2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS **9740/01**

Higher 2

Paper 1

18 AUGUST 2016
THURSDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A bakery sells strawberry, blueberry and walnut muffins. During a promotion, a customer purchased all 3 types of muffins and twice as many strawberry muffins as walnut muffins. The promotion price of each strawberry, blueberry and walnut muffin is \$1.60, \$1.75 and \$2.20 respectively. Given that the customer paid \$53.40 for 30 muffins, find the number of each type of muffins purchased. [4]

2 Solve the inequality $\frac{2}{x+2} \geq \frac{x+1}{3}$. [3]

Hence, find the range of values of x for which $\frac{2}{x+3} \geq \frac{x+2}{3}$. [2]

3 (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(xe^{-x}\right)$. [1]

(ii) Hence, find $\int \frac{x-x^2}{e^x}\,\mathrm{d}x$. [4]

4 It is given that $\mathrm{h}(x) = x\cos x$ for $0 \leq x \leq \frac{\pi}{2}$. It is also known that $\mathrm{h}(-x) = \mathrm{h}(x)$ and $\mathrm{h}(\pi + x) = -\mathrm{h}(x)$ for all real values of x.

(i) Sketch the graph of $y = \mathrm{h}(x)$ for $-2\pi \leq x \leq 2\pi$. [3]

(ii) On a separate diagram, sketch the graph of $y^2 = \mathrm{h}(x)$ for $-2\pi \leq x \leq 2\pi$. [2]

5 The planes p_1, p_2 and p_3 have equations $3x + 4y - 7z = 2$, $x - 2y = 4$ and $5x - 4y + az = 3$ respectively, where a is a constant. The point C has position vector $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(i) Given that $a = 2$, find the coordinates of the point of intersection of p_1, p_2 and p_3. [2]

(ii) Find the coordinates of the foot of perpendicular from C to p_2. [3]

(iii) Find the value of a such that p_1, p_2 and p_3 have no common points. [3]

6 **Do not use a calculator in answering this question.**

The complex number z is given by $z = \frac{3+i}{2-i}$.

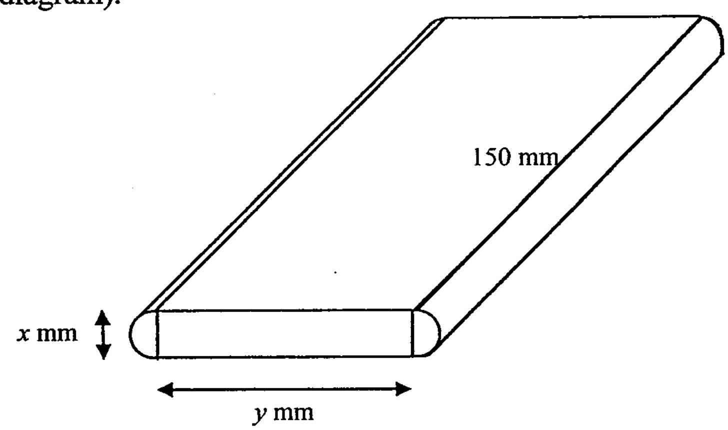
(i) Find $|z|$ and $\arg z$ in exact form. [2]

(ii) Hence, find the exact values of x and y, where $-\pi < y \leq \pi$, such that

$$e^{x+i2y} = \frac{3+i}{2-i}.$$ [3]

(iii) Find the smallest positive integer n such that $\left(\frac{z^2}{z^*}\right)^n$ is purely imaginary. [3]

7 A company manufactures a container of length 150 mm. The container has a uniform cross section made up of a rectangle y mm by x mm and 2 semi-circles of diameter x mm (see diagram).



Given that the container has a volume of 7200 mm^3, find the exact value of x which gives a container of minimum external surface area. [8]

8 (a) A bowl of hot soup is placed in a room where the temperature is a constant 20 °C. As the soup cools down, the rate of decrease of its temperature θ °C after time t minutes is proportional to the difference in temperature between the soup and its surroundings. Initially, the temperature of the soup is 80 °C and the rate of decrease of the temperature is 4 °C per minute. By writing down and solving a differential equation, show that $\theta = 20 + 60\,e^{-\frac{1}{15}t}$. [6]
Find the time it takes the soup to cool to half of its initial temperature. [2]

(b) The gradient of a curve C is given by

$$\frac{dy}{dx} = (x+y)^2.$$

Use the substitution $u = x + y$ to show that the above equation reduces to

$$\frac{du}{dx} = 1 + u^2.$$ [2]

Hence find y in terms of x given that C passes through the origin. [2]

9 (a) Given that $y = \ln(\cos x)$, show that

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -1.$$ [2]

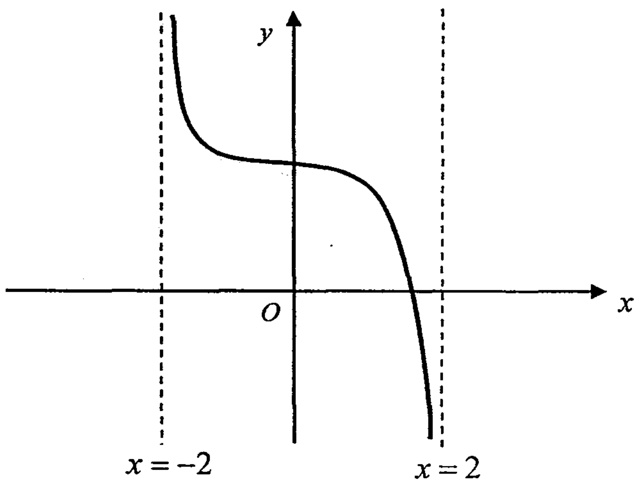
(i) By further differentiation of this result, find the Maclaurin series for y, up to and including the term in x^4. [5]

(ii) Hence, find the Maclaurin series for $\tan x$, up to and including the term in x^3. [2]

(b) Using an appropriate expansion from MF15, find the first three terms of the Maclaurin series for $\ln(k+x)^n$, where n and k are positive constants. [3]

10 **(i)** Find $\displaystyle\int \frac{1}{1+(3-y)^2}\,\mathrm{d}y$. [1]

(ii)



The diagram shows the curve with equation $y=3-\dfrac{x}{\sqrt{4-x^2}}$. Find the exact volume of revolution when the region bounded by the curve, the line $y = 1$ and the y-axis is rotated completely about the y-axis. [4]

By using the substitution $x=2\sin\theta$, find the exact area of the region bounded by the curve, the line $x=1$ and the axes. [4]

11 In a training session, athletes run from a starting point S towards their coach in a straight line. When they reach the coach, they run back to S along the same straight line. A lap is completed when athletes return to S. At the beginning of the training session, the coach stands at A_1 which is 25 m away from S. After the first lap, the coach moves from A_1 to A_2 and after the second lap, he moves from A_2 to A_3 and so on. The points A_1, A_2, A_3, ..., are increasingly further away from S in a straight line where A_iA_{i+1} = 1 m, $i\in\mathbb{Z}^{+}$. The training session will stop only when the athletes have run more than 1500 m.

An athlete completes his first lap in 20 seconds but the time for each subsequent lap is 15% more than the time for the preceding lap. Given that the athlete must complete each lap he runs and there is no resting time between laps, find the least amount of time to complete the training session, giving your answer correct to the nearest minute. [5]

Assuming that the athlete runs at a constant speed for each lap, find the number of complete laps when he has run for 15 minutes. [2]
Hence, find the distance from S and the direction of travel of the athlete after he has run for exactly 15 minutes. [3]

12 Planes p and q are perpendicular to each other. Plane p has equation $\mathbf{r}\cdot\begin{pmatrix}-2\\3\\1\end{pmatrix}=-4$ and plane q contains the line l with equation $x=0,\ y+1=\dfrac{z-2}{4}$.

(i) Find a cartesian equation of q. [4]

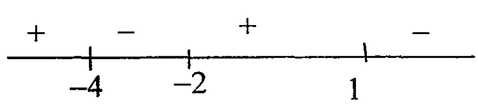
(ii) Find a vector equation of the line m where p and q meet. [2]

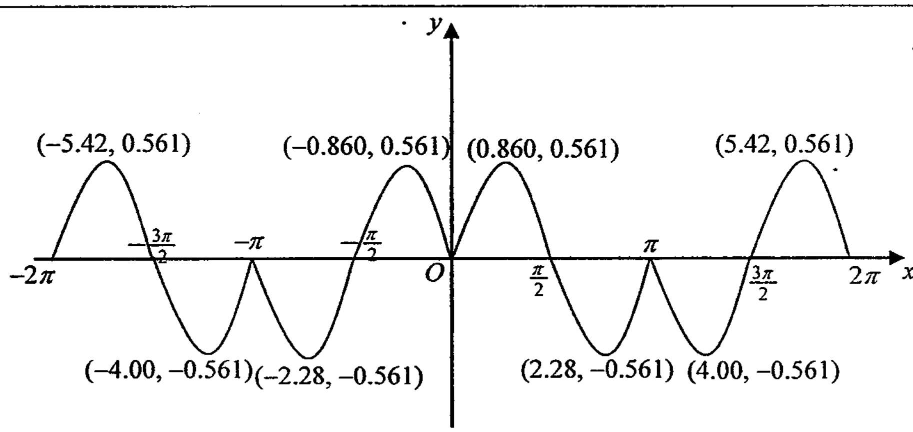
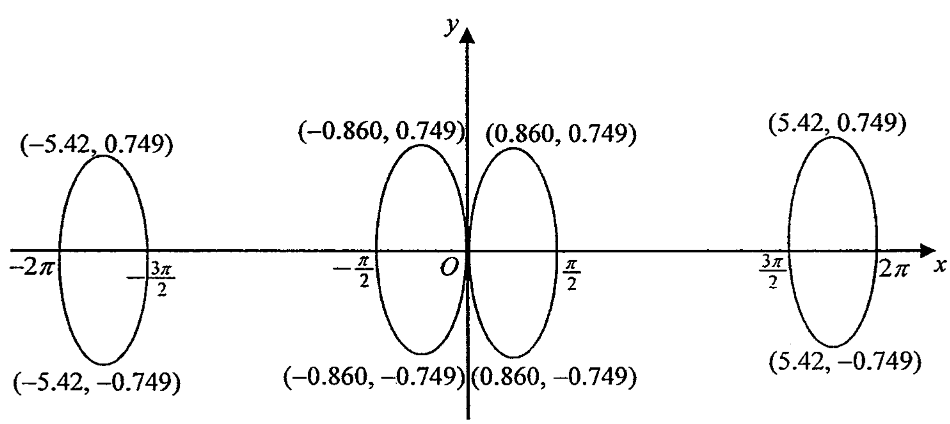
(iii) Find the coordinates of the point C at which l intersects m. [3]

(iv) The points A and B have coordinates $(0, -1, 2)$ and $(x, 0, 0)$. If the area of triangle ABC is increasing at a rate of 17 units^2 per second, find the rate of change of x when $x=\sqrt{5}$. [5]

~ End of Paper ~

2016 JC2 PRELIMINARY EXAM PAPER 1
H2 MATHEMATICS
SOLUTIONS

Qn	Solution
1	Let S, B and W be the number of strawberry, blueberry and walnut muffins purchased respectively. $S+B+W=30$ $1.6S+1.75B+2.2W=53.40$ $S=2W \Rightarrow S-2W=0$ From GC, $S=12,\ B=12,\ W=6$
2	$\dfrac{2}{x+2} \geq \dfrac{x+1}{3}$ $\dfrac{2}{x+2}-\dfrac{x+1}{3} \geq 0$ $\dfrac{6-(x+1)(x+2)}{3(x+2)} \geq 0$ $\dfrac{-x^2-3x+4}{3(x+2)} \geq 0$ $\dfrac{-(x+4)(x-1)}{3(x+2)} \geq 0$  Hence, $x \leq -4$ or $-2 < x \leq 1$ For $\dfrac{2}{x+3} \geq \dfrac{x+2}{3}$ From above, $x+1 \leq -4$ or $-2 < x+1 \leq 1$ $x \leq -5$ or $-3 < x \leq 0$

3(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(xe^{-x}\right)=-xe^{-x}+e^{-x}$ $=e^{-x}\left(1-x\right)$
(ii)	$\int \frac{x-x^2}{e^x}\,\mathrm{d}x=\int x\frac{\left(1-x\right)}{e^x}\,\mathrm{d}x$ $=x^2e^{-x}-\int xe^{-x}\mathrm{d}x$ $=x^2e^{-x}-\left[-xe^{-x}+\int e^{-x}\mathrm{d}x\right]$ $=x^2e^{-x}+xe^{-x}+e^{-x}+C$
4(i)	
4(ii)	

5(i)	$3x+4y-7z=2$ $x-2y=4$ $5x-4y+2z=3$ From GC, $x=-\frac{15}{31}, y=-\frac{139}{62}, z=-\frac{55}{31}$ $\therefore$ the point of intersection is $\left(-\frac{15}{31},-\frac{139}{62},-\frac{55}{31}\right)$
(ii)	Let F be the foot of perpendicular from C to plane. Equation of CF: $\mathbf{r}=\begin{pmatrix}-1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\0\end{pmatrix}, \lambda\in\mathbb{R}$ $\overrightarrow{OF}=\begin{pmatrix}-1\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\0\end{pmatrix}$, for some $\lambda\in\mathbb{R}$ F also lies on plane p_2 $\overrightarrow{OF}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}=4$ $\begin{pmatrix}-1+\lambda\\2-2\lambda\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}=4$ $-1+\lambda-4+4\lambda=4$ $5\lambda=9$ $\lambda=\frac{9}{5}$ $\therefore\overrightarrow{OF}=\begin{pmatrix}-1\\2\\1\end{pmatrix}+\frac{9}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix}=\begin{pmatrix}4/5\\-8/5\\1\end{pmatrix}$ the coordinates is $\left(\frac{4}{5},-\frac{8}{5},1\right)$
(iii)	$3x+4y-7z=2$ $x-2y=4$ From GC, p_1 and p_2 intersect at the line $\mathbf{r}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\mu\begin{pmatrix}14\\7\\10\end{pmatrix}, \mu\in\mathbb{R}$ No common points $\Rightarrow$ p_3 must be parallel to the line.

	$\begin{pmatrix}5\\-4\\a\end{pmatrix} \cdot \begin{pmatrix}14\\7\\10\end{pmatrix}=0$ $42+10a=0$ $a=-4.2$
6(i)	$z=\dfrac{3+\mathrm{i}}{2-\mathrm{i}}=\dfrac{(3+\mathrm{i})(2+\mathrm{i})}{2^2+1}=\dfrac{1}{5}(5+5\mathrm{i})=1+\mathrm{i}$ Therefore, $\lvert z\rvert=\sqrt{2}$ [Or $\lvert z\rvert=\left\lvert\dfrac{3+\mathrm{i}}{2-\mathrm{i}}\right\rvert=\dfrac{\sqrt{10}}{\sqrt{5}}=\sqrt{2}$] $\arg z=\dfrac{\pi}{4}$
(ii)	$\mathrm{e}^{x+\mathrm{i}2y}=z$ $\mathrm{e}^{x}\mathrm{e}^{\mathrm{i}2y}=\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ $\Rightarrow \mathrm{e}^{x}=\sqrt{2}$ or $\mathrm{e}^{\mathrm{i}2y}=\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ or $\mathrm{e}^{\mathrm{i}\left(-\frac{7\pi}{4}\right)}$ $\Rightarrow x=\ln\sqrt{2}=\dfrac{1}{2}\ln 2$ or $y=\dfrac{\pi}{8}$ or $-\dfrac{7\pi}{8}$
(iii)	For $\left(\dfrac{z^2}{z^*}\right)^n$ to be purely imaginary, $\arg\left(\dfrac{z^2}{z^*}\right)^n=(2k+1)\dfrac{\pi}{2}, k\in\mathbf{Z}$ $n[2\arg z-\arg z^*]=(2k+1)\dfrac{\pi}{2}$ $n\left[\dfrac{\pi}{2}+\dfrac{\pi}{4}\right]=(2k+1)\dfrac{\pi}{2}$ $n=\dfrac{2}{3}(2k+1)$ Hence, the smallest positive integer $n=2$

7	Let A and V be the external surface area and volume of the container respectively. $A=300y+150\pi x+2xy+2\pi\left(\frac{x}{2}\right)^2$ $V=150xy+150\pi\left(\frac{x}{2}\right)^2$ $7200=150xy+\frac{75}{2}\pi x^2$ $y=\frac{48}{x}-\frac{\pi x}{4}$ Substitute $y=\frac{48}{x}-\frac{\pi x}{4}$ into surface area equation: $A=300\left(\frac{48}{x}-\frac{\pi x}{4}\right)+150\pi x+2x\left(\frac{48}{x}-\frac{\pi x}{4}\right)+\frac{\pi x^2}{2}$ $=\frac{14400}{x}+96+75\pi x$ $\frac{\mathrm{d}A}{\mathrm{d}x}=-14400x^{-2}+75\pi$ To find least surface area, $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ $-\frac{14400}{x^2}+75\pi=0$ $x=\sqrt{\frac{192}{\pi}}$ or $x=-\sqrt{\frac{192}{\pi}}$ (rejected $\because x>0$) By 2nd Derivative Test $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=\frac{28800}{x^3}$ When $x=\sqrt{\frac{192}{\pi}}$, $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}>0$, A is minimum.
8(a)	Note that $\theta\geq 20$ or $\theta-20\geq 0$ $\frac{\mathrm{d}\theta}{\mathrm{d}t}=-k(\theta-20)$, $k>0$ Given when $t=0$, $\theta=80$, $\frac{\mathrm{d}\theta}{\mathrm{d}t}=-4$. $-4=-k(80-20)\Rightarrow k=\frac{1}{15}$ $\frac{\mathrm{d}\theta}{\mathrm{d}t}=-\frac{1}{15}(\theta-20)$

	$\int \frac{1}{\theta-20}\,d\theta = -\frac{1}{15}\int 1\,dt$ $\ln(\theta-20) = -\frac{1}{15}t + C$ $(\because \theta-20>0)$ $\theta-20 = e^{-\frac{1}{15}t+C}$ $\theta-20 = Ae^{-\frac{1}{15}t}$, where $A=e^{c}$ $\Rightarrow \theta = 20 + Ae^{-\frac{1}{15}t}$ When $t=0$, $\theta=80$, $\Rightarrow 80 = 20 + A$ i.e. $A = 60$ $\therefore \theta = 20 + 60e^{-\frac{1}{15}t}$ $40 = 20 + 60e^{-\frac{1}{15}t}$ $e^{-\frac{1}{15}t} = \frac{1}{3}$ $-\frac{1}{15}t = \ln\frac{1}{3} = -\ln 3$ $t = 15\ln 3$	Alternatively: $\ln\lvert\theta-20\rvert = -\frac{1}{15}t + C$ $\lvert\theta-20\rvert = e^{-\frac{1}{15}t+C}$ $\theta-20 = Ae^{-\frac{1}{15}t}$, where $A = \pm e^{c}$ $\Rightarrow \theta = 20 + Ae^{-\frac{1}{15}t}$
(b)	$u = x + y$ $\Rightarrow \frac{du}{dx} = 1 + \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$ $\frac{dy}{dx} = (x+y)^2$ $\Rightarrow \frac{du}{dx} - 1 = u^2$ $\Rightarrow \frac{du}{dx} = 1 + u^2$ $\int \frac{1}{1+u^2}\,du = \int 1\,dx$ $\tan^{-1}(u) = x + C$ $\tan^{-1}(x+y) = x + C$ When $x=0$, $y=0$, $C=0$ $\therefore x + y = \tan x$ Hence, $y = \tan x - x$	

9(a)	$y=\ln(\cos x)$ $\frac{dy}{dx}=\frac{-\sin x}{\cos x}=-\tan x$ $\frac{d^2y}{dx^2}=-\sec^2 x=-(1+\tan^2 x)$ $\frac{d^2y}{dx^2}=-\left[1+\left(\frac{dy}{dx}\right)^2\right]$ $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-1$
(i)	$\frac{d^3y}{dx^3}+2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}=0$ $\frac{d^4y}{dx^4}+2\left(\frac{dy}{dx}\right)\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}\frac{d^2y}{dx^2}=0$ When $x=0, y=0, \frac{dy}{dx}=0$, $\frac{d^2y}{dx^2}=-1$, $\frac{d^3y}{dx^3}=0$, $\frac{d^4y}{dx^4}=-2$ $y=\ln(\cos x)=\frac{x^2}{2!}(-1)+\frac{x^4}{4!}(-2)+...$ $=-\frac{1}{2}x^2-\frac{1}{12}x^4+...$
(ii)	$\tan x=-\frac{dy}{dx}$ $=-\frac{d}{dx}\left[-\frac{1}{2}x^2-\frac{1}{12}x^4+...\right]$ $=-\left(-x-\frac{1}{3}x^3+...\right)$ $=x+\frac{1}{3}x^3+...$
(b)	$f(x)=\ln(k+x)^n$ $=n\ln(k+x)$ $=n\ln\left(k\left(1+\frac{x}{k}\right)\right)$ $=n\left[\ln k+\ln\left(1+\frac{x}{k}\right)\right]$ $=n\ln k+n\ln\left(1+\frac{x}{k}\right)$ $=n\ln k+n\left(\frac{x}{k}-\frac{1}{2}\left(\frac{x}{k}\right)^2+\cdots\right)$ $=n\ln k+\frac{nx}{k}-\frac{nx^2}{2k^2}+\cdots$

10 (i)	$\int \frac{1}{1+(3-y)^2}\,\mathrm{d}y = (-1)\tan^{-1}\left(\frac{3-y}{1}\right)+C$ $= -\tan^{-1}(3-y)+C$
(ii)	$y = 3 - \frac{x}{\sqrt{4-x^2}}$ $\frac{x}{\sqrt{4-x^2}} = 3-y$ $\frac{x^2}{4-x^2} = (3-y)^2$ $x^2 = (3-y)^2(4-x^2)$ $x^2 = 4(3-y)^2 - x^2(3-y)^2$ $x^2 + x^2(3-y)^2 = 4(3-y)^2$ $x^2 = \frac{4(3-y)^2}{1+(3-y)^2}$ $= 4 - \frac{4}{1+(3-y)^2}$ Volume of revolution about the y-axis $= \pi\int_1^3 x^2\,\mathrm{d}y$ $= \pi\int_1^3 4 - \frac{4}{1+(3-y)^2}\,\mathrm{d}y$ $= \pi\int_1^3 4\,\mathrm{d}y - 4\pi\int_1^3 \frac{1}{1+(3-y)^2}\,\mathrm{d}y$ $= \pi[4y]_1^3 - 4\pi\left[-\tan^{-1}(3-y)\right]_1^3$ $= 8\pi - 4\pi\tan^{-1}(2)$
	Using the substitution $x = 2\sin\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$

	When $x=1, \sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$ When $x=0, \sin\theta=0 \Rightarrow \theta=0$ Area under the curve $=\int_0^1 3-\frac{x}{\sqrt{4-x^2}}\,\mathrm{d}x$ $=\int_0^{\frac{\pi}{6}} \left(3-\frac{2\sin\theta}{\sqrt{4-4\sin^2\theta}}\right)(2\cos\theta)\,\mathrm{d}\theta$ $=\int_0^{\frac{\pi}{6}} 6\cos\theta-\frac{4\sin\theta\cos\theta}{2\cos\theta}\,\mathrm{d}\theta$ $=\left[6\sin\theta+2\cos\theta\right]_0^{\frac{\pi}{6}}$ $=1+\sqrt{3}$
11	Distance travelled per lap is in AP: $a_1=50, d=2\times 1=2$. Given total distance travelled > 1500 $\frac{n}{2}[\,2(50)+(n-1)2] > 1500$ $n^2+49n-1500>0$ $(n+70.33)(n-21.33)>0$ $n<-70.33$ or $n>21.33$ Since $n \in \mathbf{Z}^+$, least $n=22$ Time taken per lap is in GP:

	$a_1 = 20, r = 1.15$ Required least time taken $= \dfrac{20\left((1.15)^{22} - 1\right)}{1.15 - 1}$ ≈ 2752.6 s ≈ 46 min $\dfrac{20\left((1.15)^{n} - 1\right)}{1.15 - 1} \geq 900$ $(1.15)^n \geq 7.75$ $n \geq \dfrac{\ln 7.75}{\ln 1.15}$ $n \geq 14.65$ Number of complete laps = 14 $S_{14} = \dfrac{20\left((1.15)^{14} - 1\right)}{1.15 - 1}$ $= 810.094$ s He needs to run for another $900 - 810.094 = 89.906$ s Distance $T_{15} = 50 + (15 - 1)2 = 78$ $\dfrac{89.906}{20(1.15)^{14}} \times 78 = 49.555$ He is running <u>towards</u> S and at a distance 78 – 49.555≈ <u>28.4 m</u> away from S.

12 **(i)**	$l:\mathbf{r}=\begin{pmatrix}0\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\4\end{pmatrix},\lambda\in\mathbb{R}$. $\mathbf{n}_2=\begin{pmatrix}-2\\3\\1\end{pmatrix}\times\begin{pmatrix}0\\1\\4\end{pmatrix}$ $=\begin{pmatrix}11\\8\\-2\end{pmatrix}$ $q:\mathbf{r}\cdot\begin{pmatrix}11\\8\\-2\end{pmatrix}=\begin{pmatrix}0\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}11\\8\\-2\end{pmatrix}$ $=-12$ $\therefore$ the Cartesian equation is $11x+8y-2z=-12$
(ii)	$-2x+3y+z=-4$ $11x+8y-2z=-12$ From GC, $x=-\frac{4}{49}+\frac{2}{7}\mu, y=-\frac{68}{49}-\frac{1}{7}\mu, z=\mu$ $\therefore\mathbf{r}=\begin{pmatrix}-\frac{4}{49}\\-\frac{68}{49}\\0\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\7\end{pmatrix},\mu\in\mathbb{R}$
(iii)	$\begin{pmatrix}0\\-1+\lambda\\2+4\lambda\end{pmatrix}=\begin{pmatrix}-4/49+2\mu\\-68/49-\mu\\7\mu\end{pmatrix}$ From GC, $\lambda=-\frac{3}{7},\mu=\frac{2}{49}$ Substitute $\lambda=-\frac{3}{7}$ into l: $\mathbf{r}=\begin{pmatrix}0\\-1\\2\end{pmatrix}-\frac{3}{7}\begin{pmatrix}0\\1\\4\end{pmatrix}=\begin{pmatrix}0\\-\frac{10}{7}\\\frac{2}{7}\end{pmatrix}$ $\therefore$ the coordinates of C is $\left(0,-\frac{10}{7},\frac{2}{7}\right)$.

(iv)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $=\begin{pmatrix}x\\0\\0\end{pmatrix}-\begin{pmatrix}0\\-1\\2\end{pmatrix}$ $=\begin{pmatrix}x\\1\\-2\end{pmatrix}$ $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$ $=\begin{pmatrix}0\\-\frac{10}{7}\\\frac{2}{7}\end{pmatrix}-\begin{pmatrix}0\\-1\\2\end{pmatrix}$ $=\begin{pmatrix}0\\-\frac{3}{7}\\-\frac{12}{7}\end{pmatrix}$ Area of ABC, $R=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}x\\1\\-2\end{pmatrix}\times\begin{pmatrix}0\\-3/7\\-12/7\end{pmatrix}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}-18/7\\12x/7\\-3x/7\end{pmatrix}\right\|=\frac{3}{14}\left\|\begin{pmatrix}-6\\4x\\-x\end{pmatrix}\right\|$ $=\frac{3}{14}\sqrt{(-6)^2+(4x)^2+(-x)^2}$ $=\frac{3}{14}\sqrt{36+16x^2+x^2}$ $=\frac{3}{14}\sqrt{36+17x^2}$

	$\frac{dR}{dx}=\frac{3}{14}\times\frac{1}{2}\left(36+17x^2\right)^{-\frac{1}{2}}(34x)$ $=\frac{102x}{28\sqrt{36+17x^2}}$ $=\frac{51x}{14\sqrt{36+17x^2}}$ when $x=\sqrt{5},\frac{dx}{dt}=\frac{dx}{dR}\times\frac{dR}{dt}$ $=\frac{14(11)}{51\sqrt{5}}\times17$ $=\frac{154}{3\sqrt{5}}$ units per second

2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS — 9740/02

Higher 2

Paper 2

24 August 2016
WEDNESDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

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Section A: Pure Mathematics [40 marks]

1 A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_1 = 1 \quad \text{and} \quad u_{n+1} = nu_n + 1 \quad \text{for } n \geq 1.$$

Use the method of induction to prove that

$$u_n = (n-1)!\sum_{r=0}^{n-1} \frac{1}{r!}. \qquad [4]$$

Hence, find the exact value of $\lim\limits_{n \to \infty} \dfrac{u_n}{(n-1)!}$. [1]

2 Show that $\mathrm{f}(x) = \dfrac{5x-2}{x(x-1)(x+2)}$ can be expressed as $\dfrac{A}{x-1} + \dfrac{B}{x} + \dfrac{C}{x+2}$, where A, B and C are constants to be determined. [1]

Hence, find $\sum\limits_{r=2}^{n} \mathrm{f}(r)$. (There is no need to express your answer as a single algebraic fraction.) [3]

Explain, with the aid of a sketch of $y = \mathrm{f}(x)$, $x > 1$, why $\sum\limits_{r=2}^{n} \mathrm{f}(r) > \int_2^{n+1} \mathrm{f}(x)\, \mathrm{d}x$ for $n \geq 2$. [2]

3 The parametric equations of a curve C are $x = t - a\sin t$, $y = t\cos t$, where $-\dfrac{\pi}{2} \leq t \leq \dfrac{\pi}{2}$ and a is a constant. It is given that the normal to C at $x = 0$ is parallel to the x-axis.

(i) Show that $a = 1$. [3]

(ii) Sketch C, giving the coordinates of any points of intersection with the axes. [2]

(iii) Find the area of the region enclosed by C and the x-axis. [3]

4 The functions f and g are defined as follows.

$$\mathrm{f} : x \mapsto \frac{x^2+1}{2x}, \quad x > 0,$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \quad x > 0.$$

(i) Determine whether the composite function gf exists. If it exists, define gf in a similar form and find the range of gf. [4]

(ii) Give a reason why f does not have an inverse function. [1]

(iii) If the domain of f is further restricted to $x \geq k$, state the least value of k for which the function f^{-1} exists. Find $\mathrm{f}^{-1}(x)$ and write down the domain of f^{-1}. [4]

5 **(i)** Solve the equation

$$z^6 + 64 = 0,$$

giving the roots in the form $re^{i\alpha}$, where $r > 0$ and $-\pi < \alpha \leq \pi$. [3]

(ii) The roots in part **(i)** represented by z_n, where $1 \leq n \leq 6$, are such that $-\pi < \arg(z_1) < \arg(z_2) < \cdots < \arg(z_6) \leq \pi$. Show the roots on an Argand diagram and describe geometrically the relationship between the roots. [4]

The complex number w satisfies the equation $\left|iw + 4 + 4\sqrt{3}i\right| = 2$.

(iii) On the same Argand diagram, sketch the locus $\left|iw + 4 + 4\sqrt{3}i\right| = 2$. [3]

(iv) Hence, find the maximum possible value of $|w - z_n|$. [2]

Section B: Statistics [60 marks]

6 The CEO of a company with 40 000 employees wishes to investigate employees' opinions about the food stalls in the staff canteen. 2% of the employees will be chosen to take part in the survey. Explain briefly how the CEO could carry out a survey using

(i) random sampling, [2]

(ii) quota sampling. [2]

7 In a certain town, every car license plate number is a 4-digit number where the digits are chosen from 1 to 9 and cannot be repeated.

Find the number of different car license plate numbers if

(i) there are no restrictions, [1]

(ii) the digits of the car license plate number must not be in ascending order from left to right, [2]

(iii) exactly one of the digits is an even number. [2]

Due to an increasing population, it is decided that the digits used in the car license plate number can be repeated.

(iv) Find the number of different car license plate numbers where no digits can be larger than the third digit. [2]

8 **(a)** Given that events X and Y are independent, prove that events X and Y' are independent. [2]

(b) For events A and B, it is given that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A' \mid B') = 0.3$.

Find

(i) $P(A \cap B)$, [3]

(ii) $P(B' \mid A)$. [2]

Stating your reason, determine if events A and B are

(iii) mutually exclusive, [1]

(iv) independent. [1]

9 Rickie takes the train home after work on weekdays.

(i) The number of days in a week where Rickie finds a seat on the train is denoted by A. State, in context, two assumptions needed for A to be well modelled by a binomial distribution. [2]

Assume now that A has the distribution B(5, 0.65).

(ii) Rickie is contented if he finds a seat on two or three days in a week. Using a suitable approximation, find the probability that in a year (52 weeks), Rickie is contented in no more than 30 weeks. [5]

10 In a certain country, it is to be assumed that the number of drug trafficking cases per week can be modelled by the distribution Po(0.2) and the number of cigarette trafficking cases per week can be modelled by the independent distribution Po(0.7).

(i) Find the probability that, in a randomly chosen period of 8 weeks,

(a) the country has more than 6 drug trafficking cases, [2]

(b) the total number of drug and cigarette trafficking cases is fewer than 5. [2]

(ii) The probability that the country sees fewer than 2 drug trafficking cases in a period of n weeks is less than 0.01. Express this information as an inequality in n, and hence find the smallest possible integer value of n. [3]

(iii) Give two reasons in context why the assumptions made at the start of the question may not be valid. [2]

11 In this question, you should state clearly the values of the parameters of any normal distribution you use.

The masses, in grams, of towels manufactured by companies Alpha and Bravo are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Alpha	μ	20
Bravo	275	15

(i) Given that 6.68% of the towels from Alpha have mass more than 380 grams, show that the value of μ is 350 grams, correct to 3 significant figures. [2]

Towels from Alpha and Bravo are soaked in water to investigate their absorbency. A soaked towel from Alpha is 60% heavier than its dry towel, while a soaked towel from Bravo is 50% heavier than its dry towel.

(ii) Find the probability that the total mass of 4 soaked towels from Alpha and 2 soaked towels from Bravo exceeds 3 kilograms. [4]

12 A new Burger Chain, Burger Queen, claims that the mean waiting time for a burger is at most 4 minutes. The CEO of Burger Jack decides to record the waiting time, x minutes, for a burger at Burger Queen at 80 different locations. The results are summarised by

$$\sum(x-4)=25\ ,\ \sum(x-4)^2=140\,.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 1% level of significance, whether there is any evidence to doubt Burger Queen's claim. [4]

(iii) It is assumed that the standard deviation of the waiting time for a Burger Queen burger is 1.5 minutes. Given that the mean waiting time at another 80 locations is $\bar{x}$, use an algebraic method to find the set of values of $\bar{x}$ for which Burger Queen's claim would not be rejected at the 10% level of significance. [3]

13 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A), (B), (C) below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive. The letters a, b, c, d, e and f represent constants.

(A) $y=a+bx^2$, where a is positive and b is negative,

(B) $y=c+\dfrac{d}{x}$, where both c and d are positive,

(C) $y=e+fx$, where e is positive and f is negative. [3]

An archaeologist found an unknown substance on an excavation trip. Research is being carried out to investigate how the mass of the substance varies with time, measured from when it is placed in a cooled chamber. Observations at successive times give the data shown in the following table.

Time (x hours)	100	800	1500	3000	6000	8000
Mass (y grams)	25	8	5	4	3.5	3.3

(ii) Draw the scatter diagram for these values, labelling the axes. [1]

(iii) Explain which of the three cases in part **(i)** is the most appropriate for modelling these values, and calculate the product moment correlation coefficient for this case. [2]

(iv) Use the case that you identified in part **(iii)** to find the equation of a suitable regression line and estimate the time when the mass of the substance is 10 grams. Comment on the reliability of the estimate. [3]

~ End of Paper ~

2016 JC2 PRELIMINARY EXAM PAPER 2
H2 MATHEMATICS
SOLUTIONS

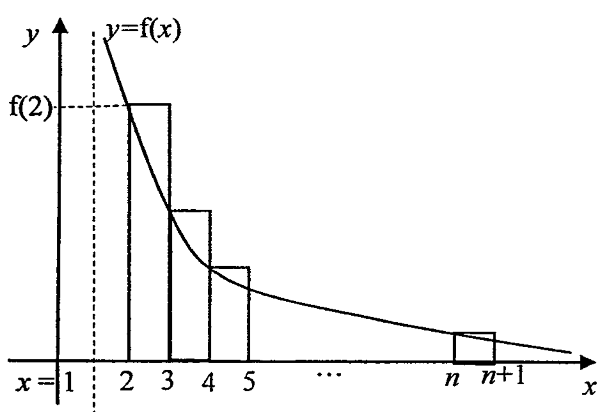
Qn	Solution
1	Let P_n be the statement "$u_n=(n-1)!\sum_{r=0}^{n-1}\frac{1}{r!}$" When $n=1$, L.H.S. $= u_1 = 1$ (Given) R.H.S $= (1-1)!\left(\frac{1}{0!}\right) = 1$ Hence, P_1 is true. Assume that P_k is true for some $k \in \mathbf{Z}^+$, ie, $u_k=(k-1)!\sum_{r=0}^{k-1}\frac{1}{r!}$. To show P_{k+1} is true, ie, $u_{k+1}=(k)!\sum_{r=0}^{k}\frac{1}{r!}$. $u_{k+1} = ku_k+1$ $=k(k-1)!\sum_{r=0}^{k-1}\frac{1}{r!}+1$ $=k!\sum_{r=0}^{k-1}\frac{1}{r!}+\frac{k!}{k!}$ $=k!\left(\sum_{r=0}^{k-1}\frac{1}{r!}+\frac{1}{k!}\right)$ $=k!\sum_{r=0}^{k}\frac{1}{r!}$ Therefore, P_k is true $\Rightarrow$ P_{k+1} is true By Mathematical Induction, P_n is true for all $n \geq 1$. $\therefore \frac{u_n}{(n-1)!}=\sum_{r=0}^{n-1}\frac{1}{r!}$ $\lim_{n\to\infty}\frac{u_n}{(n-1)!}=\sum_{r=0}^{\infty}\frac{1}{r!}=\left.e^x\right\|_{x=1}=e$

2

$$f(x)=\frac{5x-2}{x(x-1)(x+2)}=\frac{1}{x-1}+\frac{1}{x}-\frac{2}{x+2}$$

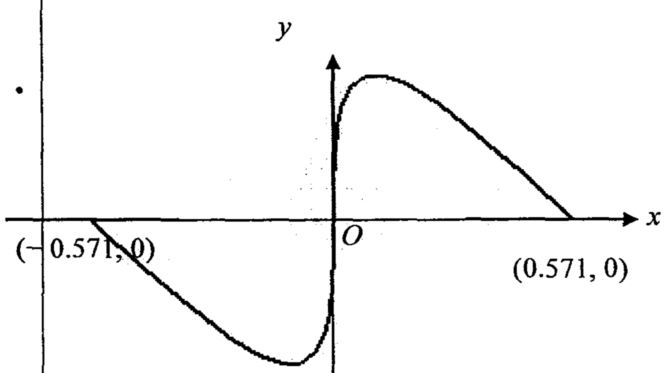
$$\sum_{r=2}^{n} f(r)=\sum_{r=2}^{n}\left(\frac{1}{r-1}+\frac{1}{r}-\frac{2}{r+2}\right)$$

$$\begin{aligned}
&=\frac{1}{1}+\frac{1}{2}-\frac{2}{4}\\
&+\frac{1}{2}+\frac{1}{3}-\frac{2}{5}\\
&+\frac{1}{3}+\frac{1}{4}-\frac{2}{6}\\
&+\frac{1}{4}+\frac{1}{5}-\frac{2}{7}\\
&+\frac{1}{5}+\frac{1}{6}-\frac{2}{8}\\
&+\cdots\\
&+\frac{1}{n-4}+\frac{1}{n-3}-\frac{2}{n-1}\\
&+\frac{1}{n-3}+\frac{1}{n-2}-\frac{2}{n}\\
&+\frac{1}{n-2}+\frac{1}{n-1}-\frac{2}{n+1}\\
&+\frac{1}{n-1}+\frac{1}{n}-\frac{2}{n+2}\\
&=\frac{8}{3}-\left(\frac{1}{n}+\frac{2}{n+1}+\frac{2}{n+2}\right)
\end{aligned}$$



From the above diagram,
Sum of areas of $(n-1)$ rectangles
> Area under graph of $y=f(x)$ from $x=2$ to $n+1$

$$(1)f(2)+(1)f(3)+(1)f(4)+\ldots+(1)f(n) > \int_{2}^{n+1} f(x)\,dx \;\therefore\; \sum_{r=2}^{n} f(r) > \int_{2}^{n+1} f(x)\,dx$$

3(i)	$x = t - a\sin t \quad y = t\cos t$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 - a\cos t \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -t\sin t + \cos t$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$ $= \frac{-t\sin t + \cos t}{1 - a\cos t}$ normal parallel to x-axis $\Rightarrow 1 - a\cos t = 0$ $\Rightarrow a = \frac{1}{\cos t}$ --- (1) When $x = 0$, $t - a\sin t = 0$ ----- (2) Sub (1) into (2): $t - \tan t = 0$ From GC, $t = 0$ (since $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$) Sub into (1): $a = \frac{1}{\cos 0} = 1$ (shown)
(ii)	
(iii)	Area of region $= \left\| \int_{1-\frac{\pi}{2}}^{0} y \, \mathrm{d}x \right\| + \int_{0}^{\frac{\pi}{2}-1} y \, \mathrm{d}x$ $= \left\| \int_{-\frac{\pi}{2}}^{0} t\cos t(1-\cos t) \, \mathrm{d}t \right\| + \int_{0}^{\frac{\pi}{2}} t\cos t(1-\cos t) \, \mathrm{d}t$ $= 0.408$

4(i)

gf exists when $R_f \subseteq D_g$.

Since $R_f = [1,\infty) \subseteq (0,\infty) = D_g$, therefore gf exists.

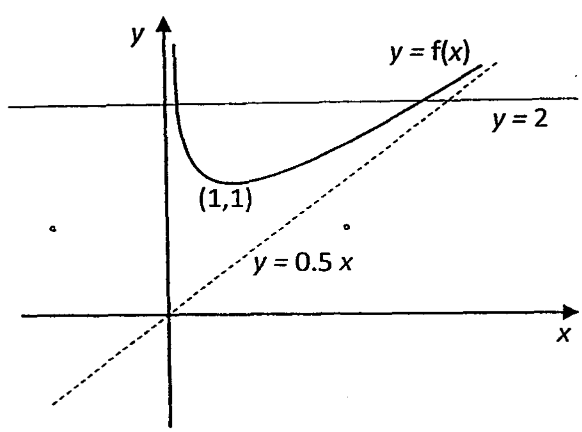
$$\text{gf}(x) = \text{g}\left(\frac{x^2+1}{2x}\right)$$
$$= \frac{2x}{x^2+1}$$

$$\text{gf} : x \mapsto \frac{2x}{x^2+1}, \quad x > 0$$

$$(0,\infty) \xrightarrow{\text{f}} [1,\infty) \xrightarrow{\text{g}} (0,1]$$

$$\therefore R_{gf} = (0,1]$$

4(ii)



The line $y = 2$ cuts the graph of f more than once. Hence, f is not a one–one function. Therefore, f does not have an inverse.

4(iii)

From the graph, least value of $k = 1$

$$x^2 - 2yx + 1 = 0$$

$$x = \frac{2y \pm \sqrt{4y^2 - 4}}{2} = y \pm \sqrt{y^2 - 1}$$

Since $x \geq 1$, $x = y + \sqrt{y^2 - 1}$

$$\text{f}^{-1}(x) = x + \sqrt{x^2 - 1}, \; x \geq 1$$

5(i)

$z^6 = -64 = 64\,\mathrm{e}^{\mathrm{i}\pi}$

$\quad = 64\,\mathrm{e}^{\mathrm{i}(2k+1)\pi},\ k \in \mathbf{Z}$

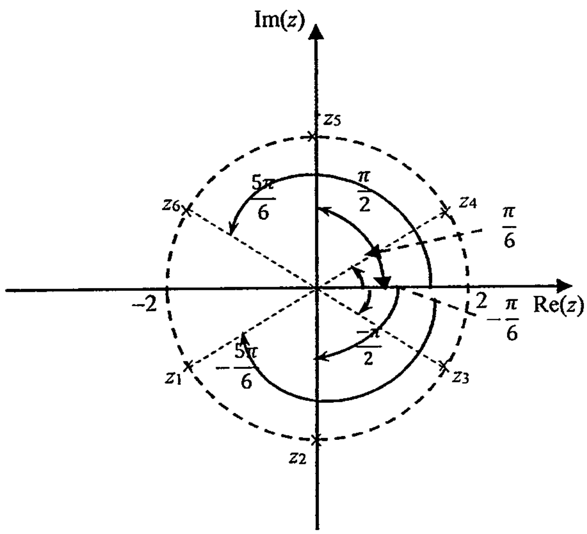
$z = 2\,\mathrm{e}^{\mathrm{i}\left(\frac{2k+1}{6}\right)\pi},\ k = 0, \pm 1, \pm 2, -3$

$z = 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}\right)},\ 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{2}\right)},\ 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{6}\right)},\ 2\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{6}\right)},\ 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{2}\right)},\ 2\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}\right)}$

(ii)

$z_1 = 2\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}\right)},\ z_2 = 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{2}\right)},\ z_3 = 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{6}\right)}$

$z_4 = 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}\right)},\ z_5 = 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{2}\right)},\ z_6 = 2\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{6}\right)}$



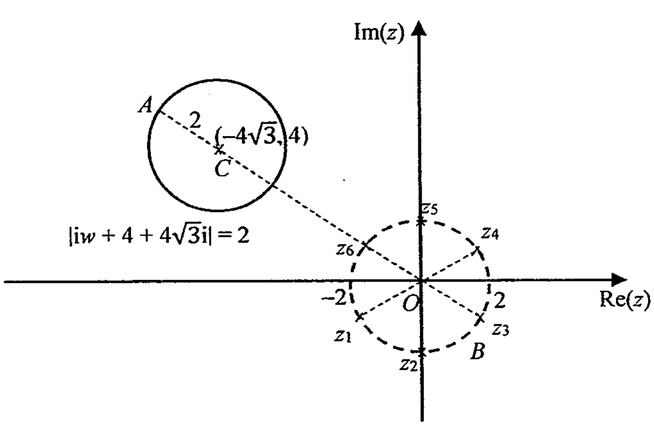
The 6 points representing the 6 roots are the vertices of a regular hexagon.

(iii)

$|\mathrm{i}w + 4 + 4\sqrt{3}\,\mathrm{i}| = 2$

$|\,\mathrm{i}\,|\,|w - \mathrm{i}(4 + 4\sqrt{3}\,\mathrm{i})| = 2$

$|w - (-4\sqrt{3} + 4\mathrm{i})| = 2$

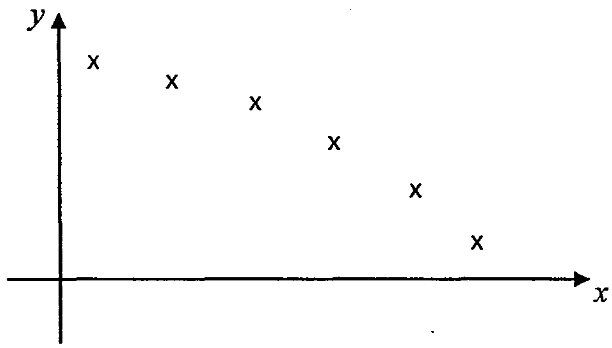
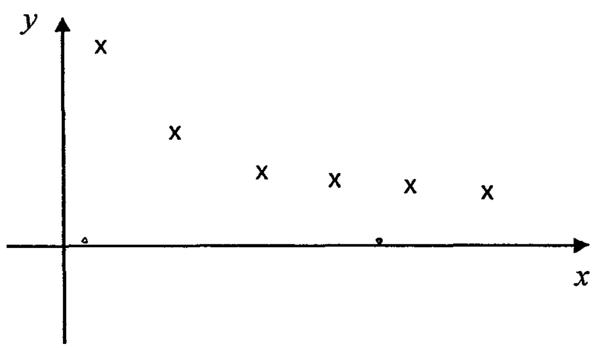
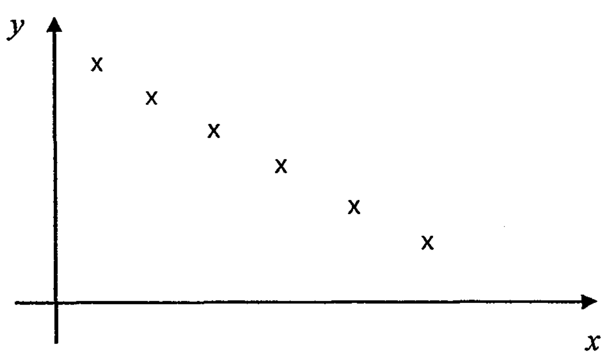


(iv)	As $\arg(-4\sqrt{3}+4i)=\dfrac{5\pi}{6}$, O, C and B are collinear. Hence, maximum $\lvert w-z_n\rvert=\lvert w-z_3\rvert=AB$ $=OC+CA+OB$ $=\sqrt{\left(4\sqrt{3}\right)^2+4^2}+2+2$ $=12$
6(i)	Number the employees from 1 to 40 000. Randomly select 800 numbers using a random number generator. The employees corresponding to these 800 numbers are selected for the survey.
(ii)	The manager can survey the first 400 male employees and first 400 female employees who step into the canteen on a particular day.
7(i)	No of ways $=9\times8\times7\times6$ $=3024$
(ii)	No of ways $=3024-C_4^9$ $=2898$
(iii)	No of ways $=C_1^4\times C_3^5\times4!$ $=960$
(iv)	No of ways$=\sum_{r=1}^{9}r^3$ $=2025$
8(a)	$P(X\cap Y')=P(X)-P(X\cap Y)$ $=P(X)-P(X)\times P(Y)$ since X and Y are ind. $=P(X)[1-P(Y)]$ $=P(X)\times P(Y')$ Since $P(X\cap Y')=P(X)\times P(Y')$, events X and Y' are independent.
(b)(i)	P(A'|B')=0.3 $\dfrac{P(A'\cap B')}{P(B')}=0.3$ $P(A'\cap B')=0.3\times(1-0.6)$ $=0.12$ $P(A\cup B)=1-0.12=0.88$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $P(A\cap B)=0.5+0.6-0.88$ $=0.22$

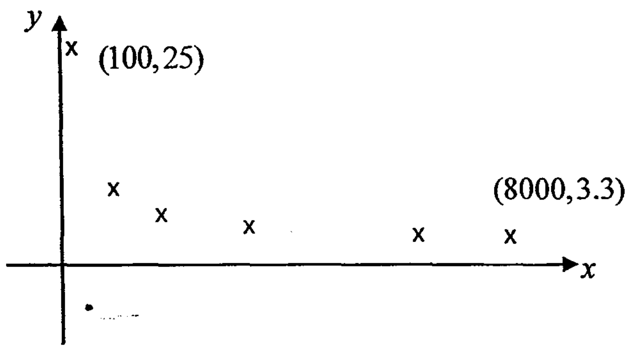
(ii)	$P(B'\cap A)=P(A)-P(A\cap B)$ $=0.5-0.22=0.28$ $P(B'\mid A)=\dfrac{P(B'\cap A)}{P(A)}$ $=\dfrac{0.28}{0.5}=0.56$
(iii)	Since $P(A\cap B)=0.22\neq 0$, events A and B are not mutually exclusive
(iv)	$P(A\cap B)=0.22$ $P(A)\times P(B)=0.5\times 0.6=0.3$ Since $P(A\cap B)\neq P(A)\times P(B)$, events A and B are not independent.
9(i)	(1) The probability of Rickie finding a seat on the train is constant every weekday. (2) The event of Rickie finding a seat on the train on one weekday is independent of the other weekdays.
(ii)	A~B(5, 0.65) $P(A=2 \text{ or } 3)=P(A\leq 3)-P(A\leq 1)$ $=0.51756$ Let X be the number of weeks, out of 52, that Rickie is contented. Then $X\sim B(52,\ 0.51756)$ Since $n=52$ is large, $np=52(0.51756)=26.91312>5$, $nq=52(1-0.51756)=25.08688>5$, $X\sim N(26.91312,\ 52\times 0.51756\times(1-0.51756))$ approx. i.e. $X\sim N(26.91312, 12.98397)$ approx. $P(X\leq 30)\rightarrow P(X<30.5)$ using continuity correction ≈ 0.840 (3 s.f)

10(i) (a)	Let X and Y be the r.v. "number of drug trafficking and cigarette trafficking cases in 8 weeks respectively". Then $X \sim \text{Po}(1.6)$, $Y \sim \text{Po}(5.6)$ $\text{P}(X > 6) = 1 - \text{P}(X \le 6)$ ≈ 0.00134 (3 s.f)
(i) (b)	$X + Y \sim \text{Po}(7.2)$ $\text{P}(X + Y < 5) = \text{P}(X + Y \le 4)$ ≈ 0.156 (3 s.f)
(ii)	Let W be the number of drug trafficking cases in a period of n weeks. Then $W \sim \text{Po}(0.2n)$ $\text{P}(W < 2) < 0.01$ $\text{P}(W \le 1) < 0.01$ $e^{-0.2n} + 0.2n(e^{-0.2n}) < 0.01$ Using GC, If $n = 33$, $\text{P}(W \le 1) = 0.0103 > 0.01$ If $n = 34$, $\text{P}(W \le 1) = 0.00869 < 0.01$ $\therefore$ smallest possible integer value of n is 34. Alternative method: Plot graph Using GC, $n > 33.19176$ $\therefore$ smallest possible integer value of n is 34.
(iii)	The number of drug and cigarette trafficking cases per week may decrease after police's raids so the trafficking cases may not occur with a constant average rate. A person can be both a drug and cigarette trafficker so the drug trafficking cases and cigarette trafficking cases may not be independent.
11(i)	Let A and B be the mass of a randomly chosen towel from Alpha and Bravo respectively. $\text{P}(A > 380) = 0.0668$ $\text{P}(A \le 380) = 0.9332$ $\text{P}(Z \le \frac{380 - \mu}{20}) = 0.9332$ Using GC, $\frac{380 - \mu}{20} = 1.5000556$

	$380-\mu=30.001112$ $\mu=349.998888$ $=350$ (3 s.f)
(ii)	$T=1.6(A_1+A_2+A_3+A_4)+1.5(B_1+B_2)$ $\sim N(1.6\times4\times350+1.5\times2\times275,\ 1.6^2\times4\times20^2+1.5^2\times2\times15^2)$ i.e. $T\sim N(3065,\ 5108.5)$ $P(T>3000)=0.81843...$ ≈ 0.818 (3 s.f)
12(i)	Unbiased estimate of population mean, $\bar{x}=\dfrac{\sum(x-4)}{80}+4$ $=\dfrac{25}{80}+4=4.3125$ Unbiased estimate of population variance, $s^2=\dfrac{1}{80-1}\left(140-\dfrac{25^2}{80}\right)$ $=1.673259...$ ≈ 1.67 (3 s.f)
(ii)	$H_0:\mu=4$ $H_1:\mu>4$ Under H_0, the test statistic is $Z=\dfrac{\bar{X}-\mu}{S/\sqrt{n}}\sim N(0,1)$ approx. (by CLT), where $\mu=4,\ \bar{x}=4.3125,\ s^2=1.6733,\ n=80$ Level of significance=1% Using GC, p-value=0.015357 (5 s.f) Since p-value=0.015357>0.01, we do not reject H_0 and conclude that at the 1% level, there is no sufficient evidence to doubt Burger Queen's claim.
(iii)	Critical value=1.28155 In order not to reject H_0, $\dfrac{\bar{x}-4}{1.5/\sqrt{80}}<1.28155$ $\bar{x}<4.2149...$ Required set = $\{\bar{x}\in\mathbb{R}^+:\bar{x}<4.21\}$

13(i)	(A) $y=a+bx^2$, where a is positive and b is negative  (B) $y=c+\frac{d}{x}$, where both c and d are positive  (C) $y=e+fx$, where e is positive and f is negative 

(ii)



(iii)

Case (B) is the most appropriate because as x increases, y decreases at a decreasing rate.

From GC, $r = 0.994981\ldots$

$$\approx 0.995 \text{ (3s.f)}$$

(iv)

From GC, $y = 3.62896 + \dfrac{2154.915}{x}$

$$y = 3.63 + \frac{2150}{x} \text{ (3 sf)}$$

When $y = 10$, $10 = 3.62896 + \dfrac{2154.915}{x}$

$$x \approx 338 \text{ (3s.f)}$$

The estimate is likely to be reliable as $y = 10$ is within the range of given y values and r value is close to 1.