JC 2
Preliminary Examination
Higher 1

MATHEMATICS **8864/01**

Paper 1 **30 Aug 2016**

Additional Materials: Answer paper **3 hours**
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your *Civics Group* and *Name* on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

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The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [35 marks]

1 Find the exact solution of the equation $9^x - 3^{2-2x} = 6$. [4]

2 The curve C has equation $y = x^2 - \ln(x+1)$.

(i) Sketch the graph of C, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

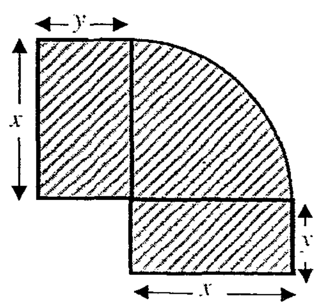
(ii) Deduce the range of values of m such that the equation $\ln(x+1) = x^2 - m$ has two distinct real roots. [2]

3 The curve C has equation $y = 2x^3 + kx^2 + kx - 5$, where k is a real constant.

(i) Find the range of values of k for which C has no stationary point. [4]

(ii) Find the value of k if the normal to C at the point where $x = -1$ is parallel to the line $x + 4y = 1$. [3]

4



The figure above shows the design for an earring consisting of a quarter circle with two identical rectangles attached to the two straight edges of the quarter circle. The quarter circle has radius x cm and each of the rectangles has dimensions x cm by y cm. The earring is assumed to have negligible thickness and treated as a two-dimensional object with an area of 12.25 cm^2.

(i) Show that the perimeter, P cm, of the earring is given by $P = 2x + \dfrac{49}{2x}$. [4]

(ii) Find the value of x and the corresponding value of y that makes the perimeter of the earring a minimum, fully justifying that this value of x produces a minimum perimeter. [5]

5 The equations of curves C_1 and C_2 are given by:

$$C_1 : y = 16 - (x-2)^2 \quad \text{and} \quad C_2 : y = 11 + e^{x-4}$$

(i) Sketch the graphs of C_1 and C_2 on the same diagram, stating the coordinates of any stationary points and points of intersection with the axes, and the equations of any asymptotes. [4]

(ii) Find the exact area of the region in the first quadrant bounded by C_1, C_2 and the y-axis. [4]

(iii) On your diagram in **(i)**, shade the region whose area is represented by the expression $\displaystyle\int_{12}^{16} \left(2 - \sqrt{16-y}\right) dy$. [2]

Section B: Statistics [60 marks]

6 A director of a company wants to obtain his employees' views on work-life balance. The company has 500 employees, both male and female, in 10 different departments. Describe clearly how you would choose a systematic random sample of 50 employees. Describe briefly one disadvantage of this sampling method in this context. [4]

7 The continuous random variable X has $\mathrm{E}(X) = 5$ and $\mathrm{Var}(X) = \dfrac{25}{3}$.

The random variable Y is defined by $Y = aX + b$, where a and b are positive constants. It is given that $\mathrm{E}(Y) = 20$ and $\mathrm{Var}(Y) = 75$. Find a and b. [4]

Find an approximate value for the probability that the sum of 50 random observations of Y is greater than 900. [3]

8 Ten people signed up for a weight-loss exercise programme. At the end of three months, the amount of time, t hours, they have spent on the programme and their weight loss, w kg, are recorded in the table below.

t (h)	50	64	110	95	75	44	135	98	100	66
w (kg)	2.0	2.3	1.2	4.8	4.1	1.8	5.5	5.2	4.7	3.5

(i) Give a sketch of the scatter plot for the data. Identify the outlier and indicate it as P on your diagram. Suggest a reason why this data pair may actually have been recorded correctly. [3]

Remove the identified data pair in part **(i)** for calculations from part **(ii)** onwards.

(ii) Calculate the product moment correlation coefficient. [1]

(iii) Find the equation of the regression line of w on t. Draw the line on the scatter diagram in **(i)**. [2]

(iv) A new participant intends to spend 80 hours on the programme in three months. Use the regression line in **(iii)** to predict the weight loss of this participant at the end of three months, giving your answer correct to 1 decimal place. Comment on the reliability of this prediction. [2]

9 A bag contains 5 white balls and n black balls.

(i) Two balls are drawn randomly from the bag with replacement. Find, in terms of n, the probability that the two balls are of different colours. [2]

It is now given that $n = 10$.

An ordinary fair die is rolled. If a '1' or '6' is obtained, two balls are drawn randomly from the bag with replacement. Otherwise, two balls are drawn randomly from the bag without replacement.

(ii) Find the probability that two white balls are drawn. [3]

(iii) Given that the two balls drawn are not both white, find the probability that a '1' or '6' is obtained on the die. [4]

10 Every week a restaurant manager buys a large quantity of a certain type of sushi. Recently, he suspected that the mean weight of a piece of sushi, μ g, is less than the stated value of 16.0 g. In a particular week, the restaurant manager randomly weighed 100 pieces of sushi and found that $\sum x = 1590$ and $\sum x^2 = 25381.4$, where x g denotes the weight of a piece of sushi.

(i) Calculate unbiased estimates for the population mean and variance of the weight of a piece of sushi. [2]

(ii) Test, at the 4% level of significance, whether the restaurant manager's suspicion is justified. [4]

(iii) Using the same set of data, a 2-tailed test is carried out at the 1% level of significance. Find the range of possible values of μ_0 if the null hypothesis $\mu = \mu_0$ is rejected. [4]

11 At a supermarket, customers can choose to pay for their purchases with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customer's method of payment. From past records, it is known that 60% of customers choose to pay with a credit card.

(i) There are 6 customers queueing at one cashier counter of the supermarket. Find the probability that

(a) fewer than three customers pay with a credit card, [2]

(b) the first three customers pay with a credit card and the next three pay with cash. [2]

(ii) There are n customers queueing at another cashier counter. Given that the probability that at least one customer pays with cash is greater than 0.995, find the minimum value of n. [3]

(iii) On a Sunday afternoon, there is a total of sixty customers queueing at the cashier counters in the supermarket. Using a suitable approximation, find the probability that more than half of these customers pay with a credit card. [4]

12 In this question, you should state clearly the values of the parameters of any normal distributions that you use.

(a) A student scores 75 marks in an examination and is ranked at the 75^{th} percentile. Given that the marks follow a normal distribution with mean 58, calculate the standard deviation of the distribution. [4]

(b) An owner of a factory uses a machine to dispense milk into bottles. The bottles are sold in 2 sizes, small and standard. The amount of milk dispensed in each type of bottle is normally distributed with means and standard deviations as shown in the table below.

	Mean (ml)	Standard deviation (ml)
Small	450	30
Standard	1000	35

(i) Find the probability that the mean amount of milk dispensed in 5 randomly chosen small bottles is more than 480 ml. [3]

(ii) Find the probability that the total amount of milk dispensed in 2 randomly chosen standard bottles exceeds 4 times the amount of milk dispensed in a randomly chosen small bottle by more than 250 ml. [4]

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[Turn over

Section A: Pure Mathematics [35 marks]

1 Find the exact solution of the equation $9^x - 3^{2-2x} = 6$. [4]

[Solution]

$$9^x - 3^{2-2x} = 6 \Rightarrow 9^x - \frac{9}{9^x} = 6$$

$$(9^x)^2 - 6(9^x) - 9 = 0$$

Let $y = 9^x$ $\quad \therefore y^2 - 6x - 9 = 0$

$$y = \frac{6 \pm \sqrt{36+36}}{2} = \frac{6 \pm 6\sqrt{2}}{2} = 3 \pm 3\sqrt{2}$$

Since $y = 9^x > 0$, $9^x = 3 + 3\sqrt{2}$

$$x \ln 9 = \ln(3 + 18\sqrt{2})$$

$$x = \frac{\ln(3+3\sqrt{2})}{\ln 9}$$

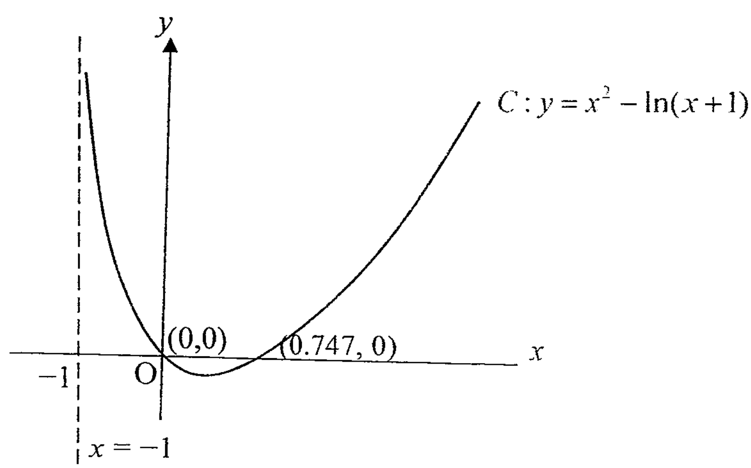
2 The curve C has equation $y = x^2 - \ln(x+1)$.

(i) Sketch the graph of C, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) Deduce the range of values of m such that the equation $\ln(x+1) = x^2 - m$ has two distinct real roots. [2]

[Solution]

(i)



(ii) $\ln(x+1) = x^2 - m \Rightarrow x^2 - \ln(x+1) = m$

Minimum point of C: $(0.366, -0.178)$

For the equation to have two distinct real roots, the line $y = m$ should cut the curve C at two points.

Hence $m > -0.178$

3 The curve C has equation $y = 2x^3 + kx^2 + kx - 5$, where k is a real constant.

(i) Find the range of values of k for which C has no stationary point. [4]

(ii) Find the value of k if the normal to C at the point where $x = -1$ is parallel to the line $x + 4y = 1$. [3]

[Solution]

(i) $y = 2x^3 + kx^2 + kx - 5$

$\frac{dy}{dx} = 6x^2 + 2kx + k$

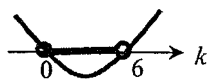
For stationary points, $\frac{dy}{dx} = 0 \Rightarrow 6x^2 + 2kx + k = 0$

If the curve has no stationary point, this equation has no real solution.

Discriminant $(2k)^2 - 4(6)k < 0$

$k^2 - 6k < 0$

$k(k - 6) < 0$



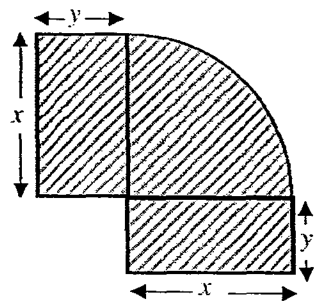
$0 < k < 6$

(ii) $x + 4y = 1 \Rightarrow y = -\frac{1}{4}x + \frac{1}{4}$ Gradient $= -\frac{1}{4}$

Hence gradient of tangent to C at $x = -1$ is 4. i.e. $\left.\frac{dy}{dx}\right|_{x=-1} = 4$

$6(-1)^2 + 2k(-1) + k = 4$

$6 - k = 4 \Rightarrow k = 2$



The figure above shows the design for an earring consisting of a quarter circle with two identical rectangles attached to the two straight edges of the quarter circle. The quarter circle has radius x cm and each of the rectangles has dimensions x cm by y cm. The earring is assumed to have negligible thickness and treated as a two-dimensional object with an area of 12.25 cm^2.

(i) Show that the perimeter, P cm, of the earring is given by $P = 2x + \dfrac{49}{2x}$. [4]

(ii) Find the value of x and the corresponding value of y that makes the perimeter of the earring a minimum, fully justifying that this value of x produces a minimum perimeter. [5]

[Solution]

(i) Area $= 2xy + \dfrac{1}{4}\pi x^2 = 12.25$

$$\Rightarrow 8xy + \pi x^2 = 49 \Rightarrow y = \frac{49 - \pi x^2}{8x} \quad \text{---------(1)}$$

$$P = 4y + 2x + \frac{1}{4}(2\pi x) \quad \text{----------(2)}$$

Sub (1) into (2): $P = 4\left(\dfrac{49 - \pi x^2}{8x}\right) + 2x + \dfrac{1}{2}\pi x$

$$= \frac{49 - \pi x^2}{2x} + 2x + \frac{1}{2}\pi x$$

$$= \frac{49}{2x} - \frac{\pi x}{2} + 2x + \frac{\pi x}{2} = 2x + \frac{49}{2x}$$

(ii) $\dfrac{dP}{dx} = 2 - \dfrac{49}{2x^2}$

For stationary points, $\dfrac{dP}{dx} = 0 \Rightarrow 2 - \dfrac{49}{2x^2} = 0$

$$x^2 = \frac{49}{4}$$

$$x = \frac{7}{2} \text{ or } -\frac{7}{2} \text{ (rejected as } x \text{ is positive)}$$

$\dfrac{d^2P}{dx^2} = \dfrac{49}{x^3} > 0$ at $x = \dfrac{7}{2}$, $\therefore$ P is minimum at $x = \dfrac{7}{2}$.

When $x = \dfrac{7}{2}$, $y = \dfrac{49 - \pi\left(\frac{49}{4}\right)}{8\left(\frac{7}{2}\right)} = 0.376$

5 The equations of curves C_1 and C_2 are given by:

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(i) Sketch the graphs of C_1 and C_2 on the same diagram, stating the coordinates of any stationary points and points of intersection with the axes, and the equations of any asymptotes. [4]

(ii) Find the exact area of the region in the first quadrant bounded by C_1, C_2 and the y-axis. [4]

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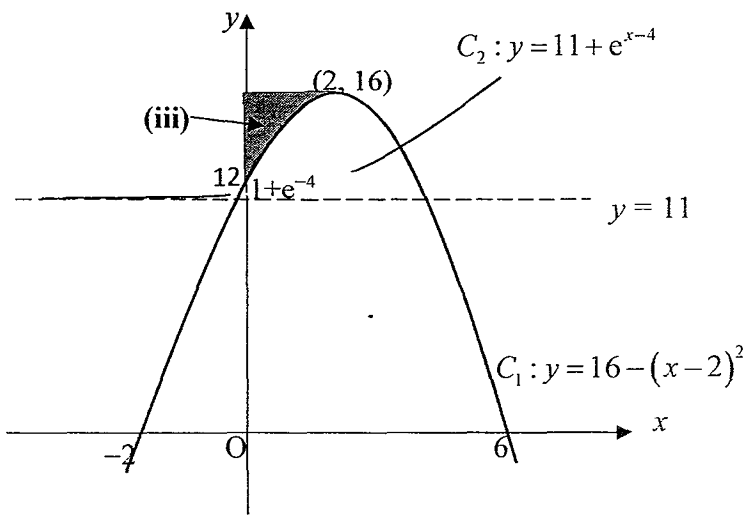
[Solution]

(i)



(ii) x-coordinate of the point of the intersection = 4

$$\text{Area} = \int_0^4 [16 - (x-2)^2 - 11 - e^{x-4}]\,\mathrm{d}x$$

$$= \int_0^4 [5 - (x-2)^2 - e^{x-4}]\,\mathrm{d}x$$

$$= \left[5x - \frac{(x-2)^3}{3} - e^{x-4}\right]_0^4$$

$$= \left(20 - \frac{8}{3} - 1\right) - \left(0 + \frac{8}{3} - e^{-4}\right)$$

$$= \frac{41}{3} + e^{-4} \text{ square units}$$

(iii) $y = 16 - (x-2)^2 \Rightarrow (x-2)^2 = 16 - y$

For $x \le 2$, $x = 2 - \sqrt{16-y}$

$\int_{12}^{16} \left(2 - \sqrt{16-y}\right) \mathrm{d}y$ = shaded area shown

Section B: Statistics [60 marks]

6 A director of a company wants to obtain his employees' views on work-life balance. The company has 500 employees, both male and female, in 10 different departments. Describe clearly how you would choose a systematic random sample of 50 employees. Describe briefly one disadvantage of this sampling method in this context. [4]

[Solution]

Use the list with all the employees' names arranged in alphabetical order and number them from 1 to 500.

Determine the sampling interval, $k = \frac{500}{50} = 10$

Randomly select the first person from the first 10 people on the list, then select every 10^{th} person subsequently until a sample of 30 employees is obtained.

The sample obtained may not be a good representative of the population as it may be over represented by a particular gender or department.

7 The continuous random variable X has $\mathrm{E}(X) = 5$ and $\mathrm{Var}(X) = \frac{25}{3}$.

The random variable Y is defined by $Y = aX + b$, where a and b are positive constants. It is given that $\mathrm{E}(Y) = 20$ and $\mathrm{Var}(Y) = 75$. Find a and b. [4]

Find an approximate value for the probability that the sum of 50 random observations of Y is greater than 900. [3]

[Solution]

$\mathrm{E}(Y) = a\,\mathrm{E}(X) + b$

$\mathrm{E}(Y) = 20 \Rightarrow 5a + b = 20$

$\mathrm{Var}(Y) = a^2\,\mathrm{Var}(X)$

$\mathrm{Var}(Y) = 75 \Rightarrow \frac{25}{3}a^2 = 75$

$\Rightarrow a^2 = 9 \Rightarrow a = 3$

Hence $b = 20 - 5(3) = 5$

Let $S = Y_1 + Y_2 + \ldots + Y_{50}$

Since $n = 50$ is large, by Central Limit Theorem,

$S \sim \mathrm{N}(50\times20,\ 50\times75) = \mathrm{N}(1000, 3750)$ approximately.

$\mathrm{P}(S > 900) = 0.949$

8 Ten people signed up for a weight-loss exercise programme. At the end of three months, the amount of time, t hours, they have spent on the programme and their weight loss, w kg, are recorded in the table below.

t (h)	50	64	110	95	75	44	135	98	100	66
w (kg)	2.0	2.3	1.2	4.8	4.1	1.8	5.5	5.2	4.7	3.5

(i) Give a sketch of the scatter plot for the data. Identify the outlier and indicate it as P on your diagram. Suggest a reason why this data pair may actually have been recorded correctly. [3]

Remove the identified data pair in part **(i)** for calculations from part **(ii)** onwards.

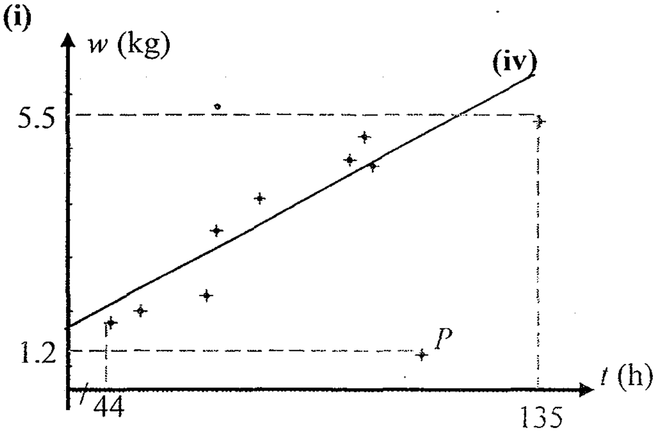
(ii) Calculate the product moment correlation coefficient. [1]

(iii) Find the equation of the regression line of w on t. Draw the line on the scatter diagram in **(i)**. [2]

(iv) A new participant intends to spend 80 hours on the programme in three months. Use the regression line in **(iii)** to predict the weight loss of this participant at the end of three months, giving your answer correct to 1 decimal place. Comment on the reliability of this prediction. [2]

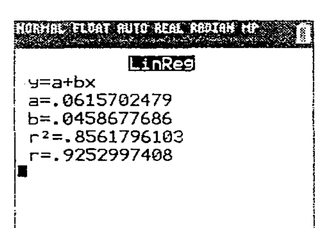
[Solution]

(i)



The data pair (110, 1.2) may have been recorded correctly as the participant may not have followed a proper diet plan even though he/she has committed 110 h to the exercise programme. Hence the weight loss is very low as compared to the rest.

(Any other reasonable answer)



(ii) From the G.C., $r = 0.925$

(iii) Regression line of w on t: $w = 0.061570 + 0.045868t$
i.e. $w = 0.0616 + 0.0459\,t$ (3 s.f.)

(iv) When $t = 80$ hours, $w = 3.7$ kg (correct to 1 d.p.)

As r is close to 1, it indicates a strong linear correlation between w and t, and since $t = 80$ h is within the data range, the prediction is reliable.

9 A bag contains 5 white balls and n black balls.

(i) Two balls are drawn randomly from the bag with replacement. Find, in terms of n, the probability that the two balls are of different colours. [2]

It is now given that $n = 10$.

An ordinary fair die is rolled. If a '1' or '6' is obtained, two balls are drawn randomly from the bag with replacement. Otherwise, two balls are drawn randomly from the bag without replacement.

(ii) Find the probability that two white balls are drawn. [3]

(iii) Given that the two balls drawn are not both white, find the probability that a '1' or '6' is obtained on the die. [4]

[Solution]

(i) The probability that the 2 balls are of different colours

$$= \text{P(BW)} + \text{P(WB)} = 2\left(\frac{5}{5+n}\right)\left(\frac{n}{5+n}\right) = \frac{10n}{(5+n)^2}$$

(ii) P(two white balls are drawn)

= P('1' or '6' is obtained) P(2 white balls are drawn with replacement)

+ P(neither '1' nor '6' is obtained) P(2 white balls are drawn without replacement)

$$= \frac{2}{6} \times \frac{5}{15} \times \frac{5}{15} + \frac{4}{6} \times \frac{5}{15} \times \frac{4}{14}$$

$= 0.100529 = 0.101$ (3 s.f.)

(iii) P('1' or '6' is obtained | 2 balls drawn are not both white)

$$= \frac{\text{P('1' or '6' is obtained and 2 balls drawn are not both white)}}{\text{P(2 balls drawn are not both white)}}$$

$$= \frac{\frac{2}{6}\left(1 - \frac{5}{15} \times \frac{5}{15}\right)}{1 - 0.100529}$$

$= 0.329$ (3 s.f.)

10 Every week a restaurant manager buys a large quantity of a certain type of sushi. Recently, he suspected that the mean weight of a piece of sushi, μ g, is less than the stated value of 16.0 g. In a particular week, the restaurant manager randomly weighed 100 pieces of sushi and found that $\sum x = 1590$ and $\sum x^2 = 25381.4$, where x g denotes the weight of a piece of sushi.

(i) Calculate unbiased estimates for the population mean and variance of the weight of a piece of sushi. [2]

(ii) Test, at the 4% level of significance, whether the restaurant manager's suspicion is justified. [4]

(iii) Using the same set of data, a 2-tailed test is carried out at the 1% level of significance. Find the range of possible values of μ_0 if the null hypothesis $\mu = \mu_0$ is rejected. [4]

[Solution]

(i) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{1590}{100} = 15.9$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right) = \frac{1}{99}\left(25381.4 - \frac{(1590)^2}{100}\right) = 1.01414 \approx 1.01$$

(ii) $H_0: \mu = 16.0$

$H_1: \mu < 16.0$

Level of significance: 4%

Since n (=100) is large, by **Central Limit Theorem**,

$$\frac{\overline{X} - \mu}{\sqrt{1.01414/100}} \sim N(0,1) \text{ approximately.}$$

If H_0 is true, $\mu = 16$.

p-value = 0.160 = 16% > 4%

Since p-value > level of significance, H_0 is not rejected. Hence there is insufficient evidence at 4% level of significance to justify the restaurant manager's suspicion.

(iii) $H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

$P(Z < -2.5758) = 0.005$

To reject H_0 at 1% level of significance,

$\Rightarrow z < -2.5758$ or $z > 2.5758$

$$\Rightarrow \frac{15.9 - \mu_0}{\sqrt{\frac{1.014}{100}}} < -2.5758 \text{ or } \frac{15.9 - \mu_0}{\sqrt{\frac{1.014}{100}}} > 2.5758$$

$\Rightarrow \mu_0 > 16.2$ or $\mu_0 < 15.6$

11 At a supermarket, customers can choose to pay for their purchases with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customer's method of payment. From past records, it is known that 60% of customers choose to pay with a credit card.

(i) There are 6 customers queueing at one cashier counter of the supermarket. Find the probability that

(a) fewer than three customers pay with a credit card, [2]

(b) the first three customers pay with a credit card and the next three pay with cash. [2]

(ii) There are n customers queueing at another cashier counter. Given that the probability that at least one customer pays with cash is greater than 0.995, find the minimum value of n. [3]

(iii) On a Sunday afternoon, there is a total of sixty customers queueing at the cashier counters in the supermarket. Using a suitable approximation, find the probability that more than half of these customers pay with a credit card. [4]

[Solution]

(i) **(a)** Let X be the number of customers out of 6 who pay with a credit card.

$$X \sim B(6, 0.6)$$

$$P(X < 3) = P(X \leq 2) = 0.179$$

(b) P (first 3 customers pay with credit card and next 3 pay with cash)

$= 0.6^3 \times 0.4^3$

$= 0.0138$

(ii) Let Y be the number of customers out of n who pay with cash.

$$Y \sim B(n, 0.4)$$

$P(Y \geq 1) > 0.995$

$\Rightarrow P(Y = 0) < 0.005$

Using GC,

When $n = 10$, $P(Y = 0) = 0.00605 > 0.005$

When $n = 11$, $P(Y = 0) = 0.00363 < 0.005$

Minimum value of n is 11.

Alternative Method:

$0.6^n < 0.005$

$n \ln 0.6 < \ln 0.005$

$n > 10.4$

Minimum value of n is 11.

(iii) Let W be the number of customers out of 60 who pay with a credit card.

$$W \sim B(60, 0.6)$$

Since $n = 60$ is large, $np = 36 > 5$ and $n(1-p) = 24 > 5$

$W \sim N(36, 14.4)$ approximately

$P(W > 30) \approx P(W > 30.5)$ by continuity correction

$= 0.926$ (0.943 without CC)

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(a) A student scores 75 marks in an examination and is ranked at the 75th percentile. Given that the marks follow a normal distribution with mean 58, calculate the standard deviation of the distribution. [4]

(b) An owner of a factory uses a machine to dispense milk into bottles. The bottles are sold in 2 sizes, small and standard. The amount of milk dispensed in each type of bottle is normally distributed with means and standard deviations as shown in the table below.

	Mean (ml)	Standard deviation (ml)
Small	450	30
Standard	1000	35

(i) Find the probability that the mean amount of milk dispensed in 5 randomly chosen small bottles is more than 480 ml. [3]

(ii) Find the probability that the total amount of milk dispensed in 2 randomly chosen standard bottles exceeds 4 times the amount of milk dispensed in a randomly chosen small bottle by more than 250 ml. [4]

[Solution]

(a) Let M be the marks a student scores in the examination.

$$M \sim \mathrm{N}\left(58,\ \sigma^2\right)$$

$$\mathrm{P}(M \le 75) = 0.75$$

$$\mathrm{P}\left(Z \le \frac{75-58}{\sigma}\right) = 0.75$$

From GC, $\dfrac{75-58}{\sigma} = 0.6744897$

$\Rightarrow \sigma = 25.2$

(b) Let X and Y be the amount of milk dispensed in a randomly chosen small and standard bottles respectively.

$$X \sim \mathrm{N}\left(450,\ 30^2\right), \qquad Y \sim \mathrm{N}\left(1000,\ 35^2\right)$$

(i) $\overline{X} = \dfrac{X_1 + X_2 + \ldots\ldots + X_5}{5} \sim \mathrm{N}\left(450,\ \dfrac{30^2}{5}\right)$

$\mathrm{P}\left(\overline{X} > 480\right) = 0.0127$

(ii) $\mathrm{E}\left(Y_1 + Y_2 - 4X\right) = 2(1000) - 4(450) = 200$

$\mathrm{Var}\left(Y_1 + Y_2 - 4X\right) = 2\left(35^2\right) + 4^2\left(30^2\right) = 16850$

$Y_1 + Y_2 - 4X \sim \mathrm{N}(200,\ 16850)$

$\mathrm{P}\left(Y_1 + Y_2 - 4X > 250\right) = 0.350$