Preliminary Examinations

MATHEMATICS
Higher 2

9740 / 1

30 August 2016

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

1 In the finals of a General Knowledge Quiz, a team is required to answer 25 questions. Each question that is correctly answered scores 5 points, while a question that is wrongly answered is deducted 3 points. If the answer is partially correct, the team scores 2 points.
After 24 questions, the results are shown in the following table.

Number of Questions			
Correct	**Partially Correct**	**Wrong**	**Points**
a	b	c	79

If the team answers the last question wrongly, then the total number of questions answered correctly and partially correct is four times the number of questions answered wrongly. By forming a system of linear equations, find the values of a, b and c. [4]

2 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = \frac{1}{3}$ and

$$u_{r+1} = u_r - \frac{1}{(2r-1)(2r+3)}, \quad \text{for all } r \geq 1.$$

(i) Use the method of mathematical induction to prove that $u_n = \dfrac{n}{4n^2-1}$. [4]

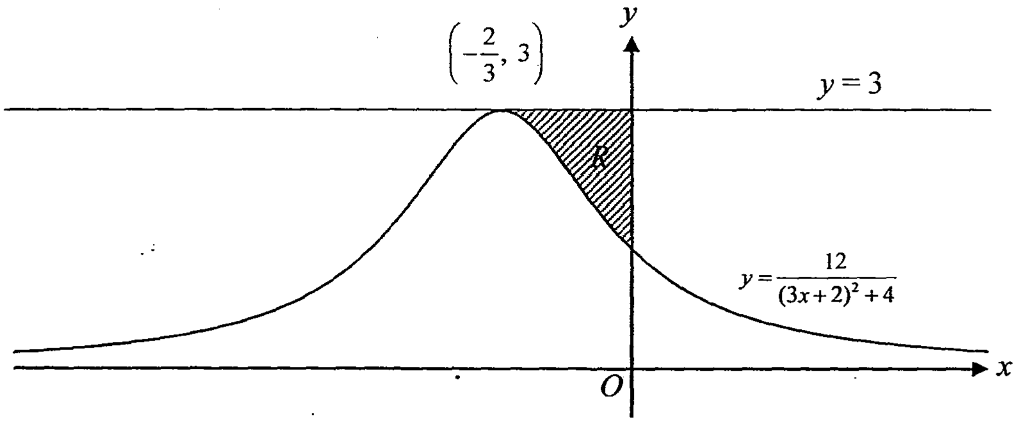
(ii) Hence prove that the sum of the first n terms of the series

$$\frac{1}{5\times 9} + \frac{1}{7\times 11} + \frac{1}{9\times 13} + \cdots$$

is $\dfrac{3}{35} - \dfrac{n+3}{4(n+3)^2-1}$. [3]

(iii) Give a reason why the series in part **(ii)** is convergent and state the sum to infinity. [2]

3 The diagram shows the curve C with equation $y=\dfrac{12}{(3x+2)^2+4}$ which has a turning point at $\left(-\dfrac{2}{3},\ 3\right)$. The region R is bounded by C, the y-axis and the line $y=3$.



(i) Find the exact area of R. [5]

(ii) R is rotated through 2π radians about the y-axis. Find the volume of the solid of revolution formed, giving your answer to 4 decimal places. [3]

4 Let $y=\tan\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)$.

(i) Show that $\left(1+x^2\right)\dfrac{dy}{dx}=2\left(1+y^2\right)$. [2]

(ii) Hence find the Maclaurin series for y, up to and including the term in x^2. [4]

Denote the answer to part **(ii)** of the Maclaurin series by $g(x)$ and $f(x)=\tan\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)$.

(iii) Find, for $-0.4\le x\le 0.4$, the set of values of x for which the value of $g(x)$ is within ± 0.5 of the value of $f(x)$. [2]

5 A curve C has parametric equations

$$x = 2\sin 2t, \qquad y = \cos 2t, \qquad \text{for } 0 \leq t < \pi.$$

(i) Show that the equation of the normal to C at the point P with parameter θ is

$$(2\cos 2\theta)x - (\sin 2\theta)y = m\sin 2\theta \cos 2\theta,$$

where m is an integer to be determined. [3]

(ii) The normal to C at the point P cuts the x-axis and y-axis at points A and B respectively. By finding the mid-point of AB, determine a cartesian equation of the locus of the mid-point of AB as θ varies. [5]

6 A function f is said to be self-inverse if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

The function g is defined by

$$\mathrm{g} : x \mapsto \sqrt{\frac{x^2+2}{x^2-1}}, \quad x > 1.$$

(i) Sketch the curve $y = \mathrm{g}(x)$, stating the equations of the asymptotes clearly. [2]

(ii) Define g^{-1} in a similar form and show that g is self-inverse. [4]

(iii) Show that $\mathrm{g}^2(x) = x$ and that $\mathrm{g}^3(x) = \mathrm{g}(x)$. Hence find the values of x for which

$$4 - \mathrm{g}^{50}(x) = \left[\mathrm{g}^{51}(x)\right]^2.$$ [4]

7 (a) Find $\int \frac{\cos(\ln x)}{x^2}\,\mathrm{d}x$. [4]

(b) Using the substitution $u = \sqrt{x+3}$, find $\int_1^6 \frac{x-2}{x\sqrt{x+3}}\,\mathrm{d}x$, giving your answer in the form

$$a + \frac{b}{\sqrt{3}}\ln\left(\frac{c-\sqrt{d}}{c+\sqrt{d}}\right),$$

where a, b, c and d are constants to be determined. [6]

8 The complex number z satisfies the following inequalities:

$$|z|^2 \leq 4 \quad \text{and} \quad -\frac{\pi}{6} \leq \arg(z+\sqrt{3}-\mathrm{i}) \leq 0.$$

(i) On an Argand diagram, sketch the region R in which the point representing z can lie. [4]

(ii) Find exactly the minimum and maximum possible values of $|z-2\mathrm{i}|$. [3]

(iii) Determine the number of roots of the equation $z^{100} = 2^{100}$ that lie in the region R. [3]

9 It is given that $\mathrm{f}(x) = x + \dfrac{m^2}{x-2}$, where $0 < m < 1$.

(i) Sketch the graph of $y = \mathrm{f}(x)$, showing clearly the coordinates of the turning points and the equation(s) of any asymptote(s). [5]

(ii) By inserting a suitable graph to your sketch in **(i)**, find the set of values of k, in terms of m, for which the equation $x^2 - (2+k)x + (m^2 + 2k) = 0$ has two distinct positive roots. [4]

(iii) The curve $y = \mathrm{f}(x)$ undergoes the transformations A, B and C in succession:

A: A translation of -2 units in the direction of x-axis,

B: A stretch parallel to the x-axis with scale factor of $\dfrac{1}{2}$, and

C: A translation of -2 units in the direction of y-axis.

Given that the resulting curve is $y = 2x + \dfrac{1}{8x}$, find the value of m. [2]

10 The point A has position vector $\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, where $\lambda \in$ □ .

(i) Find the position vector of the foot of the perpendicular from A to l. [4]

(ii) Show that a cartesian equation of the plane π_1 which contains A and l is

$$x + y + 2z = 1.$$ [2]

The equation of the plane π_2 is $x + 7z = c$, where c is a constant.

(iii) Given that π_1 and π_2 intersect in a line L, show that a vector equation of L is

$$\mathbf{r} = \begin{pmatrix} c \\ 1-c \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix}, \text{ where } \mu \in \text{□} .$$ [2]

Another plane π_3 has equation $2x - y + dz = 5$, where d is a constant.

(iv) Find the values of c and/or d if all three planes π_1, π_2 and π_3

(a) meet in the line L, [3]

(b) have only one point in common. [1]

11 At the beginning of May 2016, Sam borrowed \$50 000 from a bank that charges him a special rate of 0.2% interest at the end of every month. Sam pays back \$1 000 for every instalment at the beginning of every month, starting from June 2016.

(i) Show that the total amount with interest that Sam still owes the bank at the end of the month after the nth instalment is paid is

$$\$\left[50\,000(1.002^{n+1}) - 501\,000(1.002^{n} - 1)\right].$$ [4]

(ii) Find the number of instalments required for Sam to settle all the amount owed. [2]

(iii) How much does he pay on his last instalment? [2]

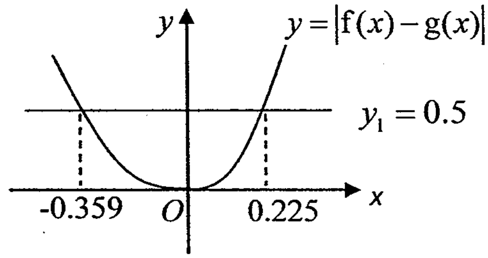
(iv) If Sam wishes to settle all the amount owed after paying 19 instalments, what is the minimum amount (to the nearest dollar) he should pay each month? [2]

2016 JC2 H2 Mathematics Prelim Paper 1 Solutions

Qn	Solution
1	$a+b+c=24$ $5a+2b-3c=79$ $a+b=4(c+1)$ $a=17, b=3, c=4$
2(i)	Let P_n be the statement that $u_n=\dfrac{n}{4n^2-1}$ for all $n\in\mathbb{Z}^+$ When $n=1$, LHS = $u_1=\dfrac{1}{3}$ (given) RHS $=\dfrac{1}{4\times1^2-1}=\dfrac{1}{3}$ Hence P_1 is true.
	Assume P_k is true for some $k\in\mathbb{Z}^+$, i.e. $u_k=\dfrac{k}{4k^2-1}$. We want to prove that P_{k+1} is true, i.e. $u_{k+1}=\dfrac{k+1}{4(k+1)^2-1}$ LHS $=u_{k+1}$ $=u_k-\dfrac{1}{(2k-1)(2k+3)}$ $=\dfrac{k}{(2k-1)(2k+1)}-\dfrac{1}{(2k-1)(2k+3)}$ $=\dfrac{k(2k+3)-(2k+1)}{(2k-1)(2k+1)(2k+3)}$ $=\dfrac{2k^2+k-1}{(2k-1)(2k+1)(2k+3)}$ $=\dfrac{(2k-1)(k+1)}{(2k-1)(2k+1)(2k+3)}$ $=\dfrac{k+1}{(2k+1)(2k+3)}$ $=\dfrac{k+1}{4k^2+8k+3}$ $=\dfrac{k+1}{4(k+1)^2-1}=$ RHS Hence P_k is true $\Rightarrow P_{k+1}$ is true. Since P_1 is true & P_k is true $\Rightarrow P_{k+1}$ is true, by mathematical induction, P_n is true for all $n\in\mathbb{Z}^+$.

(ii)	Sum of 1st n terms of $\frac{1}{5\times9}+\frac{1}{7\times11}+\frac{1}{9\times13}+\cdots\cdots\cdots+\frac{1}{(2n+3)(2n+7)}$ $=\sum_{r=3}^{n+2}\frac{1}{(2r-1)(2r+3)}$ $=\sum_{r=3}^{n+2}[u_r-u_{r+1}]$ $= u_3-u_4$ $+u_4-u_5$ $+u_5-u_6$ $+\cdots\cdots\cdots$ $+\cdots\cdots\cdots$ $+u_{n+2}-u_{n+3}$ $=u_3-u_{n+3}$ $=\frac{3}{35}-\frac{n+3}{4(n+3)^2-1}$.
(iii)	As $n\to\infty, \frac{n+3}{4(n+3)^2-1}\to 0$ Hence, the series is convergent and $\sum_{r=3}^{\infty}\frac{1}{(2r-1)(2r+3)}=\frac{3}{35}$
3 (i)	Required Area $=\int_{-\frac{2}{3}}^{0}\left(3-\frac{12}{(3x+2)^2+4}\right)\mathrm{d}x$ $=\int_{-\frac{2}{3}}^{0}\left(3-\frac{12}{(3x+2)^2+2^2}\right)\mathrm{d}x$ $=\left[3x-2\tan^{-1}\left(\frac{3x+2}{2}\right)\right]_{-\frac{2}{3}}^{0}$ $=0-2\left(\frac{\pi}{4}\right)-[-2-0]$ $=2-\frac{\pi}{2}$

(ii)	$y=\dfrac{12}{(3x+2)^2+4} \quad \Rightarrow \quad 3x=-2\pm\sqrt{\dfrac{12}{y}-4}=-2\pm\sqrt{\dfrac{12-4y}{y}}$ Since $x\geq-\dfrac{2}{3}$, $x=-\dfrac{2}{3}+\dfrac{1}{3}\sqrt{\dfrac{12-4y}{y}}$ Required volume = $\pi\int_{\frac{3}{2}}^{3}x^2\ \mathrm{d}y$ $=\pi\int_{\frac{3}{2}}^{3}\left(-\dfrac{2}{3}+\dfrac{1}{3}\sqrt{\dfrac{12-4y}{y}}\right)^2\ \mathrm{d}y$ $=0.5125$
4(i)	$y=\tan\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec^2\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)\dfrac{2}{1+x^2}$ $\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\left(1+\tan^2\left(2\tan^{-1}x+\dfrac{\pi}{4}\right)\right)$ $\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\left(1+y^2\right)$
	<u>Alternative Method</u> $\tan^{-1}y=2\tan^{-1}x+\dfrac{\pi}{4}$ Differentiate with respect to x, $\dfrac{1}{1+y^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2}{1+x^2}$ $\Rightarrow\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\left(1+y^2\right)$
(ii)	Differentiate with respect to x, $\Rightarrow\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x\dfrac{\mathrm{d}y}{\mathrm{d}x}=4y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2x-4y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ When $x=0$, $y=\tan\dfrac{\pi}{4}=1$ $(1+0)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2(1+1)\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=4$ $(1+0)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+(0-4)(4)=0\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=16$ $y=\tan\left[2\tan^{-1}x+\dfrac{\pi}{4}\right]$ $=1+4x+\dfrac{16}{2!}x^2+.....$ $=1+4x+8x^2+....$

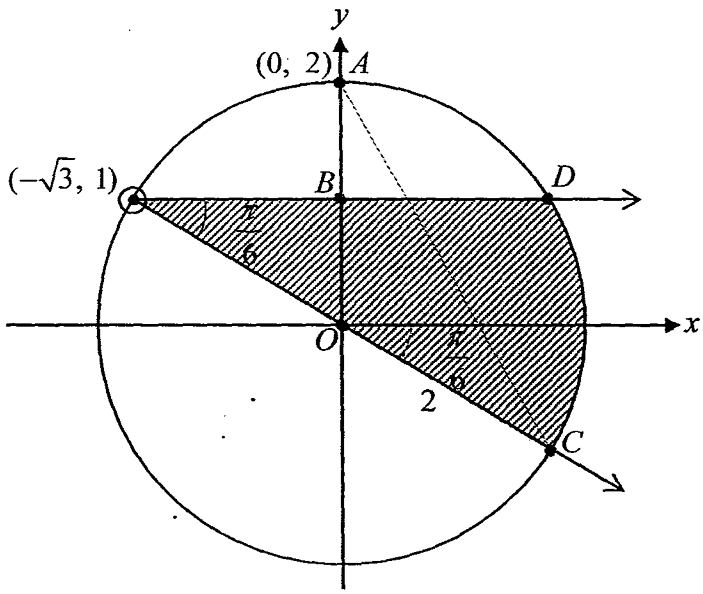
(iii)	Sketch $y=\lvert f(x)-g(x)\rvert$ and $y_1=0.5$ 
	For $\lvert f(x)-g(x)\rvert<0.5$, $\quad -0.359<x<0.225$
5(i)	$x=2\sin 2t$, $\qquad y=\cos 2t$, $\qquad$ for $0\le t<\pi$. $\frac{dx}{dt}=4\cos 2t \qquad \frac{dy}{dt}=-2\sin 2t$ $\frac{dy}{dx}=-\frac{\sin 2t}{2\cos 2t}$ Equation of normal at P, $t=\theta$: $$y-\cos 2\theta=\frac{2\cos 2\theta}{\sin 2\theta}(x-2\sin 2\theta)$$ $(\sin 2\theta)y-\cos 2\theta\sin 2\theta=(2\cos 2\theta)x-4\cos 2\theta\sin 2\theta$ $(2\cos 2\theta)x-(\sin 2\theta)y=3\cos 2\theta\sin 2\theta$ (shown) i.e. $m=3$
(ii)	At the x-axis, $y=0$ $(2\cos 2\theta)x=3\sin 2\theta\cos 2\theta$ $x=\frac{3}{2}\sin 2\theta \quad$ i.e. $A\left(\frac{3}{2}\sin 2\theta,\ 0\right)$ At the y-axis, $x=0$ $-(\sin 2\theta)y=3\sin 2\theta\cos 2\theta$ $y=-3\cos 2\theta \quad$ i.e. $B(0,\ -3\cos 2\theta)$ mid-point of AB: $\left(\frac{3}{4}\sin 2\theta,-\frac{3}{2}\cos 2\theta\right)$ $x=\frac{3}{4}\sin 2\theta \Rightarrow \sin 2\theta=\frac{4}{3}x$ $y=-\frac{3}{2}\cos 2\theta \Rightarrow \cos 2\theta=-\frac{2}{3}y$ Cartesian equation of the locus of the mid-point of AB: $\sin^2 2\theta+\cos^2 2\theta=1$ $\frac{16x^2}{9}+\frac{4y^2}{9}=1 \quad$ i.e. $16x^2+4y^2=9$

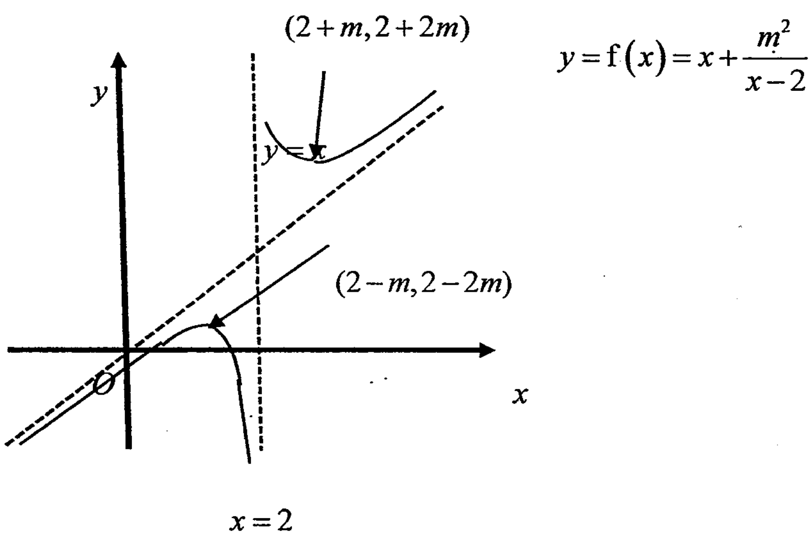
Qn	Solution
6(i)	y $y=1$ x O $x=1$
(ii)	Let $y=\mathrm{g}(x)=\sqrt{\frac{x^2+2}{x^2-1}},\quad x>1$ Then $y^2=\frac{x^2+2}{x^2-1}=1+\frac{3}{x^2-1}$ $y^2-1=\frac{3}{x^2-1}$ $x^2-1=\frac{3}{y^2-1}$ $x^2=1+\frac{3}{y^2-1}=\frac{y^2+2}{y^2-1}$ $x=\mathrm{g}^{-1}(y)=\sqrt{\frac{y^2+2}{y^2-1}}$ since $x>1>0$ $\Rightarrow \mathrm{g}^{-1}: x\mapsto\sqrt{\frac{x^2+2}{x^2-1}}\quad, x>1$ g is self-inverse as $\mathrm{g}(x)=\mathrm{g}^{-1}(x)$ and $D_{\mathrm{g}^{-1}}=D_{\mathrm{g}}$
(iii)	$\mathrm{g}^2(x)=\mathrm{gg}(x)=\mathrm{gg}^{-1}(x)=x.$ $\mathrm{g}^3(x)=\mathrm{gg}^2(x)=\mathrm{g}(x).$ (shown) It follows that $\mathrm{g}^{50}(x)=x$ and $\mathrm{g}^{51}(x)=\mathrm{g}(x),\quad x>1$ For $4-\mathrm{g}^{50}(x)=\left[\mathrm{g}^{51}(x)\right]^2$ Then $4-x=\frac{x^2+2}{x^2-1},\quad x>1$ $(4-x)(x^2-1)=x^2+2,\quad x>1$ $x^3-3x^2-x+6=0,\quad x>1$ $(x-2)(x^2-x-3)=0$ $x=2$ or $x=\frac{1\pm\sqrt{1+12}}{2}$ since $x>1,\quad\Rightarrow x=2$ or $x=\frac{1+\sqrt{13}}{2}$ (ans)

Qn	Solution
7 (a)	$u = \cos(\ln x) \qquad \dfrac{dv}{dx} = \dfrac{1}{x^2}$ $\dfrac{du}{dx} = -\dfrac{\sin(\ln x)}{x} \qquad v = -\dfrac{1}{x}$ $\displaystyle\int \frac{\cos(\ln x)}{x^2}\,dx = -\frac{\cos(\ln x)}{x} - \int \frac{\sin(\ln x)}{x^2}\,dx$ ⟶ $\displaystyle = -\frac{\cos(\ln x)}{x} + \frac{\sin(\ln x)}{x} - \int \frac{\cos(\ln x)}{x^2}\,dx$
	$\displaystyle\Rightarrow\ 2\int \frac{\cos(\ln x)}{x^2}\,dx = \frac{\sin(\ln x)}{x} - \frac{\cos(\ln x)}{x}$ $\displaystyle\therefore\ \int \frac{\cos(\ln x)}{x^2}\,dx = \frac{1}{2x}\left[\sin(\ln x) - \cos(\ln x)\right] + c$
(b)	$u = \sqrt{x+3} \quad \Rightarrow \quad u^2 = x+3$ Differentiating w.r.t. x, $2u\dfrac{du}{dx} = 1$ When $x = 1$, $u = 2$; When $x = 6$, $u = 3$ $\displaystyle\int_1^6 \frac{x-2}{x\sqrt{x+3}}\,dx = \int_2^3 \frac{u^2-5}{(u^2-3)u}\,(2u\,du)$ $\displaystyle = 2\int_2^3 \left(1 - \frac{2}{u^2-3}\right) du$ $\displaystyle = \left[2u - \frac{4}{2\sqrt{3}}\ln\left(\frac{u-\sqrt{3}}{u+\sqrt{3}}\right)\right]_2^3$ $\displaystyle = 2(3) - \frac{2}{\sqrt{3}}\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right) - 2(2) + \frac{2}{\sqrt{3}}\ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)$ $\displaystyle = 2 + \frac{2}{\sqrt{3}}\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)$ i.e. $a = b = 2$, $c = d = 3$

$$u = \sin(\ln x) \qquad \frac{dv}{dx} = \frac{1}{x^2}$$

$$\frac{du}{dx} = \frac{\cos(\ln x)}{x} \qquad v = -\frac{1}{x}$$

Qn	Solution
8 (i)	$\lvert z\rvert^2 \le 4 \Rightarrow \lvert z\rvert \le 2$ $-\frac{\pi}{6} \le \arg(z+\sqrt{3}-\mathrm{i}) \le 0 \Rightarrow -\frac{\pi}{6} \le \arg[z-(-\sqrt{3}+\mathrm{i})] \le 0$ 
(ii)	Minimum value of $\lvert z-2\mathrm{i}\rvert$ $= AB$ $= 1$ Maximum value of $\lvert z-2\mathrm{i}\rvert$ $= AC$ $= \sqrt{2^2+2^2-2(2)(2)\cos\frac{2\pi}{3}}$ $= 2\sqrt{3}$
(iii)	$z^{100} = 2^{100} = 2^{100}\mathrm{e}^{\mathrm{i}0}$ $\Rightarrow z = 2\mathrm{e}^{\mathrm{i}\left(\frac{0+2k\pi}{100}\right)}, \ k = 0, \ \pm 1, \ \pm 2, \ \ldots, \ \pm 49, \ 50$ $\Rightarrow z = 2\mathrm{e}^{\mathrm{i}\frac{k\pi}{50}}$ Roots are found in region R (along the minor arc CD) if $-\frac{\pi}{6} \le \frac{k\pi}{50} \le \frac{\pi}{6}$. $\Rightarrow -8\frac{1}{3} \le k \le 8\frac{1}{3}$ $\Rightarrow k = -8, \ -7, \ -6, \ \ldots, \ 8$ $\therefore$ Number of roots found in region $R = 17$.

Qn	Solution
9(i)	$\mathrm{f}(x)=x+\dfrac{m^2}{x-2}$ Let $\dfrac{\mathrm{df}}{\mathrm{d}x}=1-\dfrac{m^2}{(x-2)^2}=0$ $(x-2)^2-m^2=0$ $x=2\pm m$ When $x=2+m$, $\mathrm{f}(x)=2+m+\dfrac{m^2}{2+m-2}=2+2m$ When $x=2-m$, $\mathrm{f}(x)=2-m+\dfrac{m^2}{2-m-2}=2-2m$ The stationary points are $(2+m,2+2m)$ and $(2-m,2-2m)$
	
(ii)	$\mathrm{f}(x)=\dfrac{x(x-2)+m^2}{x-2}$ when $x=0$, $\mathrm{f}(x)=-\dfrac{m^2}{2}$ $x^2-(2+k)x+(m^2+2k)=0$ $x^2-2x+m^2=k(x-2)$ $x+\dfrac{m^2}{x-2}=k$ By inserting a horizontal line $y=k$ on the graph of C, by observation, to have two distinct positive roots, then $-\dfrac{m^2}{2}<k<2-2m$ or $k>2+2m$ (ans)

(iii)	$y = \mathrm{f}(x) = x + \frac{m^2}{x-2}$ After A: $y = \mathrm{f}(x+2) = x + 2 + \frac{m^2}{x}$ After B: $y = \mathrm{f}(2x+2) = 2x + 2 + \frac{m^2}{2x}$ After C: $y = \mathrm{f}(2x+2) - 2 = 2x + \frac{m^2}{2x}$ Given that $2x + \frac{m^2}{2x} = 2x + \frac{1}{8x}$ $\Rightarrow \frac{m^2}{2} = \frac{1}{8}$ $\Rightarrow m = \frac{1}{2}$ since $0 < m < 1$ (ans)

Qn	**Solution**
10 (i)	Let F be the foot of the perpendicular. $\overrightarrow{AF} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$ $\Rightarrow \left[\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$ $\Rightarrow -2 + 3\lambda - 1 = 0$ $\Rightarrow \lambda = 1$ $\therefore \quad \overrightarrow{OF} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$.
(ii)	Let B be $(2, -3, 1)$. A normal to π_1 is $\overrightarrow{BA} \times \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. $\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 1$ A cartesian equation of π_1 is $x + y + 2z = 1$.

(iii)	$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ -1 \end{pmatrix}$ A direction vector of L is $\begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix}$. $\pi_1: x+y+2z=1$ $\pi_2: x+7z=c$ Let $z=0$. Then $x=c$ and $y=1-c$. A point on L is $(c,\ 1-c,\ 0)$. $\therefore$ A vector equation of L is $\mathbf{r} = \begin{pmatrix} c \\ 1-c \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix}$, where $\mu \in \mathbb{R}$.
(iv) **(a)**	For the 3 planes to meet in the line L, $\begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ d \end{pmatrix} = 0$ and $\begin{pmatrix} c \\ 1-c \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ d \end{pmatrix} = 5$. $\Rightarrow -14-5+d=0$ and $2c-1+c=5$ $\Rightarrow d=19$ and $c=2$
(b)	For the 3 planes to have only one point in common, $\begin{pmatrix} -7 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ d \end{pmatrix} \neq 0$. $\Rightarrow d \neq 19$

Qn	Solution

11 (i)

Instalment	Outstanding amount at the beginning of month	Total amount with interest owed at the end of month
	50000	50000×1.002
1	$50000\times1.002-1000$	$50000\times1.002^2-1000\times1.002$
2	50000×1.002^2 $-1000\times1.002-1000$	$50000\times1.002^3-1000\times1.002^2$ -1000×1.002
....	....	....

Amount owed at the end of n instalments

$$=50000\times1.002^{n+1}-1000\times1.002^{n}-1000\times1.002^{n-1}-...-1000\times1.002$$

$$=50000\times1.002^{n+1}-\frac{1000\times1.002\times\left[1.002^{n}-1\right]}{1.002-1}$$

$$=50000\times1.002^{n+1}-1000\times501\times(1.002^{n}-1) \quad \text{------------(*)}$$

$$=50000\times1.002^{n+1}-501000\times(1.002^{n}-1) \text{ (shown)}$$

(ii)

$$50000\times1.002^{n+1}-501000\times(1.002^{n}-1)\leq0$$

By using G.C., least integer $n=53$

i.e. no of instalments required = 53

(iii)

Amount paid at the 53th instalment

= Amount owed at the end of 52 instalments

$$=50000\times1.002^{53}-501000\times\left(1.002^{52}-1\right)$$

=733.12 (to 2 d.p.)

(iv)

Let the amount need to be paid for each instalment be k.

Then from (*) in (i)

$$50000\times1.002^{20}-k\times501\left[1.002^{19}-1\right]\leq0$$

$$k\geq\frac{50000\times1.002^{20}}{501\left[1.002^{19}-1\right]}=2684.53$$

Least value of $k=2685$ (to the nearest dollars)

Preliminary Examinations

MATHEMATICS
Higher 2

9740 / 2

31 August 2016

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

Section A: Pure Mathematics [40 marks]

1 **(a)** Given that x and y are related by $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 y$ and that $y = 0$ when $x = 1$, find x in terms of y. [4]

(b) A medical researcher is investigating the rate of spread of a virus in a group of people of size n at time t weeks. He suggests that n and t are related by the differential equation $\frac{\mathrm{d}^2 n}{\mathrm{d}t^2} = \mathrm{e}^{-\frac{t}{5}}$.

(i) Find the general solution of the differential equation, giving your answer in the form $n = \mathrm{f}(t)$. [2]

(ii) Explain why all solution curves of the differential equation are concave upwards. [1]

(iii) It is given that initially, the number of people infected with the virus is 50. Sketch on a single diagram, two distinct solution curves for the differential equation to illustrate the following two cases for large values of t:

I. the population of infected people increases indefinitely,

II. the population of infected people stabilizes at a certain positive number.

[3]

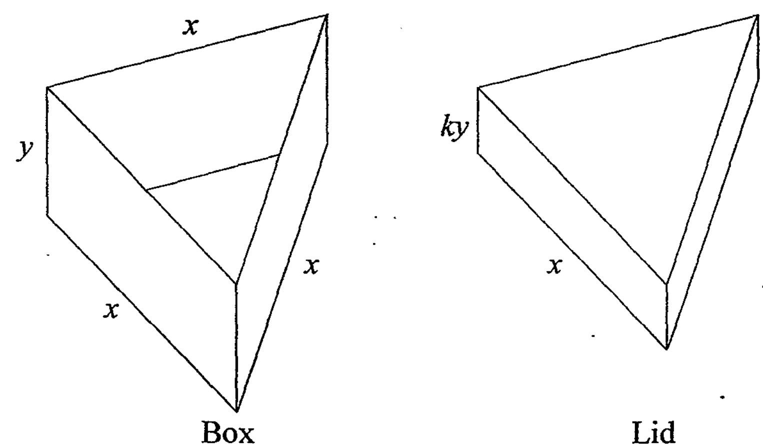
2 **(a)** A parallelogram has two adjacent sides defined by the vectors $\mathbf{a}$ and $2\mathbf{a} + 3\mathbf{b}$. Given that the magnitudes of $\mathbf{a}$ and $\mathbf{b}$ are 4 and 5 respectively and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $30°$, find the area of the parallelogram. [4]

(b) A point P has coordinates $(2, -1, -2)$ and a line l has equation $\frac{x-1}{2} = 1 - z, \; y = 3$.

(i) Find the perpendicular distance from P to l. [4]

(ii) Find the acute angle between l and the line L that is parallel to the z-axis. [2]

3 A box with volume 250 cm^3 is made of cardboard of negligible thickness. It has a height of y cm and an equilateral triangular base of side x cm. Its lid has depth ky cm, where $0 < k \leq 1$ (see diagram).



(i) Show that the total external surface area of the box and lid can be expressed as

$$\frac{1000\sqrt{3}(1+k)}{x}+\frac{\sqrt{3}}{2}x^2.$$ [4]

(ii) Use differentiation to find, in terms of k, the value of x that gives a minimum total external surface area of the box and lid. [3]

(iii) Find the ratio $\frac{y}{x}$ in this case, in terms of k, simplifying your answer. [2]

(iv) Find the values for which $\frac{y}{x}$ must lie. [2]

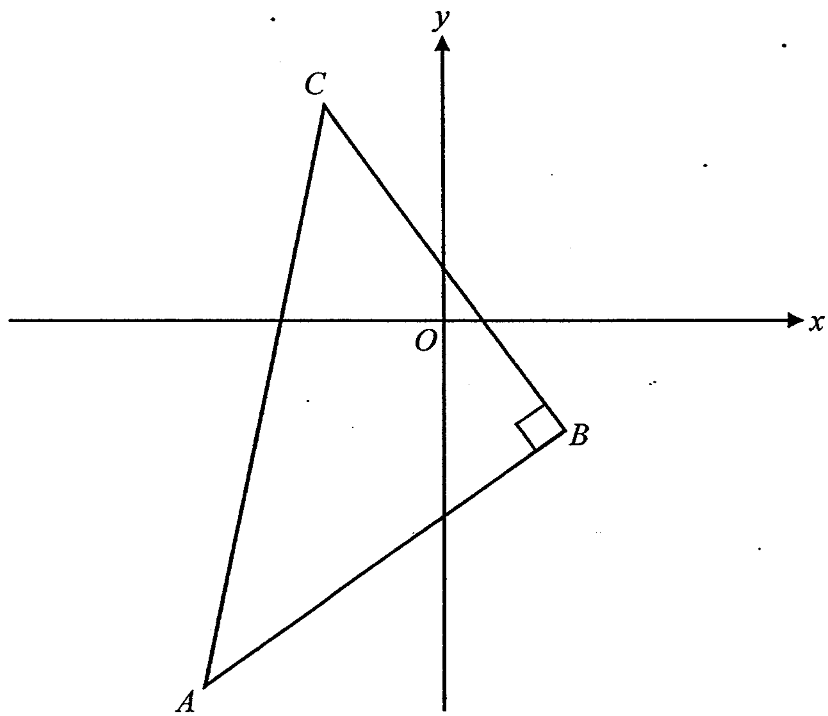
4 The complex numbers a and b are given by $a = -(1+\sqrt{3}\mathrm{i})$ and $b = \frac{1}{2}(1-\mathrm{i})$.

(i) Without using a calculator, find the value of a^2b in the form $x + \mathrm{i}y$. [2]

(ii) By using the moduli and arguments of a and b, find the modulus and argument of a^2b. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to show that $\sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$. [2]

(iv) The diagram below shows an isosceles right triangle ABC, where the points A, B and C represent the complex numbers a, b and c respectively. Find the exact value of c. [2]



Section B: Statistics [60 marks]

5 A group of 11 people consists of 6 men and 5 women, 3 of whom are sisters. A committee consisting of six people is to be selected. Find the number of ways the committee can be formed if

(i) it consists of exactly two men, [1]

(ii) it includes at least one of the sisters. [2]

Given that the chosen committee consists of 2 sisters, Sue and Suzy, together with 3 other men, Muthu, Mark, Michael and 1 other woman, Wina. They are seated at a round table meant for six people. Find the number of possible arrangements if

(iii) one of the men is to be seated between the two sisters, [2]

(iv) the two sisters are sitting directly opposite each other. [2]

6 The table below shows the number of male and female students studying Chemistry, Physics and Biology at a private school.

	Chemistry	**Physics**	**Biology**
Male	200	130	70
Female	250	300	50

One of the students is chosen at random. Events C, B and M are defined as follows:

C : The student chosen is studying Chemistry.

B : The student chosen is studying Biology.

M : The student chosen is a male.

Find

(i) $\text{P}(C \mid M)$, [1]

(ii) $\text{P}(M \cup C)$, [1]

(iii) $\text{P}(M' \cap B')$. [1]

Determine whether C and M are independent. [2]

It is given that 20% of Chemistry students, 30% of Physics students and 5% of Biology students are international students.

(iv) One of the students selected at random is an international student. What is the probability that this student studies Chemistry? [2]

(v) Three students are chosen at random. Find the probability that there is exactly one international student who studies Physics. [2]

7 In order to investigate whether there is a correlation between rainfall and crop yields, the total rainfall, x mm, and the weights of a particular crop per square metre, y kg, were recorded in a number of fields. The data are shown below.

x	36	72	44	74	64	50
y	2.2	8.4	1.8	7.4	4.3	2.2

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between x and y is $y = a + bx$. [2]

(iii) By comparing the product moment correlation coefficients, explain whether $y = a + bx$ or $y = c + dx^2$ is a better model. [2]

(iv) Using a suitable regression line, estimate the yield of crop per square metre when the total rainfall is 55mm. Comment on the reliability of your estimation. [3]

8 It is known that 8% of the population of a large city use a particular web browser called Voyager. A researcher wishes to interview people from the city who use Voyager and selects people at random, one at a time.

(i) Find the probability that the first person that he finds uses Voyager is the third person selected. [2]

A random sample of n people is now selected.

(ii) State two conditions needed for the number of people in the sample who use Voyager

to be well modelled by binomial distribution. [2]

(iii) Given that $n = 80$, use a suitable approximation to find the probability that, fewer than 10 people use Voyager. [3]

(iv) Find the least value of n such that the probability of at least 10 people use Voyager is more than 0.2. [3]

9 A supermarket sells boxes of a particular brand of biscuits in two flavours, chocolate and strawberry. The mean number of boxes of chocolate biscuits sold in a day is 2.2.

(i) Find the probability that in a day, no boxes of chocolate biscuits were sold. [1]

(ii) In a week of 7 days, find the expected number of days that no boxes of chocolate biscuits were sold. [2]

The mean number of boxes of strawberry biscuits sold in a day is denoted by λ.

(iii) Given that the probability of less than 2 boxes of strawberry biscuits sold in a day is 0.6, write down an equation for the value of λ, and find λ numerically, correct to 1 decimal place. [3]

(iv) Find the probability that in a week of 7 days, the total number of boxes of chocolate and strawberry biscuits sold exceeds 25 boxes. [2]

(v) Use a suitable approximation to find the probability that, in a month of 30 days, the number of boxes of chocolate biscuits sold is more than the number of boxes of strawberry biscuits. [4]

10 A researcher is running a trial of a new variety of potato. A field contains 20 rows of the new variety of potato plants, with 80 plants in each row. A researcher intends to dig up 8 plants and measure the mass of potatoes produced by each plant.

(i) Describe how he could choose a systematic sample of 8 plants from a single row of 80 plants and state the advantage of this sampling method. [3]

The researcher claims that the average mass of the new variety of potato is at least 150g. The mass of a new variety of potato is denoted by X grams. The masses of a random sample of 80 new variety potatoes are summarized by

$$\sum(x-150)=-160, \ \sum(x-150)^2=5520.$$

(ii) Calculate the unbiased estimates of the population mean and variance. [2]

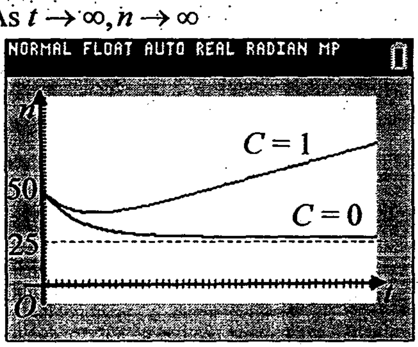
(iii) Test at the 1% significance level, whether the researcher's claim is valid. [4]

(iv) Explain what you understand by the phrase "at the 1% significance level" in the context of this question. [1]

Another random sample of 8 potatoes was chosen with mean mass 148.5g and standard deviation k g. Find the range of values that k can take such that at 1% level of significance, this sample would indicate that the researcher's claim is invalid. [3]

BLANK PAGE

2016 JC2 H2 Mathematics Prelim Paper 2 Solutions

Qn	Solution
1(a)	$\frac{dy}{dx}=\sec^2 y$ $\int \cos^2 y \, dy=\int 1 \, dx$ $\int \frac{\cos 2y+1}{2} \, dy=\int 1 \, dx$ $\frac{1}{2}\left[\frac{\sin 2y}{2}+y\right]=x+c$ When $y=0$, $x=1 \Rightarrow c=-1$ $\therefore x=\frac{1}{4}\sin 2y+\frac{1}{2}y+1$
1(b) (i)	$\frac{d^2n}{dt^2}=e^{-\frac{t}{5}}$ $\frac{dn}{dt}=\int e^{-\frac{t}{5}}dt=-5e^{-\frac{t}{5}}+C$ $n=25e^{-\frac{t}{5}}+Ct+D$
1(b) (ii)	$\frac{d^2n}{dt^2}=e^{-\frac{t}{5}}>0$ for all values of t. Solution curves are concave upwards.
1(b) (iii)	When $t=0, n=50$ $50=25e^0+C(0)+D$ $D=25$ $n=25e^{-\frac{t}{5}}+Ct+25$ When $C=0$, $n=25e^{-\frac{t}{5}}+25$. As $t\to\infty, n\to 25$ When $C=1$, $n=25e^{-\frac{t}{5}}+t+25$. As $t\to\infty, n\to\infty$ 

Qn	Solution
2(a)	Area of parallelogram $= \lvert \mathbf{a} \times (2\mathbf{a} + 3\mathbf{b}) \rvert$ $= \lvert 2(\mathbf{a} \times \mathbf{a}) + 3(\mathbf{a} \times \mathbf{b}) \rvert$ $= 3\lvert \mathbf{a} \times \mathbf{b} \rvert$ $= 3\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \sin 30°$ $= 3(4)(5)\frac{1}{2}$ $= 30$
(b) (i)	A vector equation of l is $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$. Let A be the point $(1,\ 3,\ 1)$ on l. Perpendicular distance from P to l $= \dfrac{\left\lvert \overrightarrow{AP} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\rvert}{\left\lvert \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\rvert}$ $= \frac{1}{\sqrt{5}} \left\lvert \begin{pmatrix} 1 \\ -4 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\rvert$ $= \frac{1}{\sqrt{5}} \left\lvert \begin{pmatrix} 4 \\ -5 \\ 8 \end{pmatrix} \right\rvert$ $= \frac{\sqrt{105}}{\sqrt{5}} = \sqrt{21}$
(ii)	Acute angle between l and L $= \cos^{-1} \dfrac{\left\lvert \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rvert}{\left\lvert \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rvert}$ $= \cos^{-1} \frac{1}{\sqrt{5}}$ $= 63.4°$

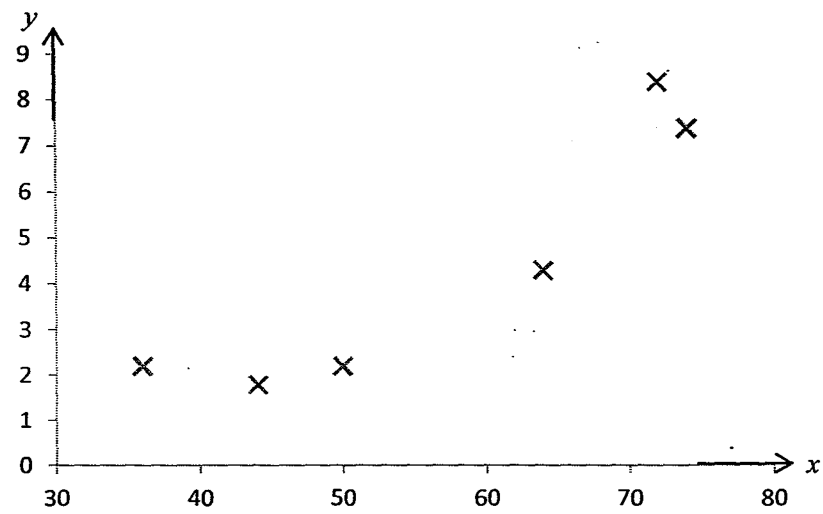
Qn	Solution
3(i)	Area of equilateral $\Delta = \frac{1}{2}x^2 \sin 60° = \frac{\sqrt{3}}{4}x^2$ Given that the volume of the box is 250 cm^3 $V = \frac{\sqrt{3}}{4}x^2 y = 250$ $y = \frac{1000}{\sqrt{3}x^2}$ Surface Area $A = 3xy + 3kxy + 2\left(\frac{\sqrt{3}}{4}x^2\right)$ $= 3xy(1+k) + \frac{\sqrt{3}}{2}x^2$ $= 3x(1+k)\frac{1000}{\sqrt{3}x^2} + \frac{\sqrt{3}}{2}x^2$ $= \frac{1000\sqrt{3}(1+k)}{x} + \frac{\sqrt{3}}{2}x^2$ (shown)
(ii)	For stationary points, $\frac{dA}{dx} = -\frac{1000\sqrt{3}(1+k)}{x^2} + \sqrt{3}x = 0$ $x^3 = 1000(1+k)$ $x = 10(1+k)^{\frac{1}{3}}$ $\frac{d^2A}{dx^2} = \frac{2000\sqrt{3}(1+k)}{x^3} + \sqrt{3} > 0$ Thus, $x = 10(1+k)^{\frac{1}{3}}$ gives a minimum surface area.
(iii)	Since $y = \frac{1000}{\sqrt{3}x^2}$ $\frac{y}{x} = \frac{1000}{\sqrt{3}x^3} = \frac{1000}{\sqrt{3}(1000)(1+k)}$ $= \frac{1}{\sqrt{3}(1+k)}$

(iv)	Since $0 < k \le 1$ $1 < 1 + k \le 2$ $\frac{1}{2} \le \frac{1}{1+k} < 1$ $\frac{1}{2\sqrt{3}} \le \frac{1}{\sqrt{3}(1+k)} < \frac{1}{\sqrt{3}}$ i.e. $\frac{1}{2\sqrt{3}} \le \frac{y}{x} < \frac{1}{\sqrt{3}}$

Qn	**Solution**
4(i)	$a^2b = \frac{1}{2}(1+\sqrt{3}\mathrm{i})^2(1-\mathrm{i})$ $= \frac{1}{2}(1+2\sqrt{3}\mathrm{i}-3)(1-\mathrm{i})$ $= (-1+\sqrt{3}\mathrm{i})(1-\mathrm{i})$ $= (\sqrt{3}-1)+(\sqrt{3}+1)\mathrm{i}$
(ii)	$\lvert a^2b\rvert = \lvert a\rvert^2 \lvert b\rvert$ $= 2^2\left(\frac{1}{\sqrt{2}}\right)$ $= 2\sqrt{2}$ $\arg(a^2b) = 2\arg(a)+\arg(b)$ $= 2\left(-\frac{2\pi}{3}\right)-\frac{\pi}{4}$ $= -\frac{19\pi}{12}$ $\therefore \arg(a^2b) = -\frac{19\pi}{12}+2\pi = \frac{5\pi}{12}$.
(iii)	Considering the imaginary part of a^2b, we have $2\sqrt{2}\sin\frac{5\pi}{12} = \sqrt{3}+1$ $\Rightarrow \sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(iv)	$\overrightarrow{BA}$ can be obtained by rotating $\overrightarrow{BC}$ through 90° in the anticlockwise direction about B. $i(c-b)=a-b$ $\Rightarrow c=-i(a-b)+b$ $=-ia+b(1+i)$ $=i(1+\sqrt{3}i)+\frac{1}{2}(2)$ $=(1-\sqrt{3})+i$
5 **(i)**	${}^6C_2\times{}^5C_4=75$ ways
(ii)	Number of ways if at least one of the sisters are included = number of ways without restriction – number of ways if none of the sisters is included $={}^{11}C_6-{}^8C_6$ $=434$ **Alternative Method** ${}^3C_1\times{}^8C_5+{}^3C_2\times{}^8C_4+{}^3C_3\times{}^8C_3=434$
(iii)	Select a man to be between the 2 sisters and group the 3 of them as one unit and arrange 4 units round a table Number of ways $={}^3C_1\times3!\times2$ $=36$
(iv)	First arrange the other 4 persons round the table. There are 4 ways to insert the sisters. Number of ways = $3!\times4$ $=24$

Qn	Solution
6 (i)	$\text{P}(C \mid M) = \dfrac{\text{P}(C \cap M)}{\text{P}(M)}$ $= \dfrac{200}{400} = \dfrac{1}{2}$
(ii)	$\text{P}(M \cup C) = \text{P}(M) + \text{P}(C) - \text{P}(M \cap C)$ $= \dfrac{400}{1000} + \dfrac{450}{1000} - \dfrac{200}{1000}$ $= \dfrac{650}{1000} = \dfrac{13}{20}$
(iii)	$\text{P}(M' \cap B') = \dfrac{250+300}{1000} = \dfrac{11}{20}$ $P(C) = \dfrac{9}{20}$ $P(C \mid M) = \dfrac{1}{2} \neq P(C)$ C and M are not independent.
(iv)	No. of international studens in the sample $= 0.2(200+250) + 0.3(130+300) + 0.05(120) = 225$ $\text{P}(C \mid \text{international student}) = \dfrac{\text{P}(C \cap \text{international student})}{\text{P}(\text{international student})}$ $= \dfrac{\frac{(200+250)0.2}{1000}}{\frac{225}{1000}}$ $= 0.4$
(v)	Number of international students studying Physics $= 0.3(430) = 129$ $\text{P(exactly one international student studying Physics)} = \dfrac{{}^{129}C_1 \, {}^{871}C_2}{{}^{1000}C_3}$ $= 0.294$
	Alternative method Required Probability $= \dfrac{129 \;\; 871 \;\; 870}{1000 \;\; 999 \;\; 998} \times 3$ $= 0.294$

7 **(i)**	
(ii)	$r = 0.914099 \approx 0.914$ (to 3 s.f) Though the value of r shows a strong positive linear correlation, from the scatter diagram, it is possible that x and y may have a curvilinear relationship.
(iii)	For $y = c + dx^2$, $r = 0.93986 \approx 0.940$ Since the value of r for $y = c + dx^2$ is closer to the value of 1, $y = c + dx^2$ is a better model.
(iv)	$y = -0.88934 + 0.0015441x^2$ When $x = 55$, $y = -0.88934 + 0.0015441(55)^2$ $y = 3.7816 \approx 3.8$ (to 1 d.p)
	Since $x = 55$ is within the range of data given and $r \approx 0.940$ is close to 1, the estimation is reliable.

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>8
(i)</td><td>P(first person that uses Voyager is the third person selected)

$= 0.92 \times 0.92 \times 0.08$

$= 0.067712$</td></tr>
<tr><td>(ii)</td><td>1. Whether a person uses Voyager is independent of another person.
2. The probability that a person uses Voyager is constant for every person in the sample.</td></tr>
<tr><td>(iii)</td><td>Let Y be the number of people who use Voyager out of 80 people.

$Y \sim \mathrm{B}(80, 0.08)$

Since $n = 80 > 50,\ np = 6.4 > 5,\ nq = 73.6 > 5$,

$Y \sim \mathrm{N}(6.4, 5.888)\, approx$

$\mathrm{P}(Y < 10) \xrightarrow{c.c} \mathrm{P}(Y \le 9.5)$

$= 0.899295$

$= 0.899$ (to 3 s.f.)</td></tr>
<tr><td>(iv)</td><td>Let V be the number of people who use Voyager out of n people.

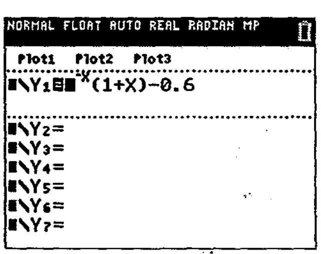
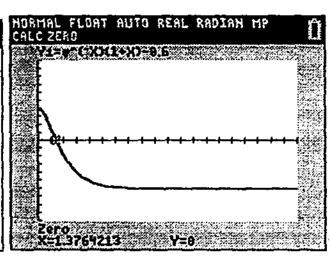
$V \sim \mathrm{B}(n, 0.08)$

$\mathrm{P}(V \ge 10) > 0.2$

$1 - \mathrm{P}(V \le 9) > 0.2$
$\mathrm{P}(V \le 9) < 0.8$

Using GC,

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl
<table>
<tr><th>X</th><th>Y1</th></tr>
<tr><td>90</td><td>.81786</td></tr>
<tr><td>91</td><td>.80902</td></tr>
<tr><td>92</td><td>.79999</td></tr>
<tr><td>93</td><td>.79078</td></tr>
<tr><td>94</td><td>.7814</td></tr>
</table>
Least value of $n = 92$</td></tr>
</table>

9 (i)	Let C be the number of boxes of chocolate biscuits sold in a day. $C \sim \text{Po}(2.2)$ $\text{P}(C=0)=0.11080$ $=0.111$ (to 3 s.f.)
(ii)	Let D be the number of days that no boxes of chocolate biscuits were sold out of 7 days. $D \sim \text{B}(7, 0.11080)$ $\text{E}(D)=7\times 0.11080$ $=0.77562$ $=0.776$
(iii)	Let S be the number of boxes of strawberry biscuits sold in a day. $S \sim \text{Po}(\lambda)$ $\text{P}(S<2)=0.6$ $\text{P}(S=0)+\text{P}(S=1)=0.6$ $\text{e}^{-\lambda}\left(\frac{\lambda^0}{0!}\right)+\text{e}^{-\lambda}\left(\frac{\lambda^1}{1!}\right)=0.6$ $\text{e}^{-\lambda}(1+\lambda)=0.6$
	Using GC,   $\lambda=1.376$ $=1.4$ (to 1 d.p)

(iv)	Let T be the total number of boxes of chocolate and strawberry biscuits sold in 7 days. $T \sim \text{Po}(7\times2.2+7\times1.376)=\text{Po}(25.032)$ $\text{P}(T>25)=1-\text{P}(T\le25)$ $=0.44962$ $=0.450$ (to 3 s.f)
(v)	Let X be number of boxes of chocolate biscuits sold in 30 days. $X\sim\text{Po}(30\times2.2)=\text{Po}(66)$ Since $\lambda=66>10$, $X\sim\text{N}(66,66)$approx Let Y be number of boxes of strawberry biscuits sold in 30 days. $Y\sim\text{Po}(30\times1.376)=\text{Po}(41.28)$ Since $\lambda=41.28>10$, $Y\sim\text{N}(41.28,41.28)$approx $X-Y\sim\text{N}(24.72,107.28)$approx $\text{P}(X-Y>0)\xrightarrow{c.c}\text{P}(X-Y>0.5)$ $=0.99032$ $=0.990$ (to 3 s.f.)

10 (i)	Choose a plant randomly from the first 10 plants, say the 5th plant. Choose every 10th plant thereafter until 8 plants are selected i.e. 5th, 15th, 25th, ... The 8 plants selected will be *evenly spread out* across the row of 80 plants.
(ii)	Unbiased estimate of the population mean, $\hat{\mu}$ $=\frac{\sum(x-150)}{80}+150$ $=-\frac{160}{80}+150$ $=148$ Unbiased estimate of the population variance, s^2 $=\frac{1}{80-1}\left[\sum(x-150)^2-\frac{\left(\sum(x-150)\right)^2}{80}\right]$ $=\frac{1}{79}\left[5520-\frac{(-160)^2}{80}\right]$ $=\frac{5200}{79}$
(iii)	$H_0:\mu=150$ $H_1:\mu<150$ Under H_0, since $n=80>50$, by Central Limit Theorem, $\overline{X}\sim N\left(150,\frac{5200}{79(80)}\right)$ approx. Test statistic $Z=\frac{\overline{X}-150}{\sqrt{\frac{5200}{79(80)}}}\sim N(0,1)$ approx. From GC, p–value = 0.013731 = 0.0137 (to 3 s.f.) $\alpha=0.01$ Since p–value $=0.0137>\alpha=0.01$, we do not reject H_0 at 1% level of significance and conclude that there is insufficient evidence that the researcher's claim is invalid.

(iv)	It means that there is a probability of 0.01 of concluding that the population mean mass of a new variety of potato is less than 150g given that the population mean mass of a new variety of potato is in fact 150g.
	Unbiased estimate of the population variance $=\frac{8}{7}k^2$ $H_0 : \mu = 150$ $H_1 : \mu < 150$ Under H_0, test statistic $T = \frac{\overline{X}-150}{\sqrt{\frac{S^2}{8}}} \sim t(7)$ $\alpha = 0.01$ Researcher's claim is invalid at 1% level of significance $\Rightarrow H_0$ is rejected at 1% level of significance $\Rightarrow t \le -2.9980$ $\Rightarrow \frac{148.5-150}{\sqrt{\frac{k^2}{7}}} \le -2.9980$ $\Rightarrow k \le 1.3238$ $\therefore k \le 1.32$ (to 3 s.f)