YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS **8864/01**
HIGHER 1

28 AUGUST 2017
MONDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [35 marks]

1 **(i)** Differentiate with respect to x,

(a) $5\ln\left(1-3x^2\right)$, [2]

(b) $\dfrac{1}{\left(2x+3\right)^2}$. [2]

(ii) Use a non-calculator method to find $\displaystyle\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\ \mathrm{d}x$. [4]

2 Find the range of values of k for which $kx^2+4k-kx-2x$ is always negative. [4]

3 Research has found that the concentration R of a drug in the bloodstream, in micrograms per litre, decreases according to the function $R=366e^{-0.0998t}$, $t\geq 0$, where t is measured in minutes after the drug is administered.

(i) Sketch the graph of $R=366e^{-0.0998t}$ for $t\geq 0$. [2]

(ii) Find the rate of decrease of R at the instant when t = 20. [2]

(iii) How long, to the nearest minute, will the concentration of the drug in a patient be 40 micrograms per litre? [2]

The company that manufactures the drug estimates that the profit, in millions of dollars, from the sale of the drug over the years can be modelled by the equation

$$P=-0.03x^3+0.1x^2+x-0.1,\quad 0\leq x\leq 5,$$

where x is the number of bottles of the drug, in 10 thousands, produced.

Use differentiation to find the number of bottles that corresponds to the maximum profit, giving your answer correct to 5 significant figures. [5]

4 Sketch the graph of the curve C with equation $y=\dfrac{1}{x-2}+1$, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]

Without the use of a calculator, find the area of the region bounded by C, the line $y=x+3$ and the y-axis. [6]

5 The curve C has equation $y=x^3-2\mathrm{e}^{-x}$. Without using a calculator, find the equation of the tangent to C at the point where $x=1$, giving your answer in the form $y=ax+b$, where a and b are constants. [4]

Section B: Statistics [60 marks]

6 **(a)** A game is played with a deck of n cards, each distinctly numbered from 1 to n, where n is an even number. A player randomly picks a card from the deck. Events A and B are defined as follows:

A: The card shows an even number.

B: The card shows a multiple of 17.

It is given that $\mathrm{P}(A' \cap B') = \dfrac{8}{17}$ and $\mathrm{P}(A \cap B) = \dfrac{1}{34}$.

(i) Show that $\mathrm{P}(B) = \dfrac{1}{17}$ and hence state the smallest value of n. [5]

(ii) Find $\mathrm{P}(A \mid B)$. [2]

(b) The probability that a train service breaks down is $\dfrac{1}{15}$. When the train service functions normally, 5% of the people travelling to work by train are late. It is also found that 90% of the people travelling to work by train are punctual.

(i) Draw a tree diagram to represent this situation, showing all possible outcomes and the associated probabilities. [3]

(ii) Given that a randomly chosen person travelling to work by train is late, find the probability that the train service functions normally. [2]

7 In a survey on usage of internet security software conducted with a large number of smartphone users, it was found that 37% of them had anti-virus software A installed on their smartphones, 56% had anti-virus software B installed and 7% did not have any of them installed.

(i) A random sample of twenty smartphone users was chosen. Find the probability that at least eight of the users had anti-virus software A installed on their smartphones. [2]

(ii) Fifty such samples of twenty smartphone users was randomly chosen. Using a suitable approximation, find the probability that there were fewer than thirty samples with at least eight users with anti-virus software A installed on their smartphones. [4]

(iii) Another random sample of n smartphone users was chosen. Given that the probability that at most one user did not have any anti-virus software installed was less than 0.5, show that n would satisfy the inequality $0.93^{n-1}(0.93 + 0.07n) < 0.5$. Hence find the least value of n. [3]

8 An online blog shop owner has compiled a list of 3000 customers. A sample of 60 customers is chosen to take part in a survey. Describe how the sample could be chosen using systematic sampling. [2]

The purpose of the survey is to find out the customers' opinions about the products sold at the blog shop. Give a reason why a stratified sample might be preferable in this context. [1]

9 An electric power service company keeps records of the installation time of its electricity meters in the houses in a new estate. The time taken, X minutes, to install an electricity meter may be assumed to be normally distributed with mean 50 minutes and standard deviation 10 minutes.

(i) Two randomly chosen houses have their electricity meters installed by the company. Find the probability that each installation takes more than an hour. [2]

(ii) The company wishes to improve its processes such that at least 95% of the installations will take less than 60 minutes. Suppose the population standard deviation is still 10 minutes, find the maximum population mean time taken to install an electricity meter. [3]

Each month, the amount of electricity, measured in kilowatt-hours (kWh), used by a particular household in the estate is normally distributed with mean 522 kWh and standard deviation 26 kWh. The company charges households for electricity used at $0.21 per kWh.

(iii) Find the probability that, in a randomly chosen month, the electricity charge for the household is between $100 and $120. [2]

(iv) The household is billed every two months. Find the largest integral value of d such that the probability that a randomly chosen bill is at least \$$d$ is more than 0.90. State an assumption that is needed in your calculation. [4]

The company also installs gas meters in the houses in the estate. The time taken, Y minutes, to install a gas meter has mean 47 minutes and standard deviation 25 minutes.

(v) Explain why Y cannot be well modelled by a normal distribution. [1]

(vi) A random sample of 55 gas meter installations has been completed in the estate. Find the probability that the mean time taken to install a gas meter is more than 45 minutes. [2]

10 The following table gives the median monthly household income from work, $\$h$, from each year, t, between 2011 to 2016 (inclusive).

Year, t	2011	2012	2013	2014	2015	2016
Median monthly household income from work, $\$h$	7037	7566	7872	8292	8666	8846

Source: http://www.singstat.gov.sg/statistics/browse-by-theme/household-income-tables

(i) Give a sketch of the scatter diagram for the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the values of $\bar{t}$ and $\bar{h}$ and mark the point $\left(\bar{t}, \bar{h}\right)$ on your scatter diagram in part **(i)**. [2]

(iv) Find the equation of the regression line of h on t, in the form $h = mt + c$, giving the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram in part **(i)**. [2]

(v) Use a suitable regression line to estimate the year when the median monthly household income from work is \$9700. Comment on the reliability of the estimate obtained. [2]

11 A departmental store manager claims that the mean amount of time that customers spent in the shopping mall is 41 minutes. A random sample of 150 customers is taken and the time, x minutes, spent by each customer is noted. The results are summarized by

$$\sum x = 6386, \quad \sum x^2 = 277270.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% level of significance, whether the manager's claim is valid. [4]

(iii) State, with a reason, whether it is necessary to assume a normal distribution for the population for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of 'at 5% level of significance'. [1]

After several rounds of publicity for the shopping mall, the publicity manager claims that the mean time that customers spent in the shopping mall has increased. The new population variance of the time spent may be assumed to be 49.3 minutes^2. A new random sample of 40 customers is chosen and the mean of this sample is k minutes.

(v) Find the set of values within which k must lie, such that there is not enough evidence from the sample to support the publicity manager's claim at the 10% level of significance. [4]

~ ~ End of paper ~ ~

YISHUN JUNIOR COLLEG5E

Mathematics Department

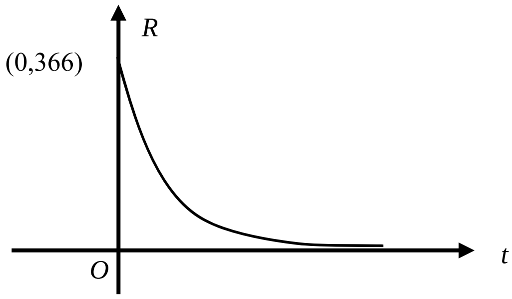
PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8864 **Date** :

Qn	Solution
1(i)	**(a)** $\frac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\frac{1}{1-3x^2}\right)\left(-6x\right)$ $=-\frac{30x}{1-3x^2}$ **(b)** $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\left(2x+3\right)^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(2x+3\right)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\ \mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_1^3$ $=\left(\frac{3^2}{2}-\frac{2(3)^3}{3}+\frac{(3)^4}{4}\right)-\left(\frac{1^2}{2}-\frac{2(1)^3}{3}+\frac{(1)^4}{4}\right)$ $=\frac{27}{4}-\frac{1}{12}$ $=\frac{20}{3}$
2	$\left(-k-2\right)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $(5k+2)(3k-2)>0$ $k<-\frac{2}{5}$ or $k>\frac{2}{3}$ Since $k<0$, $\therefore$ $k<-\frac{2}{5}$

Qn	Solution

3 (i)



(ii)

From GC, when $t = 20$,

$$\frac{dR}{dt} = -4.96$$

The rate of decrease is 4.96 micrograms/litre per min

(iii)

$$40 = 366e^{-0.0998t}$$

$$e^{-0.0998t} = \frac{40}{366}$$

$$-0.0998t = \ln\left(\frac{40}{366}\right)$$

$$t = 22.18$$

$$t \approx 22\text{mins}$$

Alternative solution:
Draw graph of y = 40 and find intersection points.

$$P = -0.03x^3 + 0.1x^2 + x - 0.1$$

$$\frac{dP}{dx} = -0.09x^2 + 0.2x + 1$$

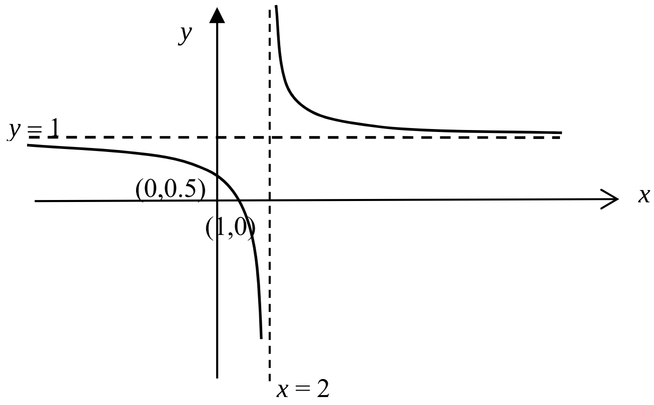
For maximum P, $\frac{dP}{dx}=0$

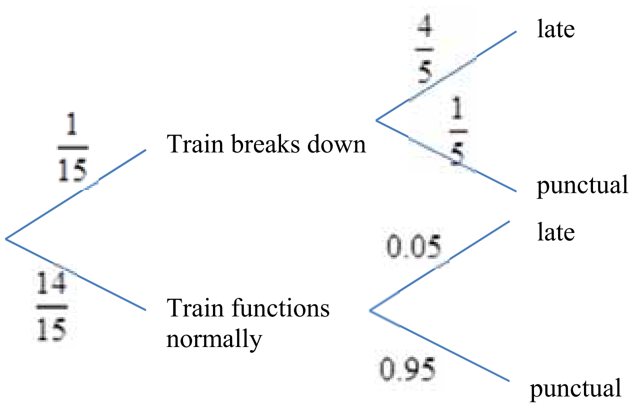
$$-0.09x^2 + 0.2x + 1 = 0$$

$$x = 4.624753 \quad (x > 0)$$

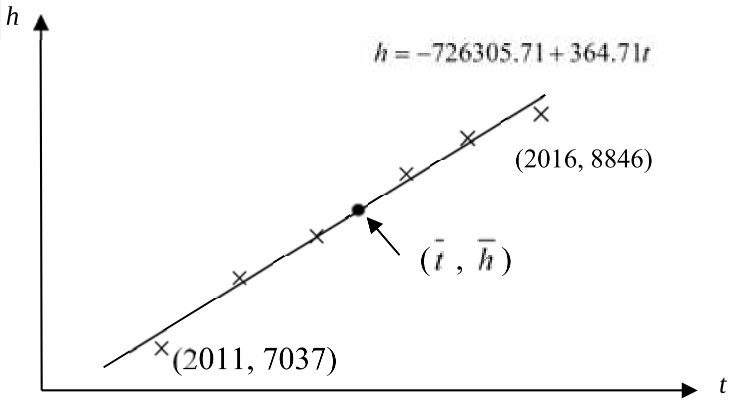
x	4.624753^-	4.624753	4.624753^+
$\frac{dP}{dx}$	+ve	0	−ve
slope			

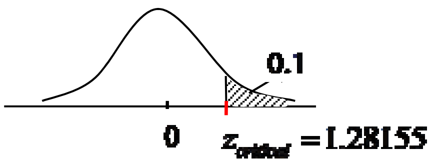
Thus, P is maximum when the number of bottles is 46248.

Qn	Solution
4	 $\frac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region $=\int_{-\sqrt{5}}^{0}\left(x+3-\left(\frac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}+2x-\ln\|x-2\|\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\|-\sqrt{5}-2\|\right)$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\frac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\frac{2}{e}\right)=\left(3+\frac{2}{e}\right)(x-1)$ $y=\left(3+\frac{2}{e}\right)x-2-\frac{4}{e}$

Qn	Solution
6(a)(i)	$\mathrm{P}(A\cup B)=1-\mathrm{P}(A'\cap B')$ $=\dfrac{9}{17}$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $\dfrac{9}{17}=\dfrac{1}{2}+\mathrm{P}(B)-\dfrac{1}{34}$ $\Rightarrow \mathrm{P}(B)=\dfrac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$\mathrm{P}(A\mid B)=\dfrac{\frac{1}{34}}{\frac{1}{17}}=\dfrac{1}{2}$
(b)(i) (ii)	 P (train functions normally | late) $=\dfrac{\frac{14}{15}(0.05)}{0.1}$ ≈ 0.467
7(i)	Let X be the random variable 'number of smartphone users with anti-virus software A installed on their smartphones out of 20 users' $X \sim \mathrm{B}(20, 0.37)$ $\mathrm{P}(X\geq 8)=1-\mathrm{P}(X\leq 7)$ ≈ 0.47346 $=0.473$ (3 sig fig)
(ii)	Let W be the 'number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples' $W \sim \mathrm{B}(50, 0.47346)$ Since n is large, $np = 23.673 > 5$, $n(1-p) = 26.327 > 5$ $W \sim N(23.673, 12.465)$ approx $\mathrm{P}(W<30)\to \mathrm{P}(W<29.5)$ using c.c. ≈ 0.95057 $= 0.951$ (3 sig fig)

Qn	Solution
(iii)	Let Y be the 'number of smartphone users who did not have any anti-virus software installed, out of n' $Y \sim \mathrm{B}(n, 0.07)$ $\mathrm{P}(Y \leq 1) < 0.5$ $\mathrm{P}(Y = 0) + \mathrm{P}(Y = 1) < 0.5$ ${}^{n}C_0 (0.07)^0 (0.93)^n + {}^{n}C_1 (0.07)(0.93)^{n-1} < 0.5$ $(0.93)^n + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
8	Number the customers from 1 to 3000. $k = \frac{3000}{60} = 50$ Randomly choose the first customer from the first 50 customers. Thereafter, select every 50^{th} customer until 60 customers are chosen. Using a stratified sample will take into consideration the different opinions from all the different strata (for example, age group), hence resulting in a sample which is more representative of the population.
9(i)	$X \sim N(50, 10^2)$ Required Prob = $\left[\mathrm{P}(X > 60)\right]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)
(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $\mathrm{P}(X < 60) \geq 0.95$ $\mathrm{P}(Z < \frac{60-\mu}{10}) \geq 0.95$ $\frac{60-\mu}{10} \geq 1.64485$ $\mu \leq 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P\left(100 < B < 120\right) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \geq d) > 0.9$ $1 - P\left(T < d\right) > 0.9$ $P\left(T < d\right) < 0.1$ $d < 209.344$

Qn	Solution
	Largest integral value of *d* is 209. Assume that the electricity used in each month is independent for a particular household.
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, *Y* is not well modelled by a normal distribution.
(vi)	Since sample size = 55 is large, $\bar{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $P(\bar{Y} > 45) = 0.724$ (3 s.f.)
10(i)	
(ii)	$R \approx 0.992$ (3 s.f) There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5$, $\bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t$ (2 d.p.)
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 The estimate is not reliable since the estimate is obtained via extrapolation.
11(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{6386}{150} = 42.573 \approx 42.6$ (3s.f.) Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \bar{x} = 42.573, n = 150$.

Qn	Solution
	By GC, p-value $= 0.00137\left(3 \text{ s.f.}\right)$. Since p-value < 0.05, we reject H_0 and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since n is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under H_0, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\ n = 40$.  Since H_0 is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4\left(3 \text{ s.f.}\right)$ Required set = $\left\{k \in \square : 0 < k < 42.4\right\}$

YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS **8865/01**
HIGHER 1

28 AUGUST 2017
MONDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF26)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 **(i)** Differentiate with respect to x,

(a) $5\ln\left(1-3x^2\right)$, [2]

(b) $\dfrac{1}{\left(2x+3\right)^2}$. [2]

(ii) Use a non-calculator method to find $\displaystyle\int_1^3 x^3\left(\frac{1}{x}-1\right)^2 \, \mathrm{d}x$. [4]

2 Find the range of values of k for which $kx^2+4k-kx-2x$ is always negative. [4]

3 Research has found that the concentration R of a drug in the bloodstream, in micrograms per litre, decreases according to the function $R=366\mathrm{e}^{-0.0998t}$, $t \geq 0$, where t is measured in minutes after the drug is administered.

(i) Sketch the graph of $R=366\mathrm{e}^{-0.0998t}$ for $t \geq 0$. [2]

(ii) Find the concentration of the drug in the bloodstream of a patient 1 hour after it is administered. [1]

(iii) Find the rate of decrease of R at the instant when t = 20. [2]

(iv) How long, to the nearest minute, will the concentration of the drug in a patient be 40 micrograms per litre? [2]

The company that manufactures the drug estimates that the profit, in millions of dollars, from the sale of the drug over the years can be modelled by the equation

$$P=-0.03x^3+0.1x^2+x-0.1, \quad 0 \leq x \leq 5,$$

where x is the number of bottles of the drug, in 10 thousands, produced.
Use differentiation to find the number of bottles that corresponds to the maximum profit, giving your answer correct to 5 significant figures. [5]

4 Sketch the graph of the curve C with equation $y=\dfrac{1}{x-2}+1$, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]
Without the use of a calculator, find the area of the region bounded by C, the line $y=x+3$ and the y-axis. [6]

5 The curve C has equation $y = x^3 - 2e^{-x}$. Without using a calculator, find the equation of the tangent to C at the point where $x = 1$, giving your answer in the form $y = ax + b$, where a and b are constants. [4]

6 Mr Lee wants to buy 6 packets of milo, 3 packets of cereal and 4 packets of coffee. Based on the usual retail price in the supermarket, the total bill will be $93.80.
Mr Lee has a voucher which will entitle him to a 20% discount off each packet of milo, capped at 5 packets. The voucher is not to be used in conjunction with other promotions. If he uses the voucher, his total bill will be $85.30.
The supermarket is currently having a "Buy 2 get 1 free" promotion for milo and for cereal. If he uses the "Buy 2 get 1 free" promotion, his total bill will become $70.40.
Write down and solve equations to find the usual retail price of each packet of milo, cereal and coffee. [4]

Section B: Statistics [60 marks]

7 **(a)** A game is played with a deck of n cards, each distinctly numbered from 1 to n, where n is an even number. A player randomly picks a card from the deck. Events A and B are defined as follows:

A: The card shows an even number.

B: The card shows a multiple of 17.

It is given that $\mathrm{P}(A' \cap B') = \dfrac{8}{17}$ and $\mathrm{P}(A \cap B) = \dfrac{1}{34}$.

(i) Show that $\mathrm{P}(B) = \dfrac{1}{17}$ and hence state the smallest value of n. [5]

(ii) Find $\mathrm{P}(A \mid B)$. [2]

(b) The probability that a train service breaks down is $\dfrac{1}{15}$. When the train service functions normally, 5% of the people travelling to work by train are late. It is also found that 90% of the people travelling to work by train are punctual.

(i) Draw a tree diagram to represent this situation, showing all possible outcomes and the associated probabilities. [3]

(ii) Given that a randomly chosen person travelling to work by train is late, find the probability that the train service functions normally. [2]

8 A chess team of 5 players is to be selected from 15 boys. In how many ways can the team be chosen if

(i) no more than one of the three best players is to be included, [2]

(ii) at least one of the 4 youngest players is to be included? [2]

9 In a survey on usage of internet security software conducted with a large number of smartphone users, it was found that 37% of them had anti-virus software *A* installed on their smartphones, 56% had anti-virus software *B* installed and 7% did not have any of them installed.

(i) A random sample of twenty smartphone users was chosen. Find the probability that at least eight of the users had anti-virus software *A* installed on their smartphones. [2]

(ii) Fifty such samples of twenty smartphone users was randomly chosen. Find the probability that there were fewer than thirty samples with at least eight users with anti-virus software *A* installed on their smartphones. [3]

(iii) Another random sample of n smartphone users was chosen. Given that the probability that at most one user did not have any anti-virus software installed was less than 0.5, show that n would satisfy the inequality

$0.93^{n-1}(0.93+0.07n)<0.5$. Hence find the least value of n. [3]

10 The following table gives the median monthly household income from work, \$$h$, from each year, t, between 2011 to 2016 (inclusive).

Year, t	2011	2012	2013	2014	2015	2016
Median monthly household income from work, \$$h$	7037	7566	7872	8292	8666	8846

Source: http://www.singstat.gov.sg/statistics/browse-by-theme/household-income-tables

(i) Give a sketch of the scatter diagram for the data. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the values of $\bar{t}$ and $\bar{h}$ and mark the point $\left(\bar{t},\bar{h}\right)$ on your scatter diagram in part **(i)**. [2]

(iv) Find the equation of the regression line of h on t, in the form $h = mt + c$, giving the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram in part **(i)**. [2]

(v) Use a suitable regression line to estimate the year when the median monthly household income from work is \$9700. Comment on the reliability of the estimate obtained. [2]

11 An electric power service company keeps records of the installation time of its electricity meters in the houses in a new estate. The time taken, X minutes, to install an electricity meter may be assumed to be normally distributed with mean 50 minutes and standard deviation 10 minutes.

(i) Two randomly chosen houses have their electricity meters installed by the company. Find the probability that each installation takes more than an hour. [2]

(ii) The company wishes to improve its processes such that at least 95% of the installations will take less than 60 minutes. Suppose the population standard deviation is still 10 minutes, find the maximum population mean time taken to install an electricity meter. [3]

Each month, the amount of electricity, measured in kilowatt-hours (kWh), used by a particular household in the estate is normally distributed with mean 522 kWh and standard deviation 26 kWh. The company charges households for electricity used at \$0.21 per kWh.

(iii) Find the probability that, in a randomly chosen month, the electricity charge for the household is between \$100 and \$120. [2]

(iv) The household is billed every two months. Find the largest integral value of d such that the probability that a randomly chosen bill is at least \$$d$ is more than 0.90. State an assumption that is needed in your calculation. [4]

The company also installs gas meters in the houses in the estate. The time taken, Y minutes, to install a gas meter has mean 47 minutes and standard deviation 25 minutes.

(v) Explain why Y cannot be well modelled by a normal distribution. [1]

(vi) A random sample of 55 gas meter installations has been completed in the estate. Find the probability that the mean time taken to install a gas meter is more than 45 minutes. [2]

[Question 12 is printed on the next page.]

12 A departmental store manager claims that the mean amount of time that customers spent in the shopping mall is 41 minutes. A random sample of 150 customers is taken and the time, x minutes, spent by each customer is noted. The results are summarized by

$$\sum x = 6386, \qquad \sum x^2 = 277270.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% level of significance, whether the manager's claim is valid. [4]

(iii) State, with a reason, whether it is necessary to assume a normal distribution for the population for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of 'at 5% level of significance'. [1]

After several rounds of publicity for the shopping mall, the publicity manager claims that the mean time that customers spent in the shopping mall has increased. The new population variance of the time spent may be assumed to be 49.3 minutes^2. A new random sample of 40 customers is chosen and the mean of this sample is k minutes.

(v) Find the set of values within which k must lie, such that there is not enough evidence from the sample to support the publicity manager's claim at the 10% level of significance. [4]

~ ~ End of paper ~ ~

YISHUN JUNIOR COLLEGE

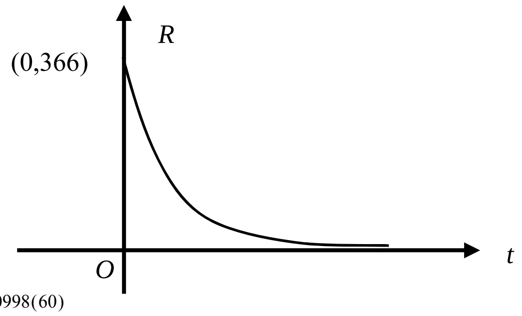
Mathematics Department

PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8865 **Date** :

Qn	Solution
1(i)	(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\frac{1}{1-3x^2}\right)(-6x)$ $=-\frac{30x}{1-3x^2}$ (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{(2x+3)^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}(2x+3)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\,\mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_1^3$ $=\left(\frac{3^2}{2}-\frac{2(3)^3}{3}+\frac{(3)^4}{4}\right)-\left(\frac{1^2}{2}-\frac{2(1)^3}{3}+\frac{(1)^4}{4}\right)$ $=\frac{27}{4}-\frac{1}{12}$ $=\frac{20}{3}$
2	$(-k-2)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $k<0$ and $(5k+2)(3k-2)>0$ $k<-\frac{2}{5}$ or $k>\frac{2}{3}$ Since $k<0$, $\therefore$ $k<-\frac{2}{5}$

3 (i)



(ii)

$R = 366e^{-0.0998(60)}$

$= 0.918$

The concentration is 0.918 micrograms/litre

(iii)

From GC, when $t = 20$,

$\frac{dR}{dt} = -4.96$

The rate of decrease is 4.96 micrograms/litre per min

(iv)

$40 = 366e^{-0.0998t}$

$e^{-0.0998t} = \frac{40}{366}$

$-0.0998t = \ln\left(\frac{40}{366}\right)$

$t = 22.18$

$t \approx 22\text{mins}$

Alternative solution:
Draw graph of y = 40 and find intersection points.

$P = -0.03x^3 + 0.1x^2 + x - 0.1$

$\frac{dP}{dx} = -0.09x^2 + 0.2x + 1$

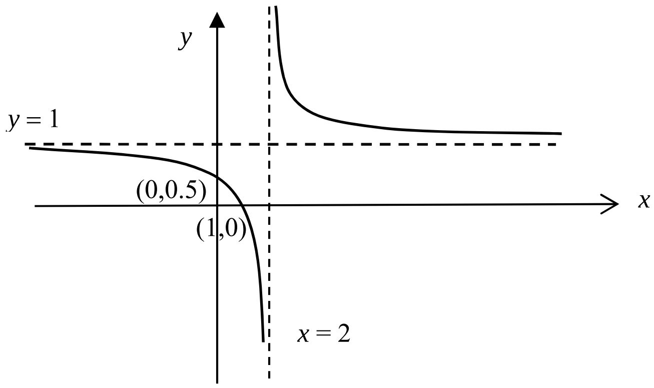
For maximum P, $\frac{dP}{dx}$=0

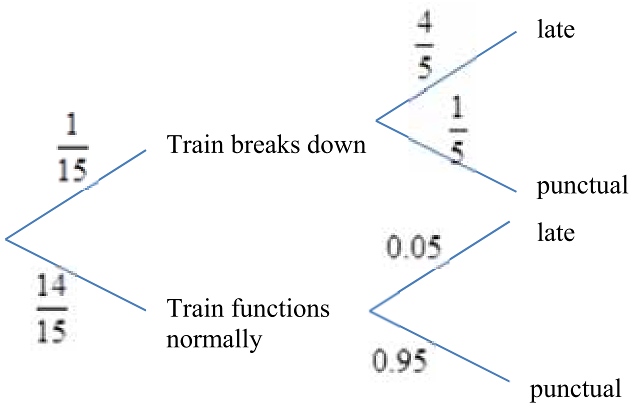
$-0.09x^2 + 0.2x + 1$=0

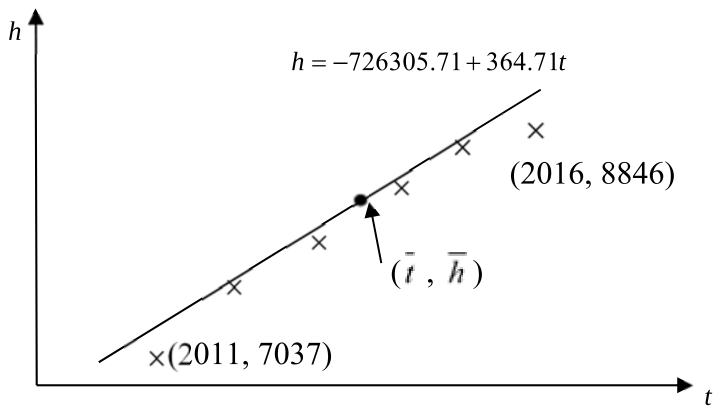
$x = 4.624753 \ \ (x > 0)$

x	4.624753^-	4.624753	4.624753^+
$\frac{dP}{dx}$	+ve	0	-ve
slope	/	—	\

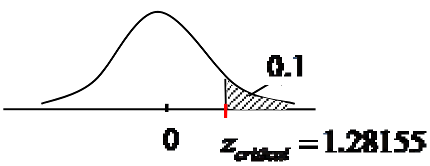
Thus, P is maximum when the number of bottles is 46248.

4	 $\frac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region $=\int_{-\sqrt{5}}^{0}\left(x+3-\left(\frac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}+2x-\ln\lvert x-2\rvert\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\lvert-\sqrt{5}-2\rvert\right)$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\frac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\frac{2}{e}\right)=\left(3+\frac{2}{e}\right)(x-1)$ $y=\left(3+\frac{2}{e}\right)x-2-\frac{4}{e}$
6	$6x+3y+4z=93.80$ $5x+3y+4z=85.30$ $4x+2y+4z=70.40$ From GC, x = 8.50, y = 6.40, z = 5.90 The usual retail prices of 1 packet of milo, 1 packet of cereal and 1 packet of coffee are \$8.50, \$6.40 and \$5.90 respectively.

7(a)(i)	$\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B')$ $= \dfrac{9}{17}$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\dfrac{9}{17} = \dfrac{1}{2} + \mathrm{P}(B) - \dfrac{1}{34}$ $\Rightarrow \mathrm{P}(B) = \dfrac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$\mathrm{P}(A \mid B) = \dfrac{\frac{1}{34}}{\frac{1}{17}} = \dfrac{1}{2}$
(b)(i)	
(ii)	P (train functions normally | late) $= \dfrac{\frac{14}{15}(0.05)}{0.1} = \dfrac{7}{15} \approx 0.467$
8(i)	No of teams = ${}^{12}C_5 + {}^{3}C_1\,{}^{12}C_4$ $= 2277$
(ii)	No of teams = ${}^{15}C_5 - {}^{11}C_5$ $= 2541$
9(i)	Let X be the random variable 'number of smartphone users with anti-virus software A installed on their smartphones out of 20 users' $X \sim \mathrm{B}(20, 0.37)$ $\mathrm{P}(X \geq 8) = 1 - \mathrm{P}(X \leq 7)$ ≈ 0.47346 $= 0.473$ (3 sig fig)
(ii)	Let W be the 'number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples' $W \sim \mathrm{B}(50, 0.47346)$ $\mathrm{P}(W < 30) = \mathrm{P}(W \leq 29)$ $\approx 0.95061 \quad = 0.951$ (3 sig fig)

(iii)	Let Y be the ‘number of smartphone users who did not have any anti-virus software installed, out of n’ $Y \sim \mathrm{B}(n, 0.07)$ $\mathrm{P}(Y \le 1) < 0.5$ $\mathrm{P}(Y = 0) + \mathrm{P}(Y = 1) < 0.5$ ${}^{n}C_0(0.07)^0(0.93)^n + {}^{n}C_1(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^n + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
10(i)	
(ii)	$r \approx 0.992\ (3\text{ s.f.})$ There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5, \quad \bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t\ (2\text{ d.p.})$
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 Since the estimate is obtained via extrapolation, the estimate is not reliable.
11(i)	$X \sim N(50, 10^2)$ Required Prob = $[\mathrm{P}(X > 60)]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)

(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $\mathrm{P}(X < 60) \geq 0.95$ $\mathrm{P}(Z < \frac{60-\mu}{10}) \geq 0.95$ $\frac{60-\mu}{10} \geq 1.64485$ $\mu \leq 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P(100 < B < 120) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \geq d) > 0.9$ $1 - P(T < d) > 0.9$ $P(T < d) < 0.1$ $d < 209.344$ Largest integral value of d is 209. Assume that the electricity used in each month is independent for a particular household
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, Y is unlikely to be normally distributed.
(vi)	Since sample size=55 is large, $\overline{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $\mathrm{P}(\overline{Y} > 45) = 0.724$ (3 s.f.)
12(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{6386}{150} = 42.573 \approx 42.6(3\text{s.f.})$ Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\overline{X} - \mu}{S / \sqrt{n}} \sim \mathrm{N}(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \bar{x} = 42.573,\ n = 150$.

	By GC, p-value $= 0.00137 \text{ (3 s.f.)}$. Since p-value < 0.05, we reject H_0 and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since n is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under H_0, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \text{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\ n = 40$.  Since H_0 is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4 \text{ (3 s.f.)}$ Required set = $\{k \in \mathbb{R} : 0 < k < 42.4\}$