8) = 0.84162$ $\text{InvNorm }(0.8) \neq 0.8$ Note the interquartile range and its related probability: $\text{P}(Y < u) - \text{P}(Y < l) = 0.5$ where $u - l = 10.8$ $\text{P}(Y < u) - \text{P}(Y < l) = 10.8$ is WRONG!
10 (c) (i)	Let M be the random variable 'marks of another school examination'. $M \sim \text{N}(52, 13^2)$ $\text{P}(50 < M) = 0.56113$ $\text{Number of passes} = (\text{total candidature}) \times 0.56113 = 289$ $\therefore\ \text{total candidature} = 289 \div 0.56113 = 515$	

10 (c) (ii)	$P(\lvert M-52\rvert < m) > 0.9 \Rightarrow P(52-m < M < 52+m) > 0.9$ where $M \sim N(52,13^2)$ $\Rightarrow P(M < 52-m) < 0.05$ $\Rightarrow \quad 52-m < 30.6$ $\Rightarrow \quad m > 21.4$ $\therefore$ Smallest integral value of $m = 22$	**P**: Missing first step **R**: *m* marks from mean, 90%, more than, etc. As 52 & 13 are given, there is no need for standardisation. The preferred method is InvNorm(0.05, 52, 13). Trial and error using GC table is not advisable.