2024 Nanyang Junior College Preliminary Examination H2 Mathematics Paper 1

1 Find the complex numbers v and w which satisfy the following simultaneous equations.

$$v^2 - \mathrm{i}w + 2 = 0$$

$$v = w + \mathrm{i} + 3$$

Give your answers in the form $a + \mathrm{i}b$, where a and b are real numbers. [4]

2 The path traced by a moving particle is a curve C, given by

$$x = a\theta - a\sin\theta \text{ and } y = a - a\cos\theta,$$

where a is a positive constant and $0 \le \theta < 2\pi$.

(a) Show that the gradient of C at a point with parameter θ, is $\cot\left(\frac{1}{2}\theta\right)$. [2]

(b) Find the equation of the tangent to C at the point where $\theta = \frac{\pi}{3}$ and show that this tangent passes through the point $\left(\frac{1}{3}\pi a,\ 2a\right)$. [4]

3 **(a)** Given that $y = \left(\sin^{-1} x\right)^2$, show that $\left(1 - x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} = 2$. [2]

(b) Hence obtain the Maclaurin expansion of y in terms of x, up to and including the term in x^4. [3]

(c) By using $x = \frac{1}{2}$, find an approximation for π in surd form. Find the percentage error of this approximation and comment on its accuracy. [3]

4 It is given that $\mathrm{f}(x) = \sqrt{3}\cos x + \sin x$.

(a) Write $\mathrm{f}(x)$ as $R\cos(x - \alpha)$, where R and α are constants to be found. [2]

(b) Find the exact value of $\int_0^{\frac{\pi}{6}} \left(\frac{1}{\mathrm{f}(x)}\right)^2 \mathrm{d}x$. [2]

(c) Find the exact value of $\int_0^{\frac{\pi}{12}} \frac{1}{\mathrm{f}(2x)} \mathrm{d}x$. [3]

5 **(a)** Find $\int \frac{4x-1}{x^2+4x+4} \mathrm{d}x$, giving your answer in simplest form. [4]

(b) Find the value of $\int_0^1 \frac{|4x-1|}{x^2+4x+4} \mathrm{d}x$, expressing your answer in the form $p \ln q + r$, where p, q and r are exact constants in simplest form to be found. [3]

6 **(a)** An arithmetic series has first term a and common difference d, where $d \neq 0$. The 1st, 6th and 14th terms of this series are the 1st, 2nd and 3rd terms of a geometric series. Find d in terms of a. [3]

(b) A geometric series has first term b, where $b > 0$, and common ratio 0.5.

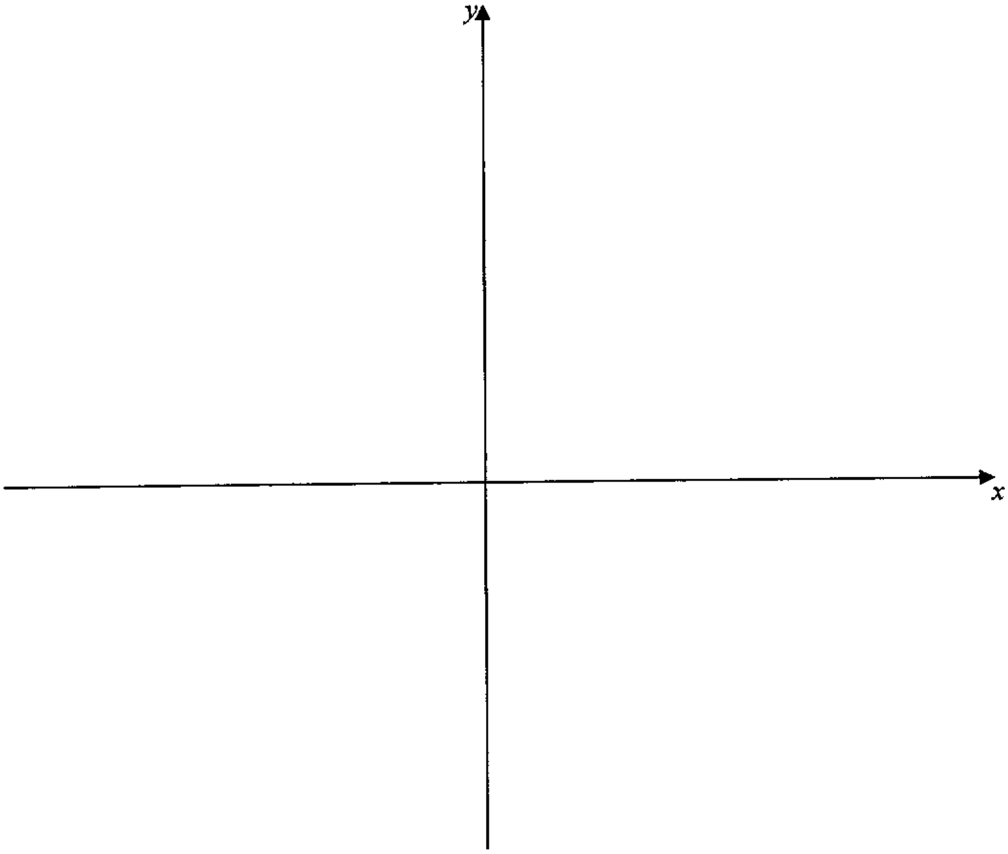
(i) Find the sum to infinity of this series in terms of b. [1]

(ii) Find the smallest possible value of n for which the sum of the first n terms of the series differs from the sum of the first $2n$ terms of the series by less than $0.004b$. [4]

7 An isosceles triangle ABC, with $AB = AC$, is inscribed in a fixed circle of radius 1 unit and centre O. It is given that angle $BOC = 2\theta$, where θ is acute. Using calculus, show that the area of the triangle ABC is a maximum when it is equilateral. State the maximum value of this area exactly. [8]

8 A curve C has equation $y = ax + b + \dfrac{1}{x-a}$, where a and b are positive real constants such that $a > 1$ and $x \neq a$.

(a) Sketch C on the axes below stating the equations of any asymptotes and the coordinates of the point where C crosses the y-axis. [4]



(b) On the same axes, sketch the graph of $(x-a)^2 + \left(y - a^2 - b\right)^2 = r^2$, where r is a positive constant, such that it intersects C at more than 2 points. $y = \pm\sqrt{r^2 - (x-a)^2} + a^2 + b$ [2]

(c) By stating the values of a and b, solve the inequality $2x+1+\frac{1}{x-2}>\sqrt{16-(x-2)^2}+5$. [3]

9 A curve R has equation $y=x^2\ln x$ for $x>0$. R crosses the x-axis at point A and has a turning point at B.

(a) State the coordinates of A. [1]

(b) Find the exact coordinates of B. [3]

(c) Find the exact area of the region bounded by R and the line segment AB. [5]

10 The functions f and g are defined by

$$\mathrm{f}:x\mapsto 2x+1, \quad x\in\mathbb{R},$$

$$\mathrm{g}:x\mapsto \frac{ax-1}{bx-a}, \quad x\in\mathbb{R}, \quad x\neq\frac{a}{b},$$

where a and b are positive constants.

(a) Find fg(1) in terms of a and b. You do not need to simplify your answer. [2]

(b) Explain why the function gf does not exist. [1]

(c) Find $\mathrm{g}^{-1}(x)$. [2]

(d) Hence, or otherwise, find $\mathrm{g}^2(x)$. [1]

(e) Given that $a>1$ and $b=1$, describe fully a sequence of transformations which transforms the curve $y=\mathrm{g}(x)$ **onto** the curve $y=\frac{1}{x}$. [4]

11 Ethylene is a gaseous plant hormone that plays an important role in inducing the ripening process for bananas. As a banana matures, ethylene is produced as a signal to induce fruit ripening. The amount of ethylene that a banana produces can be used to determine its ripeness.

Scientists set up an experiment to measure the amount of ethylene produced by a banana. The amount of ethylene, x parts per million (ppm), produced by a banana is modelled such that, at any time t hours, the rate of increase in the amount of ethylene is $k(160-x)$, for some positive constant k.

Initially no ethylene is present and it is found that there are 40 ppm of ethylene at time $t=12$.

(a) Write down and solve a differential equation involving x and t. Find the time it takes for the amount of ethylene to reach 100 ppm. [4]

To reduce the amount of ethylene produced, ethylene absorbers can be used. The scientists repeat the experiment by adding ethylene absorbers at a rate of d ppm per hour. Initially, no ethylene is present and the rate of increase in the amount of ethylene, $k(160-x)$, is the same as that found in part **(a)**.

(b) Write down and solve a differential equation, giving x in terms of t and d. [6]

(c) Find the value of d for which the amount of ethylene will be kept at a maximum of 10 ppm in the long run. [2]

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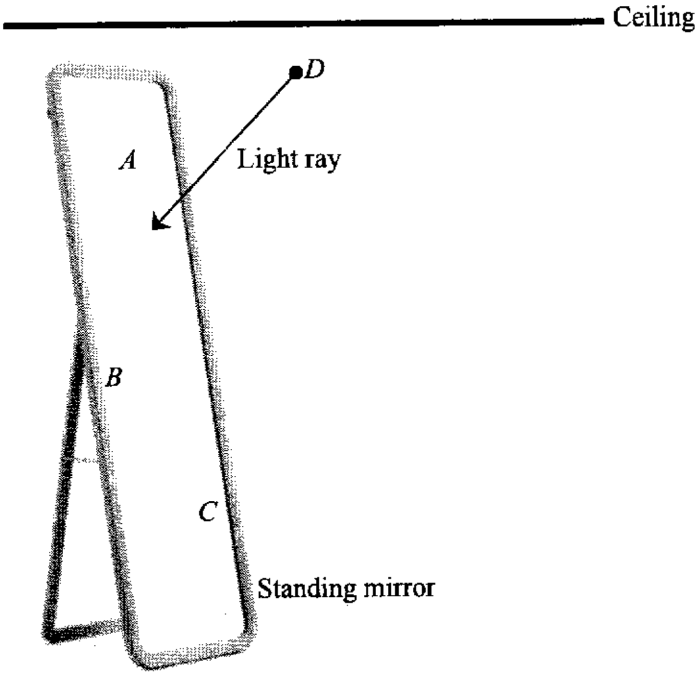


Credit: Image taken from https://www.fortytwo.sg/theola-standing-mirror-birch.html

In a room, there is a standing mirror and a light source at D below the ceiling. The three points A, B and C on the mirror, and D have position vectors $\mathbf{a}=2\mathbf{i}-\mathbf{j}+3\mathbf{k}$, $\mathbf{b}=-\mathbf{j}+6\mathbf{k}$, $\mathbf{c}=-\mathbf{i}-2\mathbf{j}+6\mathbf{k}$ and $\mathbf{d}=5\mathbf{i}-\mathbf{j}+8\mathbf{k}$ respectively with respect to a fixed point of reference.

(a) Show that the mirror can be modelled as a plane with cartesian equation $3x-3y+2z=15$. [2]

(b) Find the exact position vector of point F, the foot of perpendicular from D to the mirror. [3]

A light ray is sent in the direction of $\begin{pmatrix}-6\\-2\\-13\end{pmatrix}$ from D and hits the mirror at point E.

(c) Find the acute angle between the light ray and the mirror. [2]

(d) Show that the coordinates of E is $\left(2,-2,\frac{3}{2}\right)$. [2]

(e) The light ray from D is reflected in the mirror. The reflected ray and the light ray lie in the same plane. The angle between the light ray and the mirror is the same as the angle between the reflected ray and the mirror. Find a vector equation of the reflected ray. [3]

2024 Nanyang Junior College Preliminary Examination H2 Mathematics Paper 2

Section A: Pure Mathematics [40 marks]

1 The area bounded by the loop of the curve $y^2=(1-x)^2(x+3)$ is given by $2\int_{\alpha}^{\beta}(1-x)\sqrt{x+3}\,\mathrm{d}x$.

(a) State the values of α and β. [1]

(b) Use the substitution $u=\sqrt{x+3}$ to find the exact area bounded by this loop. [4]

2 With reference to the origin O, the points A, B and R have position vectors **a**, **b** and **r** respectively. The points A and B are fixed and R varies.

(a) Interpret geometrically the vector equation $\mathbf{r}=\lambda\mathbf{a}$ where $0\le\lambda\le1$. [1]

(b) Interpret geometrically the vector equation $\mathbf{r}\times(\mathbf{a}-\mathbf{b})=\mathbf{0}$. [2]

(c) Interpret geometrically if $\mathbf{r}\cdot(\mathbf{r}-\mathbf{a})=0$. [2]

(d) Given that $(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\mathbf{0}$. Find **r** in terms of **a** and **b**, showing your working clearly. [2]

3 **Do not use a calculator in answering the question.**

Three complex numbers are $z_1=-1+\sqrt{3}\mathrm{i}$, $z_2=-1-\mathrm{i}$ and $z_3=\sqrt{2}\left(\cos\frac{1}{12}\pi-\mathrm{i}\sin\frac{1}{12}\pi\right)$.

(a) Represent z_1, z_2 and z_3 on an Argand diagram. [2]

(b) Find $\dfrac{z_1}{z_2(z_3)^2}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [4]

A fourth complex number, z_4, is such that $\dfrac{z_1z_4}{z_2(z_3)^2}$ is purely real and $\left|\dfrac{z_1z_4}{z_2(z_3)^2}\right|=1$.

(c) Find the possible values of z_4 in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, where $r>0$ and $-\pi<\theta\le\pi$. [3]

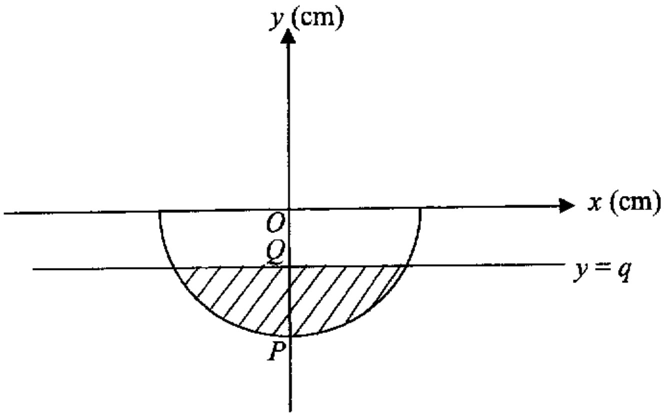
4 **(a)** Write $\dfrac{1}{4r^2-8r+3}$ in partial fractions. [2]

(b) Find an expression in terms of n for $\displaystyle\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)$. [3]

(c) Find $\displaystyle\sum_{r=2}^{\infty}\left(\frac{1}{4r^2-8r+3}\right)$. [1]

(d) Using the result in part **(b)**, find $\displaystyle\sum_{r=n+1}^{3n}\left(\frac{1}{4r^2-8r+3}\right)$. Write your answer as a simplified fraction in terms of n. [3]

5 In this question units are in centimetres (cm).



The diagram shows part of the circle $x^2 + y^2 = 256$ and the line $y = q$, where $q < 0$. The circle and the line cross the negative y-axis at points P and Q respectively.

(a) The shaded region between the circle and the line is rotated about the y-axis to form a wok of negligible thickness. The wok has depth PQ cm and a capacity of 3.3 litres. By finding the capacity of the wok in terms of q, calculate the depth of the wok giving your answer to the nearest integer. [1 litre = 1000 cm^3] [4]

(b) A flat frying pan of negligible thickness with a capacity of 1.464π litres is formed by rotating the part of the circle between the lines $y = 0$ and $y = r$, $q < r < 0$, about the y-axis. Find a cubic equation satisfied by r and hence find the value of r. [3]

(c) Water is poured from a container to the wok in part **(a)** at a rate of $\frac{3}{55}t$ litres per second, where t is the time in seconds from when pouring begins. Find the time taken to fill this wok to its full capacity of 3.3 litres. [3]

Section B: Probability and Statistics [60 marks]

6 A carpark has a row of 10 parking lots. On one day the carpark has 9 cars of different makes and 1 empty lot. Of the 9 cars, 2 are blue, 2 are red and the remaining 5 are of other distinct colours.

(a) Find the number of different arrangements of the cars in the carpark. [1]

(b) Find the number of different arrangements that can be made with both the blue cars next to each other and both the red cars next to each other. [2]

(c) Find the number of different arrangements that can be made with no two adjacent cars the same colour. [3]

7 A computer randomly chooses 10 shapes from 18 squares and 12 triangles. The number of squares chosen is denoted by S.

(a) The most probable number of squares that the computer chooses is denoted by s. By using the fact that $\mathrm{P}(S = s) > \mathrm{P}(S = s+1)$, show that s satisfies the inequality

$$(s+1)(s+3) > (a-s)(b-s),$$

where a and b are constants to be determined. Hence find the value of s. [4]

In a game, the computer randomly chooses 6 squares and 4 triangles. The squares are numbered from 1 to 6 and the triangles are numbered from 1 to 4. A square and a triangle are randomly chosen. Let X be the score which is defined as the difference (taken as always positive) between the numbers of the chosen square and triangle.

(b) Find the probability distribution of X. [3]

Three independent games are played. The total score of the games is 3.

(c) Find the probability that the score of the first game is 2. [3]

8 **In this question, you should state the parameters of any distributions you use.**

A baby quilt is made by stitching rectangular cotton fabric together. The length, in centimetres (cm), of each rectangular cotton fabric follows the distribution $\mathrm{N}\left(24, 1.5^2\right)$ and the breadth follows the distribution $\mathrm{N}\left(20, 1.2^2\right)$. Opposite sides of the rectangular cotton fabric are cut to the exact same measurement.

(a) Find the probability that the length of a randomly chosen rectangular cotton fabric is less than 23.5 cm. [1]

(b) Find the probability that the perimeter of a randomly chosen rectangular cotton fabric is more than 90 cm. [3]

(c) State a necessary assumption for your calculations to hold in part **(b)**. [1]

(d) A baby quilt is made by stitching 48 pieces of rectangular cotton fabric together. Calculate the expected number of pieces that have length between 23 and 25 cm. [2]

(e) Rectangular cotton fabrics can be paired together if their lengths differ from each other by at most k cm. Find the least value of k if 2 randomly chosen rectangular cotton fabrics have at least 90% chance of being paired together. [3]

9 A restaurant serves a signature dish. The probability that a diner orders the dish is 0.7. It may be assumed that each diner orders at most one portion of the dish and the number of diners who order the dish follows a binomial distribution.

(a) Find the probability that 10 diners order at least 3 portions of the signature dish. [2]

(b) Find the probability that 10 diners order between 3 and 8 portions of the signature dish. [2]

(c) If the restaurant has enough ingredients to prepare 80 portions of the signature dish, find the maximum possible number of diners if the restaurant has at least 90% chance of meeting the number of orders for the dish. [2]

On a particular day, the restaurant caters for a function with 40 tables of 10 diners each.

(d) Find the probability that each table orders at least 3 portions of the signature dish. [1]

(e) Find an estimate of the probability that the average number of portions of the signature dish served per table is at least 6.9. [3]

10 A bus company is contracted to ferry students at a particular school on request. Following feedback from teachers and students that the buses usually arrive, on average, more than 15 minutes **after the scheduled pick-up time**, the school administration manager conducts a study of the arrival times of the bus after the scheduled pick-up time.

The administration manager collects a random sample of 30 bus arrival times after the scheduled pick-up time. Summary data for the arrival times after the scheduled pick-up time, t minutes, of these buses is as follows.

$$n = 30 \qquad \sum t = 543 \qquad \sum t^2 = 12722$$

(a) Calculate unbiased estimates of the population mean and variance for the bus arrival time after the scheduled pick-up time. [2]

(b) State hypotheses that can be used to test if the administration manager should agree with the feedback from teachers and students, defining any symbols you use. Work out the test statistic in this case, and use it to carry out the test at the 5% level of significance, giving your answer in the context of the question. [5]

The bus company makes changes to its operations and claims that their buses will arrive, on average, 15 minutes after the scheduled pick-up time. The administration manager then takes another sample of 40 bus arrival times after the scheduled pick-up time. He carries out a 2-tail test and finds no significant evidence to reject the company's claim at the 10% level of significance.

(c) Use an algebraic method to evaluate the range of possible values of the sample mean bus arrival times after the scheduled pick-up time, $\bar{t}$. [3]

(d) State two necessary assumptions for your calculations in part **(c)**. [2]

11 Various studies suggest that plants exposed to music tend to grow taller compared to those not exposed to music. A junior biologist wants to investigate the effects of music on the growth of different plants. In one experiment, vines of a particular species were exposed to classical music. He recorded the average length of the vines, h centimetres, at age t days, in the table below.

t	7	10	13	16	19	22	25	28
h	19.2	25.4	36.5	41.4	45	k	51.9	53.8

(a) Using the data, the junior biologist calculated the least squares regression line of h on t to be $h = 1.6393t + 11.475$. Show that the value of k is 48.1, correct to 1 decimal place. [2]

The junior biologist realised that the relationship between h and t is not linear and can be modelled by either

$$h = a\sqrt[3]{t} + b \text{ or } h = c\sqrt{t} + d,$$

where a, b, c and d are constants.

(b) By calculating the relevant product moment correlation coefficients, explain why $h = a\sqrt[3]{t} + b$ is a better model than $h = c\sqrt{t} + d$. Find the equation of the better model. [4]

(c) An estimate for the average length of the vines when they are 2 months old is obtained using the better model. Explain whether the estimate is reliable. [1]

(d) Sketch a scatter diagram for h against $\sqrt[3]{t}$. Draw the line of best fit on your scatter diagram. [2]

For a line of best fit $y = \mathrm{f}(x)$, the residual is the difference between the observed and predicted value. The difference between an observed value (p,q) plotted on the scatter diagram and the predicted value $(p,\mathrm{f}(p))$ is defined as $q-\mathrm{f}(p)$.

(e) Find the value of the residual corresponding to $t = 28$, for the line of best fit found in part **(b)**. [1]

(f) Explain why the line of best fit is referred to as the 'least squares' regression line in relation to the residuals. [2]

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Q1	Suggested Answers
	Make w the subject from $w+i+3=v$: $w=v-i-3$ Sub into $v^2-iw+2=0$ $v^2-i(v-i-3)+2=0$ $v^2-iv+1+3i=0$ **Using quadratic formula,** $v=\frac{i\pm\sqrt{-1-4(1+3i)}}{2}$ $=\frac{i\pm\sqrt{-5-12i}}{2}$ $=\frac{i\pm(2-3i)}{2}$ use GC to find $\sqrt{-5-12i}$ $=\frac{i+(2-3i)}{2}$ or $\frac{i-(2-3i)}{2}$ $=1-i$ or $-1+2i$ $w=-2-2i$ or $-4+i$
	Alternative (1) Sub $v=w+i+3$ into $v^2-iw+2=0$ $(w+i+3)^2-iw+2=0$ $w^2+2(i+3)w+(i+3)^2-iw+2=0$ **Using quadratic formula,** $w=\frac{-(6+i)\pm\sqrt{(6+i)^2-4(10+6i)}}{2}$ $=\frac{-(6+i)\pm\sqrt{-5-12i}}{2}=\frac{-(6+i)\pm(2-3i)}{2}$ use GC to find $\sqrt{-5-12i}$ $w=-2-2i,-4+i$ Using $v=w+i+3$ We have $v=1-i,-1+2i$
	Alternative (2) From the equation $w+i+3=v$ Multiply by i, we have $iw-1+3i=iv\cdots(1)$ $v^2-iw+2=0\cdots(2)$ (1)+(2): $v^2-iv+1+3i=0$ **Using quadratic formula,**

$$v=\frac{\mathrm{i}\pm\sqrt{-1-4(1+3\mathrm{i})}}{2}$$

$$=\frac{\mathrm{i}\pm\sqrt{-5-12\mathrm{i}}}{2}$$

$$=\frac{\mathrm{i}\pm(2-3\mathrm{i})}{2}\quad\text{use GC to find }\sqrt{-5-12\mathrm{i}}$$

$$=\frac{\mathrm{i}+(2-3\mathrm{i})}{2}\text{ or }\frac{\mathrm{i}-(2-3\mathrm{i})}{2}$$

$$=1-\mathrm{i}\text{ or }-1+2\mathrm{i}$$

$$w=-2-2\mathrm{i}\text{ or }-4+\mathrm{i}$$

Q2 **Suggested Answers**

(a)

$$x=a\theta-a\sin\theta\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}\theta}=a-a\cos\theta$$

$$y=a-a\cos\theta\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=a\sin\theta$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a\sin\theta}{a(1-\cos\theta)}$$

$$=\frac{\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)}{\left(1-\left(1-2\sin^2\frac{\theta}{2}\right)\right)}$$

$$=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=\cot\frac{\theta}{2}$$

(b)

When $\theta=\frac{\pi}{3}$, $x=\frac{\pi}{3}a-\frac{\sqrt{3}}{2}a$, $y=\frac{1}{2}a$

and $\frac{\mathrm{d}y}{\mathrm{d}x}=\cot\frac{\pi}{6}=\frac{1}{\tan\frac{\pi}{6}}=\frac{1}{\left(\frac{1}{\sqrt{3}}\right)}=\sqrt{3}$

Equation of tangent is $y-\frac{1}{2}a=\sqrt{3}\left(x-\frac{\pi}{3}a+\frac{\sqrt{3}}{2}a\right)$

$$\therefore y=\sqrt{3}x-\frac{\sqrt{3}}{3}\pi a+2a$$

When $x=\frac{\pi}{3}a$, $y=\sqrt{3}\left(\frac{\pi}{3}a\right)x-\frac{\sqrt{3}}{3}\pi a+2a=2a$

Therefore, the tangent passes through $\left(\frac{1}{3}\pi a,\ 2a\right)$

Or

When $y=2a$, $2a=\sqrt{3}x-\frac{\sqrt{3}}{3}\pi a+2a\quad\Rightarrow x=\frac{\frac{\sqrt{3}}{3}\pi a}{\sqrt{3}}=\frac{1}{3}\pi a$

Therefore, the tangent passes through $\left(\frac{1}{3}\pi a,\ 2a\right)$

Q3	Suggested Answers
(a)	$y=\left(\sin^{-1}x\right)^2$ Differentiate wrt x: $\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\sin^{-1}x\right)\frac{1}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin^{-1}x$ Differentiate wrt x: $\sqrt{1-x^2}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{x}{\sqrt{1-x^2}}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\sqrt{1-x^2}}$ $\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}=2$ **Alternatively,** $\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin^{-1}x$ $\left(\sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\left(2\sin^{-1}x\right)^2$ $\left(1-x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4y$ Differentiate wrt x : $\left(1-x^2\right)\cdot 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4\frac{\mathrm{d}y}{\mathrm{d}x}$ $\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}=2$ $\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}-2=0$
(b)	Differentiate wrt x : $\left(1-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left(x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}\right)=0$ $\left(1-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Differentiate wrt x : $\left(1-x^2\right)\frac{\mathrm{d}^4y}{\mathrm{d}x^4}-2x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-3\left(x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)-\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ $\left(1-x^2\right)\frac{\mathrm{d}^4y}{\mathrm{d}x^4}-5x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-4\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ When $x=0$, $y=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0$ and $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=8$

	$\therefore y=2\left(\frac{x^2}{2!}\right)+8\left(\frac{x^4}{4!}\right)+\ldots$ $y=x^2+\frac{1}{3}x^4+\ldots$
(c)	$y\approx x^2+\frac{1}{3}x^4$ $\left(\sin^{-1}x\right)^2\approx x^2+\frac{1}{3}x^4$ When $x=\frac{1}{2}$, $\left(\sin^{-1}\frac{1}{2}\right)^2\approx\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^4$ $\frac{\pi^2}{36}\approx\frac{13}{48}$ $\pi\approx\frac{\sqrt{39}}{2}$ Percentage error of approximation $=\frac{\pi-\frac{\sqrt{39}}{2}}{\pi}\times 100\%$ $=0.608\%$ (3 s.f.) Approximation is quite accurate as the value of x is **close to zero** **and** the series used terms up to x^4.

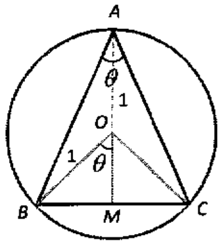
Q4	Suggested Answers
(a)	$\mathrm{f}(x)=\sqrt{3}\cos x+\sin x=R\cos(x-\alpha)$ $=R[\cos x\cos\alpha+\sin x\sin\alpha]$ $R\cos\alpha=\sqrt{3}$ --- (1) $R\sin\alpha=1$ --- (2) $(1)^2+(2)^2$: $R=\sqrt{\left(\sqrt{3}\right)^2+1^2}=2$ $\frac{(2)}{(1)}$: $\alpha=\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$
(b)	$\int_0^{\frac{\pi}{6}}\left(\frac{1}{\mathrm{f}(x)}\right)^2\mathrm{d}x=\int_0^{\frac{\pi}{6}}\frac{1}{\left(2\cos\left(x-\frac{\pi}{6}\right)\right)^2}\mathrm{d}x$ $=\frac{1}{4}\int_0^{\frac{\pi}{6}}\sec^2\left(x-\frac{\pi}{6}\right)\mathrm{d}x$ $=\frac{1}{4}\left[\tan\left(x-\frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$ $=\frac{1}{4}\left[\tan 0-\tan\left(-\frac{\pi}{6}\right)\right]$ $=\frac{1}{4\sqrt{3}}=\frac{\sqrt{3}}{12}$
(c)	$\int_0^{\frac{\pi}{12}}\frac{1}{\mathrm{f}(2x)}\mathrm{d}x=\int_0^{\frac{\pi}{12}}\frac{1}{2\cos\left(2x-\frac{\pi}{6}\right)}\mathrm{d}x$ $=\frac{1}{2}\int_0^{\frac{\pi}{12}}\sec\left(2x-\frac{\pi}{6}\right)\mathrm{d}x$ $=\frac{1}{2}\left(\frac{1}{2}\right)\ln\left[\left\|\sec\left(2x-\frac{\pi}{6}\right)+\tan\left(2x-\frac{\pi}{6}\right)\right\|\right]_0^{\frac{\pi}{12}}$ $=\frac{1}{4}\left[\ln\|\sec 0+\tan 0\|-\ln\left\|\sec\left(-\frac{\pi}{6}\right)+\tan\left(-\frac{\pi}{6}\right)\right\|\right]$ $=\frac{1}{4}\left[\ln 1-\ln\left\|\frac{2}{\sqrt{3}}+\left(-\frac{2}{\sqrt{3}}\right)\right\|\right]$ $=-\frac{1}{4}\ln\frac{1}{\sqrt{3}}$ $=\frac{1}{4}\ln\sqrt{3}=\frac{1}{8}\ln 3$

Q5	Suggested Answers
(a)	**Method 1:** $\int \frac{4x-1}{x^2+4x+4}\,\mathrm{d}x$ $=\int \frac{2(2x+4)-9}{x^2+4x+4}\,\mathrm{d}x$ $=2\int \frac{2x+4}{x^2+4x+4}\,\mathrm{d}x-\int \frac{9}{(x+2)^2}\,\mathrm{d}x$ $=2\ln\left(x^2+4x+4\right)+\frac{9}{x+2}+C$ $=4\ln(x+2)+\frac{9}{x+2}+C$
	Method 2: Use of partial fractions $\frac{4x-1}{x^2+4x+4}=\frac{4x-1}{(x+2)^2}=\frac{A}{x+2}+\frac{B}{(x+2)^2}$ $4x-1=A(x+2)+B$ Solving, $A=4$, $B=-9$ $\int \frac{4x-1}{x^2+4x+4}\,\mathrm{d}x=\int \frac{4}{x+2}-\frac{9}{(x+2)^2}\,\mathrm{d}x$ $=4\ln(x+2)+\frac{9}{x+2}+C$
(b)	$\int_0^1 \frac{\lvert 4x-1\rvert}{x^2+4x+4}\,\mathrm{d}x$ $=-\int_0^{\enclose{circle}{\frac{1}{4}}} \frac{4x-1}{x^2+4x+4}\,\mathrm{d}x+\int_{\enclose{circle}{\frac{1}{4}}}^1 \frac{4x-1}{x^2+4x+4}\,\mathrm{d}x$ $=-\left[4\ln(x+2)+\frac{9}{x+2}\right]_0^{\frac{1}{4}}+\left[4\ln(x+2)+\frac{9}{x+2}\right]_{\frac{1}{4}}^1$ $=-\left[\left(4\ln\frac{9}{4}+4\right)-\left(4\ln 2+\frac{9}{2}\right)\right]+\left[(4\ln 3+3)-\left(4\ln\frac{9}{4}+4\right)\right]$ $=-4\ln\frac{9}{4}+4\ln 2+4\ln 3-4\ln\frac{9}{4}-\frac{1}{2}$ $=4\ln\frac{2\times 3}{\left(\frac{9}{4}\times\frac{9}{4}\right)}-\frac{1}{2}$ $=4\ln\frac{32}{27}-\frac{1}{2}$

<table>
<tr><th>Q6</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>$\frac{a+5d}{a}=\frac{a+13d}{a+5d}$
$(a+5d)^2=a(a+13d)$
$a^2+10ad+25d^2=a^2+13ad$
$25d^2=3ad$
Since $d\neq 0$, $3a=25d$
$d=\frac{3a}{25}$</td></tr>
<tr><td>(b)(i)</td><td>Sum to infinity $=\frac{b}{1-0.5}=2b$</td></tr>
<tr><td>(ii)</td><td>$S_n=\frac{b\left(1-0.5^n\right)}{1-0.5}=2b\left(1-0.5^n\right)$
$S_{2n}-S_n<0.004b$ since $S_{2n}>S_n$ as all the terms are positive
$2b\left(1-0.5^{2n}\right)-2b\left(1-0.5^n\right)<0.004b$
$0.5^n-0.5^{2n}<0.002$ since $b>0$
$0.5^n-0.5^{2n}-0.002<0$

Using GC,
<table>
<tr><td>n</td><td>$0.5^n-0.5^{2n}-0.002$</td></tr>
<tr><td>8</td><td>$0.00189>0$</td></tr>
<tr><td>9</td><td>$-5.07\times10^{-5}<0$</td></tr>
</table>
Smallest $n=9$</td></tr>
</table>

Q7 **Suggested Answers**



$BM = \sin\theta$ and $OM = \cos\theta$

Length of height, $AM = 1+\cos\theta$
Length of base, $BC = 2\sin\theta$

$$\text{Area of triangle, } S = \frac{1}{2}\times(1+\cos\theta)\times 2\sin\theta$$
$$= \sin\theta + \frac{1}{2}\sin 2\theta$$

Alternatively,

$$\text{Area of triangle, } S = 2\left(\frac{1}{2}\times(1)^2\sin(\pi-\theta)\right)+\frac{1}{2}\times(1)^2\sin 2\theta$$
$$= \sin\theta + \frac{1}{2}\sin 2\theta$$

$$\frac{\mathrm{d}S}{\mathrm{d}\theta} = \cos\theta + \cos 2\theta$$

For maximum area, $\frac{\mathrm{d}S}{\mathrm{d}\theta} = 0$

$\cos\theta + \cos 2\theta = 0$
$\cos\theta + 2\cos^2\theta - 1 = 0$
$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$ (rejected since θ is acute)

Therefore, $\theta = \frac{\pi}{3}$

$$\frac{\mathrm{d}^2S}{\mathrm{d}\theta^2} = -\sin\theta - 2\sin 2\theta$$

When $\theta = \frac{\pi}{3}$, $\frac{\mathrm{d}^2S}{\mathrm{d}\theta^2} = -\frac{3\sqrt{3}}{2} < 0$

Thus S is maximum when $\theta = \frac{\pi}{3}$

Since $\angle BOC = 2\theta$, $\angle BAC = \theta = \frac{\pi}{3}$ ($\angle$ at centre = 2 $\angle$ at circumference)

As the triangle is isosceles, all the angles in the triangle are $\frac{\pi}{3}$.

Therefore, maximum area occurs when triangle ABC is equilateral, ie, when $\theta = \frac{\pi}{3}$

Maximum area $= \sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} = \frac{3}{4}\sqrt{3}$

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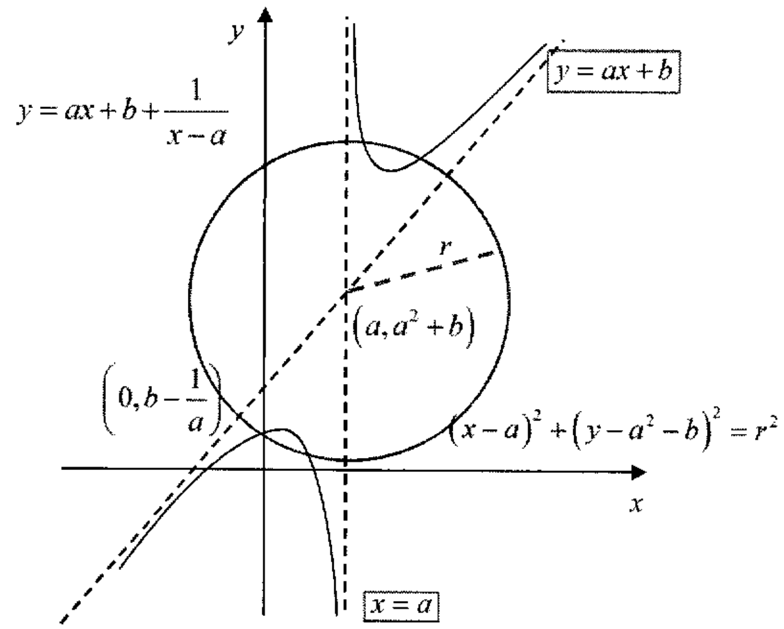
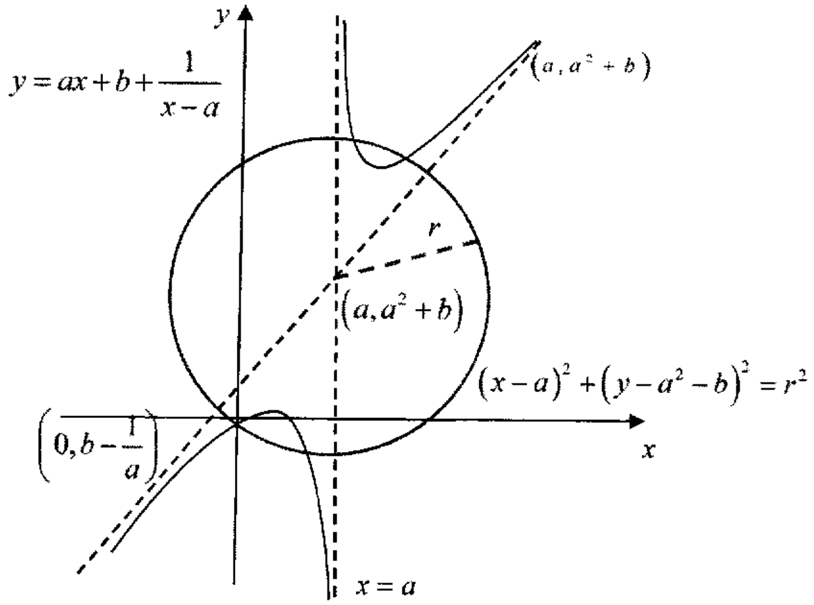
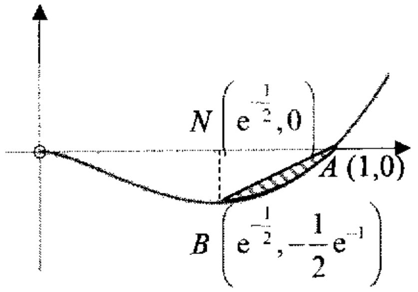
Q8	Suggested Answers
(a) (b)	y-intercept $b-\frac{1}{a}>0$  **OR** y-intercept $b-\frac{1}{a}<0$ (give one of the diagrams will do) 
(c)	$y=ax+b+\frac{1}{x-a}=2x+1+\frac{1}{x-2}$ $y=+\sqrt{r^2-(x-a)^2}+a^2+b=\sqrt{16-(x-2)^2}+5$ **(upper semicircle only)** $a=2$ and $b=1$ (and $r=4$) Using GC, the x-coordinates of the points of intersection are 2.29 and 3.52

	For $2x+1+\frac{1}{x-2}>\sqrt{16-(x-2)^2}+5$ **(upper semicircle only)** $2<x<2.29$ or $3.52<x\le 6$ (the circle is only defined for [−2, 6])

Q9	**Suggested Answers**
(a)	Coordinates of A are $(1,0)$
(b)	$y=x^2\ln x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2x\ln x+x^2\left(\frac{1}{x}\right)=x(2\ln x+1)$ $x(2\ln x+1)=0$ $x=0$ or $\ln x=-\frac{1}{2}$ Since $x>0$, $x=\mathrm{e}^{-\frac{1}{2}}$ Coordinates of B are $\left(\mathrm{e}^{-\frac{1}{2}},-\frac{1}{2}\mathrm{e}^{-1}\right)$
(c)	 $u=\ln x \quad \frac{\mathrm{d}v}{\mathrm{d}x}=x^2$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x} \quad v=\frac{1}{3}x^3$ **Method 1:** Area required $=\left(\circleddash\int_{\mathrm{e}^{-\frac{1}{2}}}^{1}\left(x^2\ln x\right)\mathrm{d}x\right)-$ area of triangle ABN $=-\left(\left[\frac{1}{3}x^3\ln x\right]_{\mathrm{e}^{-\frac{1}{2}}}^{1}-\frac{1}{3}\int_{\mathrm{e}^{-\frac{1}{2}}}^{1}x^2\,\mathrm{d}x\right)-\frac{1}{4}\mathrm{e}^{-1}\left(1-\mathrm{e}^{-\frac{1}{2}}\right)$ using integration by parts $=-\left(\left[0-\frac{1}{3}\mathrm{e}^{-\frac{3}{2}}\left(-\frac{1}{2}\right)\right]-\frac{1}{9}\left[x^3\right]_{\mathrm{e}^{-\frac{1}{2}}}^{1}\right)-\frac{1}{4}\mathrm{e}^{-1}\left(1-\mathrm{e}^{-\frac{1}{2}}\right)$ $=-\frac{1}{6}\mathrm{e}^{-\frac{3}{2}}+\frac{1}{9}\left(1-\mathrm{e}^{-\frac{3}{2}}\right)-\frac{1}{4}\mathrm{e}^{-1}\left(1-\mathrm{e}^{-\frac{1}{2}}\right)$ $=\frac{1}{9}-\frac{1}{36}\mathrm{e}^{-\frac{3}{2}}-\frac{1}{4}\mathrm{e}^{-1}$
	Method 2: (area bounded between line and curve) Equation of line joining A and B is $y=\frac{\frac{1}{2}\mathrm{e}^{-1}}{1-\mathrm{e}^{-\frac{1}{2}}}(x-1)=\frac{1}{2\mathrm{e}\left(1-\mathrm{e}^{-\frac{1}{2}}\right)}(x-1)=\frac{1}{2\left(\mathrm{e}-\mathrm{e}^{\frac{1}{2}}\right)}(x-1)$ Area required

	$=\int_{e^{-\frac{1}{2}}}^{1}\frac{1}{2\left(e-e^{\frac{1}{2}}\right)}(x-1)\,dx-\int_{e^{-\frac{1}{2}}}^{1}\left(x^{2}\ln x\right)dx$ $=\frac{1}{2\left(e-e^{\frac{1}{2}}\right)}\left[\frac{x^{2}}{2}-x\right]_{e^{-\frac{1}{2}}}^{1}-\left\{\left[\frac{1}{3}x^{3}\ln x\right]_{e^{-\frac{1}{2}}}^{1}-\frac{1}{3}\int_{e^{-\frac{1}{2}}}^{1}x^{2}\,dx\right\}$ using integration by parts $=\frac{1}{2\left(e-e^{\frac{1}{2}}\right)}\left[\left(\frac{1}{2}-1\right)-\left(\frac{e^{-1}}{2}-e^{-\frac{1}{2}}\right)\right]-\left\{\left[0-\frac{1}{3}e^{-\frac{3}{2}}\left(-\frac{1}{2}\right)\right]-\frac{1}{9}\left[x^{3}\right]_{e^{-\frac{1}{2}}}^{1}\right\}$ $=\frac{1}{2\left(e-e^{\frac{1}{2}}\right)}\left[e^{-\frac{1}{2}}-\frac{1}{2}-\frac{e^{-1}}{2}\right]-\frac{1}{6}e^{-\frac{3}{2}}+\frac{1}{9}\left(1-e^{-\frac{3}{2}}\right)$ $=\frac{1}{4\left(e-e^{\frac{1}{2}}\right)}\left[2e^{-\frac{1}{2}}-1-e^{-1}\right]-\frac{5}{18}e^{-\frac{3}{2}}+\frac{1}{9}$

Q10	Suggested Answers
(a)	$\mathrm{fg}(1)=\mathrm{f}\left(\frac{a-1}{b-a}\right)$ $=2\left(\frac{a-1}{b-a}\right)+1$
(b)	$R_{\mathrm{f}}=\mathbb{R}$ and $D_{\mathrm{g}}=\mathbb{R}\setminus\left\{\frac{a}{b}\right\}$ Since $R_{\mathrm{f}}\not\subseteq D_{\mathrm{g}}$, gf does not exist.
(c)	Let $y=\frac{ax-1}{bx-a}$ $bxy-ay=ax-1$ $bxy-ax=ay-1$ $x=\frac{ay-1}{by-a}$ $\mathrm{g}^{-1}(x)=\frac{ax-1}{bx-a}$
(d)	Hence method: Since $\mathrm{g}(x)=\mathrm{g}^{-1}(x)$ $\mathrm{gg}(x)=\mathrm{gg}^{-1}(x)$ $\mathrm{g}^{2}(x)=x$

Otherwise method:

$$\begin{aligned}
g^2(x) &= gg(x)\\
&= g\left(\frac{ax-1}{bx-a}\right)\\
&= \frac{a\left(\frac{ax-1}{bx-a}\right)-1}{b\left(\frac{ax-1}{bx-a}\right)-a}\\
&= \frac{a(ax-1)-(bx-a)}{b(ax-1)-a(bx-a)}\\
&= \frac{a^2x-a-bx+a}{abx-b-abx+a^2}\\
&= \frac{a^2x-bx}{a^2-b}\\
&= x
\end{aligned}$$

(e)

$y = g(x) = \dfrac{ax-1}{x-a} = a + \dfrac{a^2-1}{x-a}$ where $a > 0$

Replace y by $y+a$: $\quad y = \dfrac{a^2-1}{x-a}$

Replace y by $\dfrac{y}{\left(\frac{1}{a^2-1}\right)}$: $\quad y = \dfrac{1}{x-a}$

Replace x by $x+a$: $\quad y = \dfrac{1}{x}$

(1) **Translate** the graph by a units **in the negative direction** of the y-axis.

(2) **Scaling** by a **factor** of $\dfrac{1}{a^2-1}$ **parallel** to the y-axis.

(3) **Translate** a units **in the negative direction** of the x-axis.

Note: For this qn, the following order are accepted also
(1), (3), (2)
(3), (1), (2)
Transformation (1) must come before (2) if the above descriptions are used.

Q11	Suggested Answers
(a)	$\frac{dx}{dt}=k(160-x),\quad \text{where } k>0$ $\frac{1}{160-x}\frac{dx}{dt}=k$ $\int\frac{1}{160-x}\,dx=k\int 1\,dt$ $-\ln\lvert 160-x\rvert=kt+C$ $160-x=Ae^{-kt}\quad \text{where } A=\pm e^{-C}$ $t=0,x=0\Rightarrow A=160$ $160-x=160e^{-kt}$ $t=12,x=40\Rightarrow 120=160e^{-12k}$ $\frac{3}{4}=e^{-12k}$ $k=-\frac{1}{12}\ln\frac{3}{4}$ $\therefore 160-x=160e^{\left(\frac{1}{12}\ln\frac{3}{4}\right)t}\quad =160\left(e^{\ln\frac{3}{4}}\right)^{\frac{t}{12}}=160\left(\frac{3}{4}\right)^{\frac{t}{12}}$ When $x=100,\quad 60=160\left(\frac{3}{4}\right)^{\frac{t}{12}}\Rightarrow t=40.9$ (3 s.f.) The time taken is 40.9 h.
(b)	$\frac{dx}{dt}=k(160-x)-d=(160k-d)-kx$ $\frac{1}{(160k-d)-kx}\frac{dx}{dt}=1$ $-\frac{1}{k}\ln\lvert(160k-d)-kx\rvert=t+C$ $\ln\lvert(160k-d)-kx\rvert=-kt-kC$ $(160k-d)-kx=Be^{-kt}\quad \text{where } B=\pm e^{-kC}$ $t=0,x=0\Rightarrow B=160k-d$ $(160k-d)-kx=(160k-d)e^{-kt}$ $kx=(160k-d)\left(1-e^{-kt}\right)$ $x=\left(160-\frac{d}{k}\right)\left(1-e^{-kt}\right)$ $=\left(160+\frac{12d}{\ln\frac{3}{4}}\right)\left(1-e^{\left(\frac{1}{12}\ln\frac{3}{4}\right)t}\right)=\left(160+\frac{12d}{\ln\frac{3}{4}}\right)\left(1-\left(\frac{3}{4}\right)^{\frac{t}{12}}\right)$

(c)	As $t \to \infty, \left(\frac{3}{4}\right)^{\frac{t}{12}} \to 0$ Thus $x \to 160+\frac{12d}{\ln\frac{3}{4}}$ (limit in the long run) Let $160+\frac{12d}{\ln\frac{3}{4}}=10$ (x is increasing as t increases and approaches the limit) $d=3.5960$ (5 s.f.) $=3.60$ (3 s.f.)

Q12	**Suggested Answers**
(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\0\\3\end{pmatrix}$ $\overrightarrow{BC}=\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}-3\\-1\\3\end{pmatrix}$ A normal to plane is $\overrightarrow{AB}\times\overrightarrow{BC}=\begin{pmatrix}3\\-3\\2\end{pmatrix}$ Equation of plane ABC : $\mathbf{r}\bullet\begin{pmatrix}3\\-3\\2\end{pmatrix}=\begin{pmatrix}0\\-1\\6\end{pmatrix}\bullet\begin{pmatrix}3\\-3\\2\end{pmatrix}=15$ $3x-3y+2z=15$
(b)	$\mathbf{r}=\begin{pmatrix}5\\-1\\8\end{pmatrix}+\mu\begin{pmatrix}3\\-3\\2\end{pmatrix}, \mu\in\mathbb{R}$ Equation of plane ABC : $\mathbf{r}\bullet\begin{pmatrix}3\\-3\\2\end{pmatrix}=15$ At intersection, $\begin{pmatrix}5+3\mu\\-1-3\mu\\8+2\mu\end{pmatrix}\bullet\begin{pmatrix}3\\-3\\2\end{pmatrix}=15$ $34+22\mu=15\Rightarrow\mu=-\frac{19}{22}$ Position of the foot of perpendicular is $\overrightarrow{OF}=\frac{1}{22}\begin{pmatrix}53\\35\\138\end{pmatrix}$

(c)

Let ϕ be the angle between the light ray and the normal of the mirror.

Then $\cos\phi = \dfrac{\left|\begin{pmatrix}-6\\-2\\-13\end{pmatrix}\cdot\begin{pmatrix}3\\-3\\2\end{pmatrix}\right|}{\left|\begin{pmatrix}-6\\-2\\-13\end{pmatrix}\right|\left|\begin{pmatrix}3\\-3\\2\end{pmatrix}\right|}$

$\phi = 55.9^\circ$

Hence angle between the mirror and light ray $= 90^\circ - 55.9^\circ = 34.1^\circ$

Alternative method:

Let θ be the acute angle between the light ray and the mirror.

$\sin\theta = \dfrac{\left|\begin{pmatrix}-6\\-2\\-13\end{pmatrix}\cdot\begin{pmatrix}3\\-3\\2\end{pmatrix}\right|}{\left|\begin{pmatrix}-6\\-2\\-13\end{pmatrix}\right|\left|\begin{pmatrix}3\\-3\\2\end{pmatrix}\right|}$

$\sin\theta = \dfrac{|-38|}{\sqrt{209\times 22}} \quad \Rightarrow \theta = 34.1^\circ$ (or 0.595 radian)

(d)

Equation of plane ABC : $\mathbf{r}\cdot\begin{pmatrix}3\\-3\\2\end{pmatrix} = 15$

Equation of line $l : \mathbf{r} = \begin{pmatrix}5\\-1\\8\end{pmatrix} + \lambda\begin{pmatrix}-6\\-2\\-13\end{pmatrix},\ \lambda \in \mathbb{R}$

At intersection,

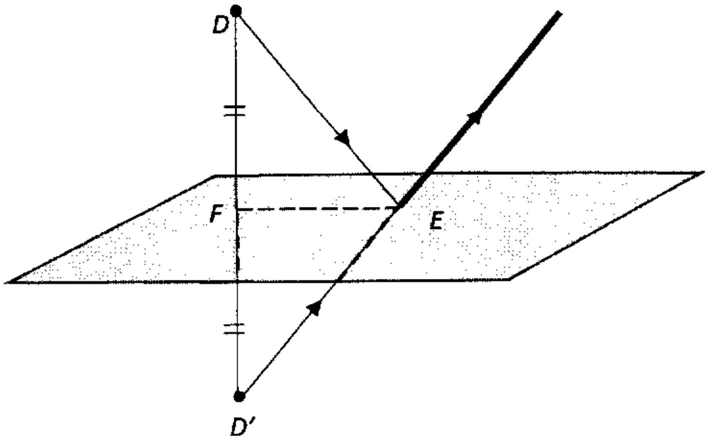
$\begin{pmatrix}5-6\lambda\\-1-2\lambda\\8-13\lambda\end{pmatrix}\cdot\begin{pmatrix}3\\-3\\2\end{pmatrix} = 15$

$34 - 38\lambda = 15 \Rightarrow \lambda = \dfrac{1}{2}$

$\overrightarrow{OE} = \begin{pmatrix}5\\-1\\8\end{pmatrix} + \dfrac{1}{2}\begin{pmatrix}-6\\-2\\-13\end{pmatrix} = \begin{pmatrix}2\\-2\\\frac{3}{2}\end{pmatrix}$

Hence coordinates of point E is $\left(2, -2, \dfrac{3}{2}\right)$

(e)



Let *D*' be the point of reflection of *D* in the mirror.

Using part (b),

$$\frac{\overrightarrow{OD}'+\overrightarrow{OD}}{2}=\overrightarrow{OF}=\frac{1}{22}\begin{pmatrix}53\\35\\138\end{pmatrix}$$

$$\overrightarrow{OD}'=\frac{1}{11}\begin{pmatrix}53\\35\\138\end{pmatrix}-\begin{pmatrix}5\\-1\\8\end{pmatrix}=\frac{1}{11}\begin{pmatrix}-2\\46\\50\end{pmatrix}$$

$$\overrightarrow{ED}'=\frac{1}{11}\begin{pmatrix}-2\\46\\50\end{pmatrix}-\begin{pmatrix}2\\-2\\\frac{3}{2}\end{pmatrix}=\begin{pmatrix}-24/11\\68/11\\67/22\end{pmatrix}=\frac{1}{22}\begin{pmatrix}-48\\136\\67\end{pmatrix}$$

Vector equation of line is $\mathbf{r}=\begin{pmatrix}2\\-2\\\frac{3}{2}\end{pmatrix}+\gamma\begin{pmatrix}-48\\136\\67\end{pmatrix},\ \gamma\in\mathbb{R}$

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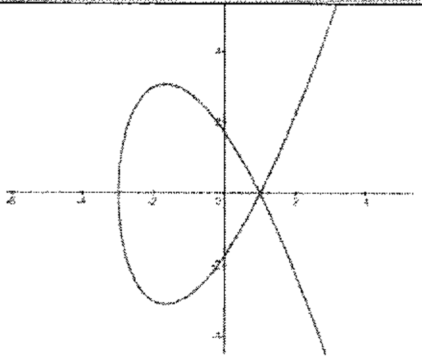
Q1	Suggested Answers
(a)	 Area of loop $= 2\int_{\alpha}^{\beta}(1-x)\sqrt{x+3}\,\mathrm{d}x$ $\alpha = -3,\ \beta = 1$
(b)	Area of loop $= 2\int_{-3}^{1}(1-x)\sqrt{x+3}\,\mathrm{d}x$ $= 2\int_{0}^{2}\left(4-u^2\right)u \times 2u\,\mathrm{d}u$ $= 4\int_{0}^{2}\left(4u^2-u^4\right)\mathrm{d}u$ $= 4\left[\frac{4}{3}u^3-\frac{1}{5}u^5\right]_0^2$ $= \frac{256}{15}$ units^2 $u=\sqrt{x+3}$ $u^2 = x+3$ $2u\frac{\mathrm{d}u}{\mathrm{d}x}=1$ or $u=\sqrt{x+3}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+3}}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2u}$ When $x=-3$, $u=0$ When $x=1$, $u=2$

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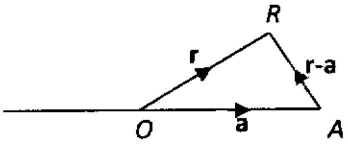
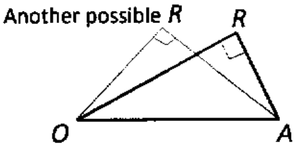
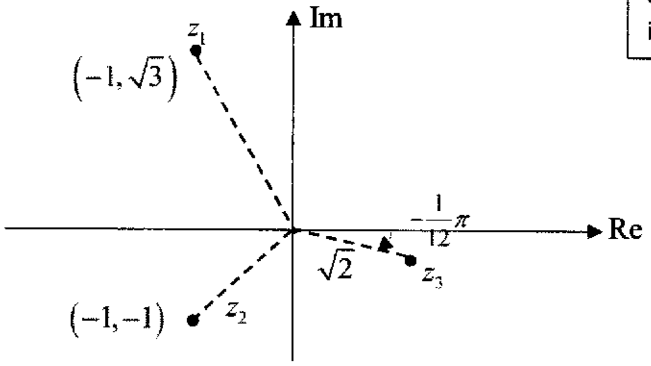
Q2	Suggested Answers
(a)	Vector equation $\mathbf{r}=\lambda\mathbf{a}$ where $0\le\lambda\le1$ The equation gives the position vector of **points on the line segment *OA***. (not line OA)
(b)	Vector equation $\mathbf{r}\times(\mathbf{a}-\mathbf{b})=\mathbf{0}$ $\Rightarrow \mathbf{r}$ is parallel to $\overrightarrow{AB}$ $\mathbf{r}=k(\mathbf{a}-\mathbf{b}),\ k\in\mathbb{R}$ The equation gives the position vector of **points on the line passing through *O* parallel to *AB***.
(c)	$\mathbf{r}\bullet(\mathbf{r}-\mathbf{a})=0$ $\overrightarrow{OR}\cdot\overrightarrow{AR}=0$ $\overrightarrow{OR}\perp\overrightarrow{AR}$   Points on a sphere (or circle) with OA as a diameter.
(d)	$(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\mathbf{0}$ $(\mathbf{r}-\mathbf{a})$ is parallel to $(\mathbf{r}-\mathbf{b})$ $(\mathbf{r}-\mathbf{a})=\mu(\mathbf{r}-\mathbf{b})$ where $\mu\neq1$ $\mathbf{r}(1-\mu)=\mathbf{a}-\mu\mathbf{b}$ $\mathbf{r}=\dfrac{\mathbf{a}-\mu\mathbf{b}}{(1-\mu)}=\left(\dfrac{1}{1-\mu}\right)\mathbf{a}+\left(\dfrac{-\mu}{1-\mu}\right)\mathbf{b}$, where $\mu\neq1$ since $\mathbf{a}\neq\mathbf{b}$ **Alternatively,** $(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\mathbf{0}$ $\overrightarrow{AR}$ is parallel to $\overrightarrow{BR}$ Since R is a common point, A, B and R are colinear i.e. R lies on the line AB $\mathbf{r}=\mathbf{a}+\alpha(\mathbf{b}-\mathbf{a})$, where $\alpha\in\mathbb{R}$

Q3 **Suggested Answers**

(a)

$z_1=-1+\sqrt{3}\mathrm{i}$, $z_2=-1-\mathrm{i}$, $z_3=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}$

Show modulus (length) and argument (angle), if possible.



(b)

$$z_1=2\mathrm{e}^{\mathrm{i}\left(\frac{2}{3}\pi\right)}$$

$$z_2=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{3}{4}\pi\right)}$$

$$z_3=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}$$

Method 1: Properties of modulus & argument

$$\boxed{\left|\frac{z_1}{z_2\left(z_3\right)^2}\right|=\frac{\left|z_1\right|}{\left|z_2\right|\left|z_3\right|^2}}=\frac{2}{\left(\sqrt{2}\right)\left(\sqrt{2}\right)^2}=\frac{1}{\sqrt{2}}$$

$$\boxed{\arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)=\arg z_1-\arg z_2-2\arg z_3}$$

$$=\frac{2}{3}\pi-\left(-\frac{3}{4}\pi\right)-2\left(-\frac{1}{12}\pi\right)$$

$$=\frac{19}{12}\pi$$

$$\therefore\arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)=\frac{19}{12}\pi-2\pi=-\frac{5}{12}\pi$$

$$\therefore\frac{z_1}{z_2\left(z_3\right)^2}=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(-\frac{5}{12}\pi\right)}$$

(b)

Method 2: Using exponential form

$$\frac{z_1}{z_2\left(z_3\right)^2}=\frac{2\mathrm{e}^{\mathrm{i}\left(\frac{2}{3}\pi\right)}}{\left(\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{3}{4}\pi\right)}\right)\left(\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}\right)^2}$$

$$=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(\frac{19}{12}\pi\right)}$$

	$=\frac{1}{\sqrt{2}}e^{i\left(\frac{19}{12}\pi-2\pi\right)}$ $=\frac{1}{\sqrt{2}}e^{i\left(-\frac{5}{12}\pi\right)}$
(c)	Let $z_4=re^{i\theta}$ **Method 1: Properties of modulus & argument** $\left\vert\frac{z_1z_4}{z_2\left(z_3\right)^2}\right\vert=1$ $\left\vert\frac{z_1}{z_2\left(z_3\right)^2}\right\vert\left\vert z_4\right\vert=1$ $\Rightarrow r=\sqrt{2}$ $\arg\left(\frac{z_1z_4}{z_2\left(z_3\right)^2}\right)=\arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)+\arg z_4$ $=-\frac{5}{12}\pi+\theta$ Since $\frac{z_1z_4}{z_2\left(z_3^*\right)^2}$ is purely real, $-\frac{5}{12}\pi+\theta=k\pi,k\in\mathbb{Z}$ $\theta=k\pi+\frac{5}{12}\pi$ $=\frac{5}{12}\pi$ or $-\frac{7}{12}\pi$ $\quad(\because -\pi<\theta\le\pi)$ $\therefore z_4=\sqrt{2}\left(\cos\frac{5}{12}\pi+i\sin\frac{5}{12}\pi\right)$ or $\sqrt{2}\left(\cos\left(-\frac{7}{12}\pi\right)+i\sin\left(-\frac{7}{12}\pi\right)\right)$
(c)	**Method 2: Using exponential form** $\frac{z_1z_4}{z_2\left(z_3\right)^2}=\left[\frac{1}{\sqrt{2}}e^{i\left(-\frac{5}{12}\pi\right)}\right]\left(re^{i\theta}\right)$ $=\frac{r}{\sqrt{2}}e^{i\left(-\frac{5}{12}\pi+\theta\right)}$ $\frac{r}{\sqrt{2}}=1\Rightarrow r=\sqrt{2}$ Since $\frac{z_1z_4}{z_2\left(z_3^*\right)^2}$ is purely real, $-\frac{5}{12}\pi+\theta=k\pi,k\in\mathbb{Z}$ $\theta=k\pi+\frac{5}{12}\pi=\frac{5}{12}\pi$ or $-\frac{7}{12}\pi$ $\quad(\because -\pi<\theta\le\pi)$ $\therefore z_4=\sqrt{2}\left(\cos\frac{5}{12}\pi+i\sin\frac{5}{12}\pi\right)$ or $\sqrt{2}\left(\cos\left(-\frac{7}{12}\pi\right)+i\sin\left(-\frac{7}{12}\pi\right)\right)$

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Q4	Suggested Answers
(a)	$\dfrac{1}{4r^2-8r+3}=\dfrac{A}{2r-3}+\dfrac{B}{2r-1}$ $A(2r-1)+B(2r-3)=1$ When $r=\dfrac{1}{2}$, $B=-\dfrac{1}{2}$ When $r=\dfrac{3}{2}$, $A=\dfrac{1}{2}$ $\dfrac{1}{4r^2-8r+3}=\dfrac{1}{2}\left(\dfrac{1}{2r-3}-\dfrac{1}{2r-1}\right)$
(b)	$\displaystyle\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)=\frac{1}{2}\sum_{r=2}^{3n}\left(\frac{1}{2r-3}-\frac{1}{2r-1}\right)$ $=\dfrac{1}{2}\left(\begin{array}{ccc} 1 & - & \cancel{\frac{1}{3}} \\ +\cancel{\frac{1}{3}} & - & \cancel{\frac{1}{5}} \\ +\cancel{\frac{1}{5}} & - & \frac{1}{7} \\ & \vdots & \\ +\cancel{\frac{1}{6n-5}} & - & \cancel{\frac{1}{6n-3}} \\ +\cancel{\frac{1}{6n-3}} & - & \frac{1}{6n-1} \end{array}\right)$ $=\dfrac{1}{2}\left(1-\dfrac{1}{6n-1}\right)\quad\left(=\dfrac{3n-1}{6n-1}\right)$
(c)	As $n\to\infty$, $\dfrac{1}{6n-1}\to 0$ and so $\displaystyle\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)\to\frac{1}{2}$ $\displaystyle\therefore\ \sum_{r=2}^{\infty}\left(\frac{1}{4r^2-8r+3}\right)=\frac{1}{2}$
(d)	$\displaystyle\sum_{r=n+1}^{3n}\left(\frac{1}{4r^2-8r+3}\right)=\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)-\sum_{r=2}^{n}\left(\frac{1}{4r^2-8r+3}\right)$ $=\dfrac{1}{2}\left(1-\dfrac{1}{6n-1}\right)-\dfrac{1}{2}\left(1-\dfrac{1}{2n-1}\right)$ $=\dfrac{1}{2}\left(\dfrac{1}{2n-1}-\dfrac{1}{6n-1}\right)$ $=\dfrac{4n}{2(2n-1)(6n-1)}$ $=\dfrac{2n}{(2n-1)(6n-1)}$

Q5	Suggested Answers
(a)	Volume of wok $= \pi \int_{-16}^{q} \left(256 - y^2\right) \mathrm{d}y$ $= \pi \left[256y - \frac{y^3}{3} \right]_{-16}^{q}$ $= \pi \left[\left(256q - \frac{q^3}{3} \right) - \left(-4096 + \frac{4096}{3} \right) \right]$ $= \pi \left(256q - \frac{q^3}{3} + \frac{8192}{3} \right)$ $\pi \left(256q - \frac{q^3}{3} + \frac{8192}{3} \right) = 3300$ Using GC, $q = -7.01245634$ Depth of wok $= -7.01245634 - (-16) = 9$ cm (correct to nearest integer)
(b)	**Method 1 (direct integration):** Volume of flat frying pan $= \pi \int_{r}^{0} \left(256 - y^2\right) \mathrm{d}y$ $= \pi \left[256y - \frac{y^3}{3} \right]_{r}^{0}$ $= \pi \left[0 - \left(256r - \frac{r^3}{3} \right) \right]$ $= \pi \left(\frac{r^3}{3} - 256r \right)$ $\pi \left(\frac{r^3}{3} - 256r \right) = 1464\pi$ $\frac{r^3}{3} - 256r = 1464$ Using GC, $r = -6$
(b)	**Method 2 (using result from part (a)):** Volume of flat frying pan $=$ Volume of hemisphere $- \pi \left(256r - \frac{r^3}{3} + \frac{8192}{3} \right)$ $= \frac{2}{3}\pi(16)^3 - \pi \left(256r - \frac{r^3}{3} + \frac{8192}{3} \right)$ $= \pi \left(\frac{r^3}{3} - 256r \right)$ $\pi \left(\frac{r^3}{3} - 256r \right) = 1464\pi$ $\frac{r^3}{3} - 256r = 1464$ Using GC, $r = -6$

(c) Let time taken to fill the wok to full capacity be T seconds

Method 1:

$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{3}{55}t$$

$$\int \mathrm{d}V=\int \frac{3}{55}t\,\mathrm{d}t$$

$$V=\frac{3t^2}{110}+C$$

When $t=0$, $V=0$, $C=0$

When $t=T$, $V=3.3$, $3.3=\dfrac{3T^2}{110}$

$$T=11$$

Method 2:

$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{3}{55}t$$

$$\int_0^{3.3} \mathrm{d}V=\int_0^{T} \frac{3}{55}t\,\mathrm{d}t$$

$$[V]_0^{3.3}=\frac{3}{55}\left[\frac{t^2}{2}\right]_0^{T}$$

$$3.3=\frac{3}{55}\left(\frac{T^2}{2}\right)$$

$T=11$

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Q6	Suggested Answers
(a)	(Treat as there is an 'invisible' car occupying a lot) Number of arrangements without restriction = $10! = 3628800$
(b)	Number of arrangements with BB and RR together $= 8! \times 2! \times 2! = 161280$
(c)	**Method 1** Number of arrangements with B1B2 together $= 9! \times 2! = 725760$ Similarly, number of arrangements with R1R2 together $9! \times 2! = 725760$ Number of arrangements with at least 2 adjacent cars $= 725760 + 725760 - 161280 = 1290240$ <u>By complement method</u>, required number of arrangements is $= 3628800 - 1290240 = 2338560$
(c)	**Method 2** Number of arrangements with B1B2 together and R1R2 not together = $7! \times 2! \times {}^8C_2 \times 2! = 564480$ Similarly, number of arrangements with R1R2 together and B1B2 not together $= 564480$ By complement, required number of arrangements is $3628800 - 161280 - 564480 - 564480 = 2338560$
(c)	**Method 3** Number of arrangements with red cars separated (no restrictions on blue cars) = $8! \times {}^9C_2 \times 2! = 2903040$ Number of arrangements with red cars separated and blue cars together = $7! \times 2! \times {}^8C_2 \times 2! = 564480$ By complement, required number of arrangements is $2903040 - 564480 = 2338560$

Q7 **Suggested Answers**

(a)

$$\mathrm{P}(S=s)>\mathrm{P}(S=s+1)$$

$$\frac{{}^{18}C_s\,{}^{12}C_{10-s}}{{}^{30}C_{10}}>\frac{{}^{18}C_{s+1}\,{}^{12}C_{9-s}}{{}^{30}C_{10}}$$

$$\frac{18!}{s!(18-s)!}\cdot\frac{12!}{(s+2)!(10-s)!}>\frac{18!}{(s+1)!(17-s)!}\cdot\frac{12!}{(s+3)!(9-s)!}$$

$$(s+1)!(17-s)!(9-s)!(s+3)!>s!(18-s)!(10-s)!(s+2)!$$

$$(s+1)(s+3)>(18-s)(10-s)$$

$$s^2+4s+3>s^2-28s+180$$

$$32s>177$$

$$s>5.53$$

Thus $s=6$.

(b) **Outcome Table**

Absolute Difference		No. on Square					
		1	2	3	4	5	6
No. on Triangle	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2

Probability Distribution

x	0	1	2	3	4	5
$\mathrm{P}(X=x)$	$\frac{4}{24}$	$\frac{7}{24}$	$\frac{6}{24}$	$\frac{4}{24}$	$\frac{2}{24}$	$\frac{1}{24}$

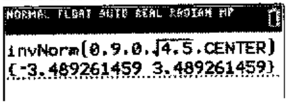
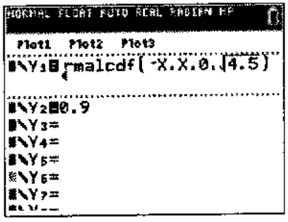
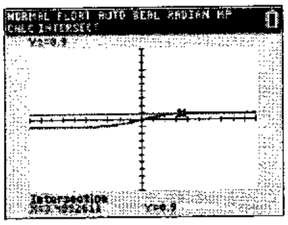
(c)

$$\mathrm{P}(X_1=2\mid X_1+X_2+X_3=3)$$

$$=\frac{\mathrm{P}(X_1=2,X_2=0,X_3=1)\times 2!}{\mathrm{P}(X_1=1,X_2=1,X_3=1)+\mathrm{P}(X_1=1,X_2=2,X_3=0)\times 3!\ +\mathrm{P}(X_1=3,X_2=0,X_3=0)\times\frac{3!}{2!}}$$

$$=\frac{\frac{6}{24}\left(\frac{4}{24}\right)\left(\frac{7}{24}\right)2!}{\left(\frac{7}{24}\right)^3+\frac{7}{24}\left(\frac{6}{24}\right)\left(\frac{4}{24}\right)3!++\frac{4}{24}\left(\frac{4}{24}\right)^2\frac{3!}{2!}}$$

$$=\frac{336}{1543}$$ or 0.218 (to 3 s.f.)

Q8	Suggested Answers
(a)	Let L be the length of a randomly chosen rectangular cotton fabric. $L \sim \mathrm{N}\left(24, 1.5^2\right)$ $\mathrm{P}(L < 23.5) = 0.36944$ (5 s.f.) $= 0.369$ (3 s.f.)
(b)	Let B be the breadth of a randomly chosen rectangular cotton fabric. $B \sim \mathrm{N}\left(20, 1.2^2\right)$ Perimeter $= 2L + 2B \sim \mathrm{N}(88, 14.76)$ $\mathrm{P}(2L + 2B > 90) = 0.30133$ (5 s.f.) $= 0.301$ (3 s.f.)
(c)	The **length and breadth** of each /a randomly chosen rectangular cotton fabric **are independent** of each other. Note that this assumption is necessary for Var(2L + 2B) = Var(2L) + Var(2B) Recall: Var(X + Y) = Var(X) + Var(Y) if and only if X and Y are independent.
(d)	$L \sim \mathrm{N}\left(24, 1.5^2\right)$ $\mathrm{P}(23 < L < 25) = 0.49502$ (5 s.f.) Let X be the number of rectangular cotton fabric (out of 48) with length between 23 and 25 cm. $X \sim \mathrm{B}(48, 0.49502)$ $\mathrm{E}(X) = 48(0.49502) = 23.761$ (5 s.f.) $= 23.8$ (3 s.f.)
(e)	$L_1 - L_2 \sim \mathrm{N}(0, 4.5)$ $\mathrm{P}\left(\lvert L_1 - L_2 \rvert \le k\right) \ge 0.9$ $\mathrm{P}\left(-k \le L_1 - L_2 \le k\right) \ge 0.9$ $k \ge 3.4893$ $k \ge 3.49$ (3 s.f.) Least $k = 3.49$ (3 s.f.) **Method 1:**  NORMAL FLOAT AUTO REAL RADIAN MP invNorm(0.9,0,√4.5,CENTER) {-3.489261459 3.489261459} **Method 2:**  NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y₁=normalcdf(-X,X,0,√4.5) Y₂=0.9 Y₃= Y₄= Y₅= Y₆= Y₇=  NORMAL FLOAT AUTO REAL RADIAN MP CALC INTERSECT Y2=0.9 Intersection X=3.4892614 Y=0.9

Q9	Suggested Answers

(a)

Let X be the number of diners (out of 10) who order the signature dish

$$X \sim \mathrm{B}(10,\ 0.7)$$

$$\begin{aligned}\mathrm{P}(X \geq 3) &= 1-\mathrm{P}(X<3)\\ &= 1-\mathrm{P}(X \leq 2)\\ &= 0.99841 \quad \text{(5 s.f.)}\\ &= 0.998 \quad \text{(3 s.f.)}\end{aligned}$$

(b)

Method 1:

$$\begin{aligned}\mathrm{P}(3<X<8) &= \mathrm{P}(X<8)-\mathrm{P}(X \leq 3)\\ &= \mathrm{P}(X \leq 7)-\mathrm{P}(X \leq 3)\\ &= 0.60663 \quad \text{(5 s.f.)}\\ &= 0.607 \quad \text{(3 s.f.)}\end{aligned}$$

(b)

Method 2 (not recommended):

$$\begin{aligned}\mathrm{P}(3<X<8) &= \mathrm{P}(X=4)+\mathrm{P}(X=5)+\mathrm{P}(X=6)+\mathrm{P}(X=7)\\ &= 0.60663 \quad \text{(5 s.f.)}\\ &= 0.607 \quad \text{(3 s.f.)}\end{aligned}$$

(c)

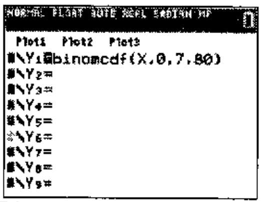
Let Y be the number of diners (out of n) who order the signature dish

$$Y \sim \mathrm{B}(n,\ 0.7)$$

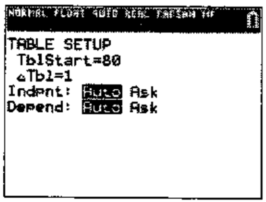
$\mathrm{P}(Y \leq 80) \geq 0.9$

n	$\mathrm{P}(Y \leq 80)$
106	$0.9112 > 0.9$
107	$0.8826 < 0.9$

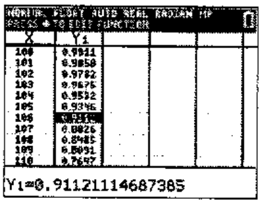
largest $n = 106$ (maximum number of diners)



```
Y1=binomcdf(X,0.7,80)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=
```



```
TABLE SETUP
 TblStart=80
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



```
X    Y1
100  0.9911
101  0.9850
102  0.9782
103  0.9675
104  0.9532
105  0.9346
106
107  0.8826
108  0.8485
109  0.8091
110  0.7697
Y1=0.91121114687385
```

(d)

$$\text{Probability} = [\mathrm{P}(X \geq 3)]^{40} = (0.99841)^{40} = 0.938 \qquad \text{(3 s.f.)}$$

OR $T \sim \mathrm{B}(40,\ 0.99841)$

$\mathrm{P}(T = 40) = 0.93833 \approx 0.938$

(e)

Let X be the number of diners (out of 10) who order the signature dish, i.e. number of portions of the signature dish served at a randomly chosen table of 10 diners

$$X \sim \mathrm{B}(10,\ 0.7)$$

$\mathrm{E}(X) = 7$, $\mathrm{Var}(X) = 2.1$

Average number of portions of the signature dish served per table,

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_{40}}{40}$$

Since $n = 40$ is large, **by the Central Limit Theorem**,

	$\bar{X} \sim \mathrm{N}\left(7, \frac{2.1}{40}\right)$ approximately $\mathrm{P}\left(\bar{X} \geq 6.9\right) = 0.66874$ (5 s.f.) $= 0.669$ (3 s.f.)

Q10	Suggested Answers
(a)	Unbiased estimate of population mean, $\bar{t} = \frac{543}{30} = 18.1$ Unbiased estimate of population variance, $s^2 = \frac{1}{29}\left[12722 - \frac{543^2}{30}\right] = 99.783 \text{ (5 s.f.)} = 99.8 \text{ (3 s.f.)}$
(b)	Let μ be the population mean bus arrival times after the scheduled pick-up time $\mathrm{H}_0: \ \mu = 15$ $\mathrm{H}_1: \ \mu > 15$ Test at 5% level of significance Under H_0, since $n = 30$ is large, **by the Central Limit Theorem,** $\bar{T} \sim \mathrm{N}\left(15, \frac{99.783}{30}\right)$ approximately Test statistic: $Z = \frac{\bar{T} - 15}{\sqrt{\frac{99.783}{30}}} \sim \mathrm{N}(0, 1)$ approximately p-value $= 0.044586$ or $z_{\text{cal}} = \frac{18.1 - 15}{\sqrt{\frac{99.783}{30}}} = 1.6998$(5 s.f.) Since p-value ≤ 0.05 (or $z_{\text{cal}} \geq 1.6449$), we reject H_0. There is sufficient evidence at 5% level of significance to conclude that the administration manager should agree with the feedback from teachers and students.
(c)	Test $\mathrm{H}_0: \ \mu = 15$ $\mathrm{H}_1: \ \mu \neq 15$ at 10 % level of significance Under H_0, since $n = 40$ is large, by the Central Limit Theorem, $\bar{T} \sim \mathrm{N}\left(15, \frac{99.783}{40}\right)$ approximately Test statistic: $Z = \frac{\bar{T} - 15}{\sqrt{\frac{99.783}{40}}} \sim \mathrm{N}(0, 1)$ approximately Since H_0 is not rejected, z-value does not lie in critical region $-1.6449 < \frac{\bar{t} - 15}{\sqrt{\frac{99.783}{40}}} < 1.6449$ $12.402 < \bar{t} < 17.598$ (5 s.f.) $12.4 < \bar{t} < 17.6$ (3 s.f.)

(d) The sample of 40 buses is a **random sample**.
There is **no change in the unbiased estimate of the population variance** bus arrival times after the scheduled pick-up time after the bus company claims to have made changes to its operation.

Q11	**Suggested Answers**

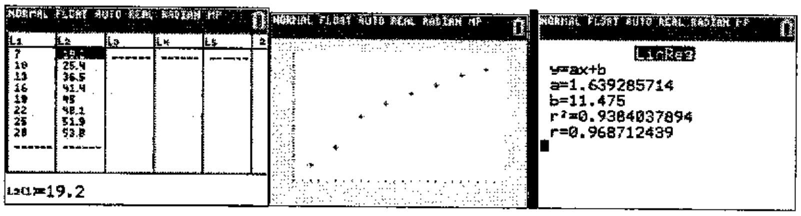
(a)

Since $\left(\bar{t},\bar{h}\right)$ lies on the regression line,

$$\bar{h}=1.6393\bar{t}+11.475$$

$$\frac{k+273.2}{8}=1.6393\left(17.5\right)+11.475$$

$k=48.1$ (to 1 d.p.)



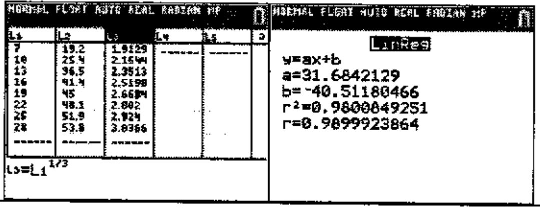
```
NORMAL FLOAT AUTO REAL RADIAN MP
L1    L2    L3    L4    L5
7     19.2
10    25.4
13    36.5
16    41.4
19    45
22    48.1
25    51.9
28    53.8
L2(1)=19.2
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg
y=ax+b
a=1.639285714
b=11.475
r²=0.9384037894
r=0.968712439
```

(b)

For $h=a\sqrt[3]{t}+b$, $r=0.98999$
For $h=c\sqrt{t}+d$, $r=0.98615$
Since *r*-value for $h=a\sqrt[3]{t}+b$ is closer to 1 than the r-value for $h=c\sqrt{t}+d$, the linear correlation between h and $\sqrt[3]{t}$ is stronger.

Using GC, the equation is $h=31.684\sqrt[3]{t}-40.512$ (5 s.f.)
$h=31.7\sqrt[3]{t}-40.5$ (3 s.f.)



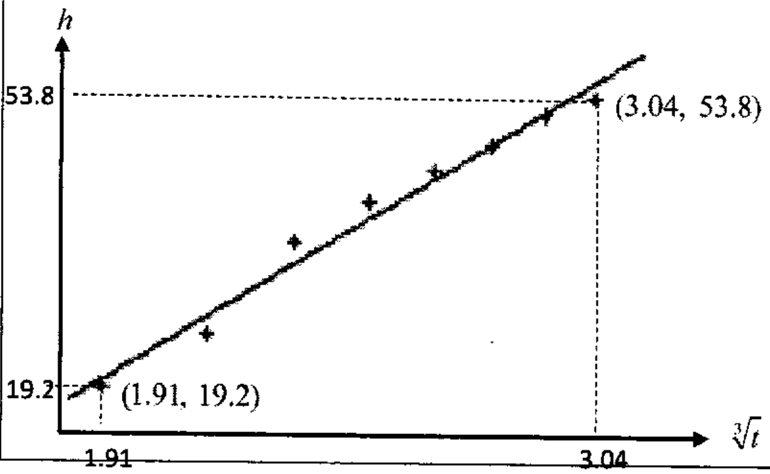
```
NORMAL FLOAT AUTO REAL RADIAN MP
L1    L2    L3      L4    L5
7     19.2  1.9129
10    25.4  2.1544
13    36.5  2.3513
16    41.4  2.5198
19    45    2.6684
22    48.1  2.802
25    51.9  2.924
28    53.8  3.0366
L3=L1^1/3
```

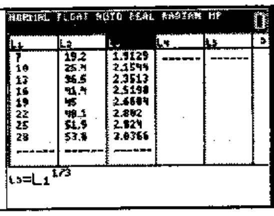
```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg
y=ax+b
a=31.6842129
b=-40.51180466
r²=0.9800849251
r=0.9899923864
```

(c)

Since the age of 2 months ($t\approx 60$) is out of the data range of t, the estimate is not reliable.

(d)

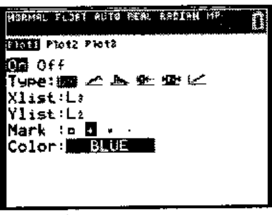




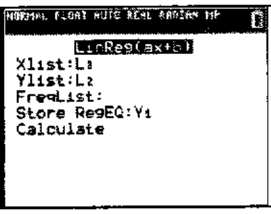
```
NORMAL FLOAT AUTO REAL RADIAN MP
```

L1	L2	L3	L4	L5
7	19.2	1.9129	------	------
10	25.4	2.1544		
13	36.5	2.3513		
16	41.4	2.5198		
19	45	2.6684		
22	48.1	2.802		
25	51.9	2.924		
28	53.8	3.0366		
------	------	------		

```
L3=L1^1/3
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L3
Ylist:L2
Mark :□ + · ·
Color: BLUE
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg(ax+b)
Xlist:L3
Ylist:L2
FreqList:
Store RegEQ:Y1
Calculate
```

(e) Value of residual = $53.8-\left(31.684\sqrt[3]{28}-40.512\right)=-1.90$ (3 s.f.)

(observed-value – predicted value)

(f) The values of the residuals could be positive or negative, and adding them up might cause the values to cancel out. By squaring the residuals, all the values will be positive.

The sum of squares of residuals has to be minimised in order to find the least squares regression line.